Effect of Viscous Dissipation on Mixed Convection in a Nanofluid Saturated Non-Darcy Porous Medium Under Convective Boundary Condition

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In this investigation, the influence of the prominent viscous dissipation effect on mixed convection transfer along a vertical plate embedded in a non-Darcy porous medium saturated with a nanofluid under convective boundary condition is studied. The model used for the nanofluid incorporates the effects of Brownian motion and thermophoresis. The nonlinear governing equations and their associated boundary conditions are initially cast into a dimensionless form of non-similarity variables. The resulting system of equations is then solved numerically by an accurate implicit finite-difference method. The effect of the physical parameters on the flow, heat transfer and nanoparticle concentration characteristics of the model are presented through graphs and the salient features are discussed. The present investigation shows that the nanoparticles can improve the heat transfer characteristics significantly for this flow problem.

KEYWORDS: Mixed Convection, Non-Darcy Porous Medium, Nanofluid, Viscous Dissipation, Convective Boundary Condition.

1. INTRODUCTION

In the recent past, the prediction of heat transfer characteristics about mixed convection of viscous fluid saturated non-Darcy porous medium has been gaining more importance due to its emerging engineering and industrial applications. One can find a detailed review of convective heat transfer in Darcian and non-Darcian porous medium in the book by Nield and Bejan.1 Conventional heat transfer fluids, for example, oil, water, and ethylene glycol mixtures, are poor heat transfer fluids because of their poor thermal conductivity. These fluids working as a cooling tool, which enhance manufacturing and operating costs in many applications. In the recent years, many attempts have been taken by several researchers to enhance the thermal conductivity of these fluids by suspending nano/micro particles in liquids. But there is no single fluid model which clearly enhances the thermal conductivity of the fluid. Therefore, during the last few years, several fluid models were proposed to enhance thermal conductivity of the fluid. One such fluid is nanofluid. The nanofluid (initially introduced by Choi2) is an advanced type of fluid containing nanometer sized particles (diameter less than 100 nm) or fibers suspended in the ordinary fluid. Undoubtedly, the nanofluids are advantageous in the sense that they are more stable and have an acceptable viscosity and better wetting, spreading and dispersion properties of a solid surface. Nanofluids are used in different engineering applications such as microelectronics, microfluidics, transportation, biomedical, solid-state lighting, and manufacturing. The research on heat transfer in nanofluids has been receiving increased attention worldwide. Many researchers have found unexpected thermal properties of nanofluids, and have proposed new mechanisms behind the enhanced thermal properties of nanofluids. A very good collection of published papers on nanofluids can be found in the book by Das et al.3 and in the review papers by Buongiorno,4 Kakac and Pramuanjaroenkij5 and Serrano et al.6

The Brownian motion of nanoparticles at molecular and nanoscale levels are the interesting to measures for a key nanoscale mechanism governing their thermal behaviors. In nanofluid systems, due to the size of the nanoparticles, the Brownian motion takes place, which can affect the heat transfer properties. As the particle size scale approaches
to the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids play an important role in the heat transfer. In view of these applications, the authors Nield and Kuznetsov, Chamkha et al., and Motsumi and Makinde have shown some interesting features and applications of convective transport in a nanofluid in the presence of different effects. Very recently, RamReddy et al. and Murthy et al. have explored the importance of inclination and/or Soret effects on the convective transport in nanofluid with or without porous medium.

We know that the viscous dissipation acts as a heat source and generates appreciable temperature in the porous medium. Hence, it is interpreted as the rate at which mechanical energy is converted into heat in a viscous fluid per unit volume. It has received little attention in the past. Viscous dissipation is of significance in mixed convection in various devices that subject to large variations of gravitational force or that operates at high rotational speeds (see Gebhart). In the study of flow past a vertical porous plate, Takhar and Beg have modeled the viscous dissipation in the porous medium and on the other hand, Murthy and Singh have modeled the flow of an incompressible fluid in a saturated porous medium, with the effect of viscous dissipation included. Indeed, it has been emphasized by Rees and Magyari, that a mixed or forced-convection boundary-layer flow generates heat everywhere when the viscous dissipation is present, including the free stream region outside the boundary-layer. For an exhaustive discussion of the mixed convective flow due to a vertical plate immersed in a non-Darcy porous medium saturated with a Newtonian fluid in the presence of viscous dissipation, the reader is referred to the works of Narayana et al. (also see the references cited therein).

In recent times, the flow analysis of convective transport in a porous medium in the presence of viscous dissipation has been a topic of extensive research due to its application, but there is limited literature available on the study of viscous dissipation in nanofluids about different surface geometries. Recently, the authors Ramiar and Ranjbar and Motsumi and Makinde have studied the free or forced convective transport over different surface geometries in a nanofluid saturated non-Darcy porous media. But, very little attention has been paid to study the significance of the effect of viscous dissipation on mixed convection in a nanofluid saturated non-Darcy porous medium. These nanofluids appear to have high thermal conductivities and may be able to meet the rising demand as an efficient heat transfer agent. In the recent past, scientists and engineers have started showing interest in the study heat transfer characteristics of these nanofluids. But a clear picture about the heat transfer through these nanofluids is yet to emerge.

Motivated by the above referenced work and the vast possible industrial applications, it is of paramount interest to consider the effect of viscous dissipation on mixed conveective flow along a vertical plate in a non-Darcy porous medium saturated by a nanofluid under convective boundary condition. The presence of convective boundary conditions make the mathematical model of the present physical system a little more complicated leading to the complex interactions of the flow, heat and mass transfer mechanism. The implicit, iterative finite-difference method discussed by Blottner is employed to solve the nonlinear system of this particular problem. The effects of viscous dissipation, mixed convection, Biot and non-Darcy parameters are examined and are displayed through graphs.

2. MATHEMATICAL FORMULATION

We choose the coordinate system such that the x-axis is along the vertical plate and y-axis is normal to the plate. The physical model and coordinate system are shown in Figure 1. Consider the mixed convection heat and nanoparticle mass transfer along a vertical plate embedded in a non-Darcy porous medium saturated by a nanofluid with free stream velocity . Also, the fluid flow is steady, laminar and two-dimensional. At this boundary the temperature , to be determined later, is the result of a convective heating process which is characterized by a temperature and a heat transfer coefficient . The nanoparticle volume fraction of the wall is . The ambient values, attained

![Fig. 1. Physical model and coordinate system.](image-url)

**Table 1.** Comparison of dimensionless similarity functions for mixed convection along a vertical flat plate in non-Darcy porous medium with \( \text{Nt} \to 0, \text{Nr} \to 0, e = 0.1, \chi^2 = 1, \text{Re}^+ = 1, \text{Bi} \to \infty \) and \( \text{S} \to 0 \) (Narayana et al.).

<table>
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<tr>
<td>0.00</td>
<td>Narayana et al. ( -0.1743 )</td>
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as \( y \) tend to infinity, of \( T \), and \( \phi \) are denoted by \( T_\infty \) and \( \phi_\infty \) respectively. Further, the porous medium is considered to be homogeneous and isotropic (i.e., uniform with a constant porosity and permeability) and is saturated with a fluid which is in local thermodynamic equilibrium with the solid matrix. The fluid has constant properties except the density in the buoyancy term of the balance of momentum equation. The fluid flow is moderate, so the pressure drop is proportional to the linear combination of fluid velocity and the square of the velocity (Forchheimer flow model (Nield and Bejan\(^1\)) the governing making use of the above assumptions and the Darcy–transformations and Magyari\(^17\) and Nield.\(^18\).

It is assumed that the nanosized particles are suspended in uniform distribution in a base fluid to form a nanofluid. This nanofluid, when passes through porous media, the suspension of the nanoparticles is maintained by using surfactant or some surface charge technology to prevent their agglomeration and to avoid of being captured by the porous matrix. By employing the Oberbeck-Boussinesq and the standard boundary layer approximations, and making use of the above assumptions and the Darcy-Forchheimer model (Nield and Bejan\(^1\)), the governing equations of the nanofluid flow problem under investigation are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\frac{\partial u}{\partial y} + \frac{c\sqrt{K}}{(\mu/\rho f_m)} \frac{\partial^2 u}{\partial y^2} = \frac{K(1 - \phi_\infty/\rho f_m)g\beta}{\mu} \frac{\partial T}{\partial y} - \frac{(\rho_p - \rho f_m)gK}{\rho f_m} K \frac{\partial \phi}{\partial y}
\]

\[
\frac{\partial T}{\partial y} + \frac{v}{K\rho} \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{J}{D_n} \frac{\partial \phi}{\partial y} + \frac{D_n}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \]

where \( u \) and \( v \) are the Darcy velocity components in the \( x \) and \( y \) directions, \( T \) is the temperature, \( \phi \) is the nanoparticle concentration, \( g \) is the acceleration due to gravity, \( K \) is the permeability, \( c \) is the empirical constant associated with the Forchheimer porous inertia term, \( \phi \) is the porosity, \( \alpha_m = k_m/(\rho c_p) \) is the thermal diffusivity of the fluid, \( v = \mu/\rho f_m \) is the fluid’s kinematic viscosity coefficient and \( J = \varphi/(\rho c_p) \). Furthermore, \( \rho f_m \) is the density of the base fluid and \( \rho, \mu, k_m \) and \( \beta \) are the density, viscosity, thermal conductivity and volumetric thermal expansion coefficient of the nanofluid, while \( \rho_p \) is the density of the nanoparticles, \( (\rho c_p) \), is the heat capacity of the fluid and \( (\rho c_p) \) is the effective heat capacity of the nanoparticle material. The coefficients that appear in Eqs. (3)–(4) are the Brownian diffusion coefficient \( D_n \), the thermophoretic diffusion coefficient \( D_b \), and the last term in Eq. (3) is the viscous dissipation term. For detailed derivation of the Eqs. (1)–(4), one can refer the papers by Buongiorno,\(^4\) Nield and Kuznetsov,\(^7,8\) and Narayana et al.\(^19\).

The associated boundary conditions are

\[
v = 0, \quad -k_n \frac{\partial T}{\partial y} = h_f(T_r - T), \quad \phi = \phi_w \text{ at } y = 0 
\]

\[
u \to \nu_\infty, \quad T \to T_\infty + \left[ 1 + \frac{c\sqrt{K}}{(\mu/\rho f_m)} \right] \frac{\nu\mu}{K\rho} \frac{\partial^2 T}{\partial y^2} \quad \phi \to \phi_\infty \text{ as } y \to \infty 
\]

where the subscripts \( w \) and \( \infty \) indicate the conditions at the wall, and at the outer edge of the boundary layer respectively. A detailed discussion of the ambient boundary conditions in mixed convection problems can be found in Rees and Magyari\(^17\) and Nield.\(^18\).

Introducing the following non-dimensional transformations

\[
\eta = \frac{y \sqrt{\chi}}{x}, \quad \psi(x, \eta) = \frac{\alpha_m \sqrt{\chi}}{\rho f_m} f(x, \eta), \quad \theta(x, \eta) = \frac{T - T_\infty}{T_r - T_\infty}, \quad S(x, \eta) = \frac{\phi - \phi_\infty}{\phi_\infty - \phi_\infty}
\]

where \( \chi = u_\infty x/\alpha_m \) is the local Peclet number, \( R_a = ((1 - \phi_\infty)\rho f_m gK\beta(T_r - T_\infty)/\mu_\alpha) \) is the local Rayleigh number and \( \chi \) is the mixed convection parameter given by \( \chi^{-1} = 1 + \sqrt{R_a}/(\chi\eta) \).

In view of the continuity Eq. (1), we introduce the stream function \( \psi \) by

\[
\frac{\partial u}{\partial y} = \frac{\partial \psi}{\partial x}, \quad \frac{\partial v}{\partial x} = -\frac{\partial \psi}{\partial y}
\]

Substituting Eq. (7) in Eqs. (2)–(4) and then using the non-dimensional transformations (6), we get the following system of non-dimensional equations:

\[
(1 + 2Re^*f')f'' = (1 - \chi)^2(\theta'' - \theta' S')
\]

\[
\theta'' + \frac{1}{2} f\theta' + N B \cdot \theta' S' + N_{\text{t}} \cdot \theta'^2
\]

\[
+ \varepsilon(1 - \chi)^{-2} f^2(1 + \text{Re}^*f') = \varepsilon \left( f \frac{\partial \theta}{\partial \varepsilon} - \theta' \frac{\partial f}{\partial \varepsilon} \right)
\]

\[
S'' + \frac{1}{N_B} \cdot \text{f}S' + N_{\text{t}} \cdot \theta'' = \text{Le} \left( f \frac{\partial S}{\partial \varepsilon} - S \frac{\partial f}{\partial \varepsilon} \right)
\]

where the primes indicate partial differentiation with respect to \( \eta \) alone, \( \text{Re}^* = c\sqrt{K}/(\mu/\rho f_m) u_\infty/\chi^2 \) is the modified Reynolds’s number, \( \text{Le} = \alpha_m/(\varphi D_p) \) is the Lewis number, \( N_r = (\rho_p - \rho f_m)(\phi_w - \phi_\infty)/(\rho f_m \beta(T_r - T_\infty)) \) is the buoyancy parameter, \( \text{Nt} = JD\phi_\infty/\alpha_m \) is the Brownian motion parameter, \( \text{Nt} = JD\phi_\infty/(\alpha_m T_\infty) \) is the thermophoresis...
parameter and $\varepsilon = (1 - \phi_a)\beta x/C_p$ is the viscous dissipation parameter (i.e., Eckert number). For most situations the Darcy number is small, so viscous dissipation is important at even modest values of the Eckert number. The circumstances in which viscous dissipation is important are those involving flows of relatively large velocity. The authors believe that the results in this paper are likely to be applicable in the context of particle bed nuclear reactors. The primary objective of this study is to estimate the first thermal boundary condition of (11a), which gives the isothermal case. This statement is also supported by the first thermal boundary condition of (11a), which gives

$$\theta(e, 0) = 1 \quad \text{as } Bi \to \infty.$$

The effects of the various parameters involved in the investigation on these coefficients are discussed in the results and discussion section.

3. HEAT AND MASS TRANSFER COEFFICIENTS

The primary objective of this study is to estimate the parameters of engineering interest in fluid flow, heat and nanoparticle mass transport problems are the Nusselt number $N_{u_1}$, nanoparticle Sherwood number $S_{h_1}$. These parameters characterize the wall heat and nanoparticle mass transfer rates, respectively.

The local heat and local nanoparticle mass fluxes from the vertical plate can be obtained from

$$q_w = -k_w \frac{\partial T}{\partial y} \bigg|_{y=0}, \quad q_m = -D_\beta \frac{\partial \phi}{\partial y} \bigg|_{y=0}$$

The dimensionless local Nusselt number $N_{u_1} = q_w x/(k_w(T_w - T_0))$ and local nanoparticle Sherwood number $S_{h_1} = q_m x/(D_\beta(\phi_w - \phi_a))$ are given by

$$N_{u_1} = -\theta'(e, 0) \quad \text{and} \quad S_{h_1} = \frac{\chi}{Pe_1^{1/2}} = -S'(e, 0)$$

4. NUMERICAL METHOD

Equations (8)–(10) represent an initial-value problem with $\varepsilon$ playing the role of time. This general non-linear problem cannot be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. The implicit, tri-diagonal finite-difference method similar to that discussed by Blottner has proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present work. All first-order derivatives with respect to $\varepsilon$ are replaced by two-point backward-difference formulæ when marching in the positive $\varepsilon$ direction. Then, all second-order differential equations in $\eta$ are discretized using three-point central difference quotients. This discretization process produces a tri-diagonal set of algebraic equations at each line of constant $\varepsilon$ which is readily solved by the well known Thomas algorithm (see Blottner). During the solution, iteration is employed to deal with the
nonlinearity aspect of the governing differential equations. The problem is solved line by line starting with line $\varepsilon = 0$ where similarity equations are solved to obtain the initial profiles of velocity, temperature and nanoparticles volume fraction and marching forward in $\varepsilon$ until the desired line of constant $\varepsilon$ is reached. Variable step sizes in the $\eta$ direction with $\Delta \eta_0 = 0.001$ and a growth factor $G = 1.035$ such that $\Delta \eta_n = G \Delta \eta_{n-1}$ and constant step sizes in the $\varepsilon$ direction with $\Delta \varepsilon = 0.01$ are employed. These step sizes are arrived at after many numerical experimentations performed to assess grid independence. The convergence criterion employed in the present work is based on the difference between the current and the previous iterations. When this difference reached $10^{-5}$ for all points in the $\eta$ directions, the solution was assumed converged and the iteration process was terminated. The above step sizes and convergence criterion were found to give accurate grid-independent results as verified by the comparison mentioned below.

With $N_b \to 0$, $N_t = N_r = 0$, $Bi \to \infty$ and $S(\eta) \to 0$ (i.e., for the regular Newtonian fluid), Eqs. (8)–(10) governing the present investigation of nanofluid saturated non-Darcy porous medium in the presence of viscous dissipation reduces to those limiting case of mixed convection flow Narayana et al.\textsuperscript{19} who investigated non-Darcy mixed convection from vertical isothermal surfaces in saturated porous media. Also, the results have been compared with Narayana et al.\textsuperscript{19} and it is found that they are in good agreement as shown in Table I. Therefore, the developed code can be used with great
5. RESULTS AND DISCUSSION
We have computed the solutions for the dimensionless velocity, temperature and nanoparticle volume fraction functions and heat and nanoparticle mass transfer rates as shown graphically in Figures 2–13. The effects of viscous dissipation parameter $\varepsilon$, mixed convection parameter $\chi$, Biot number $Bi$, modified Reynold’s number $Re^*$, Brownian motion parameter $Nb$, thermophoresis parameter $Nt$, Lewis number $Le$ and buoyancy ration $N\rho$ have been discussed.

Figure 2(a) depicts the dimensionless velocity distribution for different values of modified Reynold’s number $Re^*$ with $\varepsilon = 0.5$, $\chi = 0.5$, $Bi = 0.5$, $N\rho = 0.5$, $Nt = 0.1$, $Nb = 0.3$ and $Le = 10$. The velocity is maximum in this case due to the total absence of inertial drag. Since $Re^* = 0$ represents the case where the flow is Darcian. Figure 2(b) is prepared to bring the effect of $Re^*$ on dimensionless temperature for the fixed values of other parameters. An increase in modified Reynold’s number $Re^*$, produces an increase in the thermal boundary layer thickness. As such the temperature is minimized for the lowest value of $Re^*$ and maximized for the highest value of $Re^*$ as shown in Figure 2(b). It can be seen from the Figure 2(c) that the volume fraction profile increases with increasing $Re^*$.

Fig. 5. Variation of non-dimensional heat and nanoparticle mass transfer coefficients with $\varepsilon$ for different values of $\chi$.

Fig. 6. Effects of modified Brownian motion parameter $Nb$ on (a) velocity, (b) temperature, and (c) volume fraction profiles.
the values $Re^*$. Further, the non-dimensional heat and nanoparticle mass transfer coefficients plotted against the viscous dissipation parameter $\varepsilon$ for different values of modified Reynold’s number $Re^*$ with $\chi = 0.5$, $Bi = 0.5$, $Nr = 0.5$, $Nt = 0.1$, $Nb = 0.3$ and $Le = 10$ in Figure 3. The results indicated that increases in Bi decreases the heat transfer rate but increases nanoparticle mass transfer rate. Hence the non-Darcy parameter has an important role in controlling the flow field.

The variations of the non-dimensional velocity, the temperature and nanoparticle concentration distributions with respect to the mixed convection parameter $\chi$ are illustrated in Figure 4. We note that $\chi$ gives rise to two limiting cases, namely, free convection when $\chi \to 0$ and forced convection when $\chi \to 1$. It should be noted that $\chi^2(1 - \chi)^{-2}(1 + Re^*\chi^2) \to \infty$ as $\chi \to 1$. It is observed that the momentum boundary layer thickness increases with the increasing values of $\chi$. The Figure 4(b) shows that the thermal boundary layer thickness increases while the Figure 4(c) shows that nanoparticle concentration boundary layer thickness decreases with increasing the values of $\chi$. The heat and nanoparticle mass transfer coefficients are illustrated in Figure 5 for various values of $\chi$. It is evident that an increasing of the mixed convection parameter $\chi$ reduces the heat transfer rate but enhances the nanoparticle mass transfer rate.

![Figure 7](image1.png)

**Fig. 7.** Variation of non-dimensional heat and nanoparticle mass transfer coefficients with $\varepsilon$ for different values of $Nb$.

![Figure 8](image2.png)

**Fig. 8.** Effects of thermophoresis parameter $Nt$ on (a) velocity, (b) temperature, and (c) volume fraction profiles.
In Figure 6(a), the dimensionless velocity distribution for different values of Brownian motion \(Nb\) with \(Re^* = 1.0, Bi = 0.5, \chi = 0.5, Nr = 0.5, Nt = 0.1, \varepsilon = 0.5\) and \(Le = 10\) is analyzed. It is noted that there is no thermal transport due to buoyancy effects created as a result of nanoparticle concentration gradients for \(Nb = 0\). It is observed that the momentum boundary layer thickness increases with the increase of \(Nb\) and finally approaching their free stream condition. As the parameter \(Nb\) increases, the temperature increases for the specified conditions. As expected, the boundary layer profile for the temperature function is essentially the same form as in the case of a regular (Newtonian) fluid but the nanoparticle volume fraction decreases with increase in \(Nb\). It is also noticed that the nanoparticle volume fraction increases with increase in \(Nb\) in the case of forced convection flow. The non-dimensional heat and nanoparticle mass transfer coefficients are plotted against the viscous dissipation parameter \(\varepsilon\) for different values of Brownian motion \(Nb\) in Figure 6. The results showed that increases in \(Nb\) decreases the heat transfer coefficient whereas increases nanoparticle mass transfer coefficient.

Figure 8 illustrates the effect of the thermophoresis \(Nt\) on the velocity, temperature and volume fraction distributions, respectively. It is observed that the momentum boundary layer thickness decreases with the increase of \(Nt\). But it is interesting to note that the velocity decreases with \(Nt\) increases in the case of free convection. In addition,
Fig. 11. Variations of non-dimensional heat and nanoparticle mass transfer coefficients with $\varepsilon$ for different values of $Nr$.

As the parameter $Nt$ increases, the temperature increases for the specified conditions. As expected, the boundary layer profile for the temperature function is essentially the same form as in the case of a regular (Newtonian) fluid. Moreover, the nanoparticle volume fraction increases with an increase in $Nt$. Also, we can notice that, positive $Nt$ indicates a cold surface while negative to a hot surface. For hot surfaces, thermophoresis tends to blow the nanoparticle volume fraction boundary layer away from the surface since a hot surface repels the sub-micron sized particles from it, thereby forming a relative particle-free layer near the surface. The non-dimensional heat and mass transfer coefficients decrease with increasing values of thermophoresis parameter as shown in Figure 9.

Figure 10 depicts the effect of the nanoparticle buoyancy ratio $Nr$ on the velocity, temperature and volume fraction distributions, respectively. It is shown that the momentum and thermal boundary layer thickness decrease with the increase of $Nr$. The nanoparticle volume fraction increases with increase in $Nr$. From Figure 11, one can see that the non-dimensional heat transfer coefficient increasing but the mass transfer coefficient decreasing with increasing values of $Nr$.

The variations of the non-dimensional velocity, temperature and nanoparticle concentration with Lewis number...
Le for fixed values of other parameters are shown in Figure 12. It is noticed that an increase in the Lewis number Le results in an increase in the velocity and temperature but decrease in the volume fraction within the boundary layer. The present analysis shows that the flow field is appreciably influenced by the Lewis number Le. The effects of a Lewis number on the wall heat and nanoparticle mass transfer rates are shown in Figure 13. The influence of a Lewis number is to reduce the heat transfer rate but enhance the nanoparticle mass transfer rate.

Figure 14(a) is prepared to analyze the non-dimensional velocity for different values of Biot number Bi with fixed values of other parameters. Increased convective heating associated with an increase in Bi is seen to thicken the momentum boundary layer. Given that convective heating increases with Biot number, Bi → ∞ simulates the isothermal surface, which is clearly seen from the Figure 14(b), where θ(0) = 1 as Bi → ∞. As a result, an increase in the Biot number leads to increase of fluid temperature efficiently, also these figures confirm this fact. In fact, a high Biot number indicates higher internal thermal resistance of the plate than the boundary layer thermal resistance. Figure 14(c) indicates the variation of dimensionless nanoparticle volume fraction for different values of Bi. A reduction in nanoparticle volume fraction
is seen with increasing values of the Biot number Bi. The non-dimensional heat and nanoparticle mass transfer coefficients are plotted in Figure 15 against the viscous dissipation parameter $\varepsilon$ for different values of Biot number Bi. The results indicated that increases in Bi increase the heat and nanoparticle concentration transfer coefficients.

The non-dimensional heat and nanoparticle mass transfer coefficients are plotted against the viscous dissipation parameter $\varepsilon$ for fixed values of the other parameters in Figures 3, 5, 7, 9, 11, 13 and 15. It is indicated that the heat and mass transfer rates decrease with the viscous dissipation parameter. To increase the fluid motion we have considered viscous dissipation term. From this term we have obtained dimensionless parameter $\varepsilon$. This parameter is called the fluid motion controlling parameter. It may be noted that $\varepsilon = 0$ corresponds to the case of absence of viscous dissipation. Furthermore, the presence of viscous dissipation in the energy equation acts as an internal heat source due to the action of viscous stresses. Therefore, the heat transfer rate is at a lower level when this effect is considered ($\varepsilon \neq 0$) than that when this effect is neglected ($\varepsilon = 0$).

6. CONCLUSIONS

In this paper, the effect of viscous dissipation on mixed convection flow along a vertical plate in a non-Darcy porous medium saturated with a nanofluid under the convective boundary condition is presented. Using the dimensionless variables, the governing equations are transformed into a set of non-linear parabolic equations where numerical solution has been presented using the implicit, iterative finite difference method discussed by Blottner for a wide range of parameters. The main findings are summarized as follows:

- An increase in modified Reynold’s number $Re^*$, leads to decrease in velocity distribution and heat transfer rate whereas causes an increase in temperature, nanoparticle volume fraction distributions and nanoparticle mass transfer rate.
- The higher values of the mixed convection parameter $\chi$ resulting in a higher velocity and temperature distributions, but lower nanoparticle volume fraction distribution in the boundary layer. Further, an increase in $\chi$ resulted the lower heat transfer rate, but higher nanoparticle mass transfer rate.
- An increase in Biot number Bi, yields an enhancement in velocity, temperature and non-dimensional heat and nanoparticle mass transfer rates, whereas yield reduction in nanoparticle volume fraction and in boundary layer.
- An increase in the viscous dissipation parameter $\varepsilon$, causes a reductions in the heat and nanoparticle mass transfer rates.

References and Notes