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Abstract
Purpose – The purpose of this paper is to report a numerical solution for the problem of steady, two dimensional boundary layer buoyant flow on a vertical magnetized surface, when both the viscosity and thermal conductivity are assumed to be temperature-dependent. In this case, the motion is governed by a coupled set of three nonlinear partial differential equations, which are solved numerically by using the finite difference method (FDM) by introducing the primitive variable formulation. Calculations of the coupled equations are performed to investigate the effects of the different governing parameters on the profiles of velocity, temperature and the transverse component of magnetic field. The effects of the thermal conductivity variation parameter, viscosity variation parameter, magnetic Prandtl number Pm_r, magnetic force parameter S, mixed convection parameter Ri and the Prandtl number Pr on the flow structure and heat transfer characteristics are also examined.
Design/methodology/approach – FDM.
Findings – It is noted that when the Prandtl number Pr is sufficiently large, i.e. $Pr = 100$, the buoyancy force that driven the fluid motion is decreased that decrease the momentum boundary layer and there is no change in thermal boundary layer is noticed. It is also noted that due to slow motion of the fluid the magnetic current generates which increase the magnetic boundary layer thickness at the surface. It is observed that the momentum boundary layer thickness is increased, thermal and magnetic field boundary layers are decreased with the increase of thermal conductivity variation parameter $=100$. The maximum boundary layer thickness is increased for $=100$ and there is no change seen in the case of thermal boundary layer thickness but magnetic field boundary layer is deceased. The momentum boundary layer thickness shoot quickly for $=40$ but is very smooth for $=50$.There is no change is seen for the case of thermal boundary layer and very clear decay for $=40$ is noted.
Originality/value – This work is original research work.
Keywords University of Sargodha, Mixed convection
Paper type Research paper

1. Introduction
The problem of boundary layer flow past magnetized and non-magnetized vertical surfaces have been discussed in detail by several authors analytically, numerically as well as experimentally. The early theoretical work on viscous, incompressible and electrically conducting fluid in which the magnetic field is assumed to be coincident with the ambient fluid velocity field was discussed by Greenspan and Carrier (1959). Later, Davies (1963a, b) was the first who investigated the magneto-hydrodynamic

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boundary layer in the two dimensional semi-infinite non-magnetized plate. Further, he investigated the behavior of drag coefficient and current density as the magnetic force parameter S increases from 0 to 1. The different cases of steady, magneto-hydrodynamic flow of an incompressible, viscous, electrically conducting fluid near a stagnation point have been studied by Gribben (1963, 1965). In this study he concluded that under the influence of an external magnetic dynamic pressure gradient the skin friction decreases with the increase of magnetic field. Chawla (1971) and Glauert (1962) studied the effect of harmonic oscillations in the magnitude of the free stream velocity on the magneto-hydrodynamic boundary layer flow past a flat plate in the presence of an aligned field. He obtained the steady state solution by using the von Karman-Pohlhausen technique and it was also concluded that the aligned magnetic field significantly affect the steady as well as the oscillating component of the skin friction. Glauert (1962) considered the uniform boundary layer flow past a magnetized plate and investigated that the velocity and magnetic field are valid for very small values of magnetic Prandtl number $\mathrm{Pm_r}$ and magnetic force parameter S. Ashraf Muhammad and Hossain (2010), Muhammad *et al.* (2010, 2012a, b, 2013, 2014) extensively studied the different cases of magnetohydrodynamics convective heat transfer boundary layer flow from vertical magnetized surface. They investigated the influence of different physical parameters such as magnetic force parameter S, magnetic Prandtl number $\mathrm{Pm_r}$, Prandtl number Pr, mixed convection parameter λ, radiation parameter R_d and surface temperature θ on skin friction, rate of heat transfer, current density and magnetic intensity at the surface of the plate.

Hossain *et al.* (2000) studied the flow of viscous incompressible fluid with temperature-dependent viscosity and thermal conductivity past a permeable wedge with uniform surface heat flux and concluded the results in terms of local skin friction coefficient and rate of heat transfer for different values of the governing parameters, such as the Prandt number Pr, the pressure gradient parameter m, the viscosity variation parameter ε and thermal conductivity variation parameter γ against the local permeability parameter ξ. Mehta and Sood (1992) discussed the flow characteristics substantially change due to the consideration of temperature-dependent viscosity. The influence of the variable viscosity and buoyancy force on boundary layer flow and heat transfer due to a continuous flat plate has been examined by Hady *et al.* (1996). Kafoussias and Williams (1997, 1998) studied the effect of temperature-dependent viscosity on an incompressible fluid in steady laminar free forced convective boundary layer flow over a isothermal vertical plate. Chamkha (1996) studied MHD free convection flow from vertical plate embedded in a thermally stratified porous medium numerically. The effect of temperature-dependent viscosity on the free convective laminar boundary layer flow past a vertical isothermal flat plate has been discussed by Kafoussias and Williams (1997). Chamkha (1997) also studied MHD free convection flow from vertical plate embedded in porous medium with Hall effects numerically. Later, Kafoussias and Rees (1998) analyzed a modified and improved numerical solution for local non-similarity boundary layer flow past a vertical flat plate with temperature-dependent viscosity. The problem of mixed convection flow along a vertical permeable plate embedded in porous medium in the presence of transverse magnetic field has been carried out by Chamkha (1998). Severin and Herwig (1999) investigated the Rayleigh-Benard convection flow with fluid having temperature-dependent viscosity using the appropriate asymptotic analysis. For a fluid with viscosity inversely proportional to the temperature the problems of mixed convection flow from a vertical heated flat plate, of natural convection flow from a vertical wavy surface and a vertical truncated cone and a wedge investigated by Hossain and Munir (2000), Hossain *et al.* (2002, 2000a, ,d). Natural convection flow of a viscous fluid about a truncated cone with temperature-dependent viscosity has been carried out by Hossain *et al.* (2000b) and obtained results in terms of local skin friction and the Nusselt number. Later, Hossain *et al.* (2001) investigated the natural convection with variable viscosity and thermal conductivity from a vertical wavy cone. Greenspan and Carrier (1959) also studied the combined convection from a vertical flat plate with temperature-dependent viscosity and thermal conductivity numerically. Takhar *et al.* (2005) discussed the unsteady mixed convection on the stagnation point flow adjacent to a vertical plate with magnetic field. Sharma and Singh (2009) predicted the solutions for effects of varying viscosity and thermal conductivity on steady MHD free convection flow and heat transfer along an isothermal plate with internal heat generation. Analysis on steady MHD flow and heat transfer past a rotating disk in a porous medium with ohmic heating and viscous dissipation has been given by Sibanda and Makinde (2010). Heat and mass transfer flow along a vertical plate under the combined buoyancy force of thermal and species diffusion under the transverse magnetic field is investigated by Hazarika and Gopal-Ch (2012). Later Chamkha *et al.* (2012) predicted the behavior of heat and mass transfer from truncated cones with variable wall temperature and concentration in the presence of chemical reaction effects. Roşca *et al.* (2014) and Pătrulescu *et al.* (2014) discussed the mixed convection boundary layer flow phenomena past a vertical plate.

The above literature survey says that the effect of temperature-dependent viscosity and thermal conductivity on mixed convection flow along a magnetized surface yet not has been studied. We propose the effect of temperature-dependent viscosity and thermal conductivity along with other parameters those are described in flow model. Chief parameters considered in this study are, magnetic Prandtl number Pm_r, Prandtl number Pr, magnetic force parameter S, mixed convection parameter or Richardson number R_i, thermal conductivity variation parameter γ and viscosity variation parameter ε. We see the effects of these parameters on skin friction, rate of heat transfer, magnetic intensity, momentum, thermal and magnetic field boundary layer numerically by using the finite difference method (FDM).

2. Formulation of the problem

The flow field considered here is a magnetized vertical surface. The flow is assumed to be steady, laminar and incompressible. The effects of temperature-dependent viscosity and thermal conductivity are included in momentum and energy equation, respectively, along with aligned magnetic field. The coordinate system and flow configuration is given in Figure 1.

The governing equations for the flow field along a magnetized surface (following Davies, 1963a; Hossain *et al.*, 2000d) can be describes as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + S\left(H_x\frac{\partial H_x}{\partial x} + H_y\frac{\partial H_x}{\partial y}\right) + \varepsilon\left(\frac{\partial \theta}{\partial y}\frac{\partial u}{\partial y}\right) + \lambda\theta \tag{2}$$

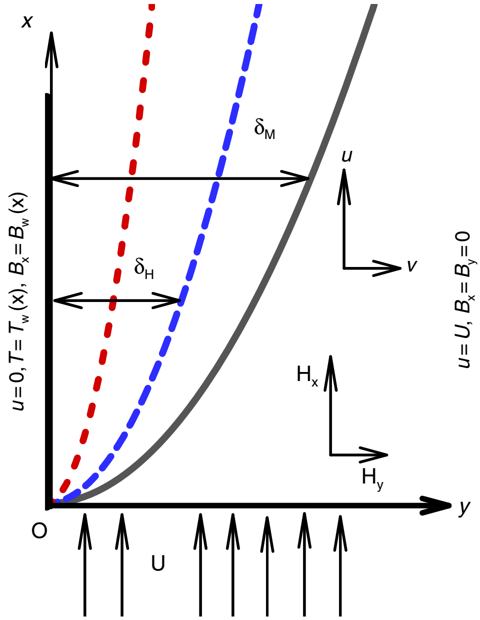


Figure 1. The coordinate system and flow configuration

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}=0 \tag{3}$$

$$-\frac{1}{Pm_r}\frac{\partial H_x}{\partial y}=\left(uH_y-vH_x\right) \tag{4}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}\left[\frac{\partial^2 \theta}{\partial y^2}+\gamma\left(\frac{\partial \theta}{\partial y}\right)^2\right] \tag{5}$$

where:

$$\begin{gathered} Re_L=\frac{U_0 L}{v},\ Gr_L=\frac{g\beta \Delta T L^3}{v^2},\ \Pr=\frac{v}{\alpha}, \\ \lambda=\frac{Gr_L}{Re_L^2},\ P_m=\frac{\upsilon}{\gamma},\ \alpha=\frac{K}{\rho C_p},\ S=\frac{\overline{\mu} H_0^2}{\rho U_0^2}, \\ \mu=\mu_\infty\left(1+\varepsilon\overline{\theta}\right),\ \text{where } \varepsilon=\gamma_1(T_w-T_\infty) \\ k=k_\infty\left(1+\gamma\overline{\theta}\right),\ \text{where } \gamma=\gamma_2(T_w-T_\infty) \end{gathered} \tag{6}$$

where Re_L is the Reynolds number, Gr_L the Grashof, number, L the reference length, λ the mixed convection parameter, Pr the Prandtl number and S the magnetic field parameter (also known as Chandrasekhar number), Pm_r the magnetic Prandtl number and α the thermal diffusion.

The corresponding boundary conditions take the form:

$$u(x,0)=0,\ v(x,0)=0,\ H_x(x,0)=1,\ \theta(x,0)=1$$

$$u(x,\infty)=1,\ H_x(x,\infty)=0,\ \theta(x,\infty)=0 \tag{7}$$

3. Numerical method

To integrate the system of equations given in (1)-(5) along with boundary conditions (7), we introduce the group of transformations that is known as primitive variable formulation to transform the given system of equations into convenient form:

$$u=U(\xi,Y),\ v=x^{\frac{1}{2}}(V),\ \varphi=x^{\frac{1}{2}}\phi,\ \theta=\theta(\xi,Y)$$

$$Y=x^{-\frac{1}{2}}y,\ \xi=x,\ H_x=\frac{\partial\varphi}{\partial y},\ H_y=-\frac{\partial\varphi}{\partial x} \tag{8}$$

By substituting the derivatives of u, v, H_x, H_y and θ from Equation (8) into (1)-(5) along with boundary conditions (7), we have the transformed system of equations:

$$\xi\frac{\partial U}{\partial\xi}-\frac{1}{2}Y\frac{\partial U}{\partial Y}+\xi\frac{\partial V}{\partial Y}=0 \tag{9}$$

$$\xi U\frac{\partial U}{\partial\xi}+\left(V-\frac{1}{2}YU\right)\frac{\partial U}{\partial Y}-\frac{\partial^2 U}{\partial Y^2}$$

$$-S\left[-\frac{1}{2}\phi\frac{\partial^2\phi}{\partial Y^2}+\xi\left(\frac{\partial\phi}{\partial Y}\frac{\partial^2\phi}{\partial\xi\partial Y}-\frac{\partial^2\phi}{\partial Y^2}\frac{\partial\phi}{\partial\xi}\right)\right]-\lambda\theta-\varepsilon\left(\frac{\partial\theta}{\partial Y}\frac{\partial U}{\partial Y}\right)=0 \tag{10}$$

$$\frac{1}{Pm_r}\frac{\partial^2\phi}{\partial Y^2}=\frac{1}{2}U\phi+\xi U\frac{\partial\phi}{\partial\xi}+\left(V-\frac{1}{2}YU\right)\frac{\partial\phi}{\partial Y} \tag{11}$$

$$\frac{1}{Pr}\left[\frac{\partial^2\theta}{\partial Y^2}+\gamma\left(\frac{\partial\theta}{\partial Y}\right)^2\right]=\xi U\frac{\partial\theta}{\partial\xi}+\left(V-\frac{1}{2}YU\right)\frac{\partial\theta}{\partial Y} \tag{12}$$

The appropriate boundary conditions satisfied by the above system of equations are:

$$U(\xi,0)=V(\xi,0)=0,\ \phi'(\xi,0)=1,\ \theta(\xi,0)=1$$

$$V(\xi,\infty)=1,\ \phi'(\xi,\infty)=0,\ \theta(\xi,\infty)=0 \tag{13}$$

The governing transformed Equations (9)-(13) are solved numerically by using the FDM. By discretizing the above system of equations given (9)-(13) we have obtained a tri-diagonal system of equations and then solved by Gaussian elimination technique with help of FORTRAN language. Most of the computations reported in this paper have been performed on 80×80 grids, which are found to model accurately the flow fields described in the results. The computation is started at $\xi=0$, and then marched downstream implicitly. The mesh sizes ΔY and $\Delta\xi$ are taken 0.01 and 0.025, respectively. It is found that the convergence reached when the maximum error is 10^{-5}.

Once we know the solutions of the above equations, we are at position to obtain the solutions in terms of skin friction, Nusselt number and magnetic intensity along the surface from the following relations:

$$Re_x^{1/2} Cf_x = f''(\xi, 0) \tag{14}$$

$$Re_x^{-1/2} Nu_x = -\theta'(\xi, 0) \tag{15}$$

and:

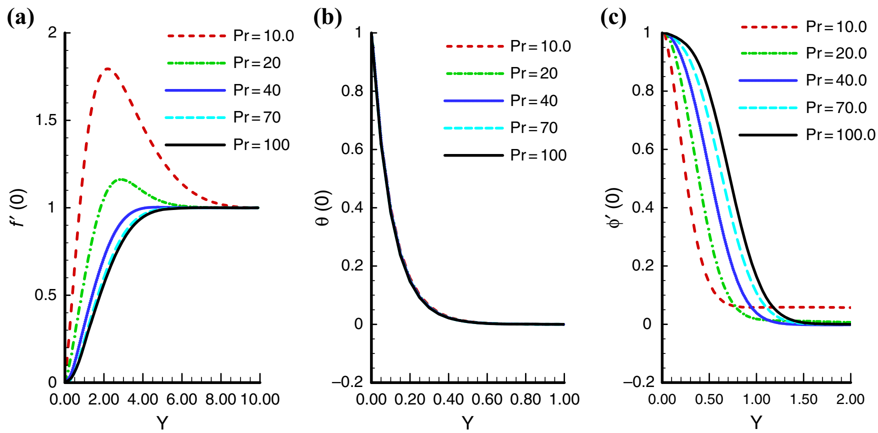
$$Re_x^{1/2} Mg_x = -\phi(\xi, 0) \tag{16}$$

4. Results and discussion

From Equations (9)-(12), it is noted that the problem under investigation is governed by the six non-dimensional parameters, magnetic force parameter S, Prandtl number Pr, and mixed convection or Richardson number R_i, magnetic Prandtl number Pm_r, viscosity variation parameter ε, and thermal conductivity variation parameter γ. Rather, the objective here is to present a sample of results in order to illustrate the effects of these parameters on velocity/momentum, temperature/thermal and magnetic boundary layer against similarity variable Y. Further, for complete study it is necessary to show the effects of these parameters on skin friction, rate of heat transfer and magnetic intensity against ξ. In the following discussion we present the numerical result of the problem under investigation graphically.

4.1 Effects of physical parameters on velocity, temperature and transverse component of magnetic field

Figure 2(a)-(c) illustrate the momentum, thermal and magnetic field boundary layers for various values of Prandtl number Pr, for strong values of magnetic Prandtl number $Pm_r = 50$, and strong magnetic force parameter $S = 5$, Richardson number $R_i = 1$, thermal conductivity variation parameter $\gamma = 20$ and viscosity variation parameter $\varepsilon = 20$.



Notes: For $Pm_r = 50$, $S = 5.0$, $\gamma = 20.0$, $\varepsilon = 20$, $R_i = 1.0$

Figure 2. Simulated results of (a) velocity (b) temperature (c) transverse components of magnetic field for various values of Prandtl mumber Pr

It is noted that when the Prandtl number Pr is sufficiently large, i.e. $\mathrm{Pr}=100$, the buoyancy force that driven the fluid motion is decreased that decrease the momentum boundary layer and there is no change in thermal boundary layer is noticed. It is also noted that due to slow motion of the fluid the magnetic current generates which increase the magnetic boundary layer thickness at the surface. It is also important to point out that the obtained numerical results in this case satisfy the boundary conditions given in Equation (13). The influence of the magnitude of thermal conductivity variation parameter γ is displayed in Figure 3(a)-(c). These figures show that the momentum boundary layer thickness is increased, thermal and magnetic field boundary layers are decreased with the increase of thermal conductivity variation parameter $\gamma=100$. Such a result mainly is because the increase in thermal variation parameter γ enhance the temperature difference which increases the fluid motion at the surface and thus decrease in thermal and magnetic boundary layer is noted. Figure 4(a)-(c) display the effects of different values of viscosity variation parameter ε by keeping other parameters constant. The maximum boundary layer thickness is increased for $\varepsilon=100$ and there is no change seen in the case of thermal boundary layer thickness but magnetic field boundary layer is deceased. The reason is that the increase in viscosity parameter ε is also increase the temperature difference which

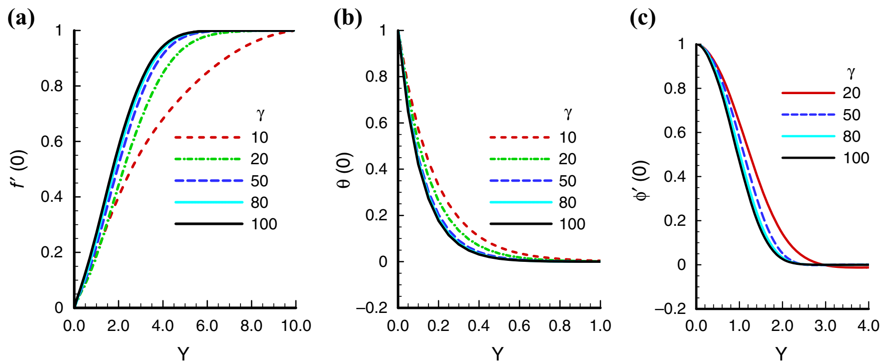


Figure 3. Simulated results of (a) velocity (b) temperature (c) transverse component of magnetic field for various values of thermal conductivity variation parameter γ

Notes: For $Pm_r=10.0$, $S=5.0$, $\mathrm{Pr}=0.71$, $\varepsilon=50$, $R_i=1.0$

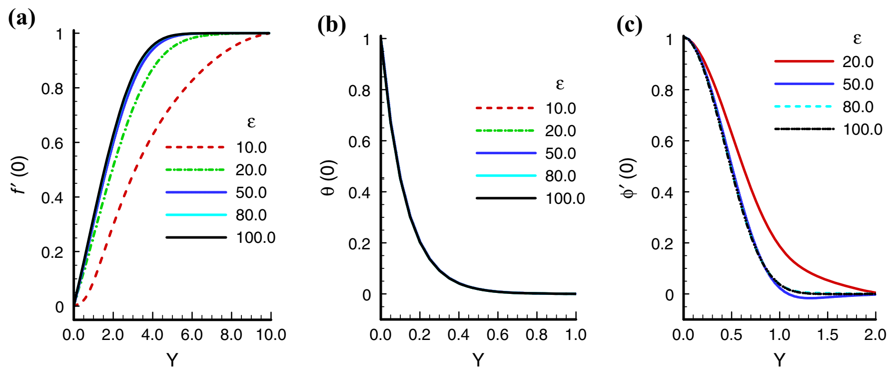


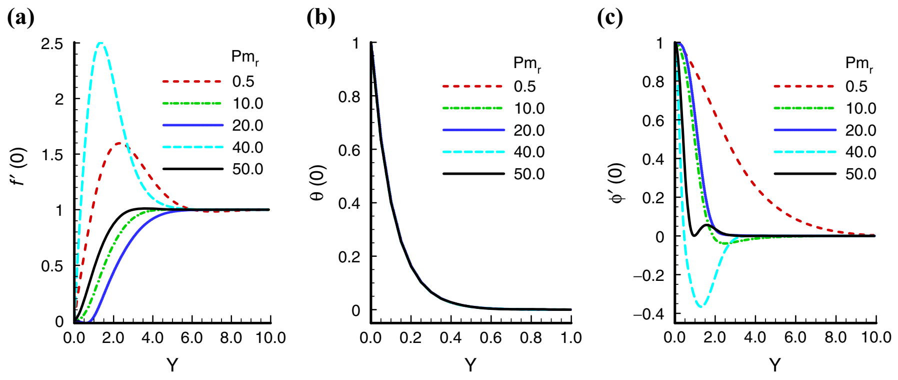
Figure 4. Simulated results of (a) velocity (b) temperature (c) transverse component of magnetic field for various values of viscosity variation parameter ε

Notes: For $Pm_r=50.0$, $S=1.0$, $\mathrm{Pr}=0.71$, $\gamma=50$, $R_i=1.0$

increase the flow-driven buoyancy force hence momentum boundary layer attain maximum growth and decrease in thermal and magnetic boundary is concluded. Figure 5(a)-(c) demonstrate the result for different and for large values of $Pm_r = 50$, and in this case very complex physical phenomena is observed. The momentum boundary layer thickness shoot quickly for $Pm_r = 40$ but is very smooth for $Pm_r = 50$. There is no change is seen for the case of thermal boundary layer and very clear decay for $Pm_r = 40$ is noted. Such complexity mainly is because of the flow structure in the natural convective regime.

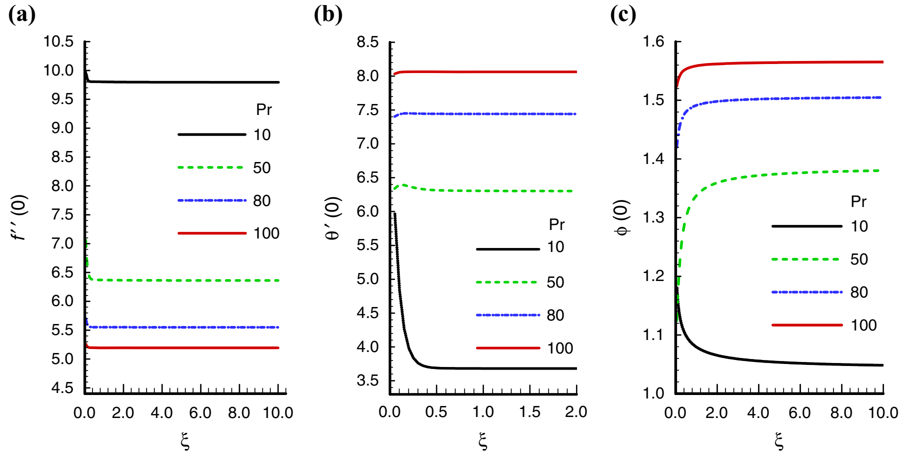
4.2 Effects of physical parameters on kin friction, rate of heat transfer and magnetic intensity

The variations in Prandtl number Pr are shown in Figure 6(a)-(c), where Figure 6(a) referred the effects of various values of Pr on skin friction and Figure 6(b) on heat transfer and Figure 6(c) on magnetic intensity. In this case the value of skin friction



Notes: For $\varepsilon = 50.0$, $S = 1.0$, $\Pr = 7.0$, $\gamma = 50$, $R_i = 1.0$

Figure 5. Simulated results of (a) velocity (b) temperature (c) and transverse component of magnetic field for various values of magnetic Prandtl parameter Pm_r



Notes: When $\varepsilon = 1.0$, $S = 0.8$, $Pm_r = 1.0$, $\gamma = 1.0$, $R_i = 50.0$

Figure 6. (a) Skin friction (b) heat transfer and (c) magnetic intensity, for various values of Prandtl number Pr

decreases with an increase in Prandtl number Pr but the values of rate of heat transfer and magnetic intensity at the surface is increased. Figure 7(a)-(c) map the effects of thermal conductivity variation parameter γ on skin friction, rate of heat transfer and magnetic intensity at the surface. In Figure 7(a), we can see that with the increase of thermal conductivity variation parameter γ the skin friction is decreased. In Figure 7(b) a dramatically change seen on rate of heat transfer for $\gamma = 1$, but for other values of γ the flow is uniform. In Figure 7(c) for $\gamma = 1$ a different profile is observed on magnetic intensity and for other values of γ the flow is developed and magnetic intensity is increased very prominently. Figure 8(a)-(c) has been produced to examine the effect of viscosity variation parameter ε on skin friction, rate of heat transfer and magnetic intensity at the surface of vertical plate. It is worth noting that for the case of skin friction the curve for value of $\varepsilon = 15$ is shows entirely different behavior, it increases at the surface and then sharply down and developed, as a result the skin friction is minimum for maximum value of $\varepsilon = 20$. We notice from Figure 8(b) that the rate of heat transfer is maximum for viscosity variation parameter $\varepsilon = 5$ and 15, and suddenly drop

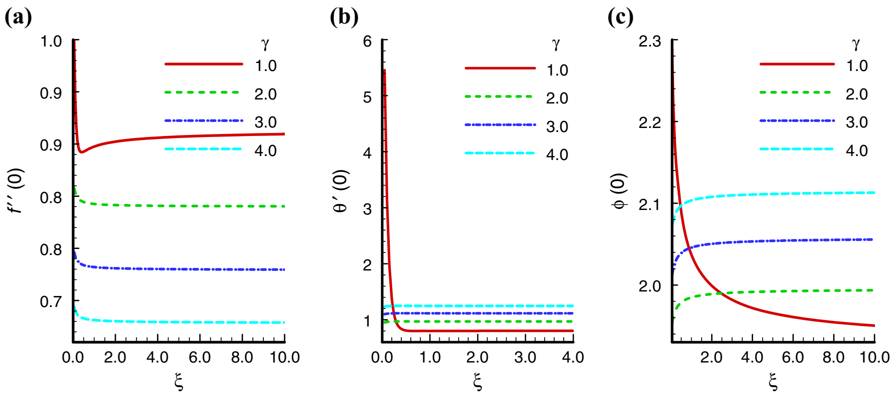


Figure 7.
(a) Skin friction
(b) heat transfer and
(c) magnetic intensity, for various values of thermal conductivity variation parameter γ

Notes: When $Pr = 0.7$, $\varepsilon = 1.0$, $S = 1.0$, $Pm_r = 0.8$, $R_i = 1.0$

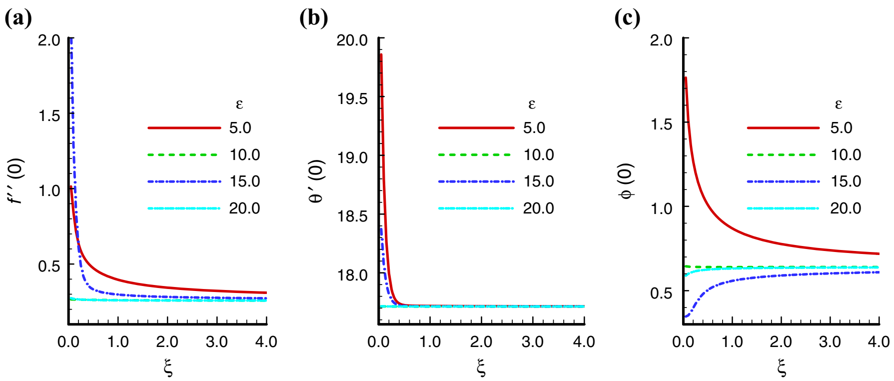


Figure 8.
(a) Skin friction
(b) heat transfer and (c) magnetic intensity for various values of viscosity variation parameter ε

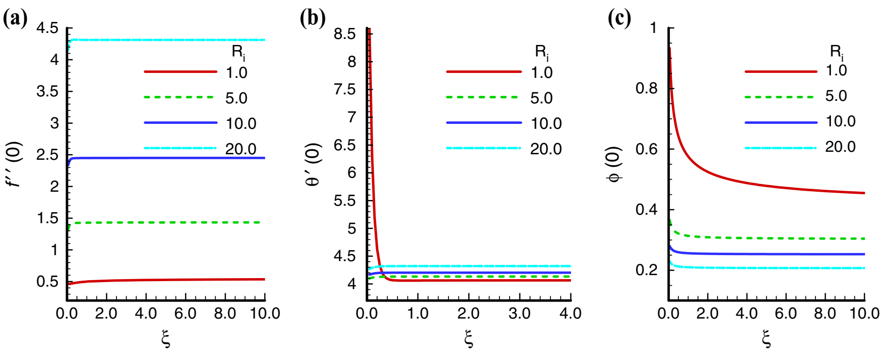
Notes: When $Pr = 0.71$ $\gamma = 1.0$, $S = 1.0$, $Pm_r = 0.8$, $R_i = 1.0$

along the vertical surface but this pattern is uniform for values of $\varepsilon = 10$ and 20. In this figure, the interactions become more complicated in boundary layer flow in both way that is for $\varepsilon = 5$, 15 and for $\varepsilon = 100$, 20. In Figure 8(c), we present the effect of viscosity variation parameter on magnetic intensity at the surface. It is worth noting that for the case of skin friction the curve for value of $\varepsilon = 15$ is shows entirely different behavior, it increases at the surface and then suddenly down and developed, as a result the skin friction is minimum for maximum value of $\varepsilon = 20$. The numerical data shown in Figure 9(a)-(c) depict the effect for various values of Richardson parameter R_i on skin friction, rate of heat transfer and magnetic intensity at the surface of magnetized vertical plate. It is observed that with the increase of Richardson parameter R_i the skin friction is increased, rate of heat transfer is increased and magnetic intensity at the surface is decreased. The reason is that with the increase of Richardson parameter R_i the buoyancy force acts like pressure gradient and increase the fluid motion for which the skin friction and heat transfer are increased and magnetic intensity at the surface is decreased.

5. Conclusion

The effects of temperature-dependent viscosity and thermal conductivity on mixed convection flow along a magnetized vertical surface has been analyzed. An accurate solution valid for entire domain has been obtained numerically by using the FDM. Major parameters considered in this study include the magnetic Prandtl number Pm_r, Prandtl number Pr, magnetic force parameter S, mixed convection parameter or Richardson number R_i, thermal conductivity parameter γ and the viscosity variation parameter ε has been performed for various combination of these physical parameters on the skin friction, rate of heat transfer, magnetic intensity, momentum, thermal and the magnetic field boundary layer. We conclude from the present problem the following comments:

It is noted that when the Prandtl number Pr is sufficiently large, i.e. Pr = 100, the buoyancy force that driven the fluid motion is decreased that decrease the momentum boundary layer and there is no change in thermal boundary layer is noticed. It is also noted that due to slow motion of the fluid the magnetic current generates which increase the magnetic boundary layer thickness at the surface. It is observed that the momentum boundary layer thickness is increased, thermal and magnetic field boundary layers are decreased with the increase of thermal conductivity variation



Notes: Where Pr = 7.0, $\varepsilon = 5.0$, $S = 2.0$, $Pm_r = 100.0$, $\gamma = 5.0$

Figure 9. (a) Skin friction (b) heat transfer and (c) magnetic intensity, for various values of Richardson parameter R_i

parameter $\gamma = 100$. The maximum boundary layer thickness is increased for $\varepsilon = 100$ and there is no change seen in the case of thermal boundary layer thickness but magnetic field boundary layer is deceased. The momentum boundary layer thickness shoot quickly for $Pm_r = 40$ but is very smooth for $Pm_r = 50$. There is no change is seen for the case of thermal boundary layer and very clear decay for $Pm_r = 40$ is noted. Dramatic changes are seen on rate of heat transfer for $\gamma = 1$, but for other values of γ the flow is uniform and for $\gamma = 1$ a different profile is observed on magnetic intensity and for other values of γ the flow is developed and magnetic intensity is increased very prominently. It is shown that for the case of skin friction the curve for value of $\varepsilon = 15$ shows entirely different behavior, it increases at the surface and then down and developed, as a result the skin friction is minimum for maximum value of $\varepsilon = 20$. It is also noted that the skin friction is minimum for maximum value of $\varepsilon = 20$. For various values of Richardson parameter R_i the skin friction, rate of heat transfer are increased and magnetic intensity at the surface of magnetized vertical plate is decreased.

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