Numerical investigation of double-diffusive convection in an open cavity with partially heated wall via heatline approach

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A B S T R A C T

Double-diffusive natural convection in an open top square cavity, partially heated and salted from the side, is studied numerically via the heatline approach. Constant temperatures and concentrations are imposed along the right and left walls, while the heat balance at the surface is assumed to obey Newton’s law of cooling. The finite difference method is used to solve the dimensionless governing equations. The governing parameters involved in this investigation are the thermal Marangoni number \(0 \leq MaT \leq 100\), the solutal Marangoni number \(0 \leq MaS \leq 100\), the Lewis number \(10 \leq Le \leq 100\), the heater size \(0.2 \leq s \leq 0.8\), Grashof number \(Gr \sim 10^5\), Prandtl number \(Pr \sim 10\), Biot number \(Bi \sim 0.1\) and aspect ratio \(l\). The numerical results are reported for the effect of the Marangoni number, Lewis number and heater size on the contours of streamlines, isotherms, isoconcentrations, masslines and heatlines. The predicted results for the average Nusselt number and Sherwood number are presented for various parametric conditions. It is shown that the heat and mass transfer mechanisms are affected by the heater segment length. A direct relation between both opposing \((N = -2)\) and aiding flow \((N = 2)\), and heat and mass transfer process is found for various values of the Marangoni and Lewis numbers.

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1. Introduction

Double-diffusive or thermosolutal convection is the flow generated by buoyancy due to simultaneous temperature and concentration gradients. These types of flows arise in a wide range of fields such as oceanography, astrophysics, the process of chemical vapor transport and crystal growth (Teamah et al. [1]). Nishimura et al. [2] studied numerically double-diffusive convection in a rectangular enclosure subject to opposing horizontal thermal and compositional buoyancies. They found that the oscillatory flow occurs in a limited range of buoyancy ratio. Costa [3] established the complete mathematical and numerical model for the calculation of double-diffusive natural convection with heat and mass diffusive walls. The heat and masslines analysis was obtained in this study. The effect of a magnetic field was studied by Chamkha and Al-Naser [4] and Teamah [5]. They obtained a good result and concluded that a magnetic field could reduce the heat transfer and fluid circulation within the enclosure. Bourich et al. [6] investigated the effect of buoyancy ratio on the dynamic behavior of the fluid and heat and mass transfer for double-diffusive convection in a porous enclosure. Baytas et al. [7] considered a non-Darcy flow in an enclosure filled with a step type porous layer. They showed that if the fluid/porous interface is not horizontal and contains a step change in height, the convection of the heat and mass is dramatically changed. The work done by Mobedi and Oztop [8] which involved thick solid ceiling in the natural convection reveals that the solid thickness has less effect on the heat transfer rate across the cavity. The Rayleigh number and thermal conductivity ratio are important factors that influenced the heat and fluid flow characteristics.

Zhao et al. [9] studied convection with partial heating and partial salting in a porous enclosure. They reported that the reversion of transitional flow to the solutal-dominated flow tends to occur at a high Lewis number for lower level segment locations. Nikhbakhti and Rahimi [10] observed that for a double-diffusive natural convection with partially active side walls, the heat and mass transfer rates are increased for the bottom-top thermally active section. The
top-bottom active section contributed to the lowest heat and mass transfer rates. A study done by Liu et al. [11] shows that the thermal Rayleigh number plays a major role in determining the transport structure of the fluid, heat and moisture in both aiding and opposite flows for a double-diffusive convection in a cavity with an inflow and outflow that exhibit the fluid flow. The study regarding different nanoparticles in a partially heated enclosure is carried out by Oztöp and Abu-Nada [12]. They reported that the heater locations and sizes affects the flow and temperature fields. In addition, the heat transfer rate is getting better for low aspect ratio compared to high aspect ratio.

If one side of the enclosure is open to air then the Marangoni/surface tension effect becomes important. Bergman [13] performed a double-diffusive Marangoni convection to understand the interaction between thermocapillary flow and diffusocapillary flow. Jue [14] observed that the Marangoni effect will alter the evolution of the flow field in the double-diffusive convection with free surface. Zhao et al. [15] studied the effect of localized heating and salting from below on convection. They found that decreasing the Darcy number will retard the flow and diffuse transport of heat and mass transfer in the enclosure. Teamah and El-Maghlany [16] found that there was no significant effect on the heat transfer for a case of double-diffusive convection with an insulated moving lid. The results of Younis et al. [17] on double-diffusive natural convection in an open top square cavity. The Prandtl number will retard the transfer rate is increased. Saleh et al. [19] studied numerically the natural convection in an open top square cavity. The Prandtl number plays a major role in determining the transport rates. A study done by Liu et al. [11] shows that the thermal Rayleigh number plays a major role in determining the transport structure of the fluid, heat and moisture in both aiding and opposite flows for a double-diffusive convection in a cavity with an inflow and outflow that exhibit the fluid flow. The study regarding different nanoparticles in a partially heated enclosure is carried out by Oztöp and Abu-Nada [12]. They reported that the heater locations and sizes affects the flow and temperature fields. In addition, the heat transfer rate is getting better for low aspect ratio compared to high aspect ratio.

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2. Mathematical formulation

Consider a steady and laminar two-dimensional fluid flow in a square cavity as shown schematically in Fig. 1. The bottom wall is adiabatic and impermeable. The right wall has higher temperature and concentrations than the left wall. Moreover the right wall is partially heated and salted, while the top of the cavity is open. The temperature gradient is applied along the upper boundary where it is assumed to obey the Newton law of cooling, insulating with respect to heat, non-diffusive with respect to solution, flat and non-deformable.

The surface tension $\sigma$ on the upper boundary is assumed to vary linearly with both temperature and concentration as

$$
\sigma = \sigma_0 \left[1 - \eta_r (T - T_0) - \eta_s (c - c_0)\right]
$$

$\sigma_0 = \sigma_0 \left[1 - \eta_r (T - T_0) + \eta_s (c - c_0)\right]
$$

where the subscript '0' denotes a reference value, $\eta_r$ and $\eta_s$ are the thermal and solutal coefficients of the surface tension, respectively. It is important for the top free surface having a linear surface tension so that there is a stress exerted at the top of the fluid ([34], [35]). This stress is also known as shear stress applied by the interface on the adjoining bulk liquid and thereby generates flow or alters an existing one. The density $\rho$ is given by

$$
\rho = \rho_0 \left[1 - \beta_r (T - T_0) - \beta_s (c - c_0)\right]
$$

$$
\rho_0 - \rho = \rho_0 \left[1 - \beta_r (T - T_0) + \beta_s (c - c_0)\right]
$$

where $\beta_r$ and $\beta_s$ is the thermal and solutal expansion coefficient of the fluid, respectively. In this study $\beta_s$ is set negative. This makes sense since an increase in temperature decreases the density, whereas an increase in concentration increases the density. The fluid is assumed to be Newtonian and incompressible. The equations of mass, momentum, energy and concentration are as follows:

$$
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
$$

$$
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
$$

Fig. 1. Schematic diagram of the model.
\[
\begin{align*}
\frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} &= -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + g \beta_T (T - T_c) + g \beta_c (c - c_c) + \frac{g}{\mu} (\rho - \rho_c) \frac{\partial T}{\partial y} \\
\frac{\partial T}{\partial x} + \nu \frac{\partial T}{\partial y} &= \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \\
\frac{\partial c}{\partial x} + \nu \frac{\partial c}{\partial y} &= D \left( \frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right)
\end{align*}
\]

where \(u, v\) are respectively the \(x\) - and \(y\) -components of the velocity, \(T\) temperature, \(p\) pressure, \(c\) concentration, \(\nu\) viscosity, \(\rho\) density, \(g\) gravitational acceleration, \(\alpha\) the thermal diffusivity and \(D\) the mass diffusivity. The boundary conditions are (Naimi et al. [38])

\[
\begin{align*}
u &= u = 0, \quad T = T_c, \quad c = c_c, \quad \text{at} \quad x = 0 \\
u &= u = 0, \quad T = T_h, \quad c = c_h, \quad \frac{r - \frac{s}{2}}{r} \leq y \leq \frac{r + \frac{s}{2}}{r} \quad \text{at} \quad x = \ell
\end{align*}
\]

otherwise, \(\frac{\partial T}{\partial x} = \frac{\partial c}{\partial x} = 0 \quad \text{at} \quad x = \ell
\]

Table 1

<table>
<thead>
<tr>
<th>Ra</th>
<th>Ref. [36]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ra = 10^4</td>
<td>2.2447</td>
<td>2.2466</td>
</tr>
<tr>
<td>Ra = 10^5</td>
<td>4.5203</td>
<td>4.5245</td>
</tr>
<tr>
<td>Ra = 10^6</td>
<td>8.8078</td>
<td>8.8413</td>
</tr>
</tbody>
</table>

Fig. 2. Grid independency study: \(\overline{Nu}\) and \(\overline{Sh}\) at the hot and cold walls for (a) \(N = 2\) and (b) \(N = -2\) versus number of grid points.
\( u = v = 0, \ \text{and} \ \frac{\partial T}{\partial y} = \frac{\partial c}{\partial y} = 0 \ \text{at} \ y = 0 \) \hspace{1cm} (11)

\( v = 0, \ -k\frac{\partial T}{\partial y} = \lambda(T - T_c) \ \text{and} \ \mu\frac{\partial u}{\partial y} = -\left(\frac{\partial \sigma}{\partial T} \frac{\partial T}{\partial x} + \frac{\partial \sigma}{\partial c} \frac{\partial c}{\partial x}\right) \ \text{at} \ y = \ell \)

\hspace{1cm} (12)

where \( k \) is the thermal conductivity, \( \lambda \) is the atmospheric convective heat transfer coefficient, \( \mu \) is the dynamic viscosity, \( \ell \) is the width and height of the cavity, \( s \) is the heater segment and \( r \) is the length from the bottom wall to the center of the heater segment.

The governing equations given above are in terms of primitive variables, i.e. \( u, v, p, T \) and \( c \). The solution procedure discussed in this study is based on equations involving the stream function, \( \psi \), the vorticity, \( \omega \), the temperature, \( T \), and the concentration, \( c \), as variables which are defined as \( u = \frac{\partial \psi}{\partial y}, \ v = -\frac{\partial \psi}{\partial x} \) and \( \omega = (\partial v/\partial x) - (\partial u/\partial y) \). Eliminating the pressure between the two momentum equations, writing in the stream function, vorticity, temperature and concentration formulation, performing nondimensionalization where the following dimensionless variables have been used:

\[ X = \frac{x}{\ell}, \ Y = \frac{y}{\ell}, \ U = \frac{u}{\ell}, \ V = \frac{v}{\ell}, \ \psi = \frac{\psi}{\ell}, \]

\[ \Omega = \frac{\omega T^2 \mu}{\ell}, \ \Theta = \frac{T - T_c}{T_h - T_c}, \ C = \frac{c - c_e}{c_h - c_e}, \ Pr = \frac{\nu}{\ell}, \]

\[ Gr = \frac{\beta \gamma(T_h - T_c)T^3}{\ell^2}, \ Ma_T = \frac{\partial_\sigma (T_h - T_c)^2}{\mu \ell}, \ Le = \frac{\alpha}{\ell}, \]

\[ Ma_c = \frac{\partial_\sigma (c_h - c_e)^2}{\mu \ell}, \ Bi = \frac{\ell}{\ell}, \]

\[ N = \frac{\beta_\gamma(T_h - T_c)^2}{\ell^2} \]

where \( Ma_T \) the thermal Marangoni number, \( Ma_c \) the solutal Marangoni number, \( Le \) the Lewis number, \( Gr \) the Grashof number, \( Bi \) the Biot number, \( Pr \) the Prandtl number and \( N \) the buoyancy ratio.

Then the Eqs. (3)–(7) become:

\[ \frac{\partial^2 \psi}{\partial X^2} + \frac{\partial^2 \psi}{\partial Y^2} = -\Omega \]

\[ \frac{\partial^2 \Omega}{\partial X^2} + \frac{\partial^2 \Omega}{\partial Y^2} = \frac{1}{Pr} \left( \frac{\partial \psi}{\partial Y} \frac{\partial \Omega}{\partial X} - \frac{\partial \psi}{\partial X} \frac{\partial \Omega}{\partial Y} \right) + GrPr \left( \frac{\partial \Theta}{\partial X} + N \frac{\partial C}{\partial X} \right) \]

\hspace{1cm} (14)

Fig. 3. Comparison of streamlines (left), isotherms (middle) and isoconcentration (right) (a) present study and (b) Jue [14] for \( Pr = 7.6, Le = 10, N = 5, Ma_T = 100, Ma_c = 0, Bi = 0 \) and \( Gr = 1.3 \times 10^4 \).
The dimensionless boundary conditions are:

\[
\begin{align*}
\Psi &= 0, \quad \Omega = \frac{\partial^2 \Psi}{\partial X^2}, \quad \Theta = 0, \quad C = 0, \quad \text{at } X = 0 \quad (15) \\
\Psi &= 0, \quad \Omega = \frac{\partial^2 \Psi}{\partial X^2}, \quad \Theta = 1, \quad C = 1, \quad r' - \frac{s'}{2} \leq Y \\
&\leq r' + \frac{s'}{2}, \quad \text{at } X = 1 \quad (16) \\
\text{otherwise, } \frac{\partial \Theta}{\partial X} = \frac{\partial C}{\partial X} = 0, \quad \text{at } X = 1
\end{align*}
\]

The dimensionless heat functions are given as:

\[
\begin{align*}
\frac{\partial H}{\partial Y} &= \frac{U \Theta}{\partial X} - \frac{\partial \Theta}{\partial X} \quad (22) \\
\frac{\partial H}{\partial X} &= \frac{V \Theta}{\partial Y} - \frac{\partial \Theta}{\partial Y} \quad (23) \\
\text{The heat function differential equation is defined as}
\end{align*}
\]

\[
\begin{align*}
\frac{\partial^2 H}{\partial X^2} + \frac{\partial^2 H}{\partial Y^2} &= \left( \frac{\partial U \Theta}{\partial Y} - \frac{\partial V \Theta}{\partial X} \right) \quad (24)
\end{align*}
\]

Fig. 4. Streamlines, isotherms, concentration, masslines and heatlines for different Marangoni numbers at \( N = 2, Le = 10 \) and \( s = 0.5 \).
considering that $H$ is a continuous function to its second-order derivatives. The boundary conditions for the dimensionless heat function are obtained from the integration of differential definition of $H$ along the considered boundary conditions:

$$H(X, 0) = H(0, 0) = 0 \quad \text{at adiabatic wall } Y = 0$$  \hspace{1cm} (25)

$$H(0, Y) = H(0, 0) + \int_{0}^{Y} \left( \frac{\partial H}{\partial X} \right) dY \quad \text{at } X = 0$$  \hspace{1cm} (26)

$$H(1, Y) = H(1, 0) + \int_{0}^{Y} \left( \frac{\partial H}{\partial X} \right) dY \quad \text{at } X = 1$$  \hspace{1cm} (27)

$$H(X, 1) = H(0, 1) + \int_{0}^{X} \left( \frac{\partial H}{\partial Y} \right) dX \quad \text{at } Y = 1$$  \hspace{1cm} (28)

The mass function that is similar to the heat function is given as:

$$\frac{\partial^2 M}{\partial X^2} + \frac{\partial^2 M}{\partial Y^2} = \frac{\partial (UC)}{\partial Y} - \frac{\partial (VC)}{\partial X}$$  \hspace{1cm} (29)

together with the boundary conditions which are in the integral form as follows:

$$M(X, 0) = M(0, 0) = 0 \quad \text{at adiabatic wall } Y = 0$$  \hspace{1cm} (30)

$$M(0, Y) = M(0, 0) + \int_{0}^{Y} \frac{\partial C}{\partial X} dY \quad \text{at } X = 0$$  \hspace{1cm} (31)

$$M(1, Y) = M(1, 0) + \int_{0}^{Y} \frac{\partial C}{\partial X} dY \quad \text{at } X = 1$$  \hspace{1cm} (32)

$$M(X, 1) = M(0, 1) + \int_{0}^{X} \frac{\partial C}{\partial Y} dX \quad \text{at } Y = 1$$  \hspace{1cm} (33)

The average Nusselt number represents the total heat transfer

Fig. 5. Streamlines, isotherms, concentration, masslines and heatlines for different Marangoni numbers at $N = -2, Le = 10$ and $s = 0.5$. 

(a) $Ma = 0$

(b) $Ma = 400$

(c) $Ma = 800$
across the cavity is obtained by integrating the local Nusselt number and is defined as

\[ \overline{Nu_h} = \int_{r_1}^{r_2} \left[ \frac{\partial \Theta}{\partial X} \right]_{X=1} \, dY \text{ at hot wall} \]  

(34)

\[ \overline{Nu_c} = \int_{0}^{1} \left[ \frac{\partial \Theta}{\partial X} \right]_{X=1} \, dY \text{ at cold wall} \]  

(35)

The average Sherwood number represents the total mass transfer across the cavity is obtained by integrating the local Sherwood number and is defined as

\[ \overline{Sh_h} = \int_{r_1}^{r_2} \left[ \frac{\partial C}{\partial X} \right]_{X=1} \, dY \text{ at hot wall} \]  

(36)

\[ \overline{Sh_c} = \int_{0}^{1} \left[ \frac{\partial C}{\partial X} \right]_{X=1} \, dY \text{ at cold wall} \]  

(37)

### 3. Numerical method and validation

The finite difference method is applied to solve the governing equations (13)–(16) subject to the boundary conditions (17)–(21). Central difference method is used to discretize the equations, while the backward and forward different schemes are applied to the boundary conditions. The vorticity at the bottom wall is given as

\[ \Omega = \frac{(\Psi_{1,j} - \Psi_{2,j})}{2(\Delta Y)^2} \]  

(38)

Similar formula is used for the right and left walls while the top free surface is given as:

\[ \Omega = -Ma_T \frac{(\Theta_{i,1,j+1} - \Theta_{i-1,j+1})}{2(\Delta X)} \frac{MaC}{Le} \frac{(C_{i,1,j+1} - C_{i-1,j+1})}{2(\Delta X)} \]  

(39)

\[ \begin{align*}
\Psi_{\text{min}} &= 0.15618, \quad \Psi_{\text{max}} = 0.78548 \\
M_{\text{min}} &= -4.7756, \quad M_{\text{max}} = 0 \\
H_{\text{min}} &= -1.0586, \quad H_{\text{max}} = 0.11969
\end{align*} \]

\[ \begin{align*}
\Psi_{\text{min}} &= 0.10425, \quad \Psi_{\text{max}} = 0.26296 \\
M_{\text{min}} &= 8.238, \quad M_{\text{max}} = 0 \\
H_{\text{min}} &= -0.9581, \quad H_{\text{max}} = 0
\end{align*} \]

\[ \begin{align*}
\Psi_{\text{min}} &= 0.10374, \quad \Psi_{\text{max}} = 0.18362 \\
M_{\text{min}} &= 10.8046, \quad M_{\text{max}} = 0 \\
H_{\text{min}} &= -0.95498, \quad H_{\text{max}} = 0
\end{align*} \]

(a) \(Le = 10\)

(b) \(Le = 50\)

(c) \(Le = 100\)

**Fig. 6.** Streamlines, isotherms, concentration, masslines and heatlines for various \(Le\) at \(s = 0.5\), \(N = 2\) and \(Ma = 1000\).
The algebraic equations are then solved by applying the Gaussian SOR iteration. The unknowns $\Psi$, $\Theta$, $\Omega$, and $C$ are calculated until the following criterium of convergence is satisfied:

\[
\frac{\sum_{ij} |\zeta_{ij}^{n+1} - \zeta_{ij}^n|}{\sum_{ij} |\zeta_{ij}^n|} \leq \epsilon
\]  

(40)

where $\zeta$ is either $\Psi$, $\Theta$, $\Omega$, and $C$. The iteration number and the convergence criterion are represented by $n$ and $\epsilon$, respectively. The integration of (34)–(37) is done by using second order Simpson’s method. For the purpose of this study, the convergence criterion is set at $\epsilon = 10^{-5}$. Uniform and regular grid distribution is used for the whole cavity. A grid resolution of 120 x 120 is used for the simulation. The effect of grid resolution was examined in order to select the appropriate grid density as shown in Fig. 2 for $Ma = 1000$, $Pr = 10$, $Le = 10$ and $Gr = 10^4$. As a validation, the average Nusselt number compare well with that obtained by Qin et al. [36] for the case $Pr = 0.71$ and $Ma = 0$ for various $Ra$ as displayed in Table 1. In addition, the contours of streamlines, isotherms and isoconcentration agree fairly with the contours obtained by Jue [14] as shown in Fig. 3.

4. Results and discussion

In the present study, we investigate the effects of partial heating and salting for double-diffusive natural convection in an open top square cavity. The numerical results for the streamline, isotherms, concentration, masslines and heatlines for various values of the associated parameters will be reported. The parameters involved in this investigation are: the thermal Marangoni number ($0 \leq Ma_T \leq 1000$), the solutal Marangoni number ($0 \leq Ma_c \leq 1000$), the Lewis number ($10 \leq Le \leq 100$), the heater size, ($0.2 \leq s \leq 0.8$), Grashof number, $Gr = 10^4$, Prandtl number, $Pr = 10$ and aspect ratio $1$. The fluid is in the form of aqueous solution by referring to the value of Prandtl and Lewis number. The buoyancy ratio used are $-2$ and $2$ that refers to the opposing and aiding flow, respectively. We remark that both $Ma_T$ and $Ma_c$ are set equal in all the simulations and denoted simply as $Ma$. In addition, the results for the average Nusselt and Sherwood number for various conditions will be presented.

![Fig. 7. Streamlines, isotherms, concentration, masslines and heatlines for various $Le$ at $s = 0.5$, $N = -2$ and $Ma = 1000$.](image-url)
4.1. Effects of the Marangoni number

Fig. 4 displays the contours of streamlines, isotherms, isoconcentrations, masslines and heatlines for the case $N = 2$, $s = 0.5$, $Le = 10$ and various Marangoni number. The fluid flow can be described as follows. The temperature of the right partially-heated wall is higher than the temperature of the fluid in the cavity which cause the wall transmit the heat to the fluid by convection. When the fluids temperatures increases, it starts flowing from the hot region to the cold part and fall along the cold wall before rises at the hot wall to create a large counter clockwise rotating rectangular cell that dominates the flow in the cavity without the presence of the shear stress ($Ma = 0$) as illustrated in Fig. 4 (a).

For the case of aiding flow where the thermal and solutal buoyancy forces are in the same direction, the fluid’s motion is decreased when the shear stress is imposed at the free surface which can be observed at the value of $\Psi_{\text{max}}$. This shows that the Marangoni number has a significant impact on the buoyancy forces.

Fig. 8. Streamlines, isotherms, concentration, masslines and heatlines for various $s$ at $N = 2$, $Le = 10$ and $Ma = 1000$. 
that dominate the convection. In the case of Marangoni effect, slower fluid motion is an indicator that heat will diffuse at a slower rate across the cavity. By taking the $Ma$ higher, the large cells still remain but a suppression is seen at the upper portion of the cavity as shown in Fig. 4(c). The contours of isotherms and iso-concentration are more twisted and distorted at the top of the left cold wall when the surface tension effect is stronger. The heat and mass path that characterize the flow of the Marangoni convection can be seen from the heatline and massline contours. It is obvious that horizontal-like lines appear at the top of the open cavity which indicates the high heat and mass transfer rates due to conduction when $Ma = 0$ as shown in Fig. 4(a). Taking $Ma$ higher, the horizontal lines for both contours are being suppressed a bit due to the cohesion-adhesion phenomena of the fluid's molecules near the free surface which decrease the rate of heat and mass transfer.

For the case when the thermal and solutal buoyancy forces are in opposite directions as shown in Fig. 5, all contours do not change significantly as $Ma$ is increased. The isotherm contours are inclined and more focused at the bottom of the left wall while the iso-concentration contours are vertically twisted near both vertical walls. The opposite is observed for the heatline and massline contours as we observe for the case $N = 2$ as $Ma$ varies. Both contours

---

**Fig. 9.** Streamlines, isotherms, concentration, masslines and heatlines for various $s$ at $N = 2$, $Le = 10$ and $Ma = 1000$. 
lines were located at the bottom region which indicates that the activity of heat and mass transfer is focused at the low portion of the cavity due to the weakening of the thermal buoyancy force. The clockwise close-loop occurring at the core of the cavity in heatlines signifies the convective heat transport due to flow circulation cells. Similar patterns of both contours were observed for higher Ma and this shows that the heat and mass is crossing the cavity nearly at the same rate. Thus, the Marangoni effect on the convection is small. It is most probably because of the fact that the effect of compositional buoyancy plays a dominant role to overcome the thermocapillary force.

4.2. Effects of the Lewis number

Various values of the Lewis number Le have been used to see its effect on the streamlines, isotherms and isoconcentration, mass-lines and heatlines. For the case Ma = 1000, s = 0.5 and N = 2, a large thermal anticlockwise recirculation occurs and dominates the core region with a little distortion at the upper surface as shown in Fig. 6(a). Higher Le made the cells separated into three parts as shown in Fig. 6(c). It is probably because the high thermally fluid tries to compete with the solutal buoyancy effect which suppressed the convection at the bottom region. At the beginning, the isotherms contours are twisted at the top portion of the cavity while the isoconcentration contours are inclined to the higher portion of the cold vertical wall. Taking the Le higher, the temperature distribution becomes stable which means the solutal buoyancy force is nearly equal to the thermocapillary force. The isoconcentration is more inclined and distorted with the existence of the thin concentration layers due to the boundary layer thickness. The boundary layer thickness becomes thinner for high value of N where it will change the flow structure and has a significant influence on the concentration field, which builds up a vertical stratification near the right and left walls of the cavity [37]. The mass path shows that for higher Le the path is focused at the upper-right wall of the cavity. This is different for heat path that being suppressed downward for Le = 10 and has wider distribution by increasing Le further as displayed in Fig. 6(b)-(c).

Fig. 7 shows the effect of Le for the case Ma = 1000, s = 0.5 and N = 2. A large rotating rectangular cell in clockwise direction dominating the whole cavity is present. As Le is increased, it is observed that there is no major changes to the cell. The fluid motion is slower which is mainly caused by the high surface tension effect that attracts the fluid particle to the upper surface. The high

![Graphs](image)

**Fig. 10.** Average Nusselt and Sherwood number at the hot and cold walls for various Le at N = 2 and s = 0.5.
thermal diffusivity rate will give the fluid enough heat to stabilize the flow of the convection and decrease the surface tension effect. The isotherms are more vertical-like, while the isoconcentration is distorted and more twisted and less denser when approaching the cold wall for \( Le > 10 \). The masslines are more focused to the lower-half region of the cavity, while the heatlines increase the distribution area for the heat across the cavity as depicted in Fig. 7(b)–(c).

We can clearly observe that the contours are more sensitive in the aiding flow case as compared to the opposing flow case as \( Le \) is increased. Higher \( Le \) will increase the thermal diffusivity of the fluid where in the aiding flow both buoyancy forces (thermal and solutal) are in the same direction. These forces will contribute to the major changes of contours patterns compared to the opposing flow, in which both buoyancy forces compete with each other although the thermal diffusivity is boosted up as \( Le \) is increased, resulting in similar contours occurs in the convection.

4.3. Effects of heater size

Fig. 8 displays the effect of the heater size on convection for \( N = 2, Ma = 1000 \) and \( Le = 10 \). For \( s = 0.2 \), a couple of cells recirculated; one small cell is near the top-free surface and another one dominates the cavity in an anti-clockwise direction. As the heater size increases, the big cell became larger, and faster fluid motion has been observed (known from \( \Psi_{\text{max}} \)). This can be explained by the fact that the large density variation due to the increase of the heater segment will lead to strong buoyant forces that eventually intensify the circulation and decrease the strength of the Marangoni effect. The isotherms and isoconcentration become more distorted to the cold wall when higher \( s \) is used. The heat and mass contours show that by increasing the heater segment, both paths always focusing on the upper portion of the cavity. The thermal buoyancy force results in the hot molecules moving towards the top surface, thus generating the heat and mass path across the cavity which also increased the heat and mass transfer rates for longer heater segment.

Fig. 9 shows that by increasing \( s \), the convective cells become larger which dominate the cavity. An opposite phenomenon occurs to the fluid motion for \( N = -2 \). The velocity of the fluid is getting slower which also indicates that the Marangoni effect made an impact on the fluid flow. The isotherms are more twisted at the bottom portion and isoconcentration become more horizontally twisted and denser near the vertical cold and hot walls when the heater size is increased. Unlike the case for the aiding flow, the heat and mass paths in the opposing flow case are focused at the bottom region of the cavity. This behavior indicates that the solutal

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Fig. 11. Average Nusselt and Sherwood number at the hot and cold walls for various \( Le \) at \( N = -2 \) and \( s = 0.5 \).
buoyancy force is the dominant factor to cause the hot molecules move downwards.

4.4. Average Nusselt and Sherwood number

Figs. 10 and 11 show the effects of the Marangoni (thermal and solutal) number on the average Nusselt and Sherwood number on both hot and cold walls for various values of \( Le \) at \( N = -2 \) and \( N = 2 \). For the aiding flow \( (N = 2) \), it is observed that the values of \( \overline{Nu} \) and \( \overline{Sh} \) on both walls tend to slightly decrease gradually with increasing value of \( Ma \) except for \( \overline{Nu}_h \) at \( Le = 10 \) as displayed in Fig. 10. The minimum values of \( \overline{Nu} \) and \( \overline{Sh} \) on both walls occur at a maximum value of \( Ma \), which means that the heat and concentration will slowly diffuse and the effect of surface tension can be neglected. This figure also shows that the \( \overline{Nu}_h \) decreases and \( \overline{Nu}_c \) increases with increasing \( Ma \) for \( Le = 10 \). The behavior of decreasing \( \overline{Nu}_h \) relates to the fact that high surface tension by increasing \( Ma \) will cause the decrement of capability of fluid to conduct heat efficiently across the hot wall. Increasing \( \overline{Nu}_c \) is mainly caused by the faster fluid flow near the cold wall as shown in Fig. 6. Meanwhile, similar patterns were observed for the opposing flow where horizontal patterns exist for \( \overline{Nu} \) and \( \overline{Sh} \) for all values of \( Le \) as depicted in Fig. 11.

This also indicates that even a strong Marangoni effect does not influence significantly the efficiency of the heat and mass transfer rate across the cavity due to the flow stabilization. It is clearly seen that for \( Le > 10 \) the values of \( \overline{Nu} \) for both flows are near to 1 which refers to the characteristics of laminar flow.

Fig. 12 displays a gradually increasing pattern of \( \overline{Nu} \) for \( s > 0.4 \), while others performed a decreasing patterns for all \( s \) for all values of \( \overline{Nu} \) and \( \overline{Sh} \). This decrement shows that the heat and mass transfer is suppressed as \( Ma \) takes higher value. By increasing the heater length, the thermocapillary effect is weaken due to the increasing of buoyancy forces that boost up the heat and mass transfer rates. Again, the horizontal pattern with little increment appears in the opposing flow case at both walls for increasing \( s \) values as shown in Fig. 13. Here, the solutal buoyancy force is the main factor that keeps the transfer rates nearly equal for all \( Ma \) values.

The effects of buoyancy ratio, \( N \), on \( \overline{Nu} \) and \( \overline{Sh} \) at hot and cold walls are illustrated in Fig. 14. Similar patterns were observed when comparing between \( \overline{Nu}_h \) and \( \overline{Nu}_c \), and \( \overline{Sh}_h \) and \( \overline{Sh}_c \). It is clearly seen that as \( N \) increased in the opposing flow \( (N < 1) \), the heat and mass transfer rates are decreased. This situation arises due to the transmission interchangeable effect of thermal and solutal buoyancy forces that reducie the heat and mass transfer rates in the

![Fig. 12. Average Nusselt and Sherwood number at the hot and cold walls for various s at N = 2 and Le = 10.](image-url)
counter clockwise rotating fluid flow. The forces of thermal and solutal buoyancy are then being neutralized and the buoyant convection is hindered at the minimum point $N = 1$. The opposite occurs for the aiding flow ($N > 1$), where the $\overline{Nu}$ and $\overline{Sh}$ are increased due to the same direction of thermal and solutal buoyancy forces that drive the increasing rates of heat and mass transfer which result in the clockwise rotation flow in the cavity.

5. Conclusions

The present study has focused on the effects of partial heating and salting for double-diffusive natural convection in an open top square cavity. The characteristics of the streamlines, isotherms, isoconcentration, masslines and heatlines for various heater size, Lewis number, thermal and solutal Marangoni number were investigated and analyzed. The study revealed the following:

1. The fluid flow, temperature, concentration and heat flow distributions within the cavity are dependent on the surface tension, the heater segment and buoyancy ratio (thermal or compositional).
2. The heat and mass transfer mechanisms are strongly influenced by the heater segment length. It was found that longer segments yield more efficient heat and mass transfer for various values of the Marangoni number.
3. For $N = -2$, all $\overline{Nu}$ and $\overline{Sh}$ display a similar stagnant trend for every value of the Marangoni number studied for $Le = 10$, while the opposite situation occurs for $N = 2$ where the increasing patterns were observed.
4. Higher $Le$ presents horizontal patterns (stable convection) for opposing flow which indicates that the high Marangoni effect on convection is not prominent, while for aiding flow the decreasing patterns are found except for the case $\overline{Nu}$.
5. In opposing flow, the heat and mass transfer rates are decreasing as $N$ increased until $N = 1$. The rotation of the flow motion is changed as $N > 1$ which refers to the aiding flow that increases the rates of heat and mass transfer. These twin buoyancy forces have significant impact on the contours especially on concentration contour where the aiding flow (both forces have same direction) will suppress the contour.

Fig. 13. Average Nusselt and Sherwood number at the hot and cold walls for various $s$ at $N = -2$ and $Le = 10$. 
downwards as compared to the opposing flow in which the thin concentration layers are formed at the vicinity of vertical walls.

6. Higher values of \( Le \) will decrease the heat transfer rates but will enhance the mass transfer rates for both flows (opposing and aiding) through the cavity.

7. In small-scale systems or in microgravity environments thermocapillary forces play a significant role in determining the dynamics of the flow besides the double buoyancy forces.

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