Conjugate heat transfer and entropy generation in a cavity filled with a nanofluid-saturated porous media and heated by a triangular solid


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A R T I C L E   I N F O

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A B S T R A C T

Entropy generation due to conjugate natural convection–conduction heat transfer in a square domain is numerically investigated under steady-state condition. The domain composed of porous cavity heated by a triangular solid wall and saturated with a CuO–water nanofluid. Equations governing the heat transfer in the triangular solid together with the heat and nanofluid flow in the nanofluid-saturated porous medium are solved numerically using the over-successive relaxation finite-difference method. A temperature dependent thermal conductivity and modified expression for the thermal expansion of nanofluid are adopted. A new criterion for assessment of the thermal performance is proposed. The investigated parameters are the nanoparticles volume fraction ϕ (0–0.05), modified Rayleigh number Ra (10–1000), solid wall to base-fluid saturated porous medium thermal conductivity ratio KΘ (0.44, 1, 23.8), and the triangular solid thickness D (0.1–1). The results show that both the average Nusselt number and the entropy generation are increasing functions of KΘ, while they are maxima at some critical values of D. It is also found that the addition of nanoparticles increases the entropy generation. According to the new proposed criterion, the results show that the largest solid thickness (D = 1.0) and the lower wall thermal conductivity ratio manifest better thermal performance.

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1. Introduction

Study of convection heat transfer in porous media is a very interesting subject because of its industrial applications. There are two advantages of using porous media. First, its dissipation area is greater than the conventional fins that enhances convection heat transfer. Second, is the irregular motion of the fluid flow around the individual beads which mixes the fluid more effectively [1]. Natural convection heat transfer in porous media is encountered in a wide variety of industrial applications such as grain storage, filtering, drying of gasses, underground pollutants, storage and cooling of radioactive waste containers, soil cleaning using steam injection, building thermal insulation, solar collector technology, electronic cooling, and many other applications [1].

Enhancement of the heat transfer is one of the main purposes in some of the mentioned engineering devices. The low thermal conductivity of conventional heat transfer fluids, commonly water, oil, ethylene glycol, have restricted designers. Fluids containing nano-sized solid particles offer a possible solution to conquer this problem. The nanofluid has greater effective thermal conductivity than a pure base fluid. Nanofluids, a name conceived by Choi [2], in Argonne National laboratory, are fluids consisting of solid nanoparticles with size less than 100 nm suspended with solid volume fraction typically less than 4%. Nanofluids can be used to improve thermal management system in many engineering applications such as transportation, microelectronics instrument and cooling devices.

A relatively few papers dealing with nanofluids saturated in porous media were published. Most of these papers studied the boundary layer flow. Nield and Kuzentsov [3] examined the influence of nanoparticles natural convection past a vertical plate. Ahmad and Pop [4] numerically studied the mixed convection boundary layer flow of the same problem of [3] using three different nanoparticles based on the conventional model of Tiwari and Das [5] which incorporates only the nanofluid volume fraction. Gorla and Chamkha [6] considered natural convection boundary layer over a non-isothermal flat plate embedded in a porous medium. The natural convection boundary layer flow about a sphere embedded in porous media was considered by Chamkha et al. [7]. More recently, Cimpean and Pop [8] studied fully developed steady-state mixed convection flow of nanofluids in an inclined porous channel. Hajipour and Dehkordi [9] considered mixed convection heat transfer of nanofluids based on the...

All the aforementioned studies are based on the first-law analysis. Recently, the second-law based investigations have gained attention for studying thermal systems. Entropy generation has been used as a gauge to evaluate the performance of thermal system. The analysis of the exergy utilization and the entropy generation has become one of the primary objectives in designing a thermal system. Bejan [13–15] focused on the different reasons behind entropy generation in applied thermal engineering. Generation of entropy destroys available work of system. Therefore, it makes good engineering sense of focus on irreversibility of heat transfer and fluid friction process. There are only very few studies that consider the second law analysis in the presence of nanofluid as a working fluid in a porous media. The effects of heat transfer in nanofluids flow over a permeable stretching wall in a porous medium are investigated by Sheikholeslami et al. [16]. They showed that an increase in the nanoparticles volume fraction decreases the momentum boundary layer thickness and entropy generation rate whereas the thermal boundary layer thickness increases. Ting et al. [17] studied the entropy generation of viscous dissipative nanofluid flow in thermal non-equilibrium porous media embedded in micro channels.

Investigation of entropy generation and natural convection heat transfer of nanofluids in a porous media has not been considered completely in the literature and this challenge is generally considered to be an open research topic that may require more study. Thus, what motivates us to continue in the field of entropy generation and natural convection in enclosures filled with nanofluids saturated porous media is the rareness of published works and hence, the incomplete views regarding this field of investigation which has an important role in biomedical applications. Moreover, the present authors are more interested in the conjugate conduction convection heat transfer features. Therefore, the present study considers steady conjugate conduction convection inside a square cavity, filled with a nanofluid-saturated porous medium and heated by a triangular solid wall occupying one corner of the square cavity making an inclined interface between the solid and the nanofluid-saturated porous medium. The numerical results of this geometry are thought to be useful in control and suppression of what is called bio-convection in controlling or suppression mixing between living and dead cells in suspensions of up swimming mobile microorganisms.

2. Mathematical modeling

Fig. 1 is a schematic illustration of the problem under consideration. It is a two-dimensional square domain with length L, the lower left corner is a solid wall like an isosceles triangular block with its bottom and vertical walls, with length d, kept isothermally at higher temperature T_h. The length d is varied in such a way to keep the overall domain as a square. The inclined wall of the triangular solid is in contact with the contents-saturated porous medium forming the remainder domain. The outer boundaries of the porous domain are kept adiabatic except the right vertical wall where it is cooled at constant temperature T_c. All of the boundaries are assumed impermeable. The pores between the solid matrix are assumed to be uniform and

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Be</td>
<td>Bejan number ((\int S_g dX dY / \int S_{GEN} dX dY))</td>
</tr>
<tr>
<td>D</td>
<td>dimensionless triangular wall thickness</td>
</tr>
<tr>
<td>g</td>
<td>gravitational field (m/s^2)</td>
</tr>
<tr>
<td>GEG</td>
<td>dimensionless global entropy generation</td>
</tr>
<tr>
<td>k</td>
<td>thermal conductivity (W/m K)</td>
</tr>
<tr>
<td>K</td>
<td>permeability of porous medium (m^2)</td>
</tr>
<tr>
<td>k_t</td>
<td>triangular wall to nanofluid thermal conductivity ratio</td>
</tr>
<tr>
<td>K_t</td>
<td>triangular wall to base fluid thermal conductivity ratio</td>
</tr>
<tr>
<td>L</td>
<td>square cavity wall length (m)</td>
</tr>
<tr>
<td>n</td>
<td>normal vector</td>
</tr>
<tr>
<td>Nu_{nf}</td>
<td>average Nusselt number over the right cooled wall</td>
</tr>
<tr>
<td>Nu_{aff}</td>
<td>average Nusselt number over the interface line</td>
</tr>
<tr>
<td>Nu_i</td>
<td>local Nusselt number along the interface line</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>s</td>
<td>segment along the interface line</td>
</tr>
<tr>
<td>S_{gen}</td>
<td>entropy generation rate (W/K m^3)</td>
</tr>
<tr>
<td>S_{GEN}</td>
<td>dimensionless entropy generation rate</td>
</tr>
<tr>
<td>T</td>
<td>temperature (K)</td>
</tr>
<tr>
<td>u</td>
<td>velocity component along x-direction (m/s)</td>
</tr>
<tr>
<td>v</td>
<td>velocity component along y-direction (m/s)</td>
</tr>
<tr>
<td>V</td>
<td>dimensionless velocity component along x-direction</td>
</tr>
<tr>
<td>x,y</td>
<td>Cartesian coordinates (m)</td>
</tr>
<tr>
<td>X,Y</td>
<td>dimensionless Cartesian coordinates</td>
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Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>(\alpha)</td>
<td>effective thermal diffusivity (m^2/s)</td>
</tr>
<tr>
<td>(\beta)</td>
<td>thermal expansion coefficient (K^-1)</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>performance criterion (GEG/Nu_{nf})</td>
</tr>
<tr>
<td>(\varphi)</td>
<td>nanoparticles volume fraction</td>
</tr>
<tr>
<td>(\mu)</td>
<td>dynamic viscosity (Pa.s)</td>
</tr>
<tr>
<td>(\theta)</td>
<td>dimensionless temperature</td>
</tr>
<tr>
<td>(\rho)</td>
<td>density (kg/m^3)</td>
</tr>
<tr>
<td>(\Psi)</td>
<td>dimensionless stream function</td>
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Subscripts

<table>
<thead>
<tr>
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<th>Description</th>
</tr>
</thead>
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<tr>
<td>c</td>
<td>cold</td>
</tr>
<tr>
<td>f</td>
<td>fluid</td>
</tr>
<tr>
<td>h</td>
<td>hot</td>
</tr>
<tr>
<td>i</td>
<td>interface</td>
</tr>
<tr>
<td>nf</td>
<td>nanofluid</td>
</tr>
<tr>
<td>p</td>
<td>solid nanoparticles</td>
</tr>
<tr>
<td>w</td>
<td>wall</td>
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</table>
undeformable. The fluid filling the pores is composed of a base fluid (water) and nanoparticles forming a nanofluid. They (the base fluid and the nanoparticles) are assumed to be in thermal equilibrium and no slip occurs between them. This nanofluid is assumed incompressible. Also, a thermal equilibrium between the nanofluid and the solid matrix is assumed. In the present study, this assumption is relied on the fact that there is no large temperature difference between the nanofluid and the solid matrix, and no sufficiently large velocities are considered to satisfy the Darcy model and the Boussinesq approximation. According to the low seepage velocity and assuming constant nanofluid and the solid matrix, and no slip occurs between them. This nanofluid is assumed incompressible and the nanoparticles are assumed to be in thermal equilibrium and thermal conductivity enhancement.

### 2.1. Governing equations of heat transfer

For numerical simulation, two approaches have been adopted in the literature to investigate the heat transfer characteristics of nanofluids, single phase model and two phase model. Another approach is to adopt the Boltzmann theory. In single phase model, uniform volume fraction distribution is assumed for nanofluids. In other words, the viscosity and thermal conductivity of nanofluids are formulated by volume fraction, temperature and nanoparticle size then continuity, momentum and energy equations are solved for nanofluids. The governing equations based on single phase model can be written as:

**Continuity:**

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

**Momentum (Darcy equation)**

\[
\frac{\partial u}{\partial y} \frac{\partial u}{\partial x} - \frac{gK(\rho_\beta \beta \beta_p + (1 - \varphi) \rho_f \beta_f)}{\mu_{nf}} \frac{\partial T_{nf}}{\partial x}
\]

**Energy for nanofluid**

\[
\frac{\partial^2 T_{nf}}{\partial x^2} + v \frac{\partial T_{nf}}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T_{nf}}{\partial x^2} + \frac{\partial^2 T_{nf}}{\partial y^2} \right)
\]

**Energy for the triangular solid**

\[
\frac{\partial^2 T_w}{\partial x^2} + \frac{\partial^2 T_w}{\partial y^2} = 0
\]

where \(\beta\) is the thermal expansion coefficient, \(\rho\) is the density, \(K\) is the permeability of the porous medium, \(\mu\) is the dynamic viscosity, \(\alpha\) is thermal diffusivity of the porous medium and \(\varphi\) is nanoparticles volume fraction. The subscripts \(p, f, nf\) and \(w\) stand for solid nanoparticles, base fluid, nanofluid, and triangular solid wall, respectively.

The nanofluid thermal properties are written as:

**Thermal diffusivity** (Chamkha and Abu-Nada):

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}
\]

**Heat capacity** (Khanafer et al.):

\[
(\rho C_p)_{nf} = (1 - \varphi) (\rho C_p) + \varphi (\rho C_p)
\]

**Thermal conductivity:**

Chon et al. [23] established an empirical correlation finding the role of temperature and particle size for nanofluid (\(Al_2O_3\)-water) thermal conductivity enhancement.

\[
k_{nf} / k_f = 1 + 64.7 \varphi^{0.764} \left( \frac{d_f}{d_p} \right)^{0.369} \left( \frac{k_f}{k_f} \right)^{0.7476} Pr_{nf}^{0.9955} Re_f^{1.2321}
\]

\[
Pr_T = \frac{\mu_f}{\rho_f \alpha_f}, \quad Re_f = \frac{\rho_f k_e T}{\partial^2 T_{nf} / \partial x^2}
\]

where \(k_e\) is the Boltzmann constant. Although, this model was established for \(Al_2O_3\)-water, Minsta et al. [24] conducted new temperature and thermal conductivity data for water-based nanofluid, they proved the validity of this model for CuO. However, the confirmation of [24], the Chon et al. model was widely used to calculate the thermal conductivity of CuO in many studies as in Haddad et al. [25] and Mahmoudi and Abu-Nada [26].

**Viscosity (Brinkman):**

\[
\mu_{nf} = \frac{\mu_f}{(1 - \varphi)^2}
\]

Although there exists many modified models for the dynamic viscosity of nanofluids, the Brinkman model still gives reasonable results but for low volume fractions of nanoparticles, Khanafer and Vafai [28].

**Thermal expansion:**

Recently, a modified expression for thermal expansion is given by Bourantas et al. [29]. It contains the natural dependence of the density on temperature:

\[
(\rho \beta)_{nf} = (1 - \varphi) (\rho \beta)_{f} + \varphi (\rho \beta)_{p} - \varphi (1 - \varphi) (\rho_p - \rho_f) (\beta_p - \beta_f)
\]

Introducing the following dimensionless set:

\[
D = d/L, \quad X = x/L, \quad Y = y/L, \quad U = u/L/\alpha_f, \quad V = v/L/\alpha_f, \quad \theta_{nf} = (T_{nf} - T_c)/(T_h - T_c), \quad \theta_w = (T_w - T_c)/(T_h - T_c).
\]

and the dimensionless definition of the stream function as: \(U = \frac{\partial \psi}{\partial X}, \quad V = -\frac{\partial \psi}{\partial Y}\). The set of Eqs. (2)–(4) can be rewritten in dimensionless form for the nanofluid-saturated porous medium as:

\[
\left[ \frac{1}{(1 - \varphi)^2} \nabla^2 \Psi \right] = -Ra \frac{(\rho \beta)_{nf}}{\rho_f \beta_f} \frac{\partial \theta_{nf}}{\partial X}
\]

\[
\left[ \frac{\partial \Psi}{\partial X} \frac{\partial \theta_{nf}}{\partial Y} - \frac{\partial \Psi}{\partial Y} \frac{\partial \theta_{nf}}{\partial X} \right] = \alpha_{nf} \nabla^2 \theta_{nf}
\]

and for the solid triangular wall

\[
\nabla^2 \theta_w = 0
\]

Eqs. (11)–(13) are subjected to the following boundary conditions:

\(\psi = 0\) on the solid boundaries.

\(\theta_{nf} = 0\) on the right vertical wall, \(X = 1, \quad 0 \leq Y \leq 1\).

\(\partial \theta_{nf}/\partial x = 0\) on the top horizontal wall \(0 \leq X \leq 1, \quad Y = 1\) and on the horizontal segment \(D \leq X \leq 1, \quad Y = 0\).

\(\partial \theta_{nf}/\partial x = 0\) on the left wall segment, \(X = 0, \quad D \leq X \leq 1\).

\(\theta_w = 0\) on the horizontal triangular wall, \(0 \leq X \leq D, \quad Y = 0\) and on the vertical triangular wall, \(X = 0, \quad 0 \leq Y \leq D\).

At the interface between the solid triangular inclined wall and the fluid-saturated porous medium, the equilibrium state is assumed to be verified. Therefore, both the temperatures and the heat fluxes are the same;

\[
\theta_w = \theta_{nf} \quad \text{and} \quad k_w \frac{\partial \theta_w}{\partial n} = k_{nf} \frac{\partial \theta_{nf}}{\partial n} \quad \text{or:}
\]

\[
\frac{\partial \theta_{nf}}{\partial n} = k_e \frac{\partial \theta_w}{\partial n}
\]
Δx
Δy
Porous medium nodes Solid nodes Interface nodes

Fig. 2. Computational domain and discretization.

where \( \mathbf{n} \) is the vector normal to the interface (\( \mathbf{n} = \sqrt{\partial X^2 + \partial Y^2} \)) and \( K_r \) is the thermal conductivity ratio,

\[
K_r = \frac{k_w}{k_{nf}}
\]

The local Nusselt number along the interface within the nanofluid-saturated porous medium side can be written as:

\[
N_{ui} = -\frac{k_{nf}}{k_f} \frac{\partial \theta_{nf}}{\partial \mathbf{n}} \bigg|_i
\]

The average Nusselt numbers of interest are calculated on the interface (for the porous medium) and on the vertical right wall, respectively as:

\[
N_{unfi} = \frac{1}{\sqrt{2}D} \int_0^{\sqrt{2}D} N_{ui} dS
\]

\[
N_{unf} = -\frac{k_{nf}}{k_f} \int_0^1 \frac{\partial \theta_{nf}}{\partial X} dY
\]

where \( S \) is a segment along the interface as shown in Fig. 1.

Due to energy balance, the overall heat transfer entering the porous cavity from the interface must be equal to that leaving the cavity from the right wall. Hence, the following energy balance can be employed for checking the accuracy of the numerical solution:

\[
\sqrt{2}D N_{unfi} = N_{unf}
\]

2.2. Governing equations for entropy generation

Based on the Darcy model, the entropy generation relation is given by Woods [30]:

\[
S_{gen} = \frac{k}{T_0} \left( \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right) + \frac{\mu_{nf}}{K_T} (u^2 + v^2)
\]

Eq. (20) is to be rearranged to include the nanofluid properties:

\[
S_{gen} = \frac{k_{nf}}{T_0} \left( \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right) + \frac{\mu_{nf}}{K_T} (u^2 + v^2)
\]

In dimensionless form, local entropy generation can be expressed as:

\[
S_{GEN} = \frac{k_{nf}}{k_f} \left( \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right) + \frac{\mu_{nf}}{\mu_f} N_{\mu} \left( \left( \frac{\partial \psi}{\partial Y} \right)^2 + \left( \frac{\partial \psi}{\partial X} \right)^2 \right)
\]

where, \( N_{\mu} = \frac{\mu_T}{k_f} \left( \frac{\sigma^2}{K_T} \right) \) is the irreversibility distribution ratio and \( S_{GEN} = S_{gen} \frac{\Delta T^2}{T_0} \) is the dimensionless local entropy generation.

The terms of Eq. (22) can be separated to the following form:

\[
S_{GEN} = S_\theta + S_\psi
\]

where \( S_\theta \) and \( S_\psi \) are the entropy generation due to heat transfer irreversibility HTI and fluid friction irreversibility FFI, respectively.

\[
S_\theta = \frac{k_{nf}}{k_f} \left( \left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2 \right)
\]

\[
S_\psi = \frac{\mu_{nf}}{\mu_f} N_{\mu} \left( \left( \frac{\partial \psi}{\partial Y} \right)^2 + \left( \frac{\partial \psi}{\partial X} \right)^2 \right)
\]

Integrating Eq. (22) over the porous domain, the global entropy generation \( GEG \) for the present two-dimensional problem is obtained:

\[
GEG = \int S_{GEN} dX dY = \int S_\theta dX dY + \int S_\psi dX dY
\]

It is appropriate to mention Bejan number in order to determine which is the dominant, heat transfer or fluid friction irreversibility. Bejan number is defined as:

\[
Be = \frac{\int S_\theta dX dY}{\int S_{GEN} dX dY}
\]

When \( Be > 0.5 \), the HTI is the dominant while when \( Be < 0.5 \), the FFI is the dominant

3. Numerical solution and validations

The dimensionless governing Eqs. (11)–(13) are discretized uniformly (\( \Delta X = \Delta Y \)) over the square domain using the finite-difference method. The values of the dimensionless solid wall \( D \) are varied with care in such a way that the interface must be localized on grid nodes
as shown in Fig. 2. The interface boundary condition Eq. (14) is interpreted numerically by taking three points backward temperature gradient within the triangular solid wall and three points forward temperature gradient through the porous medium and since \( \theta_{nf} = \theta_w \). Hence, the following difference equation is invoked to compute the interface temperature:

\[
\theta_{nf}(i, j) = \frac{4\theta_{nf}(i + 1, j + 1) - \theta_{nf}(i + 2, j + 2) + K_f [4\theta_{nf}(i - 1, j - 1) - \theta_{nf}(i - 2, j - 2)]}{3(1 + K_r)}
\]  
(28)

The Gauss–Seidel iteration procedure with Over Successive Relaxation (OSR) method is followed in the solution. The iteration is terminated when the following criterion is satisfied:

\[
\max \left| \frac{X_{new}(i, j) - X_{old}(i, j)}{X_{old}(i, j)} \right| \leq 10^{-5}
\]  
(29)

\( X \) denotes any variable, \( \Psi, \theta_{nf} \) or \( \theta_w \). The suitable grid size was based not only on the independence of Nusselt number on grid size, but also on the verification of energy balance through the domain i.e. Eq. (19) and on the global entropy generation Eq. (26). It is found that these conditions are sensitive to the value of the Rayleigh number. Fig. 3 presents grid dependency behavior for \( Ra = 1000, D = 0.1, K_{nf} = 23.8 \), \( \psi = 0.03 \). The thermo-physical properties of base fluid and CuO nanoparticles are presented in Table 1 [31]. For these conditions \( K_{nf} = 23.8 \). Ascendingly, a grid size of 121 \( \times \) 121 was chosen in the numerical solution. Relatively, this grid size is very fine, but we found it is necessary to interpret the normal gradient on the inclined interface surface and then holding the condition of Eq. (19). Once the stream function and dimensionless temperature are calculated, the distribution of volumetric entropy generation is obtained.

The numerical solution was achieved on the basis of an in-house computer code in FORTRAN. To check its validity, a comparison with selective data from the published literature was carried out. The comparison was made by adopting the present FORTRAN code to resolve three different cases namely: conjugate Darcy–Bénard convection (Saleh et al. [32]) and conjugate horizontally heating (Saeid [33]).

The third case is that of triangular enclosure filled with a Cu–Water nanofluid-saturated porous medium of Sun and Pop [11] with heater length of 0.8 times of the vertical wall. The work of Sun and Pop [11] was based on Maxwell–Garnett model for thermal conductivity and on the common definition of thermal expansion model. Accordingly, we used these models and the Cu-nanoparticles properties for comparison purpose. The results are documented in Tables 2 and 3. It is obvious that good agreement is obtained. Different numerical techniques were followed in these three different works, and this is the reason behind the relatively noticeable discrepancy (\( \geq 2\% \)) incorporated in Table 3 for \( Ra = 1000 \) and \( \psi = 0.03 \). As a result, the confidence in the present numerical solution is verified.

### 4. Results and discussion

Selective results represented by streamlines, isotherms, and isentropic lines (the local dimensionless entropy generation \( S_{GEN} \), local and average Nusselt numbers, global entropy generation and Bejan number are illustrated in this section. The effect of four pertinent parameters are discovered and discussed, these parameters are: triangular solid wall thickness \( D \) (0.1–1), thermal conductivity ratio \( K_{nf} = k_w/k_f (23.8 \) stainless steel–water, 1.0 brickwork–water, 0.44 epoxy–water), nanoparticles volume fraction \( \psi (0–0.05) \), and modified Rayleigh number \( Ra (10–1000) \), for brevity, the “modified” in-name will be dropped. It is worth mentioning that the value of \( Nu \) depends on fluid properties. The present base fluid is water, hence, \( \mu = 10^{-3} \) Pa s, \( k_f = 0.613 \) W/m K, \( \alpha = 10^{-7} \) m²/s. Taking \( (T_0/\Delta T)^2 = 10 \) K⁻¹, and assuming the porosity \( K = 10^{-14} \) m². Therefore the value of \( N_{mu} \) is fixed in this study at \( 10^{-2} \). This value was used by Basak et al. [34], and Varol et al. [35].

Fig. 4 depicts the effect of the triangular solid wall thickness \( D \) on the streamlines, isotherms, and the isentropic contours, for \( Ra = 100 \) and \( K_{nf} = 23.8 \). The effects of nanoparticles are shown also in this figure by dashed lines. Generally, the effects of buoyancy together with the imposed boundary conditions make the fluid (and nanofluid) to rotate clockwise (negative streamlines) forming a single-cell circulation. The core shape of this cell is transformed from horizontally extended to mostly vertically-extended when \( D \) increased from 0.2 to 1. The streamlines labels imply that the strongest rotation occurs when \( D = 0.5 \). The explication of this refers to the entente between the heating effect of the solid wall and the available space for the vortex rotation. The isotherms tend to be horizontal within the middle of

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**Table 1**

<table>
<thead>
<tr>
<th>Physical property</th>
<th>Base fluid (water)</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p (\text{J/kg/K}) )</td>
<td>4179</td>
<td>540</td>
</tr>
<tr>
<td>( \rho (\text{kg/m}^3) )</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>( \beta \times 10^{-3} (1/\text{K}) )</td>
<td>21</td>
<td>0.85</td>
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</table>

**Table 2**

Comparison of the average Nusselt numbers with other works—conjugate cases, pure fluid (\( \psi = 0 \)).

<table>
<thead>
<tr>
<th>( Ra )</th>
<th>D</th>
<th>( Kr )</th>
<th>Saeid [33]</th>
<th>Saleh et al. [32]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.1</td>
<td>0.44</td>
<td>2.333</td>
<td>-</td>
<td>2.334</td>
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<tr>
<td>1000</td>
<td>0.4</td>
<td>2.4</td>
<td>3.511</td>
<td>-</td>
<td>3.49</td>
</tr>
<tr>
<td>1000</td>
<td>0.2</td>
<td>0.1</td>
<td>-</td>
<td>0.446</td>
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<td>-</td>
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**Table 3**


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<th>( Ra )</th>
<th>( \psi )</th>
<th>( Nu_{nf} )</th>
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Fig. 4. Contours of streamfunction, isotherm, and isentropic for $Ra = 100, K_{ro} = 23.8$, and different $D$. Solid lines, $\psi = 0$ and dashed lines, $\psi = 0.05$. 
cavity. Their crowdedness being intensified with $D$ close to the lower edge of the wall-porous interface and the upper part of the left vertical wall. It is seen that these two positions, where the isotherms intensified, become as concentrators for entropy generation. The entropy is spread out from these concentrators toward the remainder of the cavity. Diagonally, the symmetry of isentropic lines is lost with increasing the value of $D$. The effect of nanoparticles addition is highly distinguishable in the isentropic lines contours.

Fig. 5 is presented for the same purpose and parameters of Fig. 4, but for higher Rayleigh number ($Ra = 500$). In contrary with $Ra = 100$ (Fig. 4), the streamlines become stronger, the isotherms are mostly horizontal within the cavity center, with steeper gradient close to the interface and the left vertical walls. This is an indication to the convection dominance. The entropy concentrators look very strong and propagate along both the solid wall-porous interface and the left vertical walls with increasing $D$. 

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**Fig. 5.** Contours of streamfunction, isotherm, and isentropic for $Ra = 500$, $K_n = 23.8$, and different $D$. Solid lines, $\psi = 0$ and dashed lines, $\psi = 0.05$. 
The effect of the conductivity ratio $K_{ro}$ is shown in Fig. 6 for $Ra = 500$ and $D = 0.4$. With changing the solid wall type (changing $K_{ro}$ from 23.8 to 0.44), the streamlines are characterized by a decrease in their strength and crowding close to both the interface and the left vertical walls. The mostly horizontal pattern and the steep gradient of the isotherms are lost with decreasing $K_{ro}$. A reduction in $K_{ro}$ leads to irregularity in the temperature distribution within the solid wall and especially close to its two corners. This increases the irreversibility in heat transfer there. The dimensionless entropy behavior with $K_{ro}$ can be characterized by an intensity reduction close to, except its two corners, the wall-porous interface and most of the vertical left wall. The aforementioned characteristics associated with changing $K_{ro}$ are
due to the decrease of heat transferred to the nanofluid saturated in porous media because of increasing the thermal resistance of the triangular solid wall.

Fig. 7 illustrates the behavior of contours maps with Rayleigh number for $D = 0.7$ and $K_{ro} = 23.8$. For a very low Rayleigh number, $Ra = 10$ (Fig. 7a), as expected, a relaxed weak streamlines behavior is recorded. The streamlines are strengthened and crowded significantly close to the interface and the vertical left wall with increasing $Ra$. This is an expected effect as the convection mode dominates over the conduction. The isotherms pattern supports this well-known fact, where the pattern transforms from a vertically uniform distribution to mostly horizontal with a thin boundary layer close to the interface and the vertical left wall. Regarding the isentropic lines, at $Ra = 0$ (Fig. 7a), the source of entropy generation is concentrated at the
lower edge of the interface wall only and therefore, a relatively low spread out values are recorded. Increasing $Ra$ to 100 (Fig. 7b), another concentrator appears at the upper part of the left vertical wall. Further increasing $Ra$ to 1000 (Fig. 7c) leads to strong concentrators along the interface wall and the left vertical wall. Accordingly, stronger values of entropy generation within the cavity are observed. It is worth mentioning that in most cases above, the heat transfer irreversibility within the cavity core can be ignored, forming what is called the idle region for entropy generation [36].

The local Nusselt number $Nui$ along the interface is shown in Fig. 8 for $Ra = 100$. Unless zooming a selected area in Fig. 8a, the effect of nanoparticles addition is difficult to be seen where increasing $\phi$ enhances the local heat transfer. This scenario is seen for all covered ranges of $Ra$, they are not presented herein for brevity. Fig. 8b shows the variations of $Nui$ with the wall thermal conductivity ratio $Kro$ for $Ra = 500$ and $D = 0.4$. A distinct effect of $Kro$ on enhancing the natural convection is evident. Although the local Nusselt number gives no comprehensive concept about the heat transfer, but it reflects the existence of the thermal boundary layer at the lower edge of the solid wall-porous interface as shown in both figures (Figs. 8a and b).

The average Nusselt number $Nunf$ gives a comprehensive imagination about the overall heat transfer. Fig. 9 depicts the average Nusselt number along the isothermal right vertical wall (Eq. 18) with $D$ for the three values of $Kro$. Fig. 9a is plotted at a low value of Rayleigh number, $Ra = 50$, it shows a continuous increase of $Nunf$ with $D$. The trend of this increase is mostly identical for all $Kro$ values. More heat transfer is associated with the high thermal conductivity of solid wall due to the less thermal resistance. In Fig. 9b, for a high value of Rayleigh number, $Ra = 1000$, a different behavior of $Nunf$ is seen. When $Kro \leq 1$, an important matter can be drawn from this figure, that is, increasing the wall thickness $D$ leads to an increase in the Nusselt number up to
a critical value of $D = 0.7$. Beyond this critical value, a deterioration of $N_{	ext{uf}}$ is seen. This matter can be elucidated as follows. The increase of $D$ increases three parameters; the first is the thermal resistance of the triangular wall which in turn, reduces the temperature of the interface line. The second is the length of the interface line which in turn, heats more quantity of nanofluid-saturated porous medium, and the third is the temperature difference between the interface surface and the vertical right wall where they become closer when $D$ increases. Thus, the critical values of $D$ are those for which the latter two parameters are insufficient to overcome the effect of the thermal resistance and this is a good reason to the absence of such critical $D$ values at $K_{ro} = 23.8$.

The variation of $N_{	ext{uf}}$ with $Ra$ for different values of $K_{ro}$ are shown in Fig. 10a, and for different volume fraction $\varphi$ in Fig. 10b. The higher the conductivity triangular solid wall, the faster the increase of the Nusselt number is. On the other hand, the effect of nanoparticles becomes active for $Ra \geq 100$. This reflects the dominance of the nanofluid thermal conductivity enhancement over the inertia and viscous forces associated with increasing the value of $\varphi$.

Fig. 11 depicts the variation of the global entropy generation GEG and Bejan number $Be$ with $D$ for different values of $K_{ro}$ at low Rayleigh number, $Ra = 50$. The critical solid wall thickness, $D = 0.7$, is noticed due to GEG but at $K_{ro} = 0.44$ only, otherwise a continuous increase of GEG with $D$ is recorded (Fig. 11a). The attribution of this refers to the two concentrators of entropy generation localized at the two edges of the wall-porous interface associated with low values of $K_{ro}$. Hence, when $D$ increased more than 0.7, the available space for vortex rotation will be limited which in turn leads to a reduction of its strength, hence, less irreversibility will come from the fluid friction. At the same time, the two concentrators will be widely apart and will weaken the global entropy generation. Fig. 11b demonstrates that the heat transfer irreversibility HTI is dominant ($Be > 0.5$) over the nanofluid friction irreversibility FFI for all conductivity ratios especially at higher values of $D$, this is due to the weak convection. Beyond
Fig. 12. Variation of global entropy generation (GEG) (a) and Bejan number (b) with $D$ for $K_{ro} = 23.8$, $\phi = 0.03$ and different values of $Ra$.

Fig. 13. Variation of global entropy generation (GEG) (a) and Bejan number (b) with $Ra$ for $D = 0.5$, $\phi = 0.03$, and different values of $K_{ro}$.

$D = 0.7$, all Bejan curves become asymptotic. To inspect the effects of all parameters, Fig. 11 is reconstructed but for a high Rayleigh number, $Ra = 1000$ and the results are presented in Fig. 12. This figure indicates that for high $Ra$, the critical $D$ value is noticed even at the high thermal conductivity solid wall ($K_{ro} = 23.8$). The other conductivity ratios ($K_{ro} = 1$, 0.44) manifest lower values of critical $D$ such that at $K_{ro} = 1$, $D = 0.5$ and at $K_{ro} = 0.44$, $D = 0.3$ (Fig. 12a). This can be attributed to that the diminishing of the strong convection currents with lowering $K_{ro}$, will be significantly affected by the reduced space available for vortex rotation. On the other hand, the dominance convection in this case minimizes $Be$ values as indicated in Fig. 12b. However, a continuous increase of $Be$ with $D$ is recorded for all $K_{ro}$ values.

The variations of GEG and $Be$ with $Ra$ for different $K_{ro}$ values are depicted in Fig. 13 for $D = 0.5$ and $\phi = 0.03$. It can be observed from this figure that the effect of the thermal conductivity ratio is announced beyond $Ra = 50$ (Fig. 13a), where both the GEG and $Be$ curves asymptote when $Ra \leq 50$. Figure 13b depicts the contribution transition of HTI from effective at low Rayleigh number to inactive at high Rayleigh number. However, both figures demonstrate that the thermodynamic irreversibility is generated from the characteristic of convective heat transfer.

The effect of volume fraction $\phi$ on GEG and $Be$ number are presented as a function of Rayleigh number in Fig. 14. Two stages of nanofluid effect on GEG is observed; stage 1 is for low Rayleigh number ($Ra \leq 100$), where the nanoparticles action can be ignored as seen in Fig. 14a. Stage 2 for $Ra > 100$ where the existence of nanoparticles stimulates the global entropy generation. This is because of the enhanced viscous and inertia effects associated with the nanofluid. On the other hand, the nanoparticles poses suppression effect on the HTI and for a wide range of the studied Rayleigh numbers, $Ra > 10$ (Fig. 14b).
Eventually, the effect of the pertinent parameters is discovered and discussed. Nevertheless, it is sought that the optimum values of $D$ which gives superior thermal performance could not be exactly extracted from the aforementioned displayed results. The superior performance manifests maximum heat transfer with minimum input work, i.e. minimum thermodynamic irreversibility. Therefore, establishing a ratio of the global entropy generation $GEG$ to the average Nusselt number $Nun_f$ may clearly address the aspects of such performance, such that:

$$\varepsilon = \frac{GEG}{Nun_f}$$  \hspace{1cm} (30)

The results of the $\varepsilon$ ratio are given in Figs. 15 and 16. Fig. 15 depicts the variations of $\varepsilon$ with $D$ and $K_{ro}$ for $Ra = 50$, Fig. 15a and $Ra = 1000$, Fig. 15b. In both cases, the value of $D = 1$ manifests the best performance, according to the $\varepsilon$ criterion. However, for low Rayleigh numbers, the low thermal conductivity wall gives a minimum $\varepsilon$. On the other hand, when $Ra = 1000$, the wall type does not affect the $\varepsilon$ ratio when $D \leq 0.3$, otherwise, the lower thermal conductivity walls give the best thermal performance. Finally, Fig. 16 implies to that increasing the nanoparticles volume fraction does not necessarily lead to improvements in the thermal performance, where it is evident that the proposed $\varepsilon$ criterion increases with increasing values of the nanoparticles volume fraction $\psi$.

5. Conclusions

Steady conjugate natural convection–conduction heat transfer and entropy generation in a square porous cavity filled with a CuO–water nanofluid is studied numerically. The cavity is heated by an isosceles triangular wall which occupies the lower left corner. Three types of solid wall material were examined under a wide range of wall thickness, nanoparticles volume fraction, and the Rayleigh number. In addition, a new criterion for the thermal performance is
proposed. This new criterion determines the ratio of the thermodynamic irreversibility to the convective heat transfer. The following concluding remarks are extracted from the present study:

1. For a low Rayleigh number, the overall heat transfer is an increasing function of solid wall thickness for all solid wall types. On the other hand, the rate of entropy generation increases with solid wall thickness for a highly conductive wall only, otherwise it is slightly affected by $D$. For a high Rayleigh number, a critical value of $D = 0.7$ gives a maximum heat transfer for a poorly conductive solid wall ($K_w \leq 1$). Due to the global entropy generation, in general, a maximum irreversibility appears at $D = 0.5$.

2. For a high conductivity solid wall, the overall heat transfer and the global entropy generation rate increase faster than those of a low conductivity solid wall.

3. The effect of CuO nanoparticles enhances the overall heat transfer and increases the global entropy generation. Both effects are remarked beyond $Ra = 100$.

4. From an optimum thermal performance point of view, the largest wall thickness ($D = 1.0$) and the lower wall thermal conductivity manifest minimum $\varepsilon$ ratio, i.e., minimum global entropy generation to the average Nusselt number ratio.

References


