Heatline visualization of natural convection in a trapezoidal cavity partly filled with nanofluid porous layer and partly with non-Newtonian fluid layer

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Abstract
The problem of natural convection in a trapezoidal cavity partly filled with nanofluid porous layer and partly with non-Newtonian fluid layer is visualized by heatline. Water-based nanofluids with silver or copper or alumina or titania nanoparticles are chosen for investigation. The governing equations are solved numerically using the Finite Volume Method (FVM) over a wide range of Rayleigh number ($Ra = 10^5$ and $10^6$), Darcy number ($10^{-6} \leq Da \leq 10^3$), nanoparticle volume fraction ($0 \leq \phi \leq 0.2$), power-law index ($0.6 \leq n \leq 1.4$), porous layer thickness ($0.3 \leq S \leq 0.7$), the side wall inclination angle ($0^\circ \leq \varphi \leq 21.8^\circ$) and the inclination angle of the cavity ($0^\circ \leq \omega \leq 90^\circ$). Explanation for the influence of various above mentioned parameters on streamlines, isotherms and overall heat transfer is provided on the basis of thermal conductivities of nanoparticles, water and porous medium. It is shown that convection increases remarkably by the addition of silver–water nanofluid and the heat transfer rate is affected by the inclination angle of the cavity variation. The results have possible applications in heat-removal and heat-storage fluid-saturated porous systems.

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1. Introduction

Natural convection fluid flow and heat transfer in a closed cavity partly filled with a porous layer and partly with a fluid layer have received considerable attention over the past few years. The prominence of this issue is due to the broad spectrum of environmental problems or industrial applications such as ground-water pollution, benthic boundary layers, geothermal systems, storage of nuclear waste, dendritic solidification, drying processes, thermal insulation, spreading on porous substrates and filtration processes. In order to understand the transfers in composite domains, some authors have considered the natural convection and heat transfer in cavities where the layers are confined either vertically or horizontally.

The first attempts to study experimentally and analytically the natural convection flow and heat transfer between a porous media and a homogeneous fluid by focusing on the boundary condition at the fluid/porous interface were made by Beavers and Joseph [1]. Nield [2] applied the linear stability analysis of natural convection in a configuration composed by a fluid layer overlying a homogeneous porous medium with uniform heating from below. The Darcy model was applied to study the high value of Rayleigh in natural convection in a fluid overlaying a porous bed [3]. According to Beckermann et al. [4], the Brinkman–Forchheimer–extended Darcy was adopted to investigate the natural convection flow and heat transfer between a fluid layer and a porous layer inside a rectangular enclosure. Study of the horizontal partition was carried out by Chen and Chen [5] through investigating experimentally the convective stability in a superposed fluid and porous layer. Chen and Chen [6] discussed the problem of nonlinear computational investigation of thermal convection in a superposed fluid and porous layer using Darcy–Brinkman–Forchheimer model. Gobin et al. [7] analyzed the particular subclass of such problems where natural convection occurs in a confined enclosure, partially filled with a porous medium.

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Most of the above mentioned studies focused on the natural convection heat transfer in square/rectangular cavities. In reality, natural convection in a differentially heated enclosure is a prototype of many industrial applications and in particular, a trapezoidal enclosure has received considerable attention because of its applicability in various fields. The moderately concentrating solar energy collector is an important example involving a trapezoidal geometry. The study of convective flow in a trapezoidal geometry is more difficult than that of square/rectangular cavities due to the presence of sloping walls. In general, the mesh nodes do not lie along the sloping walls, and consequently, from a programming and computational point of view, the effort required for determining flow characteristic increases significantly. Lee [8] firstly performed a numerical and experimental flow visualization study and heat transfer in a differentially heated trapezoidal cavity filled with pure fluid. Karyakin [9] has simulated the flow and temperature fields for various inclination angles of the sloping wall and showed that the heat transfer rate increases with the increasing angle of the sloping wall. However, no work has been reported in literature on natural convection fluid flow and heat transfer in a trapezoidal cavity partially filled with a porous medium.

Thermal fluids are very important for heat transfer in many industrial applications. The low thermal conductivity of conventional heat transfer fluids such as water and oils is a primary limitation in enhancing the performance and the compactness of many engineering electronic devices. Solid typically has a higher thermal conductivity than liquids. For example, copper (Cu) has a thermal conductivity of 700 time greater than that of water and 3000 greater than engine oil. An innovative and new technique to enhance heat transfer is using solid particles in the base fluid (i.e. nanofluids) in the range of sizes 10–50 nm. Due to small sizes and very large specific surface areas of the nanoparticles, nanofluids have superior properties like high thermal conductivity, minimal clogging in flow passages, long term stability and homogeneity. Thus, nanofluids have a wide range of potential applications such as electronics, automotive, and nuclear applications where improved heat transfer or efficient heat dissipation is required. Saleh et al. [10] conducted for the first time a numerical study to solve the problem of natural convection in a trapezoidal cavity filled with water–Cu and water–Al2O3 nanofluids. Garoosi et al. [11] considered numerically the effects of several pairs of heaters and coolers on natural convection of nanofluids in a square cavity. Ganji and Malvandi [12] applied the uniform magnetic field on natural convection of nanofluids inside a vertical cavity. By using modified Buongiorno’s model, Malvandi and Ganji [13] discussed the effects of magnetic field and slip boundary on free convection inside a vertical cavity filled with alumina/water nanofluid. Very recently, Garoosi et al. [14] studied numerically the natural convection heat transfer of nanofluid in a square cavity using finite volume discretization method. Nevertheless, the study of natural convection fluid flow and heat transfer in a trapezoidal cavity partially filled with porous medium has not been undertaken yet.

Most of the previous studies have investigated the Newtonian fluids with the focus on clear fluid media as mentioned in above works. On the other hand, relatively limited work has been directed to the natural convection of non-Newtonian fluids. The flow characteristics of several industrial fluids such as multi-phase mixtures (oil–water emulsions, froths and foams, etc.), paints and biological fluids (blood, synovial fluid, saliva, etc.) can be sorted into the non-Newtonian fluids. Also, the non-Newtonian fluids have attracted a lot of attentions due to their importance in several industrial applications such as, compact heat exchangers, polymer engineering, electronic cooling systems, geophysical systems, chemical reactor design, etc. When the relation between the shear stress and the shear rate is nonlinear (the shear stress is proportional to the strain rate), a fluid converts to non-Newtonian fluid. The non-Newtonian fluids in natural convection filled with clear fluid have received considerable attention by many researchers. Bin Kim et al. [15] have studied the effect of unsteady natural convection in a square enclosure with non-Newtonian fluid. Lamsaadi et al. [16] investigated analytically and numerically the effect of steady natural convection in a shallow rectangular cavity with non-Newtonian power law fluids. Turan et al. [17] have considered the two-dimensional steady-state simulations of laminar natural convection in a square enclosure filled with non-Newtonian fluids and differentially heated sidewalls subjected to constant wall temperatures. Habibi Matin et al. [18] discussed the steady natural...
convection of non-Newtonian power-law fluid between two eccentric horizontal square ducts with constant temperature. However, no work has been done on natural convection of a non-Newtonian fluid in a trapezoidal cavity partly filled with a porous medium and partly with a fluid layer, thus the authors of the present study believe that this work is valuable.

The streamlines sufficiently depict the fluid flow whereas isotherms indicate only temperature distribution which may not be adequate for the visualization of heat transport. The heatline technique is the best way to visualize the conductive as well as convective heat transport which first introduced by Kimura and Bejan [19]. Heatlines are mathematically represented by heatfunctions which are in turn related to Nusselt number based on proper dimensionless form. The work of Costa [20] considered the functions and lines for visualization of the streamline, heatline and massline methods in two-dimensional transport phenomena. Costa [21] has studied the steady unified streamline, heatline and massline methods into two-dimensional anisotropic media. Zhao et al. [22] investigated numerically the effect of the heatline analysis on natural convection in a rectangular cavity with localized heating and salting from below using the Brinkman–extended Darcy model. Saleh et al. [23] have used the heatline concept on natural convection in a trapezoidal cavity filled with a porous medium based on heatline concept. However, the study of heatline analysis on natural convection in a trapezoidal cavity partly filled with a porous medium has not been undertaken yet. The aim of this study is to investigate the heatline analysis on natural convection in a trapezoidal cavity partly filled with porous nanofluid and partly with non-Newtonian fluid. This study is unique in the sense that it focuses on natural convection in a closed cavity consisting of two layers with the presence of non-Newtonian fluids.

2. Mathematical formulation

Consider two-dimensional natural convection in a trapezoidal cavity with length $L$, the left cavity part filled with nanofluid porous $W$, while the remainder of the cavity ($L - W$) is filled with non-Newtonian power-law fluid, as illustrated in Fig. 1. The left sloping wall of the cavity is heated to a constant temperature $T_h$ and the right sloping wall is maintained at a constant cold temperature $T_c$, while the horizontal walls are adiabatic. The outer boundaries are assumed to be impermeable, while the interface boundary (between the two layers) is assumed to be permeable. The Brinkman–extended Darcy model with the energy transport equations are applied to describe the non-Newtonian nanofluid flow and the heat transfer process in the porous/liquid layer. The pores are filled with fluid composed of water-base nanofluids containing Ag, Cu, Al$_2$O$_3$ or TiO$_2$ nanoparticles. The flow is assumed to be laminar and the non-Newtonian nanofluid physical properties are constant except for the density in the buoyancy term which follows the Boussinesq approximation. Based on these assumptions, the dimensionless governing equations for the nanofluid layer can be written as:

\[
\frac{\partial u_{nf}}{\partial x} + \frac{\partial v_{nf}}{\partial y} = 0, \tag{1}
\]

\[
u_{nf} \frac{\partial u_{nf}}{\partial x} + \nu_{nf} \frac{\partial u_{nf}}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u_{nf}}{\partial x^2} + \frac{\partial^2 u_{nf}}{\partial y^2} \right) + \frac{(\rho \beta)_{nf}}{\rho_{nf}} g \sin \phi (T_{nf} - T_c) - \frac{\mu_{nf} \nu_{nf}}{\rho_{nf} K}, \tag{2}
\]

\[
u_{nf} \frac{\partial v_{nf}}{\partial x} + \nu_{nf} \frac{\partial v_{nf}}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v_{nf}}{\partial x^2} + \frac{\partial^2 v_{nf}}{\partial y^2} \right) + \frac{(\rho \beta)_{nf}}{\rho_{nf}} g \cos \phi (T_{nf} - T_c) - \frac{\mu_{nf} \nu_{nf}}{\rho_{nf} K}, \tag{3}
\]

\[
u_{nf} \frac{\partial T_{nf}}{\partial x} + \nu_{nf} \frac{\partial T_{nf}}{\partial y} = \chi_{nf} \left( \frac{\partial^2 T_{nf}}{\partial x^2} + \frac{\partial^2 T_{nf}}{\partial y^2} \right). \tag{4}
\]

Fig. 1. Physical model of convection in a trapezoidal cavity together with the coordinate system and (a) the side wall inclination angle, (b) the inclination angle of the cavity.
The dimensional equations for the non-Newtonian fluid layer can be written as:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \text{(5)}
\]

\[
u \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho_l} \frac{\partial p}{\partial x} + \frac{1}{\rho_l} (\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}) + \mu_l \beta_l g (T_{nf} - T_c) \sin \omega, \quad \text{(6)}
\]

\[
u \frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho_l} \frac{\partial p}{\partial y} + \frac{1}{\rho_l} (\frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{yx}}{\partial x}) + \mu_l \beta_l g (T_{nf} - T_c) \cos \omega, \quad \text{(7)}
\]

\[
u \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\alpha_l}{C_{p_l}} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \quad \text{(8)}
\]

For a purely-viscous non-Newtonian fluid, which follows the Ostwald–De Waele (i.e. power-law) model, the shear stress tensor is [27]

\[
\tau = 2 \mu D \eta = \mu \varepsilon \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right), \quad \text{(9)}
\]

where \(D \eta\) indicates the rate-of-deformation tensor for the two dimensional Cartesian coordinate and \(\mu \) is the apparent viscosity that is derived for the two-dimensional Cartesian coordinate as

\[
\mu = \frac{\alpha_l}{C_{p_l}} \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 + \mu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right)^2 \quad \text{(10)}
\]

where the dimensionless parameter \(n\) is the power-law index, and \(\varepsilon\) is the consistency coefficient (consistency factor). With \(n = 1\), Eq. (10) represents a Newtonian fluid with dynamic coefficient of viscosity \(\varepsilon\). Therefore, the deviation of \(n\) from unity indicates the degree of deviation from Newtonian behavior. With \(n \neq 1\), the constitutive Eq. (10) represents pseudo-plastic fluid (shear thinning) \((n < 1)\) and for \((n > 1)\) it represents a dilatant fluid (shear thickening), respectively.

The two-domain approach is used, where the porous and fluid layers are treated separately with matching coupling conditions at the interface. These conditions express the continuity of vertical velocities, shear stress, normal stress, temperature and heat mass fluxes. At the interface, these conditions are written as:

\[
T_l |_{x=-s} = T_l |_{x=s}, \quad \text{(11)}
\]

\[
k_l \frac{\partial T_l}{\partial x} |_{x=-s} = k_f \frac{\partial T_f}{\partial x} |_{x=s}, \quad \text{(12)}
\]

\[
u_l |_{x=-s} = u_f |_{x=s}, \quad \nu_f |_{x=-s} = v_f |_{x=s}, \quad \text{(13)}
\]

\[
p_l |_{x=-s} = p_f |_{x=s}, \quad \text{(14)}
\]

\[
\mu |_{x=-s} \left( \frac{\partial u_l}{\partial x} + \frac{\partial u_f}{\partial y} \right) = \mu |_{x=s} \left( \frac{\partial u_l}{\partial x} + \frac{\partial u_f}{\partial y} \right). \quad \text{(15)}
\]

Note that Eq. (15) represents an extension of the shear stress matching condition of Neale and Naber [28] for flow which is not parallel to the fluid/porous layer interface. Obviously, matching of the stresses at the interface can only be accomplished if Brinkman’s extension is used in the momentum equations for the porous layer [4]. In the interface, \(x\) and \(y\) are the Cartesian coordinates measured in the horizontal and vertical directions respectively, \(K\) represents the permeability of the porous medium and \(g\) is the acceleration due to gravity, \(\phi\) stands for the solid volume fraction of nanoparticles, \(\alpha_l\) is the effective thermal diffusivity of the nanofluids, \(\rho_{nf}\) represents the effective density of the nanofluids and \((\rho C_p)_{nf}\) is the heat capacitance of the nanofluids, defined in [29] as

\[
\alpha_l = \frac{k_{ef}}{(\rho C_p)_{nf}}, \quad \rho_{nf} = (1 - \phi) \rho_{nf} + \phi \rho_{sp}, \quad (\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_{sp} + \phi (\rho C_p)_{sp}. \quad \text{(16)}
\]

The effective dynamic viscosity of the nanofluids is given in [30] as

\[
\mu_{nf} = \mu_l f_{2} \sin \omega, \quad \text{(17)}
\]

The thermal expansion coefficient of the nanofluids can be determined by (cf. [30])

\[
\beta_{nf} = (1 - \phi) \beta_{nf} + \phi \beta_{sp}. \quad \text{(18)}
\]

\[
(\rho \beta)_{nf} = (1 - \phi) (\rho \beta)_{sp} + \phi (\rho \beta)_{sp}. \quad \text{(19)}
\]

The thermal conductivity based on Maxwell–Garnett’s (MG) model is (cf. [31])

\[
k_{nf} = k_f + 2k_{ef} - 2\phi(k_f - k_{sp}), \quad k_{sp} = k_{sp} + 2k_{sp} \beta_{sp}. \quad \text{(20)}
\]

Now, we introduce the following non-dimensional variables:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{L u}{T}, \quad V = \frac{L v}{T}, \quad \theta = \frac{T - T_c}{T_h - T_c}, \quad \text{(21)}
\]

This then yields the dimensionless governing equations for the nanofluid (porous) layer,

\[
\frac{\partial U_l}{\partial X} + \frac{\partial V_l}{\partial Y} = 0, \quad \text{(22)}
\]

\[
U_l \frac{\partial U_l}{\partial X} + V_l \frac{\partial U_l}{\partial Y} = \frac{\rho_l}{\mu_l} \frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 U_l}{\partial X^2} + \frac{\partial^2 U_l}{\partial Y^2} \right) + \frac{(\rho \beta)_{nf}}{\rho_{nf} \Pr} \theta_{nf} \sin \omega - \frac{\mu_{nf}}{\mu_{nf}} \frac{Pr U_l}{Da}, \quad \text{(23)}
\]

\[
U_l \frac{\partial V_l}{\partial X} + V_l \frac{\partial V_l}{\partial Y} = \frac{\rho_l}{\mu_l} \frac{\partial P}{\partial X} + Pr \left( \frac{\partial^2 V_l}{\partial X^2} + \frac{\partial^2 V_l}{\partial Y^2} \right) + \frac{(\rho \beta)_{nf}}{\rho_{nf} \Pr} \theta_{nf} \cos \omega - \frac{\mu_{nf}}{\mu_{nf}} \frac{Pr V_l}{Da}, \quad \text{(24)}
\]

\[
U_l \frac{\partial \theta_{nf}}{\partial X} + V_l \frac{\partial \theta_{nf}}{\partial Y} = \frac{\alpha_l}{\mu_{nf}} \left( \frac{\partial^2 \theta_{nf}}{\partial X^2} + \frac{\partial^2 \theta_{nf}}{\partial Y^2} \right), \quad \text{(25)}
\]

and the dimensionless governing equations for the fluid layer,

\[
\frac{\partial U_f}{\partial X} + \frac{\partial V_f}{\partial Y} = 0, \quad \text{(26)}
\]

\[
U_f \frac{\partial U_f}{\partial X} + V_f \frac{\partial U_f}{\partial Y} = \frac{\partial P}{\partial X} + \left[ 2 \frac{\partial}{\partial X} \left( \frac{\mu_f}{\varepsilon} \frac{\partial u_f}{\partial X} \right) + \frac{\partial}{\partial Y} \left( \frac{\mu_f}{\varepsilon} \frac{\partial u_f}{\partial Y} + \frac{\mu_f}{\varepsilon} \frac{\partial V_f}{\partial Y} \right) \right] + Pr \left[ \frac{\partial^2 U_f}{\partial X^2} + \frac{\partial^2 U_f}{\partial Y^2} \right] \quad \text{(27)}
\]
\[ U_j \frac{\partial V_j}{\partial X} + V_j \frac{\partial U_j}{\partial Y} = -\frac{\partial P}{\partial Y} + Pr \left[ \frac{\partial}{\partial X} \left( \frac{\mu_s}{e} \frac{\partial U_j}{\partial Y} + \frac{\mu_s}{e} \frac{\partial V_j}{\partial X} + 2 \frac{\partial}{\partial Y} \left( \frac{\mu_s}{e} \frac{\partial V_j}{\partial Y} \right) \right) \right] + RaPr \theta \cos \omega, \]

\[ \frac{\partial \theta}{\partial Y} + \frac{\partial \theta}{\partial X} = \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}, \]

where \( Ra_{df} = gK_{df}\mu_{df}(T_h - T_c) L/(\mu_s x_{df}) \) is the Rayleigh number, \( Da = K/L^2 \) represents the Darcy number and \( Pr = V_j/\gamma_j \) is the Prandtl number for both the fluid and the porous layers. The dimensionless boundary conditions are

\[ \Psi = 0 \]

for all solid boundaries,

\[ \theta_{df} = \theta_l = 1 \quad \text{at} \quad X = 0, \]

\[ \theta_{df} = \theta_l = 0 \quad \text{at} \quad X = 1, \]

\[ \frac{\partial \theta_{df}}{\partial Y} = \frac{\partial \theta_l}{\partial Y} = 0 \quad \text{at} \quad Y = 0, \quad Y = 1. \]

The dimensionless form of the heatfunction \( H \) for the nanofluid layer problem can be defined as [24]:

\[ \frac{\partial H}{\partial Y} = U_0 - \frac{\partial \theta}{\partial X}, \quad \frac{\partial H}{\partial X} = V_0 - \frac{\partial \theta}{\partial Y}, \]

which yields a single equation

\[ \frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} = \frac{\partial (U_0)}{\partial Y} - \frac{\partial (V_0)}{\partial Y}. \]

For the fluid layer, the dimensionless form of the heatfunction can be defined as [23]:

\[ \frac{\partial H}{\partial Y} = U_0 - \frac{\partial \theta}{\partial X}, \quad \frac{\partial H}{\partial X} = V_0 - \frac{\partial \theta}{\partial Y}, \]

which yields a single equation

\[ \frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} = \frac{\partial (U_0)}{\partial Y} - \frac{\partial (V_0)}{\partial Y}. \]

The Neumann boundary conditions for \( H \) for the (hot or cold) vertical walls from Eq. (36) and the normal derivatives \( \langle n \cdot \nabla H \rangle \) are specified as follows:

\[ n \cdot \nabla H = 0 \quad \text{at the vertical walls}, \]

\[ n \cdot \nabla H = -\frac{k_{df}}{k_f} \int_0^1 \frac{\partial \theta_{df}}{\partial X} dY \quad \text{at the top wall}. \]

The bottom insulated wall may be represented by the Dirichlet boundary condition obtained from Eq. (36), which is simplified into \( \partial H/\partial X = 0 \) for an adiabatic wall. A reference value of \( H \) is assumed as 0 at \( X = 0, Y = 0 \) and hence \( H = 0 \) is valid for \( Y = 0, X \).

\[ H = \frac{S}{2} \langle n \cdot \nabla H \rangle \text{Nu}_{df} \]

is obtained from Eq. (36) at \( X = 0, Y = 0 \) (adiabatic wall), where \( \text{Nu}_{df} \) is the local average Nusselt number evaluated at the hot sloping wall

\[ \text{Nu}_{df} = \int_0^1 \left[ \frac{1}{k_f} \frac{\partial \theta_{df}}{\partial X} \right] dY. \]

The boundary conditions for the non-adiabatic walls are given above in Eqs. (38) and (39).

3. Numerical method and validation

The dimensionless governing Eqs. (22)–(29) subject to the boundary conditions (30)–(33) which are solved numerically based on the Finite Volume Method (FVM) using a collocated grid system. A central difference scheme is used to discretize the diffusion terms whereas a blending of upwind and central difference is adopted for the convection terms. The computational domain consists of a main non-Newtonian flow zone and a porous zone. Non-uniform grids are used in the present program, allowing a fine grid spacing near the inner horizontal and vertical boundaries with special attention to the corners of the cavity where more grids are clustered in these regions to capture the flow and thermal fields accurately. The set of conservation equations are integrated over the control volumes, leading to a balance equation for the fluxes at the interfaces. The harmonic mean formulation adopted for the interface diffusion coefficients between two control volumes can handle abrupt changes in these coefficients [4]. At the beginning of the numerical simulations, the initial values for the temperature, the stream function, the pressure, the velocity components and the heatfunction at all the interior grid points were set to zero. The resulting discretized equations have been solved iteratively through strongly implicit procedure (SIP). The SIMPLE algorithm [32] has been adopted for the pressure velocity coupling. More details of the discretization and computational procedure can be found in [32]. In this paper several grid testings are performed: 50 × 50, 70 × 70, 90 × 90, 100 × 100, 120 × 120. Table 1 shows the calculated Nusselt number at different mesh numbers for \( \varphi = 0 \).

In order to validate the numerical code, a comparison with the previous published results is necessary. The present numerical results are verified against the results obtained by Chamkha and Ismail [33] as shown in Fig. 2 for \( Ra = 10^5, Da = 10^{-5}, n = 1, S = 0.3, \varphi = 0 \). Clearly from this comparison, the present results are in excellent agreement with the corresponding results of Chamkha and Ismail [33].

4. Results and discussion

We used the grid 100 × 100 for calculating the streamlines, isothersms and heatlines and 90 × 90 for the Nusselt number. The graphical results are presented in this section for the streamlines of porous/fluid-layers, isothersms of porous/liquid-layers and heatlines of porous/liquid-layers with various Prandtl number \((0.015 < Pr < 13.4), \) Darcy number \((10^{-5} < Da < 10^{-1}), \) nanoparticle volume fractions \( (0 < \phi < 0.2), \) nanofluid types, power-law index \((0.6 < n < 1.4), \) porous layer thickness \((0.3 < S < 0.7), \) the side wall inclination angle \((0^\circ < \varphi < 21.8^\circ), \) the inclination angle of the cavity \((0^\circ < \omega < 90^\circ) \) and Rayleigh number \((Ra = 10^5 \text{ and } 10^6)). \)

The values of the average Nusselt number were calculated for various values of \( Da, \phi, \omega \) and \( n. \) Table 2 lists the water base fluid \((Pr = 6.2)\) with the thermo-physical properties of the considered nanoparticles.

The effects of Prandtl number on the streamlines (left), isothersms (middle) and heatlines (right) of 0.7 power-index (shear thinning) are illustrated in Fig. 3 for water–Cu nanofluids with Rayleigh number \((Ra = 10^5), \) Darcy number \((Da = 10^{-3}), \) porous layer thickness \((S = 0.5), \) side wall inclination angle \((\varphi = 16.7^\circ)\) and inclination angle of the cavity \((\omega = 0^\circ)\). Fig. 3(a) presents the streamlines, heatlines and isotherm patterns with lower value of
Prandtl number (Pr = 0.015) for mercury. In general, changing the flow motion with the boundary condition sets led the streamlines to generate a clockwise rotating cell located in the center of cavity (between nanofluid layer and fluid layer). The streamlines in the fluid layer appear to be higher than that within the porous layer due to the fact that the fluid layer has a stronger effective thermal conductivity compared to the porous layer. Also the flow permeates more smoothly in the fluid layer than in the porous layer. The non-homogeneous Dirichlet boundary condition was applied with the heatfunctions Eqs. (35) and (37), \( H = 0 \) for the bottom wall \( (Y = 0, \forall X) \), and for the top wall \( (Y = 1, \forall X) \) \( H = \frac{\phi}{2} \frac{Nu}{Pr} \).

Contour level labels were used to know the direction of the fluid heat flow whether a clockwise or anti-clockwise directions and also the strength of the flow. The positive sign of \( \Psi \) and \( H \) denote anti-clockwise fluid heat flow, whereas the negative sign of \( \Psi \) and \( H \) designate the clockwise fluid heat flow. \( \Psi_{\text{min}} \) and \( H_{\text{min}} \) represent the optimal heatfunction and optimal streamfunction. These values are important to show the maximum changing of the flow and heat circulation strength. When the streamlines and heatlines circulated as vortices in the clockwise direction (negative signs of \( \Psi \) and \( H \)), the strength of the flow circulation was presented by \( \Psi_{\text{min}} \) and \( H_{\text{min}} \). As the nanoparticle volume fraction is applied \( (\phi = 0.05) \), the circulation intensity increases (see \( \Psi_{\text{min}}, H_{\text{min}} \) values) due to the increase in the thermal conductivity of nanofluid. The flow circulation of the streamlines and the heatlines cells for pure fluid are bigger than that for nanofluid. The conduction heat transfer forced the isotherms patterns within the nanofluid layer to take almost a diagonal shape, while the convection mode heat transfer affecting the isotherms patterns within the fluid layer appears almost horizontally with the sloping walls, as shown in Fig. 3(a) and (b). The circulation intensity increases with the streamlines circulation cell moving to the fluid layer due to the flow vortex. At all Prandtl numbers, a competing effect occurs between the momentum and heat transfer. Thus in contrast to the Newtonian situation, the heat transfer is decreased.

**Table 2**

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>Water</th>
<th>Silver</th>
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<tr>
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Fig. 2. (a) Streamlines: (left) Chamkha and Ismael [33], (right) present study, (b) isotherms: (left) Chamkha and Ismael [33], (right) present study, for \( Ra = 10^5 \), \( Da = 10^{-5} \), \( n = 1 \), \( S = 0.3 \), \( \phi = 0 \) (solid lines) and \( \phi = 0.05 \) (dashed lines).

Prandtl number (Pr = 0.015) for mercury. In general, changing the flow motion with the boundary condition sets led the streamlines to generate a clockwise rotating cell located in the center of cavity (between nanofluid layer and fluid layer). The streamlines in the fluid layer appear to be higher than that within the porous layer due to the fact that the fluid layer has a stronger effective thermal conductivity compared to the porous layer. Also the flow permeates more smoothly in the fluid layer than in the porous layer. The non-homogeneous Dirichlet boundary condition was applied with the heatfunctions Eqs. (35) and (37), \( H = 0 \) for the bottom wall \( (Y = 0, \forall X) \), and for the top wall \( (Y = 1, \forall X) \) \( H = \frac{\phi}{2} \frac{Nu}{Pr} \).

Contour level labels were used to know the direction of the fluid heat flow whether a clockwise or anti-clockwise directions and also the strength of the flow. The positive sign of \( \Psi \) and \( H \) denote anti-clockwise fluid heat flow, whereas the negative sign of \( \Psi \) and \( H \) designate the clockwise fluid heat flow. \( \Psi_{\text{min}} \) and \( H_{\text{min}} \) represent the optimal heatfunction and optimal streamfunction. These values are important to show the maximum changing of the flow and heat circulation strength. When the streamlines and heatlines circulated as vortices in the clockwise direction (negative signs of \( \Psi \) and \( H \)), the strength of the flow circulation was presented by \( \Psi_{\text{min}} \) and \( H_{\text{min}} \). As the nanoparticle volume fraction is applied \( (\phi = 0.05) \), the circulation intensity increases (see \( \Psi_{\text{min}}, H_{\text{min}} \) values) due to the increase in the thermal conductivity of nanofluid. The flow circulation of the streamlines and the heatlines cells for pure fluid are bigger than that for nanofluid. The conduction heat transfer forced the isotherms patterns within the nanofluid layer to take almost a diagonal shape, while the convection mode heat transfer affecting the isotherms patterns within the fluid layer appears almost horizontally with the sloping walls, as shown in Fig. 3(a) and (b). The circulation intensity increases with the streamlines circulation cell moving to the fluid layer due to the flow vortex. At all Prandtl numbers, a competing effect occurs between the momentum and heat transfer. Thus in contrast to the Newtonian situation, the heat transfer is decreased.

**Table 2**

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Fig. 2. (a) Streamlines: (left) Chamkha and Ismael [33], (right) present study, (b) isotherms: (left) Chamkha and Ismael [33], (right) present study, for \( Ra = 10^5 \), \( Da = 10^{-5} \), \( n = 1 \), \( S = 0.3 \), \( \phi = 0 \) (solid lines) and \( \phi = 0.05 \) (dashed lines).

Prandtl number (Pr = 0.015) for mercury. In general, changing the flow motion with the boundary condition sets led the streamlines to generate a clockwise rotating cell located in the center of cavity (between nanofluid layer and fluid layer). The streamlines in the fluid layer appear to be higher than that within the porous layer due to the fact that the fluid layer has a stronger effective thermal conductivity compared to the porous layer. Also the flow permeates more smoothly in the fluid layer than in the porous layer. The non-homogeneous Dirichlet boundary condition was applied with the heatfunctions Eqs. (35) and (37), \( H = 0 \) for the bottom wall \( (Y = 0, \forall X) \), and for the top wall \( (Y = 1, \forall X) \) \( H = \frac{\phi}{2} \frac{Nu}{Pr} \).

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Fig. 3. Streamlines (left), isotherms (middle) and heatlines (right) evolution by Prandtl number for $Ra = 10^5$, $Da = 10^{-3}$, $n = 0.7$, $S = 0.5$, $\phi = 16.7^\circ$, $\omega = 0^\circ$, $\psi = 0$ (solid lines) and $\phi = 0.05$ (dashed lines).
The effect of nanofluid layer thickness on the streamlines (left), isotherms (middle) and heatlines (right) of 0.7 power-index (shear thinning) is shown in Fig. 4 for water–Cu nanofluids, \( Ra = 10^5 \), \( Da = 10^{-3} \), \( n = 0.7 \), \( \varphi = 16.7^\circ \), \( \theta = 0^\circ \), \( \phi = 0 \) (solid lines) and \( \phi = 0.05 \) (dashed lines).

On the other hand, as we move vertically the strength of heatlines tends to be weak at the base of the cavity. As the nanofluid layer increases (\( S = 0.5 \)), the streamlines circulation cell within the fluid layer shrinks, transforming into an oval-shape. The strength of the flow circulation decreases for both pure fluid and nanofluid with increasing values of \( S \) (see \( \Psi_{min}, H_{min} \) values), due to the resistance of the nanofluid layer hydrodynamics. Similar to the streamlines, the heatlines circulation cell tends to shrink and move to the nanofluid layer for pure fluid, while the flow circulation cell for nanofluid stays within the fluid layer. In addition, to visualize the heat transfer and the total energy flow, the study of heatlines technique is necessary to show the trajectory of heat transfer from the porous...
layer to the fluid layer, as illustrated in Fig. 4(b). At a higher $S$ value ($S = 0.7$), the streamlines are significantly affected. The streamlines circulation cell takes place between the nanofluid layer and fluid layer, as the heatlines circulation cell disappears. The strength of the flow circulation decreases by the addition of the 5% of nanoparticles, while $H_{\text{min}}$ exhibits an identical value for pure fluid and nanofluid, as presented in Fig. 4(c).

Fig. 5 shows the effects of various values of the power-index on the streamlines (left), isotherms (middle) and heatlines (right) for water–Cu, $Ra = 10^6$, $Da = 10^{-3}$, $S = 0.5$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. Fig. 5(a) illustrates the effect of changing the flow motion with the boundary condition set at a lower power-index $n = 0.6$. Similar to the previous results, the streamlines in the fluid layer appear higher than that within the porous layer due to the fact that the fluid layer has a stronger effective thermal conductivity compared to the porous layer. Also, the flow permeates more smoothly in the fluid layer than in the porous layer. Noticeably, the maximum strength of the flow circulation string occurs with the pseudoplastic (shear-thinning) fluid. The streamlines appear with a high intensity near to the sloping cold wall (fluid layer) with a single clockwise rotating cell along with the fluid layer. The strength of the flow circulation of the streamlines and heatlines tends to increase by adding 5% of Cu nanoparticles. The isotherms patterns within the nanofluid layer appear with a diagonal shape, while the isotherms patterns within the fluid layer appears with almost a horizontal line to the sloping walls affected by the conduction and convection mode heat transfer. Increasing power-index to 0.8 leads to the reduction of the strength of the flow circulation (pure fluid and nanofluid). The convective heat transport is weak compared to the viscous flow resistance, as a result the streamlines magnitude decrease by increasing the power–index. The clockwise rotating cell into the fluid layer tends to shrink towards an oval-shape. The intensity of the isotherms pattern is reduced and occupies more amplitude within the cavity, signaling weak convective heat transport. The strength of the flow circulation of the heatlines decreases with the appearance of the clockwise circulation cell, where the flow circulation cell of the heatlines cells for pure fluid are much bigger than that for nanofluid. The Newtonian fluid ($n = 1$) effect is shown in Fig. 5(c); the streamlines circulation cell is moved near to the interface. In the opposite way, Fig. 5(d) displays the dilatant fluid (shear thickening), by increasing $n$ value to 1.4 the flow slows down, as a result the circulation of the flow tends to decrease. The streamlines within the fluid layer become lower than that in the porous layer. Clearly, the strength of the flow circulation of the streamlines and heatlines tends to increase by adding 5% of Cu nanoparticles. The flow circulation cell of the heatlines for pure fluid is smaller than that for nanofluid. Further, the heatlines intensity decreases at the top of the cavity which leads to an increase in the strength of the heatlines along the cavity, as depicted in Fig. 5(d).

The effects of the side wall inclination angle on the streamlines (left), isotherms (middle) and heatlines (right) of 0.7 power-index (shear thinning) are shown in Fig. 6 for water–Cu nanofluids, $Ra = 10^6$, $Da = 10^{-3}$, $S = 0.5$ and $\omega = 0^\circ$. Fig. 6(a) depicts the effect of changing the flow motion with the boundary condition set without the side wall inclination angle ($\varphi = 0^\circ$), i.e. square cavity. Clearly, the maximum strength of the flow circulation for the streamlines and heatlines of the pure fluid occurs for the square cavity due to the higher amount of the flow velocity. Enforcing a lower $\varphi$ ($\omega = 11.3^\circ$) leads to a decrease of the intensity of streamlines and heatlines for pure fluid and nanofluid, as a result the strength of the flow circulation for both streamlines and heatlines is decreased (see $W_{\text{min}}$, $H_{\text{min}}$ values). This is due to reduction of the space for fluid circulation by increasing the slopping angle. The intensity of the isotherms patterns is increased by increasing $\varphi$ value affected by the cavity types, as presented in Fig. 6(b). Fig. 6(c) illustrates that the strength of the flow circulation for the streamlines of the pure fluid remained unchanged, while for the nanofluid, increases to the maximum value with the increase of the $\varphi$ to the maximum value ($\varphi = 21.8^\circ$). Also, the figure displays that the flow circulation cell of the heatlines for pure fluid is much bigger than that for nanofluid.

The effect of the inclination angle of the cavity on the streamlines (left), isotherms (middle) and heatlines (right) of 0.7 power-index (shear thinning) are presented in Fig. 7 for water–Cu nanofluids, $Ra = 10^6$, $Da = 10^{-3}$, $S = 0.5$ and $\varphi = 16.7^\circ$. The effect of lower $\omega$ on the flow motion with the boundary condition set is demonstrated in Fig. 7(a). The streamlines tend to appear with a high intensity into the fluid layer with a circulation spherical cell close to the right cold wall. The strength of the flow circulation for the streamlines and heatlines in the clockwise direction is increased by applying 5% of Cu nanoparticles (see $W_{\text{min}}$, $H_{\text{min}}$ values). Consistent with the previous results, isotherms patterns within the nanofluid layer tend to take diagonal shape, while the isotherms patterns within the fluid layer appear with a horizontal line to the sloping walls. The streamlines and heatlines are switched into the counterclockwise direction (positive signs of $\Psi$ and $H$) by increasing $\varphi$ value ($\omega = 45^\circ$). This is due to the configuration becoming more and more like heating from below or Bénard convection problem. The gravitational force tends to push the fluid flow circulation and eventually changes the flow direction. The streamlines circulation cell within the fluid layer tends to move near to the bottom, affected by the inclination angle of the cavity, and consequently, the strength of the flow circulation for both streamlines and heatlines is increased (see $W_{\text{min}}$, $H_{\text{min}}$ values). Considerable difference is observed on the heatlines due to the significant effect of changing the inclination angle, showing the heatlines strength (large amount of heat) transfers from the upper segment to the lower segment of the cavity. The isotherms patterns within the nanofluid layer (near to the hot wall) tend to take vertical shape, while the lines near to the interface take a diagonal shape with high intensity close to the bottom wall. The heatlines circulation cell appears with the inverse situation with higher intensity within bottom wall. Fig. 7(d) illustrates that the maximum value of the strength of the flow circulation for the streamlines and heatlines (pure fluid and nanofluid) occurs for the maximum value of $\omega$ ($\omega = 90^\circ$).

Fig. 8(a) presents the effect of the power-law index on the average Nusselt number with Darcy number of water–Cu at $Ra = 10^6$, $\varphi = 0.05$, $S = 0.5$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. At a high Darcy number ($Da \gg 10^{-3}$), the convection enhances which significantly increases the average Nusselt number. The convection remains weak at low Darcy number as $n$ increases. A non-Newtonian fluid (shear thinning) ($n < 1$) effect leads to a stronger convection, which significantly increases the average Nusselt number as a result. Fig. 8(b) demonstrates the effect of various side wall inclination angles on the average Nusselt number with Darcy number of water–Cu at $Ra = 10^6$, $\varphi = 0.05$, $n = 0.7$ and $S = 0.5$. Obviously, convection heat transfer is enhanced by increasing Darcy number, consequently the average Nusselt number increases. The significant increase in average Nusselt number occurs with increase in concentration of Darcy number between $[10^{-4}, 10^{-3}]$. Higher $\varphi$ value ($\varphi = 21.8^\circ$) leads to strong enhancement in the convection with the maximum value of the average Nusselt number.

Fig. 9(a) summarizes the variations in the average Nusselt numbers with Darcy number for different nanoparticle volume fractions of water–Cu at $Ra = 10^6$. $n = 0.7$, $S = 0.5$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. The convection heat transfer is increased by the rise of Darcy number. A higher concentration of nanoparticle volume
Fig. 5. Streamlines (left), isotherms (middle) and heatlines (right) evolution by power-law index for $Ra = 10^5$, $Da = 10^{-3}$, $S = 0.5$, $\phi = 16.7^\circ$, $\epsilon = 0^\circ$, $\phi = 0$ (solid lines) and $\phi = 0.05$ (dashed lines).
fractions ($\phi = 0.2$) leads to a higher average Nusselt number, but also this value tends to drop lower than other values ($\phi = 0, 0.05$ and 0.1) when the concentration of Darcy number is between $[10^{-4}, 10^{-3}]$, due to the structure of the cavity slopping walls. The significant enhancement in average Nusselt number appears with higher values of Darcy number ($Da > 10^{-3}$). Fig. 9(b) shows the effect of the porous layer thickness on the average Nusselt number with Darcy number for water–Cu at $Ra = 10^5$, $Da = 10^{-2}$, $n = 0.7$, $S = 0.5$, $\varphi = 0.05$, $\omega = 0^\circ$ : $\phi = 0$ (solid lines) and $\phi = 0.05$ (dashed lines).

Due to its higher thermal conductivity, Ag helps water in transporting more heat compared to what it is with Cu, Al$_2$O$_3$ and TiO$_2$ nanoparticles. The weak enhancement appeared in the convection by applying Cu nanoparticle as the nanoparticle volume fraction increases. The lower thermal conductivity of Al$_2$O$_3$ and TiO$_2$ nanoparticle decreases the heat transfer rate for higher nanoparticle volume fraction ($\phi > 0.08$). It is; however, TiO$_2$ that transports marginally more heat than Al$_2$O$_3$. A very interesting

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**Fig. 6.** Streamlines (left), isotherms (middle) and heatlines (right) evolution by side wall inclination angle for $Ra = 10^5$, $Da = 10^{-2}$, $n = 0.7$, $S = 0.5$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. $\psi$ ($\Psi_{\text{min}} = -8.75, \Psi_{\text{min}} = -8.84$), $H$ ($H_{\text{min}} = -0.43, H_{\text{min}} = -0.45$) and $\varphi$ ($\varphi = 0^\circ$).
Fig. 7. Streamlines (left), isotherms (middle) and heatlines (right) evolution by inclination angle of the cavity for $Ra = 10^5$, $Da = 10^{-3}$, $n = 0.7$, $S = 0.5$, $\varphi = 16.7^\circ$, $\phi = 0$ (solid lines) and $\phi = 0.05$ (dashed lines).
result on the effect of various side wall inclination angle on the average Nusselt number for water–Cu at $Ra = 10^6$, $Da = 10^{-3}$, $n = 0.7$, $S = 0.5$ and $\omega = 0^\circ$. We find that the average Nusselt number increases with increase in $\phi$ in $[0, 0.14]$ but decreases with increase in $\phi$ in $[0.14, 0.2]$. Further, higher side wall inclination angle strongly enhances the heat transfer rate which has the maximum value of the average Nusselt number.
The effects of various Prandtl number on the average Nusselt number with power-law index for water–Cu at $Ra = 10^6$, $Da = 10^{-3}$, $n = 0.8$, $S = 0.5$, $\phi = 0.05$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$ are illustrated in Fig. 11(a). The values of average Nusselt number clearly show that the convection heat transfer decreases systematically with increase in power-law index. It is, however, pseudoplastic fluid which transports more heat than Newtonian and dilatant fluids due to the high viscosity and high shear rate. The convection heat transfer is more influenced by higher Prandtl number ($Pr = 13.4$) for salt water with the maximum average Nusselt number attributed to the high velocity to transferring the heat. Fig. 11(b) displays the effect of various side wall inclination angle on the average Nusselt number with power-law index for water–Cu at $Ra = 10^6$, $Da = 10^{-3}$, $n = 0.8$, $S = 0.5$, $\phi = 0.05$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. The average Nusselt number significantly decreases as the power-law index increases. Stronger enhancement in the heat transfer rate is observed at higher side wall inclination angle ($\varphi = 21^\circ$), thus showing the maximum value of the average Nusselt number. Because of its sloping walls, the cavity transfers more heat.

Fig. 12(a) presents the effect of various power-law index on the average Nusselt number with power-law index for water–Cu at $Ra = 10^6$, $Da = 10^{-3}$, $n = 0.6$, $S = 0.5$, $\phi = 0.05$, $\varphi = 16.7^\circ$ and $\omega = 0^\circ$. The average Nusselt number is at a maximum value for lower $n$ ($n = 0.6$) due to the high viscosity and high shear rate of pseudoplastic fluid. The convection heat transfer increases and decreases for a fixed $\omega$, the maximum increase in the heat transfer rate occurs for $40^\circ$ inclination, whereas the minimum decrease appears with $70^\circ$ inclination for all $n$. Fig. 12(b) demonstrates the effect of various porous layer thickness on the average Nusselt number with the inclination angle of the cavity for water–Cu at $Ra = 10^6$, $Da = 10^{-3}$, $\varphi = 0.05$, $S = 0.5$, $\omega = 0^\circ$. As the inclination angle of the cavity increases, the heat transfer enhances due to the velocity changing. At lower $S$ the average Nusselt number occurs with the maximum value, increasing $\omega$ to $10^\circ$ tends to reduce the convection, consequently, the heat transfer increases and reaches to the maximum value $40^\circ$, and then drops again as a result of the effect of changing the inclination angle on the flow velocity, thus influencing the heat transfer.

5. Conclusions

The present study considered the visualization of the heatlines on natural convection in a trapezoidal cavity partly filled with a nanofluid porous layer and partly with a non-Newtonian fluid layer. The dimensionless governing equations together with the boundary conditions were solved numerically based on the Finite Volume Method (FVM) using a collocated grid system. The conclusions are made over a water-based with four nanofluids types with a wide range of nanoparticle volume fraction ($0 \leq \phi \leq 0.2$), power-law index ($0.6 \leq n \leq 1.4$), porous layer
thickness $(0.3 \leq S \leq 0.7)$, the side wall inclination angle $(0^\circ \leq \phi < 21.8^\circ)$ and the inclination angle of the cavity $(0^\circ \leq \alpha < 90^\circ)$. Some important conclusions from the study are provided below:

1. It is found that when the nanoparticle volume fraction is applied, the circulation intensity increases due to the increase in the thermal conductivity of nanofluid. The convection heat transfer pushes the isotherms patterns within the nanofluid layer to take almost a diagonal shape, while the convection mode heat transfer forces the isotherms patterns within the fluid layer to appear with almost a horizontal line to the sloping walls.

2. By the increase of the nanofluid layer thickness, the streamlines circulation cell within fluid layer shrinks and transforms to the oval-shaped form. The strength of the flow circulation decreases for both pure fluid and nanofluid, due to the resistance of the nanofluid layer hydrodynamics.

3. A higher nanoparticle volume fractions $(\phi = 0.2)$ leads to a higher overall Nusselt number, but also this value tends to drop lower than other values $(\phi = 0.05$ and $0.1)$ when the concentration of Darcy number between $[10^{-4}, 10^{-2}]$, due to the structure of the cavity slopping walls.

4. The overall Nusselt number has maximum value when the power-law index is less than one (pseudoplastic fluid), and then decreases as the power-law index increases.

5. Qualitatively, the ‘enhanced-heat transfer situation’ is seen in all the four nanofluids compared to that of the base fluid but the following general result holds:

$$Nu_{\text{water—Ag}} > Nu_{\text{water—Cu}} > Nu_{\text{water—TiO}_2 > Nu_{\text{water—Al}_2O_3}}$$

6. The ramification of this important result can be seen in the context of heat removal and heat storage systems like solar-energy systems and nuclear energy systems.

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References


