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Hydromagnetic flow of heat absorbing and radiating fluid over exponentially stretching sheet with partial slip and viscous and Joule dissipation

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Abstract

Purpose – The purpose of this paper is to investigate hydromagnetic two dimensional boundary layer flow with heat transfer of a viscous, incompressible, electrically conducting, heat absorbing and optically thick heat radiating fluid over a permeable exponentially stretching sheet considering the effects of viscous and Joule dissipations in the presence of velocity and thermal slip.

Design/methodology/approach – Using similarity transform, governing differential equations representing mathematical model of the problem are solved with the help of fourth-order Runge-Kutta method along with shooting technique. Numerical solutions of fluid velocity and fluid temperature are depicted graphically for various values of pertinent flow parameters whereas numerical values of wall velocity gradient and wall temperature gradient are displayed graphically for various values of pertinent flow parameters.

Findings – Numerical results obtained in this paper are compared with earlier published results and are found to be in excellent agreement. Magnetic field and suction tend to enhance the wall velocity gradient whereas dimensionless co-ordinate, injection and velocity slip factor have reverse effect on it. Suction and heat absorption tend to enhance wall temperature gradient whereas magnetic field, velocity slip factor, injection, thermal radiation, thermal slip factor and viscous dissipation have reverse effect on it.

Originality/value – The investigation of this problem may have bearing in several engineering processes such as extrusion of plastic sheet, annealing and tinning of copper wire, paper production, crystal growing and glass blowing, continuous casting of metals and spinning of fibers.

Keywords Magnetic field, Thermal radiation, Heat absorption, Suction/injection, Velocity and thermal slips, Viscous and Joule dissipations

Paper type Research paper

Introduction

Interest toward the investigation of steady two dimensional boundary flow with heat transfer of a viscous and incompressible fluid over a permeable or non-permeable stretching sheet has grown up rapidly during past few decades due to its numerous applications in various industrial and manufacturing processes such as extrusion of polymer sheet from a dye, glass fiber and paper production, drawing of plastic films, etc. The pioneering work on stretching sheet is due to Crane (1970) who initiated the
investigation of boundary layer flow over a stretching sheet. After this famous work, a large amount of research investigations on this topic have been carried out by several researchers. However, their investigations were based on the assumption that the velocity of the stretching sheet is linearly proportional to the distance from a fixed origin. But, it was Gupta and Gupta (1977) who analyzed fluid flow with heat and mass transfer in the boundary layer over a stretching sheet subject to suction or blowing and they also mentioned that the stretching nature of the sheet may not necessarily be linear in practical situation. Keeping view this fact, Kumaran and Ramanaiah (1996) investigated boundary layer flow over a quadratic stretching sheet. Later on, this idea encouraged several researchers to consider oscillatory, hyperbolic and exponentially stretching sheets. Magyari and Keller (1999) studied boundary layer flow with heat and mass transfer over an exponentially stretching sheet. Elbashbeshy (2001) examined fluid flow with heat transfer over an exponentially stretching sheet with constant suction. Khan and Sanjayanand (2005) studied boundary layer flow with heat transfer of a viscoelastic fluid over an exponentially stretching sheet. Chamkha et al. (2010) obtained similarity solution for unsteady heat and mass transfer from a permeable stretching surface embedded in a porous medium in the presence of chemical reaction. Moreover, imposition of an external magnetic field into the flow field has received considerable attention owing to its overwhelming and important applications in plasma confinement, boundary layer flow control, crystal growth, forging, casting, levitation, etc. Taking into consideration this fact, Al-Odat et al. (2006) investigated boundary layer flow over an exponentially stretching sheet in the presence of magnetic field. Pantakratoras (2008) examined MHD boundary layer flow over a heated stretching sheet with variable viscosity. Chamkha et al. (2011a, b) investigated melting effect on unsteady magnetohydrodynamic flow of a nanofluid over a stretching sheet in the presence of transverse magnetic field. Ferdows et al. (2012) studied hydromagnetic mixed convective boundary layer flow of a nanofluid through porous medium over an exponentially stretching sheet. Makinde et al. (2013) analyzed the combined effects of buoyancy force, convective heating, Brownian motion, thermophoresis and magnetic field on stagnation point flow and heat transfer due to nanofluid flow toward a stretching sheet. Hayat et al. (2014) analyzed MHD three-dimensional flow of a viscous, incompressible and electrically conducting fluid induced by an exponentially stretching sheet.

It is worthy to mention that there exists a considerable temperature difference between the surface of solid and ambient fluid in so many fluid flow problems of physical interest. This prompted many researchers to consider temperature dependent heat source and/or sink which may have strong influence on the heat transfer characteristics. Research studies related to the boundary layer flow over a permeable or non-permeable stretching sheet of a heat generating or absorbing fluid is of much significance in several physical problems, namely, cooling of nuclear reactors, underground disposal of nuclear waste, petroleum reservoir, building insulation, food processing, casting and welding in manufacturing processes, etc. Keeping in view the significance of such facts, Elbashbeshy and Bazid (2003) analyzed fluid flow with heat transfer over an unsteady stretching surface with internal heat generation or absorption. Eldahab and Aziz (2004) studied suction/blowing effect on hydromagnetic flow with heat transfer by mixed convection from an inclined continuously stretching surface with internal heat generation/absorption. Chamkha and Aly (2010) analyzed hydromagnetic free convection flow of a heat generating/absorbing nanofluid near a vertical plate whereas Chamkha et al. (2011a, b) considered laminar hydromagnetic mixed convection flow of a nanofluid over a permeable stretching surface in the
presence of heat generation/absorption. Malvandi et al. (2013) investigated steady two
dimensional stagnation point flow of a nanofluid over an exponentially stretching
sheet with non-uniform heat source/sink. Singh and Agarwal (2014) considered
magnetohydrodynamic flow with heat transfer of Maxwell fluid through porous
medium over an exponentially stretching sheet in the presence of non-uniform heat
source/sink with variable thermal conductivity.

The effect of thermal radiation becomes significant for several industrial processes
such as glass production, furnace design, electrical power generation, solar power
technology, etc. A good working knowledge of thermal radiation helps in designing of
important equipments such as design of fins, ceramic and glass producing units and
various propulsion devices for aircraft, missiles, satellites, space vehicles, etc. Keeping
in mind its importance, Sajid and Hayat (2008) investigated the influence of thermal
radiation on the boundary layer flow over an exponentially stretching sheet. Bidin and
Nazar (2009) obtained numerical solution for the boundary layer flow of a radiating
fluid over an exponentially stretching sheet. Ishak (2011) studied MHD boundary layer
flow near an exponentially stretching sheet with radiation effect. Reddy and Reddy
(2011) studied thermal radiation effect on hydromagnetic flow due to an exponentially
stretching sheet. Makinde (2012a) studied unsteady hydromagnetic boundary layer
flow of radiating and chemically reacting fluid past a vertical plate with constant
heat flux. Mabood et al. (in press) examined MHD boundary layer flow of a viscous,
incompressible and radiating fluid over an exponentially stretching sheet using
homotopy analysis method. However, the combined effect of thermal radiation and
internal heat generation/absorption is of much significance due to its varied and wide
application in science and technology. Taking into consideration this fact, Chen (2009)
investigated MHD mixed convection of a power law fluid past a stretching surface in
the presence of thermal radiation and internal heat generation/absorption. Elbashbeshy
et al. (2012) studied the effects of radiation and internal heat generation/absorption on
unsteady hydromagnetic mixed convection flow with heat transfer over an exponentially
stretching sheet with suction. Makinde (2012b) examined the hydromagnetic mixed
convection stagnation point flow toward a vertical plate embedded in a highly porous
medium with radiation and internal heat generation.

It is to be noted that, in all of the above investigations, the effect of viscous dissipation
is not taken into account. Although viscous dissipation effect is considered to be week,
but its effect becomes significantly important in tribology, instrumentation, food
processing, lubrication, polymer manufacturing, etc. Viscous dissipation changes the
temperature distribution by playing a key role like an energy source which leads to affect
heat transfer rate. Keeping in view its significance, Vajravelu and Hadjinicolou (1993)
analyzed heat transfer characteristics in the laminar boundary layer of a viscous and
heat absorbing fluid over a stretching sheet taking viscous dissipation into account.
Partha et al. (2006) studied the effect of viscous dissipation on the mixed convection flow
with heat transfer from an exponentially stretching surface. Sanjayanand and Khan
(2006) discussed heat and mass transfer in a viscoelastic boundary layer fluid flow over
an exponentially stretching sheet taking viscous dissipation into account. Cortell (2008)
investigated the effects of viscous dissipation and thermal radiation on the thermal
boundary layer over a non-linearly stretching sheet. Aziz (2009) studied viscous
dissipation effect on mixed convection flow of a micropolar fluid over an exponentially
stretching sheet. Pavithra and Gireesha (2013) analyzed the effect of viscous dissipation
on hydromagnetic fluid flow with heat transfer in a porous medium over an
exponentially stretching sheet with fluid particle suspension. Apart from viscous
dissipation in MHD flows, the Joules dissipation also acts as a volumetric heat source. In particular, effects of viscous and Joule dissipations find application in heat-treated materials traveling between a feed roll and wind-up roll or materials manufactured by extrusion. In view of its importance, Anjali Devi and Ganga (2009) investigated the influence of viscous and Joule dissipations on MHD flow with heat and mass transfer over a stretching porous surface embedded in a porous medium. Pal (2010) analyzed mixed convection heat transfer in the boundary layers on an exponentially stretching surface with an exponential temperature distribution in the presence of magnetic field, viscous and Joule dissipations and internal heat generation/absorption.

However, all the above mentioned research studies are carried out by assuming no-slip boundary condition. The no-slip boundary condition (the assumption that a liquid adheres to a solid boundary) is one of the main features of Navier-Stokes theory. However, there are physical situations wherein this condition does not hold. The non-adherence of the fluid to a solid boundary, also known as velocity slip, is a phenomenon that has been observed under certain circumstances. The fluids that exhibit boundary slip have important technological applications such as in the polishing of artificial heart valves and internal cavities. For some coated surfaces, such as Teflon and resist adhesion, the no-slip condition is replaced by Navier’s partial slip condition, where the slip velocity is proportional to the local shear stress. Partial velocity slip may also occur on the stretching boundary when the fluid is particulate such as emulsions, suspensions, foams and polymer solutions. It is to be noted that velocity and thermal slip conditions are adequate for the flow of liquids at the micro-scale level especially in view of the lack of data on the thermal accommodation coefficient. Among the applications of micro-devices, a number of complex micro-channels arise, e.g., the micro-ducts of rectangular, triangular or trapezoidal cross-section are very popular and easier to manufacture in the micro-scale thermal fluid system. Keeping in view of its importance, Mukhopadhyay and Gorla (2012) studied the effects of partial slip on boundary layer flow past a permeable exponential stretching sheet in presence of thermal radiation. Mukhopadhyay (2013) investigated the slip effects on MHD boundary layer flow over an exponentially stretching sheet with suction/blowing and thermal radiation. Seini and Makinde (2014) investigated MHD boundary layer flow of viscous, incompressible and electrically conducting fluid near a stagnation point on a vertical surface with slip. Sharma et al. (2014) analyzed partial slip flow of nanofluid with heat transfer over a stretching sheet. Singh and Makinde (2015) examined mixed convection flow with slip and convective heat transfer along a continuously moving vertical plate in the presence of uniform free stream.

The objective of the present investigation is to study steady two dimensional hydromagnetic boundary layer flow of a viscous, incompressible, electrically conducting, heat absorbing and optically thick heat radiating fluid over a permeable exponentially stretching sheet taking into the account the effects of viscous and Joule dissipations in the presence of partial velocity and thermal slip conditions. As per authors’ knowledge, no attempt has been made for the present study. This types of problem may occur in several engineering processes such as extrusion of plastic sheet, annealing and tinning of copper wire, paper production, crystal growing and glass blowing, continuous casting of metals and spinning of fibers.

Formulation of the problem and its solution
Consider steady two dimensional boundary layer flow of a viscous, incompressible, electrically conducting, optically thick radiating and heat absorbing fluid over a permeable non-isothermal stretching sheet. Choose Cartesian co-ordinate system in such a way that
\( x \)-axis is along the sheet, \( y \)-axis is normal to the plane of the sheet and \( z \)-axis is perpendicular to \( xy \)-plane. Leading edge of the sheet is fixed at the origin \( O \) and the fluid flow is confined in the region \( y > 0 \). The sheet is stretched in \( x \)-direction with a velocity \( U(x) \) which varies exponentially with the distance from the fixed origin \( O \). A uniform magnetic field of strength \( B_0 \) is applied in a direction which is parallel to \( y \)-axis. The effect of velocity slip and thermal slip in the presence of viscous and Joule dissipations are also taken into account. Further, it is assumed that induced magnetic field generated by fluid motion is negligible in comparison to the applied one. This is justified because magnetic Reynolds number is very small for liquid metals and partially ionized fluids which are commonly used in industrial processes (Cramer and Pai, 1973). The geometry of the problem is shown in Figure 1.

In view of the above assumptions, the governing equations for fluid flow with heat transfer are given by:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{u}{\frac{\partial}{\partial x}} + \frac{v}{\frac{\partial}{\partial y}} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho}, \tag{2}
\]

\[
\frac{\partial T}{\partial x} + \frac{v}{\frac{\partial}{\partial y}} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y} - \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\sigma B_0^2 u^2}{\rho c_p}, \tag{3}
\]

where \( u, v, \rho, \sigma, v, T, c_p, q_r, Q_0, T_\infty \) and \( k \) are, respectively, fluid velocity along \( x \)-direction, fluid velocity along \( y \)-direction, fluid density, electrical conductivity, kinematic coefficient of viscosity, fluid temperature, specific heat at constant pressure, radiating flux vector, heat absorption coefficient, free-stream temperature and thermal conductivity.
The appropriate boundary conditions for the fluid flow problem are given by:

\[ u = U(x) + Nu \frac{\partial u}{\partial y}, \quad v = -V(x), \quad T = T_w(x) + D \frac{\partial T}{\partial y} \text{ at } y = 0, \]  
\[ u \to 0, \quad T \to T_\infty \text{ as } y \to \infty, \]  

(4a)  

where \( U = U_0 e^{xL} \) is the stretching velocity, \( T_w = T_\infty + (T_0 - T_\infty) e^{ax/2L} \) is the exponential temperature distribution within the sheet, \( U_0 \) is the reference velocity, \( T_0 \) is reference temperature, \( a \) is parameter of temperature distribution in the stretching sheet, \( N = N_1 e^{-x/2L} \) is the velocity slip factor which changes with \( x \), \( N_1 \) is the initial value of velocity slip, \( D = D_1 e^{-x/2L} \) is the thermal slip factor which also changes with \( x \) and \( D_1 \) is the initial value of thermal slip factor. The no-slip case is recovered for \( N = D = 0 \).

\( V(x) = V_0 e^{xL} \) is the suction/blowing velocity where \( V_0 \) is the initial strength of suction/blowing velocity.

For an optically thick fluid, Rosseland approximation (Raptis, 2011) is used to express radiative heat flux \( q_r \) which is given by:

\[ q_r = -\frac{4\sigma^* \partial T^4}{3k^*} \frac{\partial T}{\partial y}, \]  

(5)

where \( \sigma^* \) and \( k^* \) are the Stefan Boltzmann constant and the Rosseland mean absorption coefficient, respectively.

In order to linearize \( T^4 \) it is assumed that there is a small temperature difference between fluid within the boundary layer region and the free stream. With this assumption, \( T^4 \) is expanded in Taylor series about \( T = T_\infty \) and after neglecting the second and higher order terms, we obtain:

\[ T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \]  

Using (5) and (6) in Equation (3), we obtain:

\[ \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left( 1 + \frac{16\sigma^* T_\infty^3}{3kk^*} \right) \frac{\partial^2 T}{\partial y^2} + \frac{v}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 - \frac{Q_0}{\rho c_p} (T - T_\infty) + \frac{\sigma}{\rho c_p} T_\infty^2 \frac{\partial^2 u}{\partial x^2}. \]  

(7)

Following Pal (2010), we introduce the following similarity transform:

\[ \eta(x,y) = \sqrt{\frac{Re}{2}} \left( \frac{y}{L} \right)^{e^{x/2L}}, \quad \psi(x,y) = \sqrt{2Re} e^{x/2L} f(\eta), \quad T(x,y) = T_\infty + (T_0 - T_\infty) e^{ax/2L} \theta(\eta), \]  

(8)

where \( Re \) is Reynolds number and \( \eta, \psi \) and \( \theta \) are, respectively, similarity variable, stream function and dimensionless fluid temperature.

Using (8) in Equations (2) and (7), we obtain:

\[ f'''' + f f'' - 2f'^2 - 2M e^{-x} f' = 0, \]  

(9)

\[ Pr_{eff} \theta'' + f \theta' - af' \theta - 2\phi e^{-x} \theta + E_c \left( e^x f'^2 + 2M f'^2 \right) e^{2-aX/2} = 0, \]  

(10)
where \( X = x/L \) is a dimensionless co-ordinate, \( Pr_{\text{eff}} = Pr/1+R \) is the effective Prandtl number (Magyari and Pantokratoras, 2011), \( Pr = vpc_p/k \) is Prandtl number, \( M = \sigma B_0^2 L^2/\nu_0 Re \) is magnetic parameter, \( R = 16\sigma^* T_\infty^3/3k\kappa^* \) is radiation parameter, \( E_c = U_0^2/c_p(T_0 - T_\infty) \) is Eckert number, \( \phi = Q_0 L^2/\rho_0 c_p Re \) is the heat absorption parameter and \( Re = U_0 L/v \) is Reynolds number.

The boundary conditions (4a) and (4b), in non-dimensional form, are given by:

\[
f' = 1 + \lambda f'' \quad \text{and} \quad f = S, \quad \theta = 1 + \delta \theta' \quad \text{at} \quad \eta = 0, \tag{11a}
\]

\[
f' \to 0 \quad \text{as} \quad \eta \to \infty, \tag{11b}
\]

where \( \lambda = N_1(v/L)\sqrt{Re/2} \) is the velocity slip parameter, \( \delta = D_1/L\sqrt{Re/2} \) is thermal slip parameter and \( S = V_0 L/\sqrt{2/Re} \) is the suction/blowing parameter (\( S < 0 \) for suction, \( S > 0 \) for blowing and \( S = 0 \) for impermeable sheet).

### Skin friction coefficient and Nusselt number

The physical quantities of interest in the present study are the local skin friction coefficient \( C_f \) and local Nusselt number \( Nu_x \) which are given by:

\[
C_f = \frac{\tau_w}{1/2 \rho U^2} \quad \text{and} \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)},
\]

where \( \tau_w = \mu(\partial u/\partial y)_{y=0} \) is the wall shear stress and \( q_w = -k(\partial T/\partial y)_{y=0} + (q_r)_{y=0} \) is the wall heat flux.

The local skin friction coefficient \( C_f \) and the local Nusselt number \( Nu_x \), in non-dimensional form, are given by:

\[
C_f = \sqrt{2XRe_x^{-1/2}}f''(0) \quad \text{and} \quad Nu_x = -(1+R)(X/2)^{1/2}Re_x^{1/2}\theta'(0),
\]

where \( Re_x \) is the local Reynolds number based on the surface velocity and is given by:

\[
Re_x = \frac{xU(x)}{v}.
\]

### Numerical method for solution

In order to solve the non-linear ordinary differential Equations (9) and (10) subject to the boundary conditions (11a) and (11b), fourth-order Runge-Kutta method along with shooting technique is used. Equations (9) and (10) are split into a system of five first order differential equations. To solve the resulting system of equations by fourth-order Runge-Kutta method, values of \( f''(0) \) and \( \theta'(0) \) are required, but no such values are provided. Therefore, some initial guess values are assigned to \( f''(0) \) and \( \theta'(0) \). Secant iteration technique is employed to correct the guess values. For the infinity boundary conditions, a large value of \( \eta \) (say \( \eta = 18 \)) is considered and, for numerical computation, step size is taken as 0.001. In order to obtain more accurate results, the tolerance error in results is chosen as \( 10^{-6} \). The overall process is repeated until the desired accuracy in the results is achieved.
Validation of numerical solution
In order to test the accuracy of the present numerical solution, a comparison is made for the numerical values of wall temperature gradient \( \theta'(0) \) for different values of Prandtl number \( \text{Pr} \) with those of Magyari and Keller (1999), Ishak (2011) and Mukhopadhyay and Gorla (2012) when \( M = S = \lambda = R = \phi = \delta = E_c = 0 \) and \( a = 1 \), i.e. in the absence of magnetic field, suction/blowing, velocity slip, thermal radiation, heat absorption, thermal slip and viscous dissipation is presented in Table I. It is noticed from Table I that there is an excellent agreement among the numerical solution which justifies the accuracy of the numerical solution obtained in the present investigation.

Results and discussion
In order to gain a perspective of the physics of the flow regime, we have analyzed the influence of magnetic field, dimensionless co-ordinate, velocity slip factor and suction/injection on the flow field. The numerical values of fluid velocity \( f'(\eta) \) are displayed graphically vs boundary layer co-ordinate \( \eta \) in Figures 2-5 for various values of magnetic parameter \( M \), dimensionless co-ordinate \( \eta \), velocity slip parameter \( \lambda \) and suction/injection parameter \( S \).

Figure 2 demonstrates the effect of magnetic field on the fluid velocity. It is evident from Figure 2 that an increase in \( M \) results in a reduction in the fluid velocity \( f'(\eta) \). It is well known that the presence of a magnetic field in an electrically conducting fluid flow

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Note: Comparison of values of wall temperature gradient \( \theta'(0) \) when \( M = S = \lambda = R = \phi = \delta = E_c = 0 \) and \( a = 1 \)

Figure 2. Velocity profiles when \( \eta = 1.5 \), \( \lambda = 0.3 \) and \( S = 0.1 \)
is to induce a retarding hydromagnetic body force (Lorentz force) which acts in a direction perpendicular to the direction of applied magnetic field. Since $M = \sigma B_0^2 L^2 / \nu \rho Re$ signifies the ratio of hydromagnetic body force to viscous force, higher value of $M$ leads to a stronger hydromagnetic body force which tends to decelerates the fluid flow. Therefore, application of an external magnetic field is a powerful mechanism for inhibiting the fluid motion.

Figure 3 exhibits the influence of dimensionless co-ordinate $X$ on the fluid velocity. It is revealed from Figure 3 that fluid velocity $f(\eta)$ increases on increasing $X$. This implies that an increase in dimensionless co-ordinate $X$ from leading edge $X=0$ leads to an enhancement in the fluid velocity throughout the boundary layer region.

Figure 4 illustrates the effect of velocity slip factor on the fluid velocity. It is perceived from Figure 4 that $f(\eta)$ decreases on increasing $\lambda$. When slip occurs, fluid
velocity near the sheet is no longer equal to the velocity of the stretching sheet and consequently fluid velocity decreases on increasing velocity slip factor because under slip condition, the pulling of the stretching sheet may be only partly transmitted to the fluid. This indicates that velocity slip factor $\lambda$ has a substantial effect on the fluid flow.

Figure 5 presents the influence of suction/injection at the stretching sheet on the fluid velocity. It is seen from Figure 5 that $f'(\eta)$ decreases on increasing $S(W_0)$ whereas it increases on increasing $S(o_0)$ which is in agreement with the physical behavior of suction/injection. In case of suction, momentum boundary layer adheres more closely to the sheet and thus destroys the momentum of the flow which leads to a fall in the fluid velocity. On the other hand, injection adds fluid through lateral mass flux at the sheet which in turn assists the flow momentum and hence fluid velocity is getting accelerated. Thus, suction causes strong deceleration in the flow field whereas injection has a reverse effect on it throughout the boundary layer region. It is evident from Figures 2-5 that momentum boundary layer becomes thinner due to magnetic field, velocity slip factor and suction whereas it becomes thicker due to dimensionless co-ordinate and injection.

To study the effect of magnetic field, velocity slip factor, suction/injection, heat absorption, thermal radiation, thermal slip factor and viscous dissipation on the temperature field, the numerical values of fluid temperature $\theta(\eta)$ are displayed graphically vs boundary layer co-ordinate $\eta$ in Figures 6-12 for various values of magnetic parameter $M$, velocity slip parameter $\lambda$, suction/injection parameter $S$, heat absorption parameter $\phi$, effective Prandtl number $Pr_{eff}$, thermal slip factor $\delta$ and Eckert number $Ec$, taking dimensionless co-ordinate $X = 1.5$, dimensionless constant $a = 2$ and Prandtl number $Pr = 0.71$(ionized air).

Figure 6 illustrates the influence of magnetic field on the fluid temperature. It is noticed from Figure 6 that fluid temperature $\theta(\eta)$ increases on increasing $M$. Since extra work done in dragging the fluid against the magnetic field is dissipated in the form of thermal energy within the boundary layer region, heating of fluid takes place within the boundary layer region as described by Sutton and Sherman (1965), and, therefore, there is an enhancement in fluid temperature.
Figure 6. Temperature profiles when $\lambda = 0.3$, $S = 0.1$, $\phi = 0.1$, $Pr_{eff} = 0.64$, $E_c = 0.1$ and $\delta = 0.1$

Figure 7. Temperature profiles when $M = 0.4$, $S = 0.1$, $\phi = 0.1$, $Pr_{eff} = 0.64$, $E_c = 0.1$ and $\delta = 0.1$

Figure 8. Temperature profiles when $M = 0.4$, $\lambda = 0.3$, $\phi = 0.1$, $Pr_{eff} = 0.64$, $E_c = 0.1$ and $\delta = 0.1$
Figure 9.
Temperature profiles when $M = 0.4$, $\lambda = 0.3$, $S = 0.1$, $Pr_{eff} = 0.64$, $Ec = 0.1$ and $\delta = 0.1$

Figure 10.
Temperature profiles when $M = 0.4$, $\lambda = 0.3$, $S = 0.1$, $\phi = 0.1$, $Ec = 0.1$ and $\delta = 0.1$

Figure 11.
Temperature profiles when $M = 0.4$, $\lambda = 0.3$, $S = 0.1$, $\phi = 0.1$, $Pr_{eff} = 0.64$ and $Ec = 0.1$
Figure 7 describes the effect of velocity slip factor on the fluid temperature. It is noticed from Figure 7 that fluid temperature $\theta(\eta)$ increases on increasing $\lambda$. Thus, there is an enhancement in the fluid temperature throughout the boundary layer region with the increase in velocity slip parameter $\lambda$. Figure 8 exhibits the influence of suction/injection at the stretching sheet on the fluid temperature. It is observed from Figure 8 that fluid temperature $\theta(\eta)$ decreases on increasing $S(>0)$ whereas it increases on increasing $S(<0)$. This is in agreement with the physical nature of suction/injection.

Figure 9 displays the effect of heat absorption on the fluid temperature. It is perceived from Figure 9 that fluid temperature $\theta(\eta)$ decreases on increasing $\phi$ which is in agreement with the fact that as $\phi$ increases, heat absorbing capacity of the fluid increases and hence there is a reduction in fluid temperature throughout the boundary layer region.

Figure 10 illustrates the influence of thermal radiation ($Pr_{eff} = 0.71/(1 + R)$) on the fluid temperature. It is noticed from Figure 10 that fluid temperature $\theta(\eta)$ increases on decreasing $Pr_{eff}$. Since, effective Prandtl number $Pr_{eff}$ decreases when radiation parameter $R$ increases. This implies that thermal radiation tends to enhance fluid temperature within boundary layer region. This is in agreement with the physical behavior of thermal radiation that it has the tendency to increase the conduction effect and consequently thermal boundary layer becomes thicker.

Figure 11 shows the effect of thermal slip factor on the fluid temperature. It is revealed from Figure 11 that fluid temperature $\theta(\eta)$ decreases on increasing $\delta$. Since with the increase in thermal slip parameter $\delta$, less heat is transferred to the fluid from the sheet and, therefore, temperature is found to decrease. Thus, there is a reduction in the fluid temperature throughout the boundary layer region.

Figure 12 demonstrates the influence of viscous dissipation on the fluid temperature. It is evident from Figure 12 that fluid temperature $\theta(\eta)$ increases on increasing $E_c$. Physically, Eckert number $E_c$ signifies the relation between kinetic energy and enthalpy. It embodies the conversion of kinetic energy into internal energy by work done against the viscous fluid stresses. Higher viscous dissipative heat causes a rise in the fluid temperature. Therefore, viscous dissipation has a tendency to enhance the fluid temperature throughout the boundary layer region. It is revealed from Figures 6-12 that thermal boundary layer is getting thicker due to magnetic field.
velocity slip factor, injection, thermal radiant and viscous dissipation whereas it is getting thinner due to suction, heat absorption and thermal slip factor.

The numerical values of magnitude of wall velocity gradient \( f''(0) \), which measures the shear stress at the stretching sheet, are computed for various values of \( M, X, \lambda \) and \( S \) and magnitude of wall velocity gradient profile are displayed in Figures 13 and 14. The numerical values of magnitude of wall temperature gradient \( \theta'(0) \), which measures the rate of heat transfer at the stretching sheet, are presented graphically in Figures 15-17 for several values of \( \lambda, S, M, \phi, Pr_{eff}, Ec \) and \( \delta \) taking \( X = 1.5, a = 2 \) and \( Pr = 0.71 \). It is evident from Figure 13 that magnitude of wall velocity gradient \( f'(0) \) increases on increasing \( M \) whereas it decreases on increasing \( X \). Increasing values of \( M \) result in a considerable resistance to the fluid flow due to

**Figure 13.**
Wall velocity gradient profiles when \( \lambda = 0.3 \) and \( S = 0.1 \)

**Figure 14.**
Wall velocity gradient profiles when \( M = 0.4 \) and \( X = 1.5 \)
the presence of Lorentz drag force which reduces the dimensionless fluid velocity and the momentum boundary layer thickness, and thus increases the magnitude of velocity gradient and, therefore, the shear stress at the stretching sheet is getting enhanced. On the other hand, dimensional co-ordinate $X$ has a reverse effect on the shear stress at the stretching sheet. It is noticed from Figure 14 that magnitude of $f''(0)$ increases on increasing $S(>0)$ and decreases on increasing either $S(<0)$ or $\lambda$. Thus, there is an enhancement in shear stress at the stretching sheet due to suction whereas injection and velocity slip factor have reverse effect on it.

It is revealed from Figure 15 that magnitude of wall temperature gradient $\theta'(0)$ increases on increasing $S(>0)$ and it decreases on increasing either $S(<0)$ or $\lambda$. This indicates that suction has a tendency to enhance rate of heat transfer at the stretching sheet whereas injection and velocity slip factor have reverse effect on it. It is
observed from Figure 16 that magnitude of $\theta'(0)$ decreases on increasing $M$ whereas it increases on increasing either $P_{eff}$ or $\phi$. Therefore, an increase in the strength of magnetic field and radiation leads to a reduction in the rate of heat transfer at the stretching sheet whereas heat absorption has a reverse effect on it. It is noticed from Figure 17 that magnitude of $\theta'(0)$ decreases on increasing either $E_c$ or $\delta$. Thus, viscous dissipation and thermal slip factor have tendency to reduce rate of heat transfer at the stretching sheet.

**Conclusion**
Investigation of hydromagnetic two dimensional boundary layer flow with heat transfer of a viscous, incompressible, electrically conducting, heat absorbing and optically thick heat radiating fluid over a permeable exponentially stretching sheet considering the effects of viscous and Joule dissipations in the presence of velocity and thermal slip is carried out. Significant findings of fluid flow throughout the boundary layer region are as follows:

- dimensionless co-ordinate and injection tend to accelerate the fluid velocity whereas magnetic field, velocity slip factor and suction have reverse effect on it;
- magnetic field, velocity slip factor, injection, radiation and viscous dissipation tend to enhance the fluid temperature whereas suction, heat absorption and thermal slip factor have reverse effect on it;
- magnetic field and suction tend to enhance the magnitude of wall velocity gradient whereas dimensionless co-ordinate, injection and velocity slip factor have reverse effect on it; and
- suction and heat absorption tend to enhance magnitude of wall temperature gradient whereas magnetic field, velocity slip factor, injection, thermal radiation, thermal slip factor and viscous dissipation have reverse effect on it.

**Figure 17.**
Wall temperature gradient profiles when $M = 0.4$, $\lambda = 0.3$, $S = 0.1$, $P_{eff} = 0.64$ and $\phi = 0.1$
References


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