MHD mixed convection of localized heat source/sink in a nanofluid-filled lid-driven square cavity with partial slip

A.M. Rashad, Muneer A. Ismael, Ali J. Chamkha, M.A. Mansour

Abstract

Magneto-hydrodynamic mixed convection in a lid-driven square cavity filled with Cu–water nanofluid is investigated in this paper. Partial slip effect is considered along the lid driven horizontal walls. A segment of the left wall is considered as a heat source, meanwhile a heat sink is placed on the right wall of cavity. The remainder cavity walls are thermally insulated. A control finite volume method is adopted as a numerical approach of the present study. The study is achieved by controlling the effect of a set of pertinent parameters, these are; the size and position of the heat source/sink ($B = 0.2–0.8$, $D = 0.3–0.7$, respectively), the Hartman number ($Ha = 0–100$), Richardson number ($Re = 0.001–10$), nanoparticle volume fraction ($\phi = 0.0–0.1$), partial slip parameter ($S = 1–\infty$), and the lid-direction of the horizontal walls ($\lambda = \pm 1$) where the positive sign means lid-driven to the right while the negative sign means lid-driven to the left. The results show that the shortest length of the heat source/sink localized midway of the vertical walls give the maximum convective heat transfer, and the best direction of the horizontal walls is that when they are both lid-driven to the left. For very strong applied magnetic field, the lid-direction becomes inactive.

1. Introduction

During the last 2 decades, the demands of convective heat transfer enhancement inside enclosures have become an insistent demand due to their ever increasing applications in lubrication technologies, electronic cooling, food processing and nuclear reactors [1–5].

Mixed convection flow and heat transfer in enclosures are encountered in a number of industrial applications such as, float glass manufacturing, solidification of ingots, coating or continuous reheating furnaces, and any application possesses a solid material motion inside a chamber. The mixed convection flow in lid-driven cavity or enclosure is raised from two mechanisms. The first is due to shear flow caused by the movement of one (or two) of the cavity wall(s) while the second is due to buoyancy effect induced by the nonhomogeneous thermal boundaries. The contribution of shear force caused by movement of wall and the buoyancy force by temperature difference make the heat transfer mechanism complex. Cavity flow simulation was introduced in early works of Torrance et al. [6] and Ghia et al. [7]. During those days, the performance of computers was not as accurate as it is nowadays. As the performance of the technology of hardware as well as software improves, simulation has become a simple task. As such, lid-driven cavities are found studied in various situations such as pure or nanofluid-filled cavities, and pure or nanofluid saturated porous medium cavities. Lid-driven cavities filled with pure fluids as in Koseff and Prasad [1], Mohamad and Viskanta [8], Mekrouss et al. [9] Sivasankaran et al. [10], or in nanofluid filled cavities as in Tiwari and Das [11], Talebi et al. [12], Abu-Nada and Chamkha [13], Chamkha and Abu Nada [14]. The effect of magnetic field on natural convection in nanofluid filled cavities is addressed in Sheikholeslami et al. [15] and Sheikholeslami and Rashidi [16]. Lid-driven pure fluid filled cavities are studied by Oztop [17] and Mutahhamiselvan et al. [18]. Lid-driven porous cavity is addressed in Khanafa and Chamkha [19]. Lid-driven nanofluid saturated porous cavities have been recently addressed in Sun and Pop [20], Chamkha and Ismael [21], and Bourantas et al. [22].

In some applications like fluoroplastic coating (e.g., Teflon) which resists adhesion, the no-slip boundary condition imposed on the tangential velocity cannot be held. Moreover, some surfaces are rough or porous such that equivalent slip occurs (Wang...
Also, there exists a hydrodynamic boundary slip regime for rarefied gases when the Knudsen number is small (Sharipov and Seleznev [24]). Dealing with such problems is strictly embargo to consider Navier's slip-boundary condition [25] along these surfaces. Generally, the physical interpretation of the velocity slip on solid boundary arises from the unequal wall and fluid densities, the weak wall–fluid interaction, and the high temperature [26]. However, the studies dealing with slip boundary conditions may be achieved in order to simulate engineering problems, Fang et al. [27] and Yoshimura and Prudhomme [28], or to solve the problem of nonphysical singularity resulting from meeting stationary and moving walls, Navier [25], Koplik and Banavar [29], Qian and Wang [30], Nie et al. [31], Ismael et al. [32] have considered the partial slip condition along two horizontal isothermal moving walls under steady laminar mixed convection inside lid-driven square cavity. Their results have showed that there were critical values of the partial slip parameter at which the convection is declined. These nonzero critical slips were found to be sensitive to Richardson number and the lid-driven direction. Alternatively, Soltani and Yilmazer [33] have reported that the wall slip can occur in the working fluid contains concentrated suspensions. Recently, there are some studies that consider slip boundary condition in nanofluid fill cavity, as in Malvandi and Ganji [34], and Mabood and Mastroberardino [35].

The present literature survey has led us to confirm that there is, relatively, a very little published works regarding the slip boundary condition in the lid-driven cavities. Moreover, the topics of nanofluids and magneto-hydrodynamic with partial slip have not clearly arisen yet. Accordingly, the present work is prepared as an attempt to fill this gap. The present geometry is a square cavity filled with nanofluid subjected to an externally applied magnetic field. The horizontal walls are thermally insulated (adiabatic) and lid-driven with partial slip, the vertical walls are also adiabatic but contain two segments as heat source/sink. These two segments are geometrically identical. It is thought that this work will contribute in finding new parameters arrangements those govern the performance of the lid driven cavities especially those which hold very high temperature difference where the partial slip is inevitably exist.

2. Mathematical modeling

Consider a steady two-dimensional mixed convection flow inside a square cavity of side length \( H \) filled with nanofluid, as depicted in Fig. 1. The coordinates \( x \) and \( y \) are chosen such that \( x \) measures the distance along the bottom horizontal wall, while \( y \) measures the distance along the left vertical wall, respectively. Heat and sink sources are located on a part of the left and right walls with length \( B \). The upper and bottom walls are adiabatic and move. The nanofluids used in the analysis are assumed to be incompressible and laminar, and the base fluid (water) and the solid spherical nanoparticles (Cu) are in thermal equilibrium. The thermophysical properties of the base fluid and the nanoparticles are given in Table 1 [21]. The thermophysical properties of the nanofluid are assumed constant except for the density variation, which is determined based on the Boussinesq approximation.
Table 1

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Copper (Cu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>9971</td>
<td>8933</td>
</tr>
<tr>
<td>$C_p$ (J/Kg K)</td>
<td>4179</td>
<td>385</td>
</tr>
<tr>
<td>$k$ (W/m K)</td>
<td>0.613</td>
<td>401</td>
</tr>
<tr>
<td>$\beta$ (K$^{-1}$)</td>
<td>$21 \times 10^{-5}$</td>
<td>$1.67 \times 10^{-5}$</td>
</tr>
<tr>
<td>$\sigma$ (S/m)</td>
<td>0.05</td>
<td>$5.96 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>100</td>
<td>1.94</td>
<td>2.01</td>
<td>1.93</td>
</tr>
<tr>
<td>400</td>
<td>3.84</td>
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<td>3.91</td>
</tr>
<tr>
<td>1000</td>
<td>6.33</td>
<td>6.33</td>
<td>6.31</td>
</tr>
</tbody>
</table>

Under the above assumptions, the conservation of mass, linear momentum, and also conservation of energy equations are given in [4]. We can write:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \]  

\[ u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \nu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \]  

\[ u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \nu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\sigma_{nf} B_0^2}{\rho_{nf}} \nu \]  

\[ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \]  

where $u$ and $v$ are the velocity components along the $x$- and $y$-axes respectively, $T$ is the fluid temperature, $p$ is the fluid pressure, $g$ is the gravity acceleration, $\rho_{nf}$ is the density, $\mu_{nf}$ is the dynamic viscosity, $\nu_{nf}$ is the kinematic viscosity.

The boundary conditions are:

On the left wall, $x = 0$:

\[ u = v = 0, \quad T = T_b \quad (D - 0.5B) \leq \frac{y}{H} \leq (D + 0.5B), \quad \frac{\partial T}{\partial x} = 0 \text{ otherwise} \]

On the right wall, $x = H$:

\[ u = v = 0, \quad T = T_r \quad (D - 0.5B) \leq \frac{y}{H} \leq (D + 0.5B), \quad \frac{\partial T}{\partial x} = 0 \text{ otherwise} \]

On the top wall, $y = H$ (partial slip):

\[ v = 0, \quad u = \lambda_u U_x + N \mu_{nf} \frac{\partial u}{\partial y} \quad \frac{\partial T}{\partial y} = 0 \]

On the bottom wall, $y = 0$ (partial slip):

\[ v = 0, \quad u = \lambda_b U_x + N \mu_{nf} \frac{\partial u}{\partial y} \quad \frac{\partial T}{\partial y} = 0 \]

Numerous formulations for the thermophysical properties of nanofluids are proposed in the literature. In the present study, we are adopting the relations which depend on the nanoparticles volume fraction only and which were proven and used in many previous studies (Aminossadati and Ghasemi [36] and Khanafar et al. [37]) as follows:

The effective density of the nanofluid is given as:

\[ \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_p, \]  

where $\phi$ is the solid volume fraction of the nanofluid, $\rho_f$ and $\rho_p$ are the densities of the fluid and of the solid particles respectively, and the heat capacitance of the nanofluid is given by Khanafar et al. [37] as:

\[ (\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_p. \]  

The thermal expansion coefficient of the nanofluid can be determined by:

\[ (\rho \beta)_nf = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_p \]  

where $\beta_f$ and $\beta_p$ are the coefficients of thermal expansion of the fluid and of the solid particles respectively.

Thermal diffusivity, $\alpha_{nf}$ of the nanofluid is defined by Abu-Nada and Chamkha [38] as:

\[ \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \]  

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**Fig. 2.** (a) Streamlines and (b) isotherms for $Re = 1, Ha = 10, \phi = 0.05, D = 0.5, \lambda_1 = \lambda_2 = -1, S_b = S_t = 1.$
In Eq. (9), \( k_{nf} \) is the thermal conductivity of the nanofluid which for spherical nanoparticles, according to the Maxwell–Garnett model [39], is:

\[
k_{nf} = \left( k_p + 2k_t \right) - 2\phi(k_t - k_p).
\]

(10)

The effective dynamic viscosity of the nanofluid based on the Brinkman model [40] is given by

\[
\mu_{nf} = \frac{\mu_t}{1 - \phi}.
\]

(11)

where \( \mu_t \) is the viscosity of the base fluid and the effective electrical conductivity of nanofluid was presented by Maxwell [39] as:

\[
\sigma_{nf} = \frac{1 + \frac{3}{2}(\gamma - 1)\phi}{(\gamma + 2) - (\gamma - 1)\phi} \sigma_t.
\]

(12)

where \( \gamma = \frac{\sigma_f}{\sigma_t} \).

The electrical conductivity \( \sigma_{nf} \) of the nanofluid is defined [39] by

\[
\sigma_{nf} = \frac{1 + \frac{3}{2}(\frac{\sigma_f}{\sigma_t} - 1)\phi}{(\frac{\sigma_f}{\sigma_t} + 2) - (\frac{\sigma_f}{\sigma_t} - 1)\phi} \sigma_t.
\]

(13)

Introducing the following dimensionless set:

\[
X = \frac{x}{H}, \quad Y = \frac{y}{H}, \quad U = \frac{u}{U_0}, \quad V = \frac{v}{V_0}, \quad P = \frac{p}{\rho_0 U_0^2}, \quad \theta = \frac{T - T_0}{\Delta T}.
\]

(14)

into Eqs. (1)–(4) yields the following dimensionless equations:

\[
\frac{\partial U}{\partial X} = 0
\]

(15)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right).
\]

(16)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ri \frac{\sigma_{nf}}{\rho_0 \mu_t} \theta - \left( \frac{\sigma_{nf}}{\sigma_t} \right) \frac{Ha^2}{Re} V.
\]

(17)

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{PrRe} \alpha_t \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right).
\]

(18)
where

\[ Pr = \frac{V_f}{\alpha_f}, \quad Re = \frac{\text{Re}_f}{U_f}, \quad Gr = \frac{g\beta_f H^3 \Delta T}{V_f^3}, \quad Ha = B_0 H \sqrt{\alpha_f/\mu_f} \]

are respectively the Prandtl number, the Reynolds number, the Grashof number and the Hartman number.

The dimensionless boundary conditions for Eqs. (15)-(18) are as follows:

On the left wall, \( X = 0 \)

\[ U = V = 0, \quad \theta = 0.5 \quad (D - 0.5B) \leq Y \leq (D + 0.5B), \quad \frac{\partial \theta}{\partial X} = 0 \] otherwise

On the right wall, \( X = 1 \)

\[ U = V = 0, \quad \theta = -0.5 \quad (D - 0.5B) \leq Y \leq (D + 0.5B), \quad \frac{\partial \theta}{\partial X} = 0 \] otherwise

On the top wall, \( Y = 1 \) (partial slip)

\[ V = 0, \quad U = \lambda_2 + S_5 \frac{\mu_\text{nf}}{\mu_f} \frac{\partial U}{\partial Y}, \quad \frac{\partial \theta}{\partial Y} = 0 \]

On the bottom wall, \( Y = 0 \) (partial slip)

\[ V = 0, \quad U = \lambda_3 + S_6 \frac{\mu_\text{nf}}{\mu_f} \frac{\partial U}{\partial Y}, \quad \frac{\partial \theta}{\partial Y} = 0 \]

where

\[ S_5 = S_6 = \frac{N}{H} \mu_f \]

The local Nusselt number is defined as:

\[ Nu_f = \int_{X=0}^{X=0.1} \frac{\partial \theta}{\partial X} \, dX \]

and the average Nusselt number is defined as:

\[ Nu_{m,0} = \frac{1}{B} \int_{D-0.5B}^{D+0.5B} Nu_f \, dY \quad X=0 \]

\[ Nu_{m,1} = \frac{1}{B} \int_{D-0.5B}^{D+0.5B} Nu_f \, dY \quad X=1 \]

\[ Nu_m = \frac{Nu_{m,0} + Nu_{m,1}}{2} \]

3. Numerical method and validation

Eqs. (15)-(18) with the boundary conditions (19) have been solved numerically using the collocated finite volume method. The first upwind and central difference approaches have been used to approximate the convective and diffusive terms, respectively. The resulting discretized equations have been solved iteratively, through alternate direction implicit ADI, using the SIMPLE algorithm [41]. The velocity correction has been made using the Rhie and Chow interpolation. For convergence, under-relaxation technique has been employed. To check the convergence, the mass of each control volume has been calculated and the maximum value has been used to check the convergence. 10^{-5} was set as the convergence criterion. A uniform grid resolution of 81 \times 81 is found to be suitable. In order to verify the accuracy of

![Fig. 6. (a) Streamlines and (b) Isotherms for Ri = 1, Ha = 10, \phi = 0.05, B = 0.5, \lambda_2 = \lambda_3 = -1, S_5 = S_6 = 1.](image-url)
present method, the obtained results in special cases are compared with the results obtained by Iwatsu et al. [4] and Khanaf er and Chamkha [19] in terms of the mean Nusselt number at the top wall, for different values of Re. As we can see from Table 2, the results are found in a good agreement with these results. These favorable comparisons lend confidence in the numerical results to be reported subsequently.

4. Results and discussion

The numerical results of the current study are elucidated graphically via stream function contours (streamlines), dimensionless temperature contours (isotherms), local and average Nusselt numbers, and in some situations the local horizontal and vertical velocities. The pertinent parameters those have a direct effect on the flow and thermal fields inside the considered cavity are: the size and position of the heat source/sink \( B = 0.2–0.8, D = 0.3–0.7 \), respectively, Hartman number \( Ha = 0–100 \), Richardson number \( Ri = 0.001–10 \), nanoparticles volume fraction \( \phi = 0.0 – 0.1 \), slip parameter \( S = 0–20(\infty) \), and the constant moving parameter \( \lambda = \pm 1 \). The latter two parameters govern the slip amount and the direction of the moving horizontal walls, respectively. In order to display the results of these seven independent parameters, six parameters are fixed (unless where stated) while the remainder single one is varied as gathered in the following categories.

4.1. Effect of size and position of the heat source/sink

In order to study the effect of the size (\( B \)) and the position (\( D \)) of the heat source/sink, the other parameters are fixed at \( Ri = 1, \phi = 0.05, Ha = 10, \lambda_1 = \lambda_2 = -1, S_1 = S_2 = 1 \). Fig. 2 shows the effect of \( B \) on the streamlines and isotherms while \( D \) is fixed at 0.5. The streamlines are formed in clockwise rotating vortex with double-eye core. A crowded streamlines close to the horizontal walls which both move to the left (\( \lambda_1 = \lambda_2 = -1 \)) indicate to intensified flow there. Increasing \( B \) leads to the merging of double-eye into one-eye core as shown in Fig. 2a. The isotherms are clearly distinguished with increasing \( B \) where a dense isotherms confinement to the area of the heat source/sink effect exists. The isotherms are mostly horizontal and their gradients are vertically extended with increasing \( B \) as shown in Fig. 2b.

The vertical \( V \) and horizontal \( U \) velocity components are plotted in Fig. 3 along \( Y = 0.5 \) and \( X = 0.5 \), respectively. Two peaks of \( V \) are recorded close to the vertical walls with higher velocity values associated with increasing \( B \). The horizontal velocity components reveals that the \( U \) component is negative close to the bottom wall. This is an expected result because the boundary effect coincides with the driven bottom wall (\( \lambda_2 = -1 \)). The top wall is driven to the left too (\( \lambda_1 = -1 \)), nevertheless, the \( U \) values are positive (to the right), this indicates that the buoyancy driven flow overcome the shear action exerted by the top wall and this is due to the slip effect (\( S_1 = 1 \)) which reduces the shear action.

The local Nusselt number distribution along the heat source/sink is depicted in Fig. 4. It shows that the maximum local Nusselt number occurs at the lower edge of the heat source while for the heat sink, the maximum local Nusselt occurs at its upper edge. The overall convective heat transfer is shown by the average Nusselt number in Fig. 5 with nanoparticles volume fraction \( \phi \) for different \( B \) values. The effect of volume fraction will be postponed to the relevant category. However, for a given volume fraction, the average Nusselt number increases with decreasing the heat source size and vice versa. We can refer to Fig. 2b to make use of the isotherms which elucidate that increasing the heat source size, the isotherms pattern manifests large portion of heat which is transferred by conduction, hence, a decrease of the convection heat transfer occurs accordingly.

The effect of heat source/sink position (\( D \)) on streamlines and isotherms is shown in Fig. 6 for fixed size, \( B = 0.5 \). It can be seen from Fig. 6a that the double-eye core is significantly affected by \( D \), where these two eyes merge in a single eye core with increasing \( D \). The corresponding isotherms (Fig. 6b) implies that the exchange of heat transfer by convection is shifted according to the \( D \) value forming an isothermal zone at upper of the cavity for lower value
of $D (= 0.3)$, and a lower isothermal zone for higher $D$ value ($= 0.7$). The distribution of the local Nusselt number (Fig. 7) presents similar patterns for all $D$ values, but gives no clear indication about the best $D$ value that manifest the greatest convective heat transfer rate. Hence, the average Nusselt number which is shown in Fig. 8 indicates to that the maximum convective heat transfer occurs when the heat source/sink is placed midway on the vertical walls, otherwise, a deterioration in the values of $Nu_{m}$ takes place. This can be attributed to the isothermal zones formed at low and high $D$ values which prevents a substantial amount of nanofluid to participate in convective heat transfer.

4.2. Effect of Hartman number

In this category, the Hartman number is varied from $Ha = 0$–100 while the other parameters are fixed at $Ri = 1, \phi = 0.5, B = 0.5, D = 0.5, \lambda_{A} = \lambda_{B} = -1$, and $S_{m} = S_{f} = 1$. It is well known that applying an external magnetic field, Lorentz force will be generated perpendicularly to the direction of the applied magnetic field. Therefore, in the current study, Lorentz force will act normally in the negative $Y$-direction. Accordingly, the streamlines presented in Fig. 9a weaken and the core vortex starts to split at $Ha = 50$ until they formed in two clock wise rotation vortices adjacent to moving horizontal walls. The corresponding isotherms (Fig. 9b) are transmitted from convection pattern at $Ha = 0$ to mostly vertical pattern with increasing $Ha$ which indicates to the suppression of convection due to the effect of Lorentz force. The thermal boundary layers close to the heat source/sink are also attenuated with increasing $Ha$. Due to this, the Nusselt number shown in Fig. 10 indicates to significant decrease with increasing $Ha$. As the size and position of the heat source/sink of Fig. 10 are fixed, hence there is no need to plot the average Nusselt number.

4.3. Effect of Richardson number

Richardson number is a measure to the ratio of natural to forced convection modes. Its effect is studied by fixing the other independent parameters at $Ha = 10, \phi = 0.05, B = 0.5, D = 0.5, \lambda_{A} = \lambda_{B} = -1$, and $S_{m} = S_{f} = 1$. The streamlines contours depicted in Fig. 11 shows some perturbations at the left upper and the
right lower corners when $Ri = 0.001$. This is due to the dominance of forced convection exerted by the movement of the horizontal walls. The double-eye core coincides with the diagonal of the cavity. Increasing $Ri$ to 0.01, the double-eye behavior recovers its previous patterns shown previously. When $Ri$ increased to 10, a single vortex occupying the whole cavity is obtained. This can be attributed to the counter actions of the natural and forced convection modes. The isotherms of $Ri \leq 0.01$ manifests steeper temperature gradient close to the heat source/sink with isothermal zone within mostly the entire cavity. This pattern is destroyed by increasing $Ri$ greater than 0.01, where mostly horizontal isotherms are the dominant when $Ri \geq 10$ (see Fig. 11b).

The local Nusselt number distribution presented in Fig. 12 implies that the maximum convection heat transfer is associated with the lowest value of the Richardson number. This refer to that when the horizontal walls move to the left with slip parameter $S_1 = S_2 = 1$, they generate a considerable mixing which enhance the convective heat transfer. When $Ri$ increased, the induced natural convection acts in opposite manner that promoted by the shear action which in turn decreased the Nusselt number. This fact is evident by the average Nusselt number plot shown in Fig. 13.

4.4. Effect of nanoparticles volume fraction

The nanoparticles volume fraction is increased from $\phi = 0$ (pure water) to $\phi = 0.1$ for $Ri = 1$, $Ha = 10$, $B = 0.5$, $D = 0.5$, $\lambda_1 = \lambda_2 = -1$, and $S_1 = S_2 = 1$. The vertical double-eye behavior of the streamlines associated with pure water ($\phi = 0$) is transformed into a single-eye behavior with the increase in the nanoparticles volume fraction ($\phi \geq 0.07$). The isotherms (Fig. 14b) are, generally, not affected by adding the nanoparticles with any fraction except that with increasing $\phi$ the isotherms close to the heat source/sink begin to be more scattered.

Adding the Cu nanoparticles to the pure water will generate a nanofluid with higher viscosity, higher density, and higher thermal conductivity. The first two properties increases the viscous and...
inertia forces, respectively, while the enhanced thermal conductivity increases the transferred thermal energy. However, the distribution of Nusselt number presented in Fig. 15 elucidates that the enhanced viscous and inertia forces dominant over the enhanced thermal conductivity and both buoyancy and shear effects in addition. This is a reasonable reason for the deterioration of Nusselt number with addition/increase of nanoparticles.

4.5. Effect of partial slip parameter

The partial slip parameter \( S \) of the top and bottom walls is studied for \( Ha = 10, \, Ri = 1, \phi = 0.05, \, B = 0.5, \, D = 0.5, \, \lambda_t = \lambda_b = -1, \) and \( S_t = S_b = 1. \) For both moving horizontal walls, the values of \( S \) are varied in the same magnitude i.e., \( S_t = S_b = 0, 1, 5, \) and 10, 20 (\( \infty \)). For \( S_t = 0, \) the shear action completely drive the nanofluid to the left direction (\( \lambda_t = \lambda_b = -1, \)) as such, two main counter rotating vortices are seen as shown in Fig. 16a, the upper vortex rotates counter clockwise (positive stream function), and the lower one rotates clockwise (negative stream function). Increasing \( S \), means reducing the effect of lid-driven which in turn permits to the dominance of the buoyancy force effect which forms the streamlines in single clockwise rotating vortex with double-eye for \( S = 1, \) and one eye for \( S > 1 \) as shown in Fig. 16a. The corresponding isotherms (Fig. 16b) depicts a disturbed distribution for \( S = 0 \) with steep gradients along the heat source/sink. For \( S > 0, \) the isotherms manifest the stratification pattern.

The local Nusselt number shown in Fig. 17 tell us that the maximum convection is obtained when \( S = 0, \) this is clear from imagining the area under curves of the heat source (\( X = 0 \)) plot. This is an expected result due to the efficient mixing associated when \( S = 0. \)

4.6. Effect of moving parameter

Four combinations of horizontal walls lid-driven direction is studied and shown in Fig. 18 for \( Ha = 10, \, Ri = 1, \phi = 0.05, \, B = 0.5, \, D = 0.5, \) and \( S_t = S_b = 1. \) These combinations are governed by setting alternatives senses of the moving parameters \( \lambda_t \) and \( \lambda_b. \) Setting \( \lambda_t = 1 \) (the top wall move to the right) and \( \lambda_b = -1 \) (the bottom wall moves to the left) results in the streamlines those formed in a primary clockwise rotating vortex occupies most of the cavity and a secondary counter clockwise rotating vortex localized close to the top wall. It can be concluded that the primary vortex is due to the buoyancy effect while the secondary one is due to the shear action. This conclusion is supported when we set \( \lambda_t = -1 \) and \( \lambda_b = 1 \) where the resulting patterns is an inverted image to the pattern of the previous case. The corresponding isotherms of these two cases manifest mostly vertical isotherms within the region of the secondary vortex. When \( \lambda_t \) and \( \lambda_b \) are set to be in the same right direction (\( \lambda_t = \lambda_b = 1, \)) the nanofluid in the middle cavity undergoes the boundary action which results in a clockwise primary vortex while two stagnant zones localized close to the bottom and top walls are noticed. Again, mostly vertical isotherms are observed within the regions corresponding to the

![Fig. 13](image1.png)

**Fig. 13.** Variation of the average Nusselt number for \( Ha = 10, \, D = 0.5, \, B = 0.5, \, \lambda_t = \lambda_b = -1, \) and \( S_t = S_b = 1. \)

![Fig. 14](image2.png)

**Fig. 14.** (a) Streamlines and (b) isotherms for \( Ri = 1, \, Ha = 10, \, D = 0.5, \, B = 0.5, \, \lambda_t = \lambda_b = -1, \) and \( S_t = S_b = 1. \)
Fig. 15. Profiles of the local Nusselt number for Ri = 1, Ha = 10, φ = 0.05, D = 0.5, B = 0.5, λ = λ₀ = −1, S₀ = S₁ = 1.

Fig. 16. (a) Streamlines and (b) Isotherms for Ri = 1, Ha = 10, φ = 0.05, D = 0.5, B = 0.5, λ = λ₀ = −1, S₀ = S₁.

Fig. 17. Profiles of the Local Nusselt number for Ri = 1, Ha = 10, φ = 0.05, D = 0.5, B = 0.5, λ = λ₀ = −1, S₀ = S₁.
stagnant regions. As previously discussed, the case of $\lambda_b = \lambda_t = -1$ leads to double-eye rotating vortex as shown in Fig. 18a.

Fig. 19 depicts the average Nusselt number for the four combinations of $\lambda$ with $\phi$. The minimum Nusselt number occurs when $\lambda_t = \lambda_b = 1$, while the maximum Nusselt is obtained when $\lambda_t = \lambda_b = -1$, and this is the reason behind why all previous studied cases in the current paper were at this direction of lid. Fig. 20 presents the suppression effect of the applied magnetic field on the convective heat transfer. Moreover, this figure tells that for high Hartman number ($Ha > 20$), the direction of the horizontal walls lid becomes inactive.

It is worth mentioning that various results were collected but for brevity they did not present all. However, it is noticed that the variation of the lid-direction parameter ($\lambda$) has a significant effect on the streamlines and isotherms patterns when the convection mode ratio ($Ri$) is varied. Due to this, it is appropriate to present some of these results within this category.

Fig. 21 present streamlines and isotherms for $\lambda_t = \lambda_b = 1$ and different $Ri$ number. For dominant forced convection, the effect of shear action is associated by the streamlines perturbations in the upper right and the lower left corners with double-eye counter clockwise vortex. Increasing $Ri$ number increases the effect of natural convection which generate a clockwise vortex in the middle cavity. This clockwise vortex grows up to split and displace the counter clockwise vortex into two vortices adjacent to the horizontal moving walls. For dominant natural convection, $Ri = 10$, the clockwise vortex becomes completely dominant within the whole cavity. The stages of streamlines variations are reflected on the disturbed isotherms presented in Fig. 21b.

When the horizontal walls move in counter senses, $\lambda_t = -\lambda_b = 1$, the dominant forced convection ($Ri = 0.001$) results in
two counter rotating vortices as shown in Fig. 22a. Increasing Ri number, the clockwise vortex, which is localized in the lower half of the cavity, is strengthen and extended to squeeze the counter clockwise one, which rotates in the upper half of the cavity, until completely vanished when Ri = 10. The isotherms shown in Fig. 22b manifest a random distribution for dominant forced convection (Ri ≤ 0.01), while they mostly horizontal for dominant natural convection (Ri = 10). Eventually, in both isotherms presented in Figs. 21b and 22b, a steep thermal boundary layer is noticed close to the heat source/sink for dominant forced convection (low values of Ri). Hence a decrease in convective heat transfer with increasing Ri is recorded as shown in Fig. 23. This was previously shown in Fig. 13. For a first time, decreasing Nusselt number with increasing Ri may be controversial, but as explained with the discussion of Fig. 13, the opposite actions of the natural and forced convections and due to the geometry employed in generating the natural convection (differentially heating) give the reasonability to accept such behavior.
5. Conclusions

The current paper studies the steady laminar two dimensions mixed convection inside a square cavity filled with Cu–water nanofluid and subjected to externally applied magnetic field. The horizontal walls are lid-driven by considering the partial slip effect. The numerical results have led to the following conclusions:

1. The shortest length of the heat source/sink gives the maximum convective heat transfer.
2. The best position of heat source/sink that gives maximum convective heat transfer is midway of the vertical walls.
3. The best direction of the horizontal walls is that when they are both lid-driven to the left.
4. For the best direction of lid above, increasing Richardson number decreases the convective heat transfer.
5. The applied magnetic field suppresses the convective heat transfer, i.e., it can be used as a control key as in grain growth or in material solidification.
6. For very high Hartman number, $Ha > 60$, the direction of the horizontal walls lid becomes inaccurate.
7. The existence of nanoparticles leads to the deterioration of the Nusselt number. This behavior may be altered if other models to the thermal conductivity and dynamic viscosity are adopted.
8. Including the effect of partial slip in the lid-driven walls considerably changes the patterns of the streamlines and isotherms and lead to reduction of the effect of shear direction which in turn reduce the convective heat transfer.

References


