Electrohydrodynamic free convection heat transfer of a nanofluid in a semi-annulus enclosure with a sinusoidal wall

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ABSTRACT
Natural convection heat transfer of a nanofluid in the presence of an electric field is investigated. The control volume finite element method (CVFEM) is utilized to simulate this problem. A Fe$_3$O$_4$–ethylene glycol nanofluid is used as the working fluid. The effect of the electric field on nanofluid viscosity is taken into account. Numerical investigation is conducted for several values of Rayleigh number, nanoparticle volume fraction, and the voltage supplied. The numerical results show that the voltage used can change the flow shape. The Coulomb force causes the isotherms to become denser near the bottom wall. Heat transfer rises with increase in the voltage supplied and Rayleigh number. The effect of electric field on heat transfer is more pronounced at low Rayleigh numbers due to the predomination of the conduction mechanism.

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1. Introduction
In order to improve the thermophysical characteristics of the base fluid, nano-scale metal particles can be added to it. In this way, the thermal conductivity of the nanofluid rises and in turn, heat transfer can be enhanced. Khanafer et al. [1] used the finite-volume method (FVM) to simulate nanofluid hydrothermal improvement. They indicated that the addition of nanoparticles resulted in a rise in the Nusselt number. Hydromagnetic free convection flow over an inclined plate produced by solar radiation has been studied by Chamkha [2]. Sheikholeslami Kandelousi [3] studied nanofluid hydrothermal behavior in a porous channel. He showed that the Nusselt number rises with increase in Reynolds number when the power law index equals zero. Chamkha [4] studied unsteady hydromagnetic free convection in a vertical fluid-saturated porous medium channel. Sheikholeslami and Abelman [5] utilized a two-phase model for the analysis of nanofluid flow and heat transfer in an annulus in the presence of an axial magnetic field. Nanofluid hydrothermal treatment between two horizontal parallel plates in a rotating system was studied by Sheikholeslami et al. [6]. They proved that Nusselt number increases with a rise in Reynolds number and nanoparticle volume fraction but decreases with an increase in the magnetic field, rotation parameter, and Eckert number.

Among the key active methods for improving the rate of heat transfer is utilization of an electric field. The influence of an electric field on buoyancy-induced flow was studied by Shu and Lai [7]. In their study, the electrodynamics equations are separated from the fluid dynamics equations. The influence of electrode arrangements on the rate of heat transfer was studied by Kasayapanand et al. [8]. The effect of an electric field on fluid flow over a plate was investigated by Velkoff and Godfrey [9]. Rarani et al. [10] studied the effect of an electric field on the viscosity of Fe$_3$O$_4$–ethylene glycol nanofluids. They proved that increased viscosity is more pronounced for higher nanofluid...
volume fraction. Nanofluid flow and natural convection heat transfer have been investigated by several authors [11–42].

The control volume finite element method (CVFEM) is a new numerical method which benefits from a combination of the finite element and finite volume methods. This method can be used for simulation of complex geometries with multi-physics ([43] and [44]). Sheikholeslami et al. [45] utilized an active method (magnetic field) to determine the impact of Hartmann number on natural convection of a nanofluid. They proved that the effect of Lorentz forces is greater for high Rayleigh number. Sheikholeslami and Rashidi [46] investigated the impact of space-dependent magnetic field on the hydrothermal treatment of an Fe3O4–water nanofluid. Their results verified that Lorentz forces resulted in lower Nusselt number due to flow retardation. Improved formulations of the discretized pressure equation and boundary treatments in control-volume finite-element methods were presented by Lamoureux and Baliga [47]. Sheikholeslami et al. [48] presented the impact of applying a constant magnetic field on nanofluid hydrothermal behavior in a cavity heated from below. They proved that the influence of Hartmann number and heat source on Nusselt number increases as Rayleigh number rises. This and other new numerical methods have been used in recent decades to simulate the multiphasic problem [49–59].

The objective of the current article is to investigate electrohydrodynamic nanofluid flow and forced convective heat transfer in a semi-annulus enclosure using CVFEM. The influence of the voltage supplied and Reynolds number on flow characteristics and heat transfer were examined.

2. Problem definition

Figure 1 illustrates the physical geometry along with the key parameters and mesh of the semi-annulus enclosure. The lower wall has a constant temperature $T_1$ and the temperature of the other walls is $T_0$. Also, the remaining boundary conditions are depicted in Figure 1a. The shape of the inner cylindrical profile is assumed to mimic the following pattern:

$$r = r_{in} + A \cos(N(\zeta))$$  \(1\)

where $r_{in}$ is the base circle radius, $r_{out}$ is the radius of the outer cylinder, and $A$ and $N$ are the amplitude and the number of undulations, respectively. $\zeta$ is the rotation angle. In this study, $A$ and $N$ are 0.025 and 48, respectively.

<table>
<thead>
<tr>
<th>Nomenclature</th>
</tr>
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<tbody>
<tr>
<td>$b$ ionic mobility</td>
</tr>
<tr>
<td>$D_{ce}, D$ diffusion number (= \mu_0/(\rho_0 D_0)), charge diffusion coefficient</td>
</tr>
<tr>
<td>$E_c$ Eckert number (= \rho (a/L)^2/((\rho C_p)(T_1 - T_0)))</td>
</tr>
<tr>
<td>$E, E_x, E_y$ electric field</td>
</tr>
<tr>
<td>$F_E$ Coulomb force</td>
</tr>
<tr>
<td>$f$ electric current density</td>
</tr>
<tr>
<td>$N_E$ electric field number (= q_0 L^2/(\epsilon \Delta \phi))</td>
</tr>
<tr>
<td>$p$ pressure</td>
</tr>
<tr>
<td>$Pr$ Prandtl number (= \nu/\alpha)</td>
</tr>
<tr>
<td>$Pr_E$ electric Prandtl number (= \mu/(\rho_0 \Delta \phi))</td>
</tr>
<tr>
<td>$q$ electric charge density</td>
</tr>
<tr>
<td>$Pr$ Prandtl number (= \mu(p_c p)/\rho_k \Delta \phi)</td>
</tr>
<tr>
<td>$Ra$ Rayleigh number (= g\beta T L^3/(\alpha \nu))</td>
</tr>
<tr>
<td>$S_E$ Lorentz force number (= q_0 \Delta \phi/(\rho (a/L)^2))</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Subscripts</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ave$ average</td>
</tr>
<tr>
<td>$c$ cold</td>
</tr>
<tr>
<td>$s$ solid particles</td>
</tr>
<tr>
<td>$f$ base fluid</td>
</tr>
<tr>
<td>$h$ hot</td>
</tr>
<tr>
<td>$nf$ nanofluid</td>
</tr>
</tbody>
</table>
3. Governing equations

3.1. Mathematical model

In order to simulate nanofluid hydrothermal treatment in the presence of an electric field, we combine equations of electric fields with those of hydrothermal treatment. The formulas of the electric field are:

\[ \nabla \cdot (E \varepsilon) = q \]  
\[ (-\nabla \varphi) = E \]  
\[ \nabla \cdot J + \frac{\partial q}{\partial t} = 0 \]

Two models are available for charge distribution: (1) a conductivity model [60–61] and (2) a mobility model [62]. In the first, the electro-convection relies on the temperature gradient; but in the second, electro-convection is independent of the temperature gradient in the liquid. In the case of free charge origination, the second model is more acceptable according to experimental results. The electric current density can be defined as [63]:

\[ \vec{J} = \sigma \vec{E} - D \nabla q + q \vec{V} \]
where \( \sigma \vec{E} \) represents the ionic mobility, \( D \nabla q \) represents the diffusion \([64]\), and \( q \vec{V} \) represents the convection.

According to Eqs. (4) and (5), the equation for electric charge density can be obtained as follows:

\[
\frac{\partial q}{\partial t} + \vec{V} \cdot \nabla q + \rho \frac{\partial \vec{E}}{\partial t} + \text{Pr} \left[ q \left( \frac{\partial \vec{E}_y}{\partial y} + \frac{\partial \vec{E}_x}{\partial x} \right) + E_y \frac{\partial q}{\partial y} + E_x \frac{\partial q}{\partial x} \right] = \text{Pr} \left( \frac{\partial^2 q}{\partial y^2} + \frac{\partial^2 q}{\partial x^2} \right)
\]  

(6)

According to \([63]\) the diffusion term can be taken as negligible. Also, \( D \nabla q \) in Eq. (5) can be taken as negligible and \( \sigma = b q \) \([64]\). Therefore, Eq. (5) can be considered as:

\[
\vec{J} = q \vec{V} + q b \vec{E}
\]  

(7)

In the presence of an electric field, Coulomb forces should be added to the momentum equation and the Joule heating effect should be added to the energy equation. So, we have:

\[
\begin{aligned}
\nabla \cdot \vec{V} &= 0 \\
P_{nf} \left( \frac{\partial \vec{V}}{\partial t} + \left( \vec{V} \cdot \nabla \right) \vec{V} \right) &= -\nabla p + \mu_{nf} \nabla^2 \vec{V} + q \vec{E} - \beta \vec{E} \cdot (T - T_0) \\
\left( \rho C_p \right)_{nf} \left( \frac{\partial T}{\partial t} + \left( \vec{V} \cdot \nabla \right) T \right) &= k_{nf} \nabla^2 T + \vec{J} \cdot \vec{E} \\
\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} &= 0 \\
\nabla \cdot \vec{E} &= q \\
\vec{E} &= -\nabla \phi
\end{aligned}
\]  

(8)

\( \rho_{nf} \), \( \left( \rho C_p \right)_{nf} \), \( \alpha_{nf} \), \( \beta_{nf} \), \( \mu_{nf} \), and \( k_{nf} \) are defined as \([1]\):

\[
\rho_{nf} = \rho_s \Phi + \rho_f (1 - \Phi)
\]  

(9)

\[
\left( \rho C_p \right)_{nf} = \left( \rho C_p \right)_s \Phi + \left( \rho C_p \right)_f (1 - \Phi)
\]  

(10)

\[
\alpha_{nf} = k_{nf} / \left( \rho C_p \right)_{nf}
\]  

(11)

\[
\beta_{nf} = \beta_s \Phi + \beta_f (1 - \Phi)
\]  

(12)

\[
k_{nf} = k_f - 2 \Phi (k_f - k_s) + 2 k_f + k
\]  

(13)

The thermophysical properties of the working fluid are given in Table 1 \([10]\). The electric field-dependent viscosity of \( \text{Fe}_3\text{O}_4 \)-ethylene glycol nanofluid can be obtained as follows \([10]\):

\[
\mu = A_1 + A_2 (\Delta \Phi) + A_3 (\Delta \Phi)^2 + A_4 (\Delta \Phi)^3
\]  

(14)

| Table 1. Thermophysical properties of water and nanoparticles \([10]\). |
|-----------------|-----------------|-----------------|-----------------|
|                  | \( \rho \) \( (\text{kg/m}^3) \) | \( C_p \) \( (\text{j/kg} \cdot \text{k}) \) | \( \beta \times 10^{-3} \) \( (1/\text{k}) \) | \( k \) \( (\text{W/m.k}) \) |
| Ethylene glycol  | 1,110           | 2,400           | 65              | 0.26            |
| \( \text{Fe}_3\text{O}_4 \) | 5,200           | 670             | 1.3             | 6               |
Table 2 shows the coefficient values of Eq. (14). Non-dimensional parameters are introduced as follows:

\[ \begin{align*}
\bar{t} &= \frac{t}{L^2}, \quad \bar{p} = \frac{P}{\rho(\alpha_f/L)^2}, \quad \bar{y} = \frac{y}{L}, \quad \bar{x} = \frac{x}{L}, \quad \bar{v} = \frac{v}{\alpha_f}, \quad \bar{u} = \frac{u}{\alpha_f}, \\
\theta &= \frac{T - T_0}{\nabla T}, \quad \nabla T = T_1 - T_0, \quad \bar{\varphi} = \frac{\varphi - \varphi_0}{\nabla \varphi}, \quad \nabla \varphi = \varphi_1 - \varphi_0, \quad \bar{q} = \frac{q}{q_0}, \quad \bar{E} = \frac{E}{E_0}
\end{align*} \]  

where \( \nabla T \) and \( \nabla \varphi \) are \( (T_1 - T_0) \) and \( (\varphi_1 - \varphi_0) \), respectively. In order to reach a clear formulation, the over bar will be deleted in the next equations. So, the governing equations can be considered as follows:

\[
\begin{align*}
\nabla \cdot \left( \bar{V} \right) &= 0 \\
\left( \frac{\partial \bar{V}}{\partial t} + \left( \bar{V} \cdot \nabla \right) \bar{V} \right) &= -\nabla \bar{p} + \frac{\mu_\text{eff}}{\rho_\text{eff}} \frac{\partial \bar{p}}{\partial \bar{y}} \nabla^2 \bar{V} + \frac{S}{\rho_\text{eff} \beta_\text{eff}} q \bar{E} - \text{Ra Pr} \left( \frac{\beta_{\text{eff}}}{\beta_\text{eff}} \right) \theta \\
\nabla \cdot \bar{J} + \frac{\partial \bar{c}}{\partial t} &= 0 \\
\nabla \cdot \bar{E} &\equiv 0 \\
\bar{E} &= -\nabla \bar{\varphi}
\end{align*}
\]  

Table 3. Comparison of average Nusselt number, \( \text{Nu}_{\text{ave}} \) along the left wall for different grid resolutions at \( \text{Ra} = 500, \phi = 0.05, \Delta \varphi = 6kV, \) and \( \text{Pr} = 149.54. \)

<table>
<thead>
<tr>
<th>Grid</th>
<th>( \text{Nu}_{\text{ave}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>41 x 121</td>
<td>16.352878</td>
</tr>
<tr>
<td>51 x 151</td>
<td>16.351162</td>
</tr>
<tr>
<td>61 x 181</td>
<td>16.36156</td>
</tr>
<tr>
<td>71 x 211</td>
<td>16.361219</td>
</tr>
<tr>
<td>81 x 241</td>
<td>16.363907</td>
</tr>
<tr>
<td>91 x 271</td>
<td>16.368045</td>
</tr>
</tbody>
</table>

Figure 2. Comparison of average Nusselt number between the present results and numerical results by Khanafer et al. [1] \( \text{Gr} = 10^4, \phi = 0.1 \) and \( \text{Pr} = 6.8(\text{Cu - Water}). \)
The formulas of the vorticity and the stream function are:

\[
\begin{align*}
\omega &= \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}, \\
v &= -\frac{\partial \psi}{\partial x}, \quad u = \frac{\partial \psi}{\partial y}, \\
\Omega &= \frac{\alpha v^2}{\alpha}, \quad \Psi = \frac{\psi}{\alpha}
\end{align*}
\]

(17)

It should be mentioned that the continuity equation has been satisfied by the stream function. By eliminating pressure between the \(x\)- and \(y\)-momentum equations, the vorticity equation can be obtained.

Figure 3. Electric density distribution injected by the bottom electrode.
The local Nusselt number $\text{Nu}_{\text{loc}}$ and average Nusselt number $\text{Nu}_{\text{ave}}$ along the hot wall can be obtained as:

$$\text{Nu}_{\text{loc}} = \left( \frac{k_f}{k_f} \right) \frac{\partial \Theta}{\partial Y}$$

$$\text{Nu}_{\text{ave}} = \frac{1}{L} \int_{r_{\text{in}}}^{r_{\text{out}}} \text{Nu}_{\text{loc}} \, dX$$

### 3.2. Numerical method

In order to simulate this problem, the CVFEM is applied. We used a triangular element with linear interpolation through the elements. The shape of the control volume is depicted in Figures 1b and c. We developed a FORTRAN code to solve the governing equations. More information about this method can be found in [11].

### 4. Code verification and mesh independence

In order to obtain mesh independent results, various meshes were examined for the cases of $\text{Ra} = 500$, $\phi = 0.05$, $\Delta \phi = 6kV$, and $\text{Pr} = 149.54$ as illustrated in Table 3. According to this table, a mesh size of $71 \times 211$ guarantees a grid-independent solution. Also, the convergence limitation is:

$$\max_{\text{grid}} |\Gamma^{n+1} - \Gamma^n| \leq 10^{-7}$$

![Figure 4. Effect of the supplied voltage on streamlines and isotherm when $\text{Ra} = 50$, $\phi = 0.05$.](image-url)
Figure 5. Effect of the supplied voltage on streamlines and isotherm when $Ra = 100, \phi = 0.05$.

Figure 6. Effect of the supplied voltage on streamlines and isotherm when $Ra = 500, \phi = 0.05$. 
Here $\Gamma$ is the dependent variable ($\Psi, \Omega, \Theta$). In order to verify the current results, free convection of a nanofluid in a cavity was validated as shown in Figure 2 [1]. This figure proves that the current code has good accuracy and lends confidence to subsequent presentation of the results.

5. Results and discussion

The effect of non-uniform electric field on nanofluid free convection heat transfer in an enclosure with a sinusoidal wall is presented. The working nanofluid is a mixture of ethylene glycol and Fe$_3$O$_4$. The viscosity of the nanofluid relies on the strength of the electric field. Calculations are prepared for various values of the voltage supplied ($\Delta \varphi = 0, 2, 4, \text{ and } 6 \text{kV}$), volume fraction of nanoparticles ($\phi = 0\% \text{ and } 5\%$), and Rayleigh number ($Ra = 50, 100, \text{ and } 500$). In all calculations, the Prandtl number ($Pr$) and Eckert number ($Ec$) are set to 149.54 and $1e^{-6}$, respectively.

Figure 3 depicts the distribution of the electric density distribution injected by the bottom electrode for different Rayleigh numbers and voltages supplied. It is observed that the electric density contours become more disturbed for higher values of the supplied voltage.

The influence of Rayleigh number and the voltage supplied on the streamlines and isotherms is shown in Figures 4–6. At $Ra = 50$, two main eddies exist in the streamlines and which rotate in the reverse direction. As the electric field is applied, the two main eddies combine and become one eddy. Also, the isotherm become denser near the hot wall due to the presence of these eddies. As Rayleigh number increases, the buoyancy forces increase and in turn and the rate of heat transfer increases. In order to show the effect of magnetic field, we consider a low Rayleigh number, so that the effect of
Rayleigh number on the flow and heat transfer is not sensible. Thermal plumes are generated because of the presence of different vortices near the hot wall.

Figures 7 and 8 depict the influence of Δφ and $Ra$ on the values of $Nu_{loc}$ and $Nu_{ave}$ along the hot wall. As Rayleigh number increases, Nusselt number increases due to decrease in the thermal boundary layer thickness. Increasing the voltage supplied makes the isotherms more distorted. The local Nusselt number profiles show extreme values at higher values of the voltage supplied because of the existence of the thermal plumes. The Nusselt number is an increasing function of the voltage supplied. In the absence of the electric field, The Nusselt number for $Ra = 500$ is higher than that corresponding to $Ra = 50$, while in the presence of an electric field ($Δφ = 6$) the opposite trend is observed. Also, it can be concluded that the Nusselt number at $Δφ = 6$ for $Ra = 50$, 100, and 500 is 3.13665, 2.972727, and 2.598383 times higher, respectively, than that obtained at $Δφ = 0$. This observation confirms that the impact of the electric field is more marked for lower Reynolds numbers.

6. Conclusions

Free convective heat transfer of $Fe_3O_4$-ethylene glycol nanofluid in the presence of an electric field in a semi-annulus enclosure with a left sinusoidal wall is studied. The governing equations are formulated based on the equations of the electric fields with those of the hydrothermal equations. The obtained equations are solved by a new numerical procedure based on the CVFEM. Numerical results are obtained for various values of the voltage supplied, nanoparticle volume fraction, and Rayleigh number. Results show that the voltage supplied results in a change in flow characteristics. By applying an electric field, the isotherms become denser near the hot wall and this observation is more marked for stronger electric fields. The rate of augmentation for heat transfer rises with increase in the strength of the electric field. Also, Nusselt number increases with increase in Rayleigh number. Improvements in Nusselt number due to the existence of the electric field become stronger for low Rayleigh numbers.

References


