Mixed convection in a nanofluid filled-cavity with partial slip subjected to constant heat flux and inclined magnetic field

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Abstract

Mixed convection in a lid-driven square cavity filled with Cu-water nanofluid and subjected to inclined magnetic field is investigated in this paper. Partial slip effect is considered along the lid driven horizontal walls. A constant heat flux source on the left wall is considered, meanwhile the right vertical wall is cooled isothermally. The Prandtl number is 7.07. The remaining cavity walls are thermally insulated. A control volume finite method is used as a numerical appliance of the governing equations. Six pertinent parameters were studied these; the orientation of the magnetic field ($\Phi = 0-360^\circ$), Richardson number ($RI = 0.001-1000$), Hartman number ($Ha = 0-100$), the size and position of the heat source ($B = 0.2-0.8$, $D = 0.3-0.7$, respectively), nanoparticles volume fraction ($\phi = 0.0-0.1$), and the lid-direction of the horizontal walls ($\lambda = \pm 1$) where the positive sign means lid-driven to the right while the negative sign means lid-driven to the left. The results show that the orientation and the strength of the magnetic field can play a significant role in controlling the convection under the effect of partial slip. It is also found that the natural convection decreases with increasing the length of the heat source for all ranges of the studied parameters, while it does so due to the vertical distance up to Hartman number of 50, beyond this value the natural convection decreases with lifting the heat source narrower to the top wall.

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1. Introduction

The developments in electronic technology, industry, environmental applications such as lubrication technologies, food processing and nuclear reactors have become more pronounced in the recent years [1–5]. These continually developments are cumulative accompanied with progress in heat rejection (cooling) methods. Mixed convection has been and will continue to be pivotal in improving the performance of the heat rejection in electronic sources contained in enclosures. Mixed convection flow and heat transfer in enclosures can also encountered in many other industrial applications such as, float glass manufacturing, solidification of ingots, coating or continuous reheating furnaces, and any application under goes to a solid material motion inside a chamber. The mixed convection flow in lid-driven cavity or enclosure is raised from two mechanisms. The first is due to shear flow caused by the movement of one (or two) of the cavity wall(s) while the second is due to the buoyancy flow induced by the non-homogeneous thermal boundaries. The contribution of shear force caused by movement of wall and the buoyancy force by temperature difference make the heat transfer mechanism complex. Cavity flow simulation was introduced in early works of Torrance et al. [6] and Ghia et al. [7]. Recently, lid-driven cavities are found studied in various situations such as pure or nanofluid-filled cavities, and pure or nanofluid saturated porous medium cavities. Selective studies of lid-driven cavities filled with pure fluids can be referred to Koseff [1], Mohamad and Viskanta [8], Mekroussi et al. [9], Sivasankaran et al. [10], or in nanofluid filled cavities as in Tiwari and Das [11], Talebi et al. [12], Abu-Nada and Chamkha [13], Chamkha and Abu-Nada [14]. The effect of uniform magnetic field was found to have considerable effect on the rate of heat transfer [15]. The role of magnetic field on natural convection in nanofluid filled cavities is addressed in Sheikholeslami et al. [16] and Sheikholeslami and Rashidi [17]. Lid-driven pure fluid filled cavities as in Oztop et al. [18], Muhtaminselvan et al. [19]. Lid-driven porous cavity as in Khanafer and Chamkha [20], lid-driven nanofluid saturated porous cavities as in Sun and Pop [21], Chamkha and Ismael [22], and Bourantas et al. [23].

In some applications like fluoroplastic coating (e.g. Teflon) which resists adhesion, the no-slip boundary condition imposed
on the tangential velocity cannot be held. Moreover, some surfaces are rough or porous such that equivalent slip occurs (Wang [24]). Also, there exists a hydrodynamic boundary slip regime for rarefied gases when the Knudsen number is small [Sharipov and Selezniov [25]]. Dealing with such problems is strictly embargo to consider Navier’s slip-boundary condition [26] along these surfaces. Generally, the physical interpretation of the velocity slip on solid boundary arises from the unequal wall and fluid densities, the weak wall-fluid interaction, and the high temperature [27]. However, the studies dealing with slip boundary conditions may be achieved in order to simulate engineering problems Fang et al. [28] and Yoshimura and Prudhomme [29], or to solve the problem of Navier’s slip-boundary condition [26] along these surfaces. Generally, the physical interpretation of the velocity slip on solid boundary arises from the unequal wall and fluid densities, the weak wall-fluid interaction, and the high temperature [27]. However, the studies dealing with slip boundary conditions may be achieved in order to simulate engineering problems Fang et al. [28] and Yoshimura and Prudhomme [29], or to solve the problem of non-physical singularity resulting from meeting stationary and moving walls, Navier [26], Koplik & Banavar [30], Qian and Wang [31], Nie et al. [32], and Ismael et al. [33] have considered the partial slip condition along two horizontal isothermal moving walls under steady laminar mixed convection inside lid-driven square cavity. Their results have showed that there were critical values of the partial slip parameter at which the convection is declined. These non-zero critical slips where found to be sensitive to both Richardson number and the lid direction. Alternatively, Soltani and Yilmazer [34] have reported that the wall slip can occur in the working fluid contains concentrated suspensions.

Convective heat transfer of nanofluids in circular concentric pipes under the influence of partial velocity slips on the surfaces and the resulting anomalous heat transfer enhancement were investigated by Turkylimazoglu [35]. Recently, there are some studies consider slip boundary condition in nanofluid fill cavity, as in Malvandi and Ganji [36], and Mabood and Mastroberardin [37].

The present literature survey has led us to confirm that there is, relatively, a very little published works regarding the slip boundary condition in the lid-driven cavities. Moreover, the topics of nanofluids and magnetic field with partial slip have not clearly arisen yet. Accordingly, the present work is prepared as an attempt to continue in developing the mixed convection aspects. The present geometry is a square cavity filled with Cu-Water nanofluid subjected to an inclined magnetic field. The horizontal walls are thermally insulated (adiabatic) and lid-driven with partial slip, the vertical left wall is also adiabatic but contains a segment of heat source. The right wall is isothermally cooled. It is sought that this work will contribute in finding new parameters arrangements those govern the performance of the lid driven cavities especially those hold very high temperature difference where the partial slip is inevitably exist.

### 2. Mathematical modeling

Consider a steady two-dimensional mixed convection flow inside a square cavity of side length $H$ filled with Cu-water nanofluid, as depicted in Fig. 1. The coordinates $x$ and $y$ are chosen such that $x$ measures the distance along the bottom horizontal wall,
while \( y \) measures the distance along the left vertical wall, respectively. Heat source \( (q'' = \text{constant}) \) is located on a part of the left wall with length \( B' \) at \( x = D' \). The remainder of the left wall is adiabatic, while the right wall is kept isothermally at cold temperature \( T_c \). The upper and bottom walls are adiabatic and moves to the right or left with a speed \( U_0 \). The nanofluids used in the analysis are assumed to be incompressible and laminar, and the base fluid (water) and the solid spherical nanoparticles (Cu) are in thermal equilibrium. The thermo-physical properties of the base fluid and the nanoparticles are given in Table 1 [21]. The thermo-physical properties of the nanofluid are assumed constant except for the density variation, which is calculated based on the Boussinesq approximation. Under the above assumptions, the conservation of mass, linear momentum, and also conservation of energy equations are as follow [38]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. 
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{\sigma_{nf} B^2_{nf}}{\rho_{nf}} (\nu \sin \phi \cos \phi - u \sin^2 \phi), 
\]

\[
\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\sigma_{nf} B^2_{nf}}{\rho_{nf}} (u \sin \phi \cos \phi - v \cos^2 \phi) + \frac{(\nu_{nf} \rho_{nf} g (T - T_c))}{\rho_{nf}^2}.
\]

where \( u \) and \( v \) are the velocity components along the \( x \) and \( y \) axes, \( T \) is the fluid temperature, \( p \) is the fluid pressure, \( g \) is the gravity acceleration, \( \rho_{nf} \) is the density, \( \nu_{nf} \) is the kinematic viscosity.

The boundary conditions are:

On the left wall, \( x = 0 \),

\[
u = 0, \quad \frac{\partial T}{\partial x} = -\frac{q''}{k_{nf}}, \quad (D^* - 0.5B^*) \leq y \leq (D^* + 0.5B^*),
\]

\[
\frac{\partial T}{\partial x} = 0 \text{ otherwise}
\]

On the right wall, \( x = H \),

\[
u = 0, \quad T = T_c, \quad 0 \leq y \leq H
\]

On the top wall, \( y = H \) (partial slip)

\[
u = 0, \quad u = \lambda_d \dot{U}_c + \lambda_{nf} \frac{\partial u}{\partial y} \frac{\partial T}{\partial y} = 0
\]

### Table 1

Thermo-physical properties of water and nanoparticles [22].

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Copper (Cu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho ) (kg m(^{-3}))</td>
<td>997.1</td>
<td>8933</td>
</tr>
<tr>
<td>( C_p ) (J kg(^{-1}) K(^{-1}))</td>
<td>4179</td>
<td>385</td>
</tr>
<tr>
<td>( k ) (W m(^{-1}) K(^{-1}))</td>
<td>0.613</td>
<td>401</td>
</tr>
<tr>
<td>( \beta ) (K(^{-1}))</td>
<td>( 21 \times 10^{-5} )</td>
<td>( 1.67 \times 10^{-5} )</td>
</tr>
<tr>
<td>( \sigma ) (S m(^{-1}))</td>
<td>0.05</td>
<td>5.98 \times 10^{-7}</td>
</tr>
</tbody>
</table>

### Table 2

Comparisons of the mean Nusselt number at the top wall, for different values of Re at \( Pr = 0.71, Gr = 10^2 \).

<table>
<thead>
<tr>
<th></th>
<th></th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>1.94</td>
<td>2.01</td>
<td>1.93</td>
</tr>
<tr>
<td>400</td>
<td>3.84</td>
<td>3.91</td>
<td>3.91</td>
</tr>
<tr>
<td>1000</td>
<td>6.33</td>
<td>6.33</td>
<td>6.31</td>
</tr>
</tbody>
</table>

Fig. 2. (a) Streamlines and (b) Isotherms for \( Ri = 1, Ha = 10, \phi = 0.05, D = 0.5, B = 0.5, \lambda_d = 1, \lambda_{nf} = 1. \)
Numerous formulations for the thermo-physical properties of nano fluids are proposed in the literature. In the present study, we are adopting the relations which depend on the nanoparticles volume fraction only which were proven and used in many previous studies (Aminossadati and Ghasemi [39], Khanafer et al. [40], and Tukyilmazoglu [41]). This assumption is even coinciding with Buongiorno’s nano fluid model (multi-phase) when the concentration of nanoparticles is assumed constant [42] as follows:

The effective density of the nano fluid is given as:

\[
\rho_{nf} = \rho_f (1 - \phi) + \phi \rho_p
\]

(6)

where \(\phi\) is the solid volume fraction of the nano fluid, \(\rho_f\) and \(\rho_p\) are the densities of the fluid and of the solid particles respectively, and the heat capacitate of the nanofluid is given by Khanafer et al. [40] as,

\[
C_{p,nf} = C_{p,f} (1 - \phi) + \phi C_{p,p}
\]

(7)

The thermal expansion coefficient of the nanofluid can be determined by:

\[
\beta_{nf} = \beta_f (1 - \phi) + \phi \beta_p
\]

(8)

where \(\beta_f\) and \(\beta_p\) are the coefficients of thermal expansion of the fluid and of the solid particles respectively.

Thermal diffusivity, \(\alpha_{nf}\) of the nanofluid is defined by Abu-Nada and Chamkha [43] as:

\[
\alpha_{nf} = \frac{k_{nf}}{(\rho_{nf} C_{p,nf})}
\]

(9)

In Eq. (9), \(k_{nf}\) is the thermal conductivity of the nanofluid which for spherical nanoparticles, according to the Maxwell-Garnett model [44], is:

\[
k_{nf} = \frac{k_f (k_f + 2k_p) - 2\phi (k_f - k_p)}{k_f + 2k_p + \phi (k_f - k_p)}
\]

(10)

The effective dynamic viscosity of the nanofluid based on the Brinkman model [45] is given by

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{7/3}}
\]

(11)

where \(\mu_f\) is the viscosity of the base fluid and the effective electrical conductivity of nanofluid was presented by Maxwell [44] as:

\[
\sigma_{nf} = \frac{1}{\sigma_f} + \frac{3(\gamma - 1)\phi}{(\gamma + 2) - (\gamma - 1)\phi}
\]

(12)

where
\[ \gamma = \frac{\sigma_p}{\sigma_f} \]

Introducing the following dimensionless set:

\[
(X, Y, D, B) = \left(\frac{x, y, D^2, B^2}{H}\right),
(U, V) = \left(\frac{u}{U_0}, \frac{v}{V_0}\right),
\rho = \frac{p}{\rho_{nf} U_0^2},
\theta = \frac{(T-T_0)}{qH} k_{nf},
\text{Re} = \frac{Gr}{S_{nf}},
\text{S}_{nf} = \frac{N_{nf}}{H},
\]

into Eqs. (1)-(4) yields the following dimensionless equations:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,
\]

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left( \frac{\nu_{nf}}{\nu_f} \right) \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + \frac{1}{\text{Re}} \left( \frac{\nu_{nf}}{\nu_f} \right) \frac{H \alpha^2}{\text{Re}} (V \sin \phi \cos \phi - U \cos^2 \phi),
\]

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left( \frac{\nu_{nf}}{\nu_f} \right) \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{1}{\text{Re}} \left( \frac{\nu_{nf}}{\nu_f} \right) \frac{H \alpha^2}{\text{Re}} (U \sin \phi \cos \phi - V \cos^2 \phi).
\]
$U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \left( \frac{1}{Pr \, Re} \right) \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}$

where

$$Pr = \frac{\nu}{\alpha_f}, \quad Re = \frac{U_0 H}{\nu_f}, \quad Gr = \frac{g \beta H^2 \Delta T}{\nu_f^2}, \quad Ha = B_0 H \sqrt{|\mu_f|}$$

are the Prandtl number, the Reynolds number, the Grashof number and the Hartman number, respectively.

The dimensionless boundary conditions for Eqs. (15)–(18) are as follows:

On the top wall, $X=1$

$$U = V = 0, \quad \theta = 0, \quad 0 \leq Y \leq 1$$

On the bottom wall, $Y=0$ (partial slip)

$$V=0, \quad U = \lambda_t + \frac{\mu_s}{\mu_f} \frac{\partial U}{\partial Y} \frac{\partial \theta}{\partial Y} = 0$$

where $\lambda_t = \frac{S_m}{N}$

The local Nusselt number is defined as:

$$Nu = \frac{1}{\partial \theta_{\text{long}} \, y - \theta}$$

and the average Nusselt number over the heat source is defined as:

$$Nu_{av} = \frac{1}{|B|} \int_{D-0.5B}^{D+0.5B} Nu_b \, dY$$

It is useful to recall the definition of the stream function $\psi$ to describe the fluid motion. It can be expressed as follow:

$$u = \frac{\partial \psi}{\partial X}, \quad v = - \frac{\partial \psi}{\partial Y}$$

3. Numerical method and validation

Eqs. (15)–(18) with the boundary conditions (19) have been solved numerically using the collocated finite volume method. The first upwind and central difference approaches have been used to approximate the convective and diffusive terms, respectively. The resulting discretized equations have been solved iteratively, through alternate direction implicit ADI, using the SIMPLE

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Fig. 8. Variation of the mean Nusselt number for $\Phi=0$, $\phi=0.05$, $D=0.5$, $B=0.5$, $\lambda_t=1$, $\lambda_b=-1$.

Fig. 9. (a) Streamlines and (b) Isotherms for $\Phi=0$, $Ri=1$, $\phi=0.05$, $D=0.5$, $B=0.5$, $\lambda_t=-\lambda_b=1$. 
algorithm [46]. The velocity correction has been made using the Rhie and Chow interpolation. For convergence, under-relaxation technique has been employed. To check the convergence, the mass residue of each control volume has been calculated and the maximum value has been used to check the convergence. The convergence criterion was set as $10^{-5}$. A uniform grid resolution of $81 \times 81$ is found to be suitable. In order to verify the accuracy of the present method, the obtained results in special cases are compared with the results obtained by Iwatsu et al. [4] and Khanafer and Chamkha [20] in terms of the mean Nusselt number at the top wall, for different values of $Re$. As we can see from Table 2, the results are found in a good agreement with these results. These favorable comparisons lend confidence in the numerical results to be reported subsequently.

4. Results and discussion

Streamlines, isotherms, local velocity components, and local and average Nusselt numbers are adopted in inspecting the results of mixed convection aspects of the present problem. The studied parameters which pertinently affect the flow and thermal fields inside the considered cavity are; the orientation of the magnetic field $\Phi=0^\circ$-$360^\circ$, the constant moving parameter $\lambda=\pm 1$, the size and position of the constant heat source $B=0.2$-$0.8$, $D=0.3$-$0.7$, respectively, Hartman number $Ha=0$-$100$, Richardson number $Ri$ = $0.001$-$1000$, and nanoparticles volume fraction $\phi=0.0$-$0.1$. Prandtl number and slip parameter are fixed at $Pr=6.26$, and $S_b = S_t = 1$, respectively. For the purpose of viewing the results easily, five parameters are fixed (unless where stated) while the remainder single one is altered and as illustrated in the following categories.

4.1. Effect of magnetic field orientation ($\Phi$)

To investigate the effect of the orientation angle ($\Phi$) of the applied magnetic field, the other parameters are fixed at $Ri=1$, $\phi=0.05$, $Ha=10$, $D=B=1$, $\lambda_1=1$, $\lambda_2=-1$. Fig. 2 shows the streamlines and isotherms for various values of $\Phi$. For $\Phi=45^\circ$, the streamlines are formed in a main clockwise (CW) circulation with a secondary counterclockwise (CCW) vortex formed close to the upper wall which is being lid to the right. When $\Phi$ is increased to $90^\circ$, the expected magnetic force will act horizontally, hence, this will skew the buoyancy effect, as a result, reduces the streamlines strength as can be seen in Fig. 2a. When $\Phi$ is set at $180^\circ$ the magnetic force will act downward, i.e. in opposite with the buoyancy force, the case which is identical with that of $\Phi=0$. When $\Phi$ is further increased to $270^\circ$, the magnetic force will again act horizontally, hence the main circulation is mostly similar to that of $\Phi=90^\circ$, except the absence of the secondary vortex. Fig. 2b presents the isotherm contours which manifest undisturbed patterns with varying $\Phi$, this due to the equivalence convections modes ($Ri=1$) and low Hartman number as well.

The horizontal $U$ and vertical $V$ velocity components are plotted...
in Fig. 3 along $Y=0.5$ and $X=0.5$, respectively. The $U$ component reveals negative values close to both horizontal walls (Fig. 3a). This is an expected result for the bottom wall, but due to the top one, this means that the buoyancy effect overcomes the shear effect exerted by the top wall ($\lambda_t=1$ and $\lambda_b=-1$). Increasing $\Phi$ reduces the negative speed close to the moving walls. Two peaks of vertical components $V$ (Fig. 3b) are recorded close to the vertical walls with higher velocity values associated with increasing $\Phi$.

The mean Nusselt number is plotted with the nanoparticles volume fraction for different values of $\Phi$. As depicted in Fig. 4, it is clear that for pure fluid ($\phi=0$), a maximum convection suppression can be obtained when $\Phi=45^\circ$, while for nanofluid ($\phi>0$), convection suppression vanishes with increasing $\Phi$ value. However, the effect nanoparticles volume fraction will be discussed fairly in Subsection 4.5.

It is worth mentioning here that the lid-directions of the horizontal walls was inspected for different combinations of $\lambda_t$ and $\lambda_b$ with the orientation of the magnetic field $\Phi$. The set of $\lambda_t=1$ (top wall moves to right) and $\lambda_b=-1$ (bottom wall moves to left) manifests more stable performance with $\Phi$ and gives higher Nusselt number when $\Phi\leq 25^\circ$ (see Fig. 5). All others combinations of $\lambda_t$ and $\lambda_b$ manifests sinusoidally variation of $N_{um}$ with $\Phi$. However, the stable set $\lambda_t=-\lambda_b=1$ will be adopted in the next results.

### 4.2. Effect of Richardson number ($Ri$)

The ratio of natural to forced convection modes is measured by Richardson number. Its effect is studied by fixing the other independent parameters at $Ha=10$, $\phi=0.05$, $B=0.5$, $D=0.5$, $\lambda_t=1$, $\lambda_b=-1$. For low $Ri$ range (0.001–0.01) the dominance forced convection can be characterized by the dominance shear action where two counter rotated vortices each one is guided by a moving wall (Fig. 6a). The corresponding isotherms (Fig. 6b) tend to be plumed from heat source towards the cold vertical wall with isothermal
zones localized close the moving walls and dense isotherms close 
to the heat source. For the dominance natural convection 
\( Ra = 100 \) to \( 1000 \), the streamlines are strengthened and governed 
by a single main CW circulation while the mostly horizontal iso-
therms reveal the dominance convection mode.

The local velocity components \( U \) and \( V \) shown in Fig. 7 de-
monstrate the mostly nano
fluid stagnant at low \( Ra \) number.
Whereas for high \( Ra \) number these velocity components are sig-
nificantly increased, nevertheless, the cavity center (core center) 
still appearing as a stagnant point. However, \( U \) does not reach the 
cavity walls speed \( (U_0) \) because the slip effect.

The effect of Richardson number on the mean Nusselt number 
\( Nu_m \) is shown in Fig. 8 with Hartman number. It is clear that the 
\( Nu_m \) is significantly decrease with increasing \( Ri \) up to unity where 
below this \( Ri \) value there is no noticeable variation of \( Nu_m \) with 
\( Ri \). This trend reveals that the convection is mainly due to the 
forced convection mode which comes from the shear effect resulting 
from the horizontal walls movement. The variation of \( Nu_m \) 
with Hartman number will be postponed to a next category.

4.3. Effect of Hartman number

The effect of the applied magnetic field is studied by varying 
the Hartman number \( Ha \) from 0 to 100 in the case of horizontal 
magnetic field \( (\Phi = 0) \) while the other parameters are fixed at 
\( Ri = 1, \phi = 0.05, B = 0.5, \lambda_t = -\lambda_b = 1 \). It is well known that 
applying an external magnetic field, Lorentz force will be gener-
ated perpendicularly to the direction of the applied magnetic 
field i.e. opposite to the buoyancy force (in this case). Therefore, Lorentz 
force will act normally in the negative \( Y \)-direction. Accordingly, the 
streamlines presented in Fig. 9a weaken and the main vortex 
breaks up to double-eye pattern at \( Ha = 50 \) while secondary vortex
is squeezed to be limited close to the moving top wall. The corresponding isotherms (Fig. 9b) are transmitted from convection pattern at \( \text{Ha} = 0 \) to mostly vertical pattern with increasing \( \text{Ha} \) which indicates to the suppressed convection due to the magnetic force effect. The \( U \) component is flattened with \( \text{Ha} \) while the peaks of \( V \) component are significantly vanished as shown in Fig. 10. The magnetic field suppression effect can be clearly observed by the reduction of local and mean Nusselt numbers with increasing \( \text{Ha} \) as shown in Fig. 11.

4.4. Effect of position \( (D) \) and dimension \( (B) \) of heating element

The effects of the size \( (B) \) and the position \( (D) \) of the heat source will be discussed in this category for \( Ri = 1, \Phi = 0, \phi = 0.05, Ha = 10, \lambda_t = -\lambda_b = 1 \). Fig. 12 shows the effect of \( B \) on the streamlines and isotherms while \( D \) is fixed at 0.5. When \( B \) is increased, the mostly equivalent counter rotating vortices pattern associated with small heat source, \( B = 0.2 \), is changed to a main CW circulation occupying most of the cavity with small secondary vortex close to the top wall. This fashion is almost seen with isothermally heated wall cavity. Respecting with isotherms (Fig. 12b), the plume-like and the dense isotherms fashion seen adjacent to the heat source is not seen when \( B \) is increased more than 0.4. However, the effect of \( B \) is more evidenced by the local and mean Nusselt numbers as shown in Fig. 13. This figure shows that the maximum local Nusselt number occurs at the lower edge of the heat source (Fig. 13a), and generally it is greater for smaller heat source size. This can be understood by re-examining the isotherms of Fig. 12b, where their pattern manifests that when increasing the heat source size, a large portion of heat is transferred by conduction, hence, a decrease of convection heat transfer occurs accordingly as shown in Fig. 13b.

The effect of heat source position \( (D) \) on the streamlines and isotherms is depicted in Fig. 14 for fixed source size \( (B = 0.5) \). It can
be seen that the double-eye circulation is significantly affected by increasing $D$, where the CW vortex grows up while the CCW one is completely diminished leaving a stagnant zone as shown in Fig. 14a. The isotherms behaviors with increasing $D$ are similar to those behaviors associated with $B$ increments, where the plume-like and the dense isotherms fashion disappear when $D$ is increased more than 0.4. The distribution of the local Nusselt number for $\Phi = 0$, $R_i = 1$, $H_a = 10$, $\phi = 0.05$, $B = 0.5$, $\lambda_1 = 1$, $\lambda_0 = -1$ is shown in Fig. 15a which presents similar patterns for all $D$ values, but greater under-curve area with lower heat source position. This means that the mean Nusselt number is greater for lower $D$ position which can be demonstrated by Fig. 15b. This can be attributed to that when the heat source becomes narrower to the top wall, there will no enough vertical distance for the buoyancy effect to be taken place. Hence, a substantial amount of nanofluid will not corporate in convective heat transfer. However, when the Hartman number is increased more than 40, this effect will diminishes resulting in increasing $Nu_m$ with higher $D$ values.

### 4.5. Effect of nanoparticles volume fraction

The nanoparticles volume fraction is increased from $\phi = 0$ (pure water) to $\phi = 0.1$ for $R_i = 1$, $H_a = 10$, $B = 0.5$, $D = 0.5$, $\lambda_1 = -\lambda_0 = 1$, and $\Phi = 0$. The streamlines (Fig. 16a) keep its general pattern with increasing $\phi$, but their strength and intensity are appreciably decreased. The isotherms (Fig. 16b) desolate its plume like and thermal boundary layers close the heat source and the cold wall with increasing $\phi$ as an indication to the suppression of the convective heat transfer. To understand the physical aspects of adding the Cu nanoparticles to the pure water we would worthily clarify that adding very high conductive solid nanoparticles will generate a nanofluid with higher viscosity, higher density, and higher thermal conductivity. The first two properties increase the viscous and inertia forces, respectively, while the enhanced thermal conductivity increases the transferred thermal energy. Hence, the distribution of the local Nusselt number presented in Fig. 17 elucidates that the enhanced viscous and inertia forces dominant over the enhanced thermal conductivity and both buoyancy and shear effects in addition. This is a reasonable reason for the deterioration of Nusselt number with addition of nanoparticles. Fig. 4 shows the deterioration of $Nu_m$ with $\phi$. Nevertheless, other examinations of Nusselt number were conducted and the results are presented in Figs. 18–20. Fig. 18 also tells the deterioration scenario of $Nu_m$ with $\phi$ for different values of heat source position. However, inspecting $\phi$ for different values of $Ri$ (Fig. 19) gave us an encouragement issue, where a slight increase of Nusselt number up to $\phi = 0.04$ is seen when $R_i \leq 0.01$ which means the dominance of the thermal energy enhancement over the viscous and inertia increase due to the very weak buoyancy force. This gave us an ambitious to inspect another parameters. Already, Fig. 20 presents the effect of $\phi$ for different $B$ values. An enhancement in $Nu_m$ is seen at $\phi = 0.04$ for lowest heat size, $B = 0.2$. Eventually, reexamin Fig. 11b, it can be elucidated that when $Ha \geq 50$, the suppressed natural convection can serve in a continuous increase in $Nu_m$ with increasing $\phi$.

### 5. Conclusions

The studied problem is a steady laminar two dimensional mixed convection inside a square cavity filled with Cu-water nanofluid and subjected to an externally-applied inclined magnetic field. A constant heat flux is set on a portion of the left wall while the right wall is kept isothermally cold. The remainder boundaries of the cavity are adiabatic. Horizontal walls are lid-driven by considering the partial slip effect. The following conclusions are drawn from the present results:

1. The shortest length of the constant heat flux element gives the maximum convective heat transfer.
2. For $Ha < 50$, the lowest heat source position gives maximum Nusselt number, while for $Ha > 50$, Nusselt number increases with increasing the position of the heat source.
3. The orientation of the magnetic field can be considered as a key control of the convective heat transfer where the suppression exerted by the magnetic field on the Nusselt number decreases with increasing the orientation of the applied magnetic field.
4. The most stable lid-driven direction can be attained when the top wall moves to the right while the bottom one moves to the left.
5. The natural convection is a decreasing function of the heat source length for all ranges of the studied parameters, while it is so due to the vertical distance up to Hartman number of 50 where beyond this value the Natural convection decreases with lifting the heat source narrower to the top wall.
6. The effect of the nanoparticles volume fraction on the Nusselt number manifests wide variety fashions, where little enhancements of Nusselt number can be obtained with very low values of Richardson number (dominance forced convection) at volume fraction not more than 0.04. For equivalent convection modes ($R_i = 1$), Nusselt number can be enhanced slightly with shortest heat source length and noticeably with stronger magnetic field ($Ha > 50$).