Unsteady free convection flow past a periodically accelerated vertical plate with Newtonian heating

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Received 6 May 2014
Revised 8 July 2014
24 July 2014
Accepted 24 July 2014

Abstract

Purpose – The purpose of this paper is to present an analytical study for a problem of unsteady free convection boundary layer flow past a periodically accelerated vertical plate with Newtonian heating (NH).
Design/methodology/approach – The equations governing the flow are studied in the closed form by using the Laplace transform technique. The effects of various physical parameters are studied through graphs and the expressions for skin friction, Nusselt number and Sherwood number are also derived and discussed numerically.
Findings – It is observed that velocity, concentration and skin friction decrease with the increasing values of Sc whereas temperature distribution decreases in the increase in Pr in the presence of NH.
Research limitations/implications – This study is limited to a Newtonian fluid. This can be extended for non-Newtonian fluids.
Practical implications – Heat and mass transfer frequently occurs in chemically processed industries, distribution of temperature and moisture over agricultural fields, dispersion of fog and environment pollution and polymer production.
Social implications – Free convection flow of coupled heat and mass transfer occurs due to the temperature and concentration differences in the fluid as a result of driving forces. For example, in atmospheric flows, thermal convection resulting from heating of the earth by sunlight is affected differences in water vapor concentration.
Originality/value – The authors have studied heat and mass transfer effects on unsteady free convection boundary layer flow past a periodically accelerated vertical surface with NH, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed as conjugate convective flow. The equations governing the flow are studied in the closed form by using the Laplace transform technique. The effects of various physical parameters are studied through graphs and the expression for skin friction also derived and discussed.

Keywords Newtonian heating, Unsteady flow, Free convection, Heat and mass transfer, Periodically accelerated vertical plate
Paper type Research paper

1. Introduction

In nature, there exist flows which are caused not only by the temperature differences but also the concentration differences. The rate of heat transfer is affected by these mass transfer differences especially in industries, the combined buoyancy effect has

International Journal of Numerical Methods for Heat & Fluid Flow
Vol. 26 No. 7, 2016
pp. 2119-2138

0961-5539
DOI 10.1108/HFF-05-2014-0123

been taken place through the simultaneous transport of heat and mass. The thermal diffusion, this phenomenon of heat and mass transfer frequently occurs in chemically processed industries, distribution of temperature and moisture over agricultural fields, dispersion of fog and environment pollution and polymer production. As a result of driving forces the free convection flow of coupled heat and mass transfer occurs due to the temperature and concentration differences in the fluid. For instance in an atmospheric flow, thermal convection resulting from heating of the earth by sunlight is affected differences in water vapor concentration. Chamkha *et al.* (2001) analyzed the effect of buoyancy forces on the flow and heat transfer over a heated vertical or inclined surface which moves with non-uniform velocity in an ambient fluid. Heat and mass transfer effects on MHD flow of viscous fluid through non-homogeneous porous medium in presence of temperature-dependent heat source was considered by Ravikumar *et al.* (2012). Raju and Varma (2011) considered a problem of an unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature. Reddy *et al.* (2011) investigated a problem of an unsteady MHD free convection oscillatory couette flow through a porous medium with periodic wall temperature in presence of chemical reaction and thermal radiation. Gupta *et al.* (1979) studied free convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid. Kafousias and Raptis (1981) considered mass transfer and free convection effects on the flow past an accelerated vertical infinite plate with variable suction or injection. Mass transfer effects on the flow past an impulsively started infinite vertical plate with variable temperature or constant heat flux were considered by Soundalgekar *et al.* (1984). Asogwa *et al.* (2013) investigated the effects of heat and mass transfer over a vertical plate with periodic suction and heat sink.

It is known that heat transfer is concerned with the exchange of thermal energy from one physical system to another. Merkin (1994) was the first to point out Newtonian heating (NH), where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed as conjugate convective flow. Recently, NH conditions have been used by researchers in view of their practical applications in several engineering devices, for instance in a heat exchanger where the conduction in a solid tube wall is greatly influenced by the convection in the fluid flowing over it. Free convection flow above a nearly horizontal surface in a porous medium subject to NH has been studied by Lesnic *et al.* (2004) and Sankar kumar *et al.* (2012) considered an unsteady free convective flow past a moving vertical porous plate with NH. Niu *et al.* (2010) analyzed the stability of thermal convection of an Oldroyd-B fluid in a porous medium with NH. Narahari and Ishak (2011) have studied radiation effects on free convection flow a moving vertical plate with NH. Forced convection boundary layer flow at a forward stagnation point with NH was studied by Salleh *et al.* (2009). Mebine and Adigio (2009) considered an unsteady free convection flow with thermal radiation past a vertical porous plate with NH. Unsteady free convection flow past an impulsively started vertical surface in the presence of NH was addressed by Chaudhary and Jain (2006, 2007). Das *et al.* (2012) have analyzed radiation effects on unsteady free convection flow past a vertical plate with NH. Very recently Hayat *et al.* (2012) investigated NH and magnetohydrynamic effects in flow of a Jeffery fluid over a radically stretching surface. Heat and mass transfer effects on unsteady free convection boundary layer flow past an impulsively started vertical surface with NH was considered by Raju *et al.* (2013b). Free convective heat and mass transfer transient flow past an exponentially accelerated vertical plate with NH in the presence of radiation was investigated by Raju *et al.* (2013a).

Mixed convection or forced convection boundary layer flow over a horizontal circular cylinder with NH was considered by Salleh *et al.* (2010a, 2011). Salleh *et al.* (2010b) investigated a boundary layer flow and heat transfer over a stretching sheet with NH. Narahari and Nayan (2011) considered a free convection flow past an impulsively started infinite vertical plate with NH in the presence of thermal radiation and mass diffusion. Ghosh *et al.* (2009) investigated the Hall current effects on MHD flow in a rotating system with heat transfer characteristics. In all the above studies, the flow geometry was not considered as periodic vertical plate. Hall effects on MHD flow in a rotating system with heat transfer characteristics were analyzed by Satya Narayana *et al.* (2015). Influence of variable permeability and radiation absorption on the heat and mass transfer in MHD micropolar flow over a vertical moving porous plate is investigated by Harish Babu and Satya Narayana (2013). Effects of the chemical reaction and mass transfer on MHD unsteady free convection flow past an infinite vertical plate with constant suction and heat sink were considered by Kumar *et al.* (2009). Effects of Hall current and radiation absorption on MHD micropolar fluid in a rotating system was studied by Satya Narayana *et al.* (2013). Analytical and as well as numerical methods for nanoscale flows were discussed by Liu *et al.* (2014). A convection-diffusion-reaction scheme is proposed by Sheu *et al.* (2012), in this study they simulate the high gradient electroosmotic flow behavior in microchannels. The equations governing the total electric field include the Laplace equation for the effective electrical potential and the Poisson-Boltzmann equation for the electrical potential in the electric double layer. The problem of steady, laminar, coupled heat and mass transfer by MHD natural convective boundary layer flow over a permeable truncated cone with variable surface temperature and concentration in the presence of thermal radiation and chemical reaction effects was studied by Chamkha *et al.* (2012). A unified numerical method able to address a wide class of fluid flow problems of engineering interest was presented by Delussu *et al.* (2002). Heat and mass transfer by non-Darcy free convection from a vertical cylinder embedded in porous media with a temperature-dependent viscosity was investigated by Chamkha *et al.* (2011). Natural convection flow under magnetic field in a square cavity for uniformly (or) linearly heated adjacent walls was considered by Sathiyamoorthy and Chamkha (2012).

Motivated by the above studies, in this paper we have studied heat and mass transfer effects on unsteady free convection boundary layer flow past a periodically accelerated vertical surface with NH, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature, and which is usually termed as conjugate convective flow. The equations governing the flow are studied in the closed form by using the Laplace transform technique. The effects of various physical parameters are studied through graphs and the expression for skin friction also derived and discussed.

2. Formulation of the problem

We have considered a viscous, incompressible and electrically conducting fluid past a periodically accelerated vertical plate with NH. The x^*-axis is taken along the plate in the vertically upward direction and the y^*-axis is taken to be normal to the plate. Initially the plate and the fluid are at the same temperature T^*_∞ in the stationary condition with concentration level C^*_∞ at all points (Figure 1).

At time $t^* > 0$ the plate is started with oscillatory motion in its plane with the velocity $U_c \cos \omega^* t^*$ in its own plane, where U_c is the amplitude of the plate oscillations and the plate temperature is maintained as $\partial T^*/\partial y^* = (-h/k)T^*$ and the level of

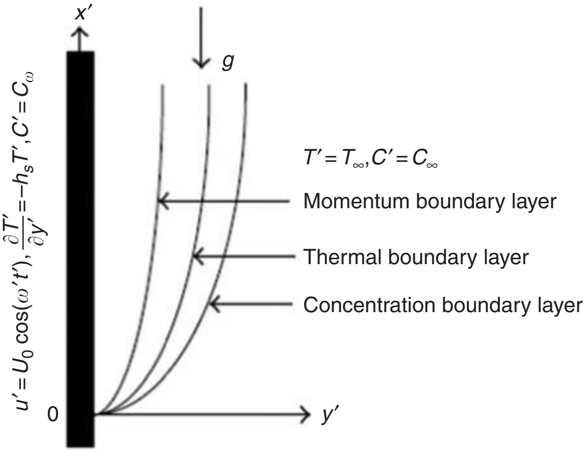

Figure 1.
Physical model and coordinate system

concentration near the plate is raised to C_w^*. It is assumed that thermal diffusion and diffusion-thermo effects are neglected. Using the above assumptions and the usual Bossinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u^*}{\partial t^*} = v\frac{\partial^2 u^*}{\partial y^{*2}} + g\beta_T(T^* - T_\infty^*) + g\beta_C(C^* - C_\infty^*) \tag{1}$$

$$\frac{\partial T^*}{\partial t^*} = \frac{\kappa}{\rho C_P}\frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{\partial C^*}{\partial t^*} = D\frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

The initial and boundary conditions are:

$$\begin{aligned} &u^* = 0, T^* = T_\infty^*, C^* = C_\infty^* \quad \text{for all } y^*, t^* \leqslant 0 \\ &u^* = U_c \cos\omega^* t^*, \quad \frac{\partial T^*}{\partial y^*} = -\frac{h}{k}T^*, \quad C^* = C_w^* \quad \text{at } y^* = 0, t^* > 0 \\ &u^* \to 0, \quad T^* \to T_\infty^*, \quad C^* \to C_\infty^* \quad \text{as } y \to \infty, t^* > 0 \end{aligned} \tag{4}$$

To reduce the above equations into non-dimensional form, we introduce the following dimensionless variables and parameters:

$$u = \frac{u^*}{U_c}, \quad y = \frac{U_c y^*}{v}, \quad \theta = \frac{T^* - T_\infty^*}{T_\infty^*}, \quad t = \frac{t^* U_c^2}{v}, \quad C = \frac{C^* - C_\infty^*}{C_\infty^*}, \quad P_r = \frac{\mu c_p}{\kappa},$$
$$G_r = \frac{vg\beta_T T_\infty^*}{U_c^3}, \quad G_m = \frac{vg\beta_c C_\infty^*}{U_c^3}, \quad S_c = \frac{v}{D}, \quad U_c = \frac{hv}{k}, \omega = \frac{\omega^* \vartheta}{U_c^2} \tag{5}$$

According to the above non-dimensional process, the characteristic velocity U_c can be defined as $U_c = h\vartheta/k$.

Substituting the above quantities (5) in Equations (1)-(3) leads to the following non-dimensional equations:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_c C \tag{6}$$

$$P_r\frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$S_C\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2} \tag{8}$$

The corresponding initial and boundary conditions in non-dimensional form are:

$$\begin{aligned}
&u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for all} \quad y^* \geqslant 0, t^* \leqslant 0 \\
&u = \cos\omega t, \quad \frac{\partial \theta}{\partial y} = -(1+\theta), \quad C = 1 \quad \text{at} \quad y = 0 \;\; t^* > 0 \\
&u \to 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as} \quad y \to \infty, t^* > 0
\end{aligned} \tag{9}$$

3. Solution of the problem

We solve the governing equations in exact form by the Laplace transform technique. The Laplace transforms of the Equations (6)-(8) and the boundary conditions (9) are given by:

$$\frac{d^2\overline{\theta}}{dy^2} - (sP_r)\overline{\theta} = 0 \tag{10}$$

$$\frac{d^2\overline{C}}{dy^2} - (sS_c)\overline{C} = 0 \tag{11}$$

$$\frac{d^2\overline{u}}{dy^2} - s\overline{u} = -G_r\overline{\theta} - G_m\overline{C} \tag{12}$$

The corresponding boundary conditions are:

$$\begin{aligned}
&\overline{u} = \frac{s}{s^2+\omega^2}, \quad \frac{\partial\overline{\theta}}{\partial y} = -\left(\frac{1}{s}+\overline{\theta}\right), \quad \overline{C} = \frac{1}{s} \quad \text{at } y = 0 \quad t < 0 \\
&\overline{u} = 0, \quad \overline{\theta} = 0, \quad \overline{C} = 0 \quad \text{as } y \to \infty \;\; t > 0
\end{aligned} \tag{13}$$

where s is the Laplace transformation parameter:

$$\overline{u}(y,s) = L[u(y,t)] = \int_0^\infty e^{-s\,t} u(y,t)dt$$

$$\overline{\theta}(y,s)=L[\theta(y,t)]=\int_0^{\infty}e^{-s\,t}\theta(y,t)dt$$

$$\overline{C}(y,s)=L[C(y,t)]=\int_0^{\infty}e^{-s\,t}C(y,t)dt$$

Solving the Equations (10)-(12) with the help of Equation (13), we get:

$$\overline{\theta}(y,s)=\left(-\frac{b}{s\left(\sqrt{s}+b\right)}\right)e^{-y\sqrt{sP_r}} \tag{14}$$

$$\overline{C}(y,s)=\left(\frac{1}{s}\right)e^{-y\sqrt{sS_c}} \tag{15}$$

$$\overline{u}(y,s)=\left(\left(\frac{s}{s^2+\omega^2}-\frac{a_1}{s^2(\sqrt{s}+b)}-\frac{a_2}{s^2}\right)e^{-y\sqrt{s}}+\left(\frac{a_2}{s^2}\right)e^{-y\sqrt{sS_c}}+\left(\frac{a_1}{s^2(\sqrt{s}+b)}\right)e^{-y\sqrt{sP_r}}\right) \tag{16}$$

The general solution of the present problem for the velocity $u(y, t)$, the temperature $\theta(y, t)$ and the concentration $C(y, t)$ for $t>0$ are given by:

$$\begin{aligned}
u(y,t)=&\frac{1}{4}\left(e^{i\omega t}\left[e^{-y\sqrt{i\omega}}erfc\left(\eta-\sqrt{i\omega t}\right)+e^{y\sqrt{i\omega}}erfc\left(\eta+\sqrt{i\omega t}\right)\right]\right)\\
&+\frac{1}{4}\left(e^{-i\omega t}\left[\begin{array}{c}e^{-y\sqrt{-i\omega}}erfc\left(\eta-\sqrt{-i\omega t}\right)\\+e^{y\sqrt{-i\omega}}erfc\left(\eta+\sqrt{-i\omega t}\right)\end{array}\right]\right)\\
&-a_1\mathrm{Pr}\left(erfc\left(\eta\sqrt{\mathrm{Pr}}\right)-erfc(\eta)-e^{\left(\frac{t}{\mathrm{Pr}}-y\right)}erfc\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{\frac{t}{\mathrm{Pr}}}\right)\right.\\
&\left.+e^{\left(\frac{t}{\mathrm{Pr}}-\frac{y}{\sqrt{\mathrm{Pr}}}\right)}erfc\left(\eta-\sqrt{\frac{t}{\mathrm{Pr}}}\right)\right)+a_1\sqrt{\mathrm{Pr}}\left(2\sqrt{\frac{t}{\pi}}e^{\left(-y^2\mathrm{Pr}/4t\right)}\right.\\
&\left.-y\sqrt{\mathrm{Pr}}\ erfc\left(\eta\sqrt{\mathrm{Pr}}\right)+y\ erfc(\eta)-2\sqrt{\frac{t}{\pi}}e^{\left(-y^2/4t\right)}\right)\\
&+a_1\left(\left(y^2\mathrm{Pr}/2+t\right)erfc\left(\eta\sqrt{\mathrm{Pr}}\right)-y\sqrt{\mathrm{Pr}}\sqrt{\frac{t}{\pi}}\ e^{\left(-y^2\mathrm{Pr}/4t\right)}\right.\\
&\left.+y\sqrt{\frac{t}{\pi}}\ e^{\left(-y^2/4t\right)}-\left(y^2/2+t\right)erfc(\eta)\right)
\end{aligned} \tag{17}$$

Case 2: $S_c = 1$

$$u(y,t) = \left(\frac{1}{4}\left(e^{i\omega t}\left[e^{-y\sqrt{i\omega}}erfc\left(\eta-\sqrt{i\omega t}\right)+e^{y\sqrt{i\omega}}erfc\left(\eta+\sqrt{i\omega t}\right)\right]\right)\right.$$

$$+\frac{1}{4}\left(e^{-i\omega t}\left[\begin{array}{c} e^{-y\sqrt{-i\omega}}erfc\left(\eta-\sqrt{-i\omega t}\right) \\ +e^{y\sqrt{-i\omega}}erfc\left(\eta+\sqrt{-i\omega t}\right)\end{array}\right]\right)\Bigg)$$

$$-a_1\Pr\left(erfc\left(\eta\sqrt{\Pr}\right)-erfc(\eta)-e^{\left(\frac{t}{\Pr}-y\right)}erfc\left(\eta\sqrt{\Pr}-\sqrt{\frac{t}{\Pr}}\right)\right.$$

$$+e^{\left(\frac{t}{\Pr}-\frac{y}{\sqrt{\Pr}}\right)}erfc\left(\eta-\sqrt{\frac{t}{\Pr}}\right)\Bigg)+a_1\sqrt{\Pr}\left(2\sqrt{\frac{t}{\pi}}e^{\left(-\frac{y^2\Pr}{4t}\right)}\right.$$

$$\left.-y\sqrt{\Pr}\ erfc\left(\eta\sqrt{\Pr}\right)+y\ erfc(\eta)-2\sqrt{\frac{t}{\pi}}e^{\left(-\frac{y^2}{4t}\right)}\right)$$

$$+a_1\left(\left(\frac{y^2\Pr}{2}+t\right)erfc\left(\eta\sqrt{\Pr}\right)\right.$$

$$\left.-y\sqrt{\Pr}\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2\Pr}{4t}\right)}+y\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)}-\left(\frac{y^2}{2}+t\right)erfc(\eta)\right)$$

$$-\frac{G_m y}{2}\left(2\sqrt{\frac{t}{\pi}}\ e^{\left(-\frac{y^2}{4t}\right)}-y\sqrt{\frac{t}{\pi}}erfc(\eta)\right) \quad (18)$$

$$\theta(y,t) = e^{\left(\frac{t}{\Pr}-y\right)}erfc\left(\eta\sqrt{\Pr}-\sqrt{\frac{t}{\Pr}}\right)-erfc\left(\eta\sqrt{P_r}\right) \quad (19)$$

$$C(y,t) = erfc\left(\eta\sqrt{S_c}\right) \quad (20)$$

4. Skin friction

Knowing the velocity field, we now study the effect of t, Pr, F, etc. on the skin friction. In the non-dimensional form, it is given by:

$$\tau = \frac{\tau'}{\rho U_c^2} = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= \frac{1}{2}\left(e^{i\omega t}\sqrt{i\omega}\ erf\left(\sqrt{i\omega t}\right)+e^{-i\omega t}\sqrt{-i\omega}\ erf\left(\sqrt{-i\omega t}\right)\right)$$

$$+\frac{G_r\sqrt{P_r}}{\sqrt{P_r}+1}\left[\left(1+2\sqrt{\frac{t}{P_r\pi}}\right)-e^{\frac{t}{P_r}}\left(1+erfc(\frac{t}{P_r\pi})\right)\right]-\frac{G_m}{S_c-1}\left(2\left(\sqrt{\frac{S_c t}{\pi}}-\sqrt{\frac{t}{\pi}}\right)\right) \quad (21)$$

5. Nusselt number

An important phenomenon in this study is to understand the effect of t, F and H on the Nusselt number. In non-dimensional form, the rate of heat transfer is given:

$$Nu = -\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \frac{1}{\theta(0,t)} + 1 = \frac{1}{e^{\frac{t}{Pr}}\left(1 + erf\left(\sqrt{\frac{t}{P_r}}\right)\right) - 1} + 1 \tag{22}$$

6. Sherwood number

An important phenomenon in this study is to understand the effect of t, S_c on the Sherwood number. In non-dimensional form, the rate of mass transfer is given by:

$$Sh = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = \sqrt{\frac{S_c t}{\pi}} \tag{23}$$

where $b = -1/\sqrt{\mathrm{Pr}}$, $a_1 = bG_r/\mathrm{Pr}-1$, $a_2 = -(G_m/(S_C-1))$, $\eta = y/2\sqrt{t}$

7. Results and discussion

Numerical evaluation of the analytical results reported in the previous section was performed and a representative set of results is reported graphically through Figures 2-16. These results are obtained to illustrate the influence of the various physical parameters like Schmidt number Sc, Grahof number Gr, modified Grashof number Gm and time t on velocity profiles.

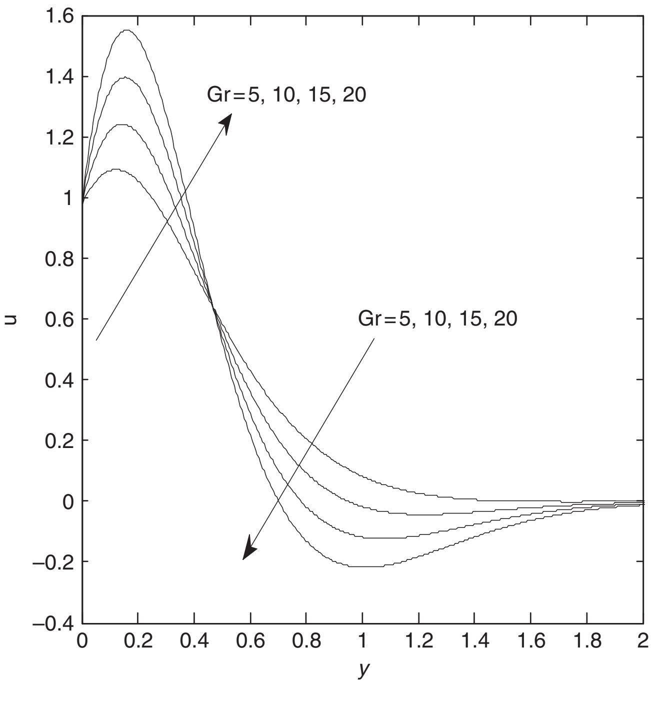

Figure 2.
Effects of Gr on velocity when $G_m = 0.5$, $Pr = 0.71$, $S_c = 0.70$, $t = 0.1$, $\omega = \pi/3$

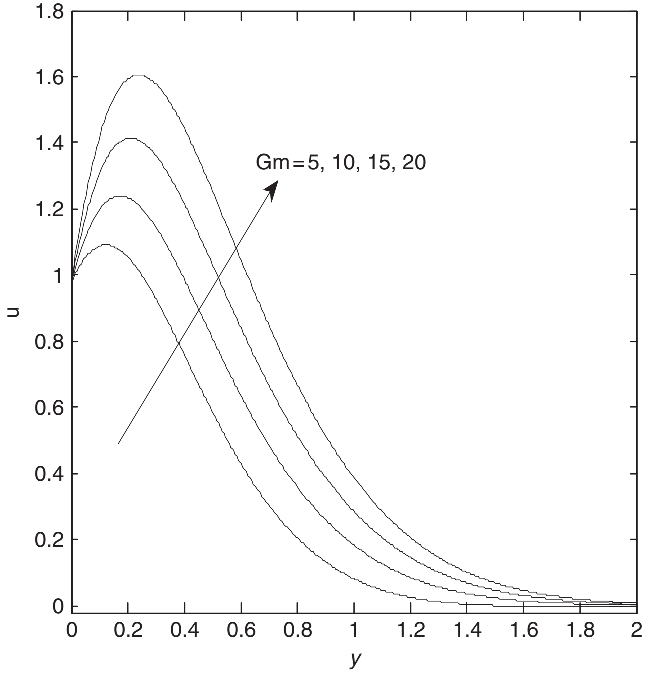

Figure 3. Effects of Gm on velocity when $G_r = 5$, $Pr = 0.71$, $S_c = 0.70$, $t = 0.1$, $\omega = \pi/3$

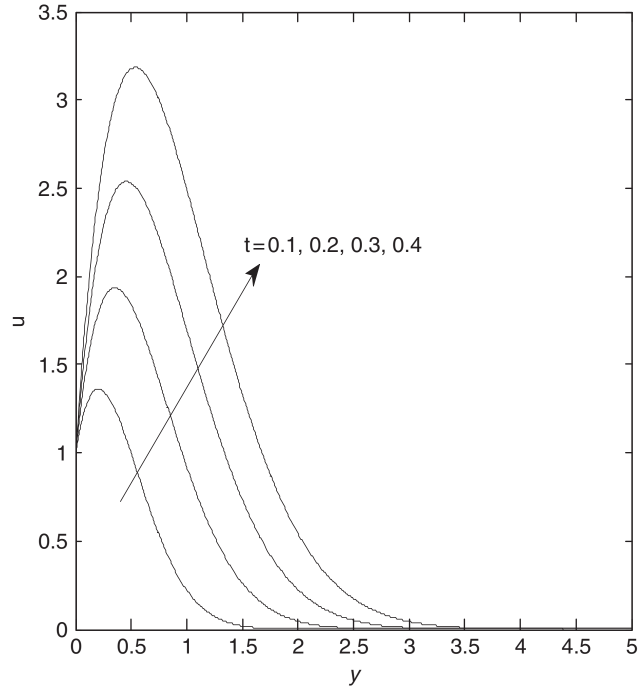

Figure 4. Effect of t on velocity when $G_m = 1$, $G_r = 5$, $P_r = 0.71$, $S_c = 0.70$, $\omega = \pi/3$

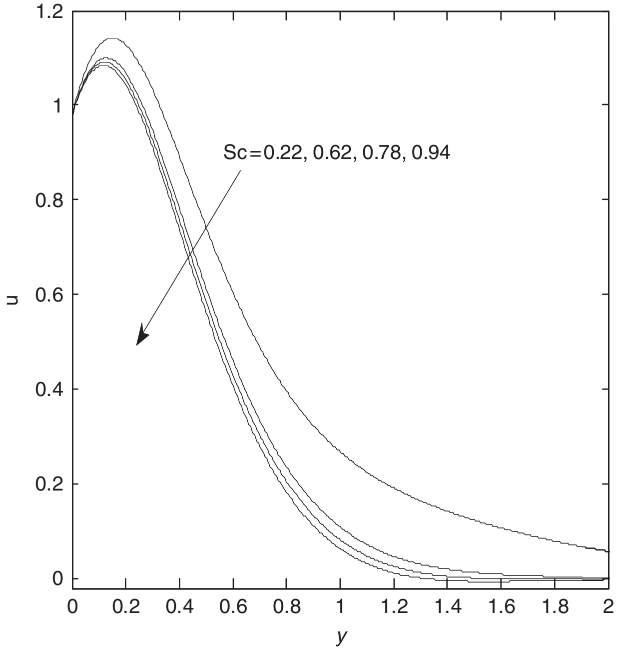

Figure 5.
Effect of Sc on velocity when $G_m = 0.5$, $Pr = 0.71$, $G_r = 5$, $t = 0.1$, $\omega = \pi/3$

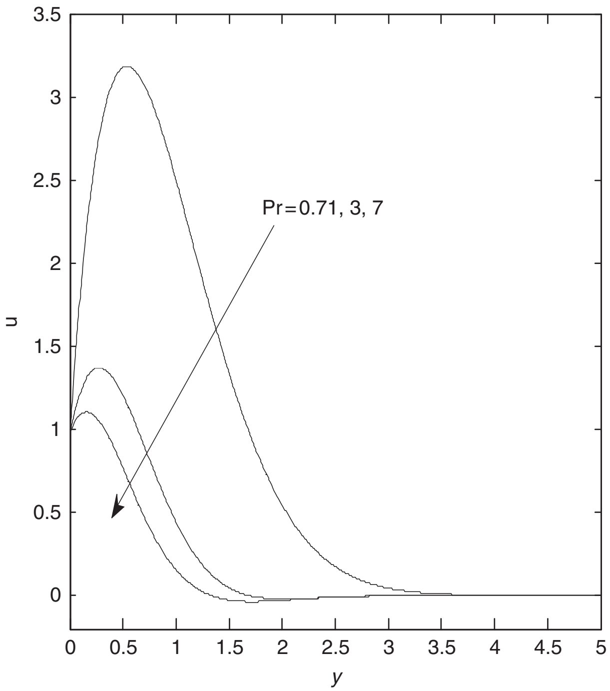

Figure 6.
Effect of P_r on velocity when $G_m = 2$, $G_r = 1$, $Sc = 0.70$, $t = 0.1$, $\omega = \pi/6$

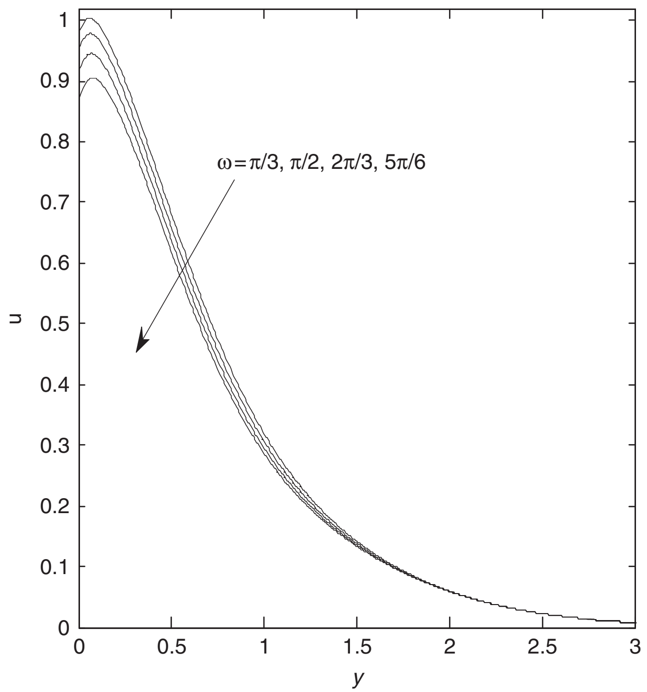

Figure 7.
Effect of ω on velocity when $G_m = 2$, $G_r = 1$, $Sc = 0.70$, $t = 0.1$, $\omega = \pi/6$

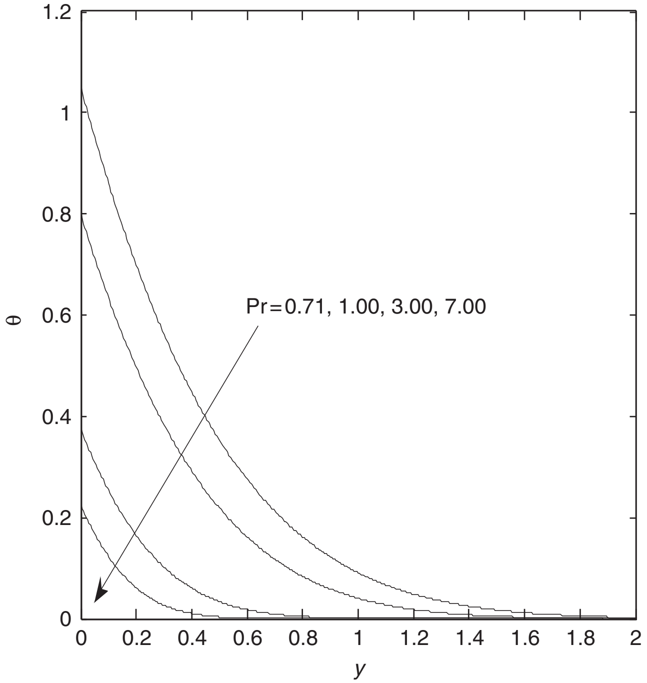

Figure 8.
Effect of P_r on temperature

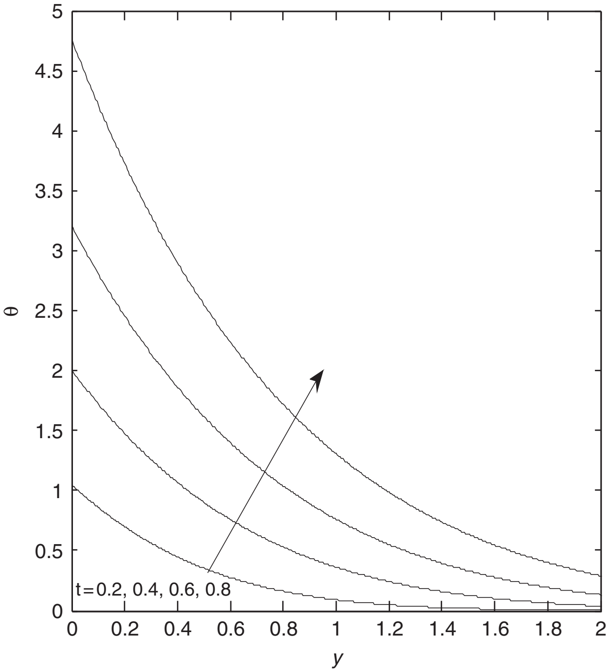

Figure 9.
Effect of t on temperature

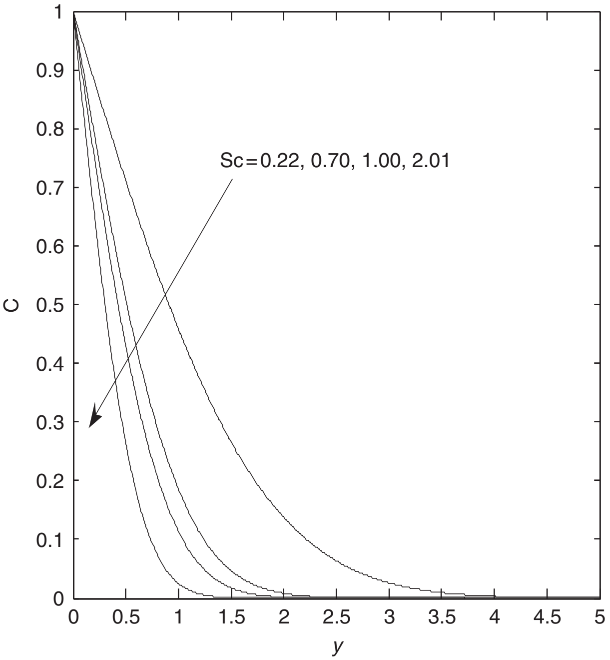

Figure 10.
Effect of Sc on concentration

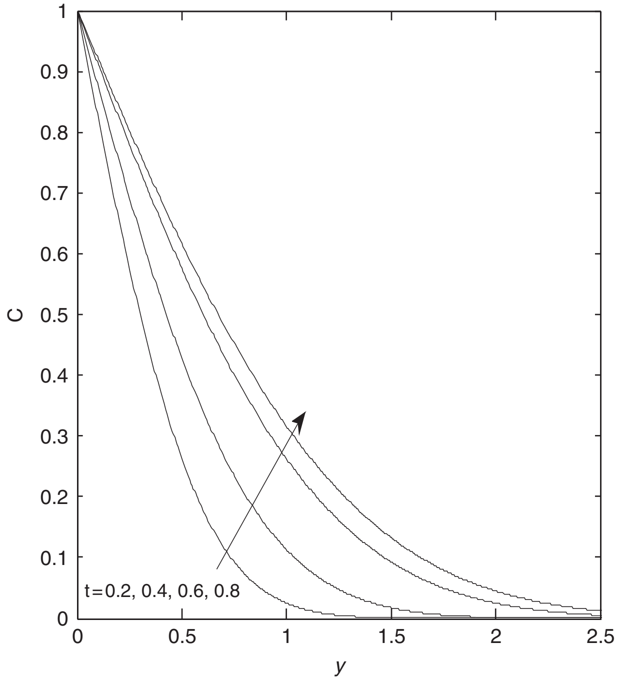

Figure 11.
Effect of t on concentration

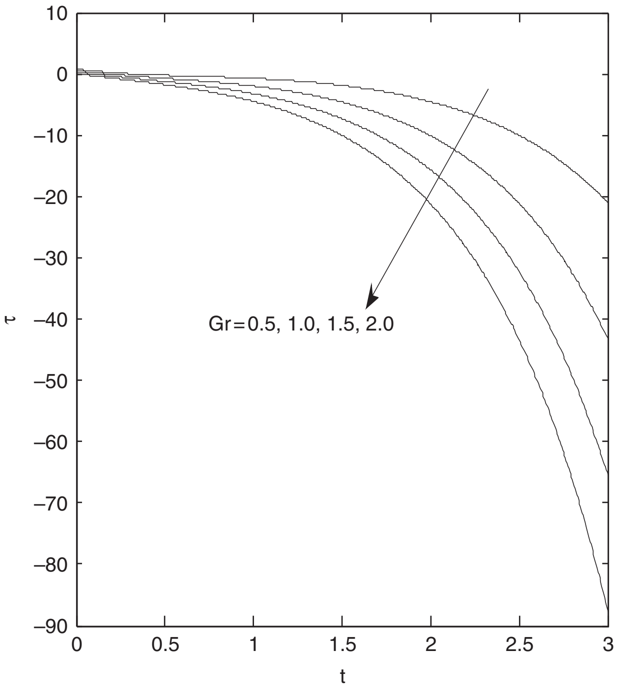

Figure 12.
Effect of Gr on skin-friction

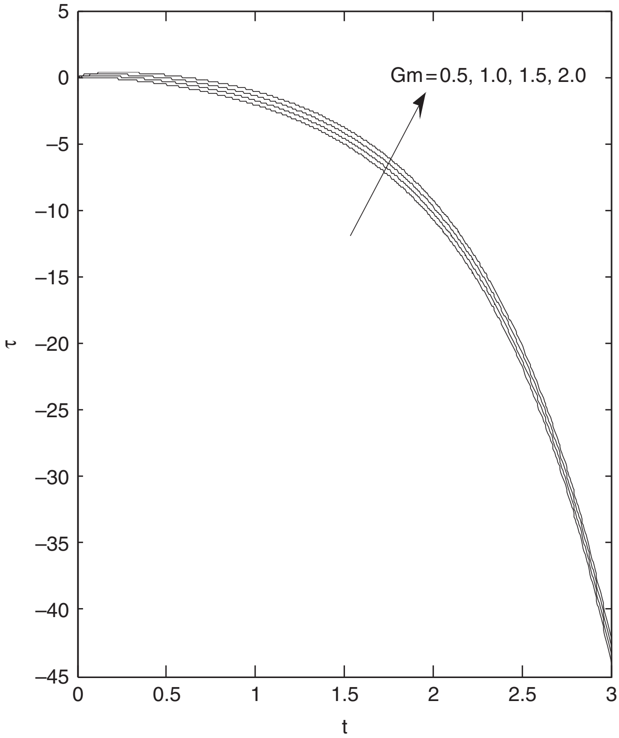

Figure 13.
Effect of Gm on skin-friction

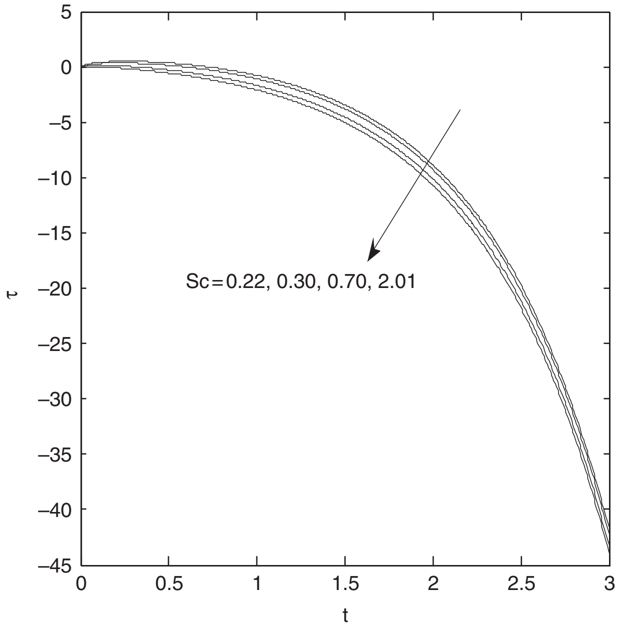

Figure 14.
Effect of Sc on skin-friction

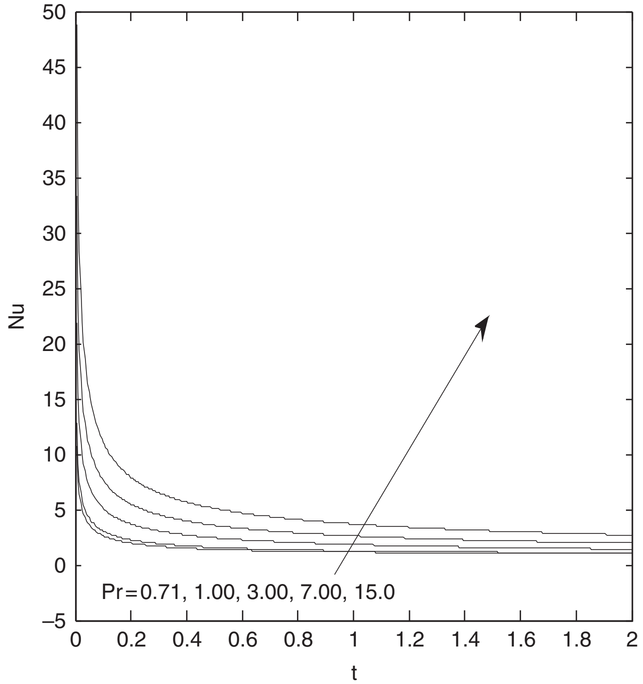

Figure 15.
Effect of Pr
Nusselt number

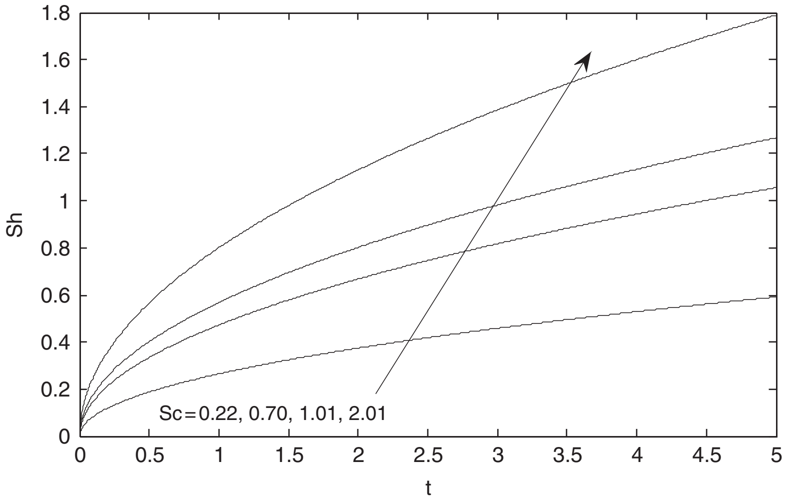

Figure 16.
Effect of Sc on
Sherwood number

Figure 2 depicts the variations in velocity distribution with the variations in Grashof number. From this figure we notice that for $0 < y < 0.5$ velocity is observed to increase with an increase in the values of Grashof number. Thereafter from $0.5 < y < 2$ in the entire region velocity is observed to decrease with an increase in the values of Grashof number. This is due the presence of buoyancy fore acting on the fluid flow. In Figure 3, velocity profiles are displayed with the variations in modified Grashof number. This figure witnesses the growth in the boundary layer with an increase in the values of modified Grashof number. Velocity profiles in the variations in t are presented through Figure 4, as expected velocity increases as t increases.

Figure 5 depicts the effect of Schmidt number on velocity and from this figure it is observed that velocity decreases as Sc increases. This is true physically, because an increase in concentration levels results a decrease in momentum boundary layer. Effect of Prandtl number on velocity is displayed in Figure 6. It is very interesting to notice that velocity decreases as Pr increases. Prandtl number is the ratio of kinematic viscosity and thermal diffusivity. If other parameters remain constant, an increase in kinematic viscosity *vis-à-vis* momentum diffusivity leads to enhance the velocity boundary layer thickness. The reason is that smaller values of Pr are equivalent to increasing the thermal conductivities, and therefore, heat is able to differ away from the heated surface more rapidly than for higher values of Pr. Hence, the boundary layer is thicker and the rate of heat transfer is reduced. Thus higher Prandtl number fluid causes lower thermal diffusivity and hence reduces the temperature at all points. Effects of ω on velocity is present in Figure 7, in which it is seen that velocity decreases as ω increases. From Figures 12-14, we observe that skin friction decreases with an increase in Schmidt number Sc and Grashof number Gr. Whereas in the case of other parameters such as modified Grashof number Gm increases the skin friction also increase. Temperature profiles with the variations in Prandtl number are displayed through Figure 8. It is seen that temperature decreases with an increase in Prandtl number, whereas temperature increases with an increase in t (Figure 9). Concentration profiles are displayed with the variations in Schmidt number Sc and time t in Figures 10-11. From these figures it is noticed that concentration decreases with an increase in Sc whereas reverse trend is observed in the case of time t. The effects of all these parameters on skin friction, Nusselt number and Sherwood number are presented in Figures 12-16. From Figure 15, it is seen that where Prandtl number Pr increases then Nusselt number also increases. It is noticed that from Figure 16, Sherwood number Sh increases when Schmidt number Sc increases.

8. Conclusions

In this paper we have studied heat and mass transfer effects on unsteady free convection boundary layer flow past a periodically accelerated vertical surface with NH, in the closed form by using the Laplace transform technique. From this study we conclude that:

- velocity increases with an increase in Gr, Gm, t where as it decreases in the presence of Sc and Pr;
- temperature increases with an increase in t and decreases in the presence of Pr, similarly concentration increases when t increases where as it decreases in the presence of Sc; and
- skin friction decreases with the increasing values of Sc and Gr but it shows reverse phenomenon in the case of Gm, whereas Nusselt number is observed to increase with the increasing values of Pr and also Sherwood number increases with an increase in Sc.

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