SORET EFFECT ON STAGNATION-POINT FLOW PAST A STRETCHING/SHRINKING SHEET IN A NANOFLUID-SATURATED NON-DARCY POROUS MEDIUM

Ch. RamReddy,1,* P. V. S. N. Murthy,2 A. M. Rashad,3 & Ali J. Chamkha4,5

1Department of Mathematics, National Institute of Technology Warangal-506004, India
2Department of Mathematics, Indian Institute of Technology, Kharagpur-721 302, India
3Department of Mathematics, Aswan University, Faculty of Science, Aswan, 81528, Egypt
4Mechanical Engineering Department, Prince Mohammad Bin Fahd University (PMUI), Al-Khobar 31952, Kingdom of Saudi Arabia
5Prince Sultan Endowment for Energy and Environment, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Kingdom of Saudi Arabia

*Address all correspondence to: Ch. RamReddy, E-mail: chittetiram@gmail.com

Original Manuscript Submitted: 10/10/2014; Final Draft Received: 9/8/2016

The significance of the Soret effect on the boundary-layer stagnation-point flow past a stretching/shrinking sheet in a nanofluid-saturated non-Darcy porous medium is investigated in this study. The nanofluid-saturated porous medium is considered by incorporating the Brownian motion and thermophoresis effects. A similarity transformation is used to reduce the governing fluid flow equations into a set of differential equations and then solved numerically by an accurate implicit finite-difference method. The flow; temperature; concentration and nanoparticle concentration fields; skin friction coefficient; and heat, mass, and nanoparticle mass transfer rates are affected by the complex interactions among the various physical parameters involved in the analysis. These profiles are illustrated graphically in order to reveal interesting phenomena.

KEY WORDS: Soret effect, stagnation-point flow, stretching/shrinking sheet, nanofluid, non-Darcy porous medium

1. INTRODUCTION

Nanotechnology is considered by many to be one of the significant forces that will drive the next major industrial revolution of this century. It represents the most relevant technological cutting edge currently being explored. Nanofluid heat transfer is an innovative technology that can be used to enhance heat transfer. Heat transfer is one of the most important processes in many industrial and consumer products. The inherently poor thermal conductivity of conventional fluids puts a fundamental limit on heat transfer. Therefore, for more than a century since the research done by Maxwell (1873), scientists and engineers have made great efforts to break this fundamental limit by dispersing millimeter- or micrometer-sized particles in liquids. However, the major problem with the use of such large particles is the rapid settling of these particles in fluids. Because extended surface technology has already been adapted to its limits in the designs of thermal management systems, once again technologies with
the potential to improve a fluid’s thermal properties are of great interest. The concept and emergence of nanofluids is related directly to trends in miniaturization and nanotechnology. Maxwell’s concept is old, but what is new and innovative in the concept of nanofluids is the idea that particle size is of primary importance in developing stable and highly conductive nanofluids. Nanomaterials have unique mechanical, optical, electrical, magnetic, and thermal properties. Nanofluids are engineered by suspending nanoparticles with average sizes below 50 nm in conventional heat transfer fluids such as water, oil, and ethylene glycol. A very small amount of guest nanoparticles, when dispersed uniformly and suspended stably in host fluids, can provide dramatic improvements in the thermal properties of host fluids. The term nanofluids (nanoparticle fluid suspensions) was first coined by Choi (1995) to describe this new class of nanotechnology-based heat transfer fluids that exhibit thermal properties superior to those of their host fluids or conventional particle fluid suspensions. The goal of nanofluids is to achieve the highest possible thermal properties at the smallest possible concentrations by uniform dispersion and stable suspension of nanoparticles in host fluids. To achieve this goal it is important to understand how nanoparticles enhance energy transport in liquids. Cooling is one of the top technical challenges facing high-tech industries such as microelectronics, transportation, manufacturing, metrology, and defense. For example, the electronics industry has provided computers with faster speeds, smaller sizes, and expanded features, leading to ever-increasing heat loads, heat fluxes, and localized hot spots at the chip and package levels. These thermal problems are also found in power electronics or optoelectronic devices. Air cooling is the most basic method for cooling electronic systems. The detailed introduction and applications of nanofluids can be found in the book by Das et al. (2007).

Problems involving fluid flow over a stretching or shrinking surface can be found in many manufacturing processes. These processes include polymer extrusion, wire and fiber coating, food stuff processing, etc. The flow field past a stretching surface was first found by Crane (1970), who presented a closed-form solution to the Navier–Stokes equations. A similarity solution to natural convective heat transfer from a vertical stretching sheet with surface mass transfer was presented by Gorla and Sidawi (1994). Takhar et al. (2001) considered the unsteady laminar boundary-layer flow of a viscous electrically conducting fluid induced by the impulsive stretching of a flat surface in two lateral directions through an otherwise quiescent fluid. Similarity and local similarity solutions for the boundary-layer flow on a linearly stretching permeable vertical surface were obtained by Ali and Al-Yousef (2002) when the buoyancy force assists or opposes the flow and the boundary-layer equations are subjected to power-law temperature and velocity variations. Mohamadine et al. (2007) presented a boundary-layer analysis to study the influence of radiation and buoyancy on the heat and mass transfer characteristics of continuous surfaces having a prescribed variable surface temperature and stretched with rapidly decreasing power-law velocities. Khedr et al. (2009) investigated the magneto-hydrodynamic (MHD) flow of a micropolar fluid past a stretched permeable surface with heat generation or absorption.

In fluid flow system, the Soret effect (thermal-diffusion process) is a thermodynamic phenomenon in which different particles respond in different ways to varying temperatures. In a simple way, we can say that mass fluxes can be created by temperature gradients known as the Soret or thermodiffusion effect. In 1879, the Swiss chemist Charles Soret first applied this effect to examine in detail Earth’s gravity. Furthermore, Soret noticed that a salt solution contained in a tube with two ends at different temperatures did not remain uniform in composition and the salt was more concentrated near the cold end than near the hot end of the tube. In particular, the strength of the effect is characterized by the Soret coefficient, which takes values of about $10^{-4}$ K$^{-1}$ for molecular mixtures. In suspensions it can be about two to three orders of magnitude higher, making the effect a phenomenon of technical importance. For all commonly used fluids, the Soret coefficient has been found to be a material constant. It plays an important role in the operation of solar ponds, biological systems, and the microstructure of oceans. In biological systems, mass transport across biological membranes induced by small thermal gradients in living matter is an important factor. Different aspects of the Soret effect have been reviewed by Platt (2006) and Rahman and Saghir (2014).

Furthermore, the Soret effect may become significant when large density differences exist in a flow regime. For example, the Soret effect can be significant when species are introduced at a surface in a fluid domain with a density lower than the surrounding fluid. In view of these important applications, several researchers have been exploring the usefulness of this effect on different fluids by considering various surface geometries. Abd El-Aziz (2008) studied the influence of the Dufour and Soret effects on the heat and mass transfer characteristics of free convection past a continuously stretching permeable sur-
face in the presence of a magnetic field, blowing/suction, and radiation. Bég et al. (2009) examined the cross-diffusion effects and porous impedance on the laminar MHD mixed-convection heat and mass transfer of an electrically conducting fluid from a vertical stretching surface in a Darcian porous medium under a uniform transverse magnetic field. The effect of cross diffusion on the double-diffusive free-convection flow in a porous medium was presented by Magyari and Postelnicu (2011). Srinivasacharya and Swamy Reddy (2012) investigated similar effects in a shear thinning fluid–saturated Darcy porous medium (pseudo plastic; $n < 1$) and a shear thickening fluid–saturated Darcy porous medium (dilatants; $n > 1$). A mathematical model was given by Murthy et al. (2013) to investigate the thermal-diffusion effect on an inclined plate in a nanofluid-saturated non-Darcy porous medium. RamReddy et al. (2013) stated that the Soret number accounts for the additional mass diffusion due to the temperature gradients and showed that the mixed-convection flow on a semi-infinite vertical flat plate in a nanofluid under the convective boundary conditions is appreciably influenced by the Soret parameter. Also, they concluded that the Soret effect enhanced the skin friction, heat, nanoparticle mass, and regular mass transfer rates in the medium.

In recent years, several authors extended the study of boundary-layer stagnation-point flow over a stretching surface to base fluids with suspended nanoparticles (nanofluids) under different physical conditions. The numerical solution for laminar fluid flow, which results from stretching a flat surface in a nanofluid, was presented by Khan and Pop (2010). A similarity solution of the steady boundary-layer flow near the stagnation-point flow on a permeable stretching sheet in a porous medium saturated with a nanofluid under the influence of internal heat generation/absorption was theoretically investigated by Hamad and Pop (2011). The problem of convective flow and heat transfer of an incompressible Newtonian nanofluid along a semi-infinite vertical stretching sheet under the effect of a magnetic field was solved analytically by Hamad (2011). Bachok et al. (2011) carried out an analysis to study the steady two-dimensional stagnation-point flow of a nanofluid past a stretching/shrinking sheet in its own plane. Chamkha and Aly (2011) focused on the numerical solution of the steady natural convection boundary-layer flow of a nanofluid consisting of a pure fluid with nanoparticles along a permeable vertical plate in the presence of magnetic field, heat generation or absorption, and suction or injection effects. A boundary-layer analysis was presented by Gorla and Chamkha (2011) for the natural convection past a horizontal plate in a porous medium saturated with a nanofluid. Hamad and Ferdows (2012) carried out heat and mass transfer analysis for boundary-layer stagnation-point flow over a stretching sheet in a porous medium saturated by a nanofluid with internal heat generation/absorption and suction/blowing. The steady MHD two-dimensional stagnation-point flow of an incompressible nanofluid toward a stretching surface subjected to convective boundary conditions in the presence of radiation was investigated numerically by Akbar et al. (2013). Stagnation-point flow and heat transfer due to nanofluid toward a stretching sheet in the presence of a magnetic field was investigated by Ibrahim et al. (2013). Kameswaran et al. (2013) discussed the effects of homogeneous–heterogeneous reactions on the stagnation-point flow of a nanofluid over a stretching or shrinking sheet. Makinde et al. (2013) analyzed the combined effects of buoyancy force, convective heating, Brownian motion, thermophoresis, and magnetic field on the stagnation-point flow and heat transfer due to nanofluid flow toward a stretching sheet. Unsteady two-dimensional stagnation-point flow of a nanofluid over a stretching sheet was investigated numerically by Malvandi et al. (2014), in which Navier’s slip condition was applied in contrast to the traditional no-slip condition at the surface. Nandy and Pop (2014) reported a numerical solution to the problem of steady two-dimensional MHD stagnation-point flow and heat transfer, with thermal radiation, of a nanofluid past a shrinking sheet.

Convective transport in porous media has been a subject of great importance and interest in recent years owing to its broad range of applications in civil, chemical, and mechanical engineering. Some studies on convection heat and mass transfer in a porous medium saturated with a nanofluid have been reported in the literature over the past few years. In many practical situations the porous medium is bounded by an impermeable wall (see Khidir and Sibanda, 2014; Uddin et al., 2015), has higher flow rates, and reveals a non-uniform porosity distribution near the wall region, making Darcy’s law inapplicable. Therefore, to better model the real physical situation, it is necessary to include the non-Darcy terms (either Brinkman or Forchheimer porous medium) in the analysis of convective transport in a porous medium. Non-Darcy models are extensions of the classical Darcy formulation, which incorporate inertial drag effects, vorticity diffusion, and combinations of these effects (see Chamkha et al., 2014; Chand and Rana, 2014; Srinivasacharya and Vijay Kumar, 2015, and citations therein). To the best of our knowledge, no studies have been reported in the
literature analyzing the Soret effect on the stagnation-point flow of a nanofluid past a vertical linearly stretching/shrinking sheet in a non-Darcy porous medium. Motivated by the aforementioned investigations, we have made an attempt to investigate the effects of the Soret effect on the boundary-layer stagnation-point flow past a stretching/shrinking sheet in a nanofluid-saturated non-Darcy porous medium. The transformed nonlinear conservation equations are solved numerically.

2. MATHEMATICAL FORMULATION

Consider a steady, incompressible two-dimensional boundary-layer flow near the stagnation point in a non-Darcy porous medium saturated with nanofluids. The Cartesian coordinates \( x \) and \( y \) are taken along the surface and normal to the surface, respectively, and \( u \) and \( v \) are the respective velocity components. The flow is generated due to stretching or shrinking of the sheet caused by the simultaneous application of two equal forces along the \( x \)-axis. Keeping the origin fixed, it is assumed that the surface is stretched/shrunken with linear velocity \( u_w(x) = bx \) as shown in Fig. 1, where \( b \) is a constant with \( b > 0 \) for a stretching sheet, \( b < 0 \) for a shrinking sheet, and \( b = 0 \) for a static sheet. It is also assumed that the ambient fluid moves with velocity \( u_e(x) = ax \), where \( a \) is a constant. The sheet is maintained at uniform wall temperature \( T_w \), nanoparticle concentration \( \phi_w \), and concentration \( C_w \). The free stream temperature, nanoparticle volume fraction, and concentration are \( T = T_\infty \), \( \phi = \phi_\infty \), and \( C = C_\infty \), respectively. In addition, the Soret effect is taken into consideration.

Under these assumptions and after making use of the Boussinesq approximation, the equations governing the flow are given by

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = u_e(x) \frac{\partial u_e(x)}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} + \frac{\varepsilon \nu}{K_P} \left[ u_e(x) - u \right] + \frac{\varepsilon^2 b}{K_P} \left[ u_e^2(x) - u^2 \right] + (1 - \phi_\infty) \times g [\beta_T (T - T_\infty) + \beta_C (C - C_\infty)] \
- \frac{(\rho_\beta - \rho f_\infty)}{\rho f_\infty} (\phi - \phi_\infty) \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + J \left[ D_B \frac{\partial \phi}{\partial y} \frac{\partial T}{\partial y} \right] + D_T \left( \frac{\partial T}{\partial y} \right)^2 \tag{3}
\]

\[
u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} = D_B \frac{\partial^2 \phi}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2} \tag{4}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_S \frac{\partial^2 C}{\partial y^2} + D_{CT} \frac{\partial^2 T}{\partial y^2} \tag{5}
\]

where \( u \) and \( v \) are the intrinsic velocity components in the \( x \) and \( y \) directions, respectively; \( T \) is the temperature; \( C \) is the concentration; \( \phi \) is the nanoparticle concentration; \( g \) is the acceleration due to gravity; \( K_P \) is the permeability; \( b \) is the empirical constant associated with the Forchheimer porous inertia term; \( \sigma \) is the electrical conductivity of the fluid; \( \varepsilon \) is the porosity; \( \alpha_m = k/(\rho c_T) \) and \( D_S \) are the thermal and solutal diffusivities of the fluid, respectively; \( \nu = \mu/\rho f_\infty \) is the kinematic viscosity coefficient; and \( J = \phi (pc)_f/(pc)_\beta \). Furthermore, \( \rho f_\infty \) is the density of the base fluid; \( \rho, \mu, k, \beta_T, \) and \( \beta_C \) are the density, viscosity, thermal conductivity, and volumetric thermal and solutal expansion coefficients of the nanofluid, respectively; \( \rho_p \) is the density of the nanoparticles; \( (pc)_f \) is the heat capacity of the fluid; and \( (pc)_\beta \) is the effective heat capacity of the nanoparticle material. The coefficients that appear in Eqs. (3)–(5) are the Brownian diffusion coefficient \( D_B \); the thermophoretic diffusion coefficient \( D_T \); and the last term in Eq. (5) is due to the Soret effect (see RamReddy, 2013, and citations therein).

The associated boundary conditions are

\[
u = u_w(x), \quad v = 0, \quad T = T_w, \quad \phi = \phi_w, \quad C = C_w \quad \text{at} \quad y = 0 \tag{6a}
\]
where subscripts \( w \) and \( \infty \) indicate the conditions at the wall and outer edge of the boundary layer, respectively; \( u_w (x) = bx \); and \( u_\infty (x) = ax \).

The following non-dimensional transformations are introduced:

\[
\eta = \sqrt{\frac{\alpha}{\nu} y}, \quad \psi (x, \eta) = \sqrt{\frac{\alpha}{\nu} x} f (\eta),
\]

\[
\theta (\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \gamma (\eta) = \frac{\phi - \phi_\infty}{\phi_w - \phi_\infty},
\]

\[
S (\eta) = \frac{C - C_\infty}{C_w - C_\infty},
\]

where \( f \) is the dimensionless stream function. In view of continuity Eq. (1), we introduce the stream function \( \psi \)

\[
u = -\frac{\partial \psi}{\partial \eta}, \quad v = \frac{\partial \psi}{\partial x}
\]

Substituting Eq. (8) in Eqs. (2)–(5) and then using non-dimensional transformations (7), we obtain the following system of non-dimensional equations:

\[
f'''' + f'' f' - f'^2 + 1 + \frac{\epsilon}{\text{Da} \cdot \text{Re}} (1 - f') + \frac{\epsilon^2 \text{Fs}}{\text{Da}} (1 - f'^2) = 0
\]

\[
\frac{1}{\text{Pr}} \theta'''' + f \theta' + N b \cdot \theta' \gamma' + N t \cdot \theta'^2 = 0
\]

\[
\gamma'' + \text{Le} \cdot f \gamma' + \frac{N t}{N b} \theta'' = 0
\]

where the primes indicate partial differentiation with respect to \( \eta \); \( \text{Gr} = [(1 - \phi_\infty) g \beta (T_w - T_\infty) x^3]/\nu^2 \) is the local Grashof number; \( \text{Re} = |u_\infty (x) x|/\nu \) is the local Reynolds number; \( \text{Re} = \text{Pr} \cdot \text{Re}^2 \) is the mixed-convection parameter; \( \text{Fs} = b/x \) is the local Forchheimer number; \( \text{Da} = K_p / x^2 \) is the local Darcy number; \( \alpha = \nu/\alpha_m \) is the Prandtl number; \( \text{Sc} = \nu/\text{Ds} \) is the Schmidt number; \( \text{Le} = \alpha_m/\phi D_B \) is the Lewis number; \( \text{Nc} = [\phi_C (C_w - C_\infty)]/\beta_T (T_w - T_\infty) \) is the regular buoyancy ration; \( \text{Nt} = [(\rho_p - \rho_f \infty) (\phi_w - \phi_\infty)] /\rho_f \infty \beta_T (1 - \phi_\infty) (T_w - T_\infty) \) is the nanoparticle buoyancy parameter; \( \text{Nb} = [J D_B (\phi_w - \phi_\infty)]/\nu \) is the Brownian motion parameter; \( \text{Nt} = (J D_T /\nu T_\infty) \times (T_w - T_\infty) \) is the thermophoresis parameter; and \( \text{Sr} = [\text{Dc}\infty (T_w - T_\infty)] /[\nu (C_w - C_\infty)] \) is the Soret number. Thus, boundary conditions (6) in terms of \( f, \theta, \gamma, \) and \( S \) become

\[
\theta (0) = 0, \quad \theta (\infty) = 1, \quad \gamma (0) = 1, \quad \gamma (\infty) = 0, \quad S (\infty) = 0
\]

and the non-dimensional skin friction \( C_f = 2 \tau_w /\rho u_\infty^2 \) is given by

\[
C_f \sqrt{\text{Re}_x} = 2 f'' (0)
\]

where \( \text{Re}_x = |u_\infty (x) x|/\nu \) is the local Reynolds number based on the surface velocity. The local heat, nanoparticle mass, and regular mass fluxes from the vertical plate can be obtained from

\[
q_w = -k \frac{\partial T}{\partial y} |_{y=0}, \quad q_m = -D_B \frac{\partial \phi}{\partial y} |_{y=0}
\]

The dimensionless local Nusselt number \( \text{Nu}_x = q_w x / [\text{Pr} k (T_w - T_\infty)] \), local nanoparticle Sherwood number \( \text{Sh}_x = q_m x / [D_B (\phi_w - \phi_\infty)] \), and regular Sherwood number \( \text{Sh}_x = q_m x / [D_S (C_w - C_\infty)] \) are given by

\[
\frac{\text{Nu}}{\text{Re}_x^{1/2}} = -\theta' (0), \quad \frac{\text{NSh}_x}{\text{Re}_x^{1/2}} = -\gamma' (0), \quad \frac{\text{Sh}_x}{\text{Re}_x^{1/2}} = -S' (0)
\]

The effects of the various parameters involved in the investigation on these coefficients are discussed in Section 4.

### 4. RESULTS AND DISCUSSION

#### 4.1 Numerical Method

Reduced flow governing Eqs. (9)–(12) are nonlinear, coupled, ordinary differential equations that possess no
closed-form solution. Therefore, they must be solved numerically subject to the boundary conditions given by Eq. (13). The implicit, iterative finite-difference method discussed by Blottner (1970) has proven to be adequate for the solution of these types of equations. For this reason, this method is employed in the present study. Equations (9)–(12) are discretized using three-point central difference quotients. This converts the differential equations into linear sets of algebraic equations, which can be readily solved by the well-known Thomas algorithm. On the other hand, Eqs. (9)–(12) are discretized and solved subject to the appropriate boundary conditions by the trapezoidal rule. The computational domain in the \( \eta \)-direction was made up of 196 non-uniform grid points. It is expected that most changes in the dependent variables occur in the region close to the plate where viscous effects dominate. However, small changes in the dependent variables are expected far away from the plate surface. For these reasons, variable step sizes in the \( \eta \)-direction are employed. The initial step size \( \Delta \eta_1 \) and growth factor \( K^* \) employed, such that \( \Delta \eta_{i+1} = K^* \Delta \eta_i \) (where subscript \( i \) indicates the grid location), were \( 10^{-3} \) and 1.0375, respectively. These values were found (by performing many numerical experimentations) to give accurate and grid-independent solutions. The solution convergence criterion employed in the present study was based on the difference between the values of the dependent variables at the current and previous iterations. When this difference reached \( 10^{-5} \), the solution was assumed converged and the iteration process was terminated.

### 4.2 Numerical Validation

With \( Nb \to 0, Nt = 0, \varepsilon = 0, Ri = 0, Le = 0, \) and \( S(\eta) \to 0 \) (i.e., for a regular Newtonian fluid), Eqs. (9)–(12) governing the present investigation of a nanofluid-saturated non-Darcy porous medium reduces to the limiting case in Mahapatra and Gupta (2002), who investigated the boundary-layer stagnation-point flow past a stretching/shrinking sheet in a viscous fluid in the absence of Soret and non-Darcy effects. Also, the results found in this study have been compared with Mahapatra and Gupta (2002) and other results and are found to be in good agreement, as shown in Tables 1 and 2. Therefore, the developed code can be used with great confidence to study the problem considered in this paper.

#### Table 1: Comparison of dimensionless similarity function \(-\theta'(0)\) for the boundary-layer stagnation-point flow past a stretching/shrinking sheet in a viscous fluid with the Mahapatra and Gupta (2002) study by taking \( Nb \to 0, Nt = 0, \varepsilon = 0, Ri = 0, Le = 0, \) and \( S(\eta) \to 0 \)

<table>
<thead>
<tr>
<th>Pr</th>
<th>( -\theta'(0) )</th>
<th>Mahapatra and Gupta (2002)</th>
<th>Present Study</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.178</td>
<td>0.178</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>0.563</td>
<td>0.563</td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>0.796</td>
<td>0.796</td>
<td></td>
</tr>
<tr>
<td>1.50</td>
<td>0.974</td>
<td>0.974</td>
<td></td>
</tr>
</tbody>
</table>

#### Table 2: Comparison of dimensionless similarity function \( f''(0) \) for the boundary-layer stagnation-point flow past a stretching/shrinking sheet in a viscous fluid with various studies by taking \( Nb \to 0, Nt = 0, \varepsilon = 0, Ri = 0, Le = 0, \) and \( S(\eta) \to 0 \)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>-0.9694</td>
<td>-0.9694</td>
<td>-0.9694</td>
<td>-0.9696</td>
<td>-0.9694</td>
</tr>
<tr>
<td>0.2</td>
<td>-0.9181</td>
<td>-0.9181</td>
<td>-0.9181</td>
<td>-0.9182</td>
<td>-0.9181</td>
</tr>
<tr>
<td>0.5</td>
<td>-0.6673</td>
<td>-0.6673</td>
<td>-0.6673</td>
<td>-0.6672</td>
<td>-0.6673</td>
</tr>
<tr>
<td>2</td>
<td>0.20175</td>
<td>0.20174</td>
<td>0.20176</td>
<td>0.20175</td>
<td>0.20175</td>
</tr>
<tr>
<td>3</td>
<td>4.7293</td>
<td>4.729</td>
<td>4.7296</td>
<td>4.7293</td>
<td>4.7293</td>
</tr>
</tbody>
</table>
4.3 Effects of the Physical Parameters

4.3.1 Brownian Motion Parameter

The nanoparticle volume fraction is a key parameter in convection in nanofluid-saturated porous media, and it will have a significant effect on the flow field, temperature, and concentration distributions. Figure 2 represents the effect of the Brownian motion parameter $N_b$ on the velocity, temperature, concentration, and nanoparticle volume fraction distributions for both aiding and opposing flows. When $N_b = 0$, there is no additional thermal transport due to buoyancy effects created as a result of nanoparticle concentration gradients. It is interesting to note that an increase in the intensity of Brownian motion parameter $N_b$ produces significant enhancement in the fluid velocity within the momentum boundary layer, thus enhancing the fluid flow, as shown in Fig. 2(a) in the case of aiding flow, and the reverse trend is observed in the case of opposing flow. It is interesting to note that the Brownian motion of nanoparticles at the molecular and nanoscale levels is a key nanoscale mechanism governing their thermal behaviors. In nanofluid systems, due to the size of the nanoparticles, Brownian motion takes place, which can affect the heat transfer properties. As the particle size scale approaches the nanometer scale, the particle Brownian motion and its effect on the surrounding liquids

FIG. 2: Variation of the (a) velocity, (b) temperature, (c) concentration, and (d) nanoparticle volume fraction profiles with the Brownian motion parameter
play an important role in the heat transfer. From Fig. 2(b), with the rise in the value of the Brownian motion parameter, enhancement in the temperature is observed, whereas a decrease in the concentration field is noticed, as depicted in Fig. 2(c) for both aiding and opposing flows. From Fig. 2(d), we notice that the nanoparticle volume fraction is slightly decreased, with a gain in the values of $Nb$ for both the aiding and opposing flow cases.

### 4.3.2 Thermophoresis Parameter

The effect of the thermophoresis parameter $Nt$ on the velocity, temperature, concentration, and nanoparticle volume fraction distributions is demonstrated in Fig. 3 for both aiding and opposing flows. Figure 3(a) reveals that for aiding flow the momentum boundary-layer thickness increases with escalating values of $Nt$, whereas the opposite orientation is noticed in the case of opposing flow. A similar trend is observed for the temperature, concentration, and nanoparticle volume fraction profiles for both the aiding and opposing flow cases when the thermophoresis parameter is increased. With an increase in values of the thermophoresis parameter, an increase in the temperature and concentration fields is noticed, as portrayed in Figs. 3(b) and 3(c), respectively. Figure 3(d) shows that with an increase in $Nt$, a depletion in the

![FIG. 3: Variation of the (a) velocity, (b) temperature, (c) concentration, and (d) nanoparticle volume fraction profiles with the thermophoresis parameter](image-url)
nanoparticle volume fraction distribution is noted. Here, $Nt > 0$ designates a cold surface, while $Nt < 0$ corresponds to a hot surface. In the case of a hot surface, thermophoresis tends to blow the nanoparticle volume fraction away from the surface since a hot surface repels submicron-sized particles from it, thereby forming a relative particle-free layer near the surface.

### 4.3.3 Varying the Stretching/Shrinking Parameter

The variation of the velocity, temperature, concentration, and nanoparticle volume fraction profiles for various values of the stretching/shrinking parameter is demonstrated in Fig. 4 for both aiding and opposing flows. In both the aiding and opposing flow cases, a similar trend is observed with increasing values of the stretching/shrinking parameter. From Fig. 4(a), it is evident that an increase in the stretching or shrinking parameter significantly enhances the momentum boundary-layer thickness. This is because in the presence of suction, a heated fluid is pushed toward the wall where the buoyancy forces can act to retard the fluid due to the high influence of the Brownian motion effects. This effect acts to decrease the wall shear stress. An increase in values of the stretching/shrinking parameter resulting in a lower temperature, concentration, and nanoparticle volume fraction is shown in Figs. 4(b)–4(d), respectively.

![FIG. 4: Variation of the (a) velocity, (b) temperature, (c) concentration, and (d) nanoparticle volume fraction profiles with the stretching/shrinking parameter](image-url)
4.3.4 Varying the Soret Number

Figure 5 shows the velocity, temperature, concentration, and nanoparticle volume fraction profiles for various values of Soret number $S_r$ for both the aiding and opposing flow cases. The Soret number accounts for the additional mass diffusion due to the temperature gradients. Figure 5(a) show that higher $S_r$ values result in a higher velocity field for aiding flow, but a reverse trend is observed for opposing flow. An increase in the Soret parameter causes a significant decrease in the temperature and concentration fields for aiding flow (increases for opposing flow) within the boundary layer, as shown in Figs. 5(b) and 5(c), respectively. Figure 5(d) reveals that the nanoparticle volume fraction is enhanced with an increase in the Soret parameter for both aiding and opposing flows. The present analysis indicates that the flow field is appreciably influenced by the Soret parameter. This analysis is in line with the observation made in RamReddy et al. (2013).

4.3.5 Varying the Forchheimer Parameter

The variation of velocity, temperature, concentration, and nanoparticle volume fraction profiles for different values of Forchheimer parameter $F_s$ is presented in Fig. 6 for both aiding and opposing flows. It can be observed from Fig. 6(a) that the velocity of the fluid decreases for aiding flow (increases in the opposing flow case) with an increase in the Forchheimer number. Since the Forch-
FIG. 6: Variation of the (a) velocity, (b) temperature, (c) concentration, and (d) nanoparticle volume fraction profiles with the non-Darcy parameter

The Forchheimer number represents the inertial drag. An increase in the $F_s$ value increases the resistance to the flow; therefore, a decrease in the fluid velocity ensues. Here, $F_s = 0$ represents the case where the flow is Darcian. The velocity is at its maximum in this case due to the total absence of inertial drag. It can be noticed from Fig. 6(b) that the temperature of the fluid increases for the aiding flow (decreases for the opposing flow) with an increase in the Forchheimer number. An increase in the $F_s$ value increases the temperature values because as the fluid is decelerated, energy is dissipated as heat and this serves to enhance the temperature in the boundary layer. The concentration and nanoparticle volume fraction distribution increase for aiding flow (decrease for opposing flow) with an increase in the Forchheimer number, as shown in Figs. 6(c) and 6(d), respectively. An increase in the non-Darcy parameter reduces the intensity of the flow but enhances the thermal, concentration, and nanoparticle volume fraction boundary-layer thicknesses.

4.3.6 Analysis of the Skin Friction, Heat, Mass, and Nanoparticle Transfer Coefficients for Different Values of the Physical Parameters

The variation in the profiles of the local skin friction, heat, mass, and nanoparticle transfer coefficients against the stretching/shrinking parameter for different values of pertinent parameters is presented graphically in Figs. 7–10 for both the aiding and opposing flows.
FIG. 7: Variation of the local (a) skin friction and heat transfer coefficients and (b) mass and nanoparticle transfer coefficients with the Brownian motion parameter

FIG. 8: Variation of the local (a) skin friction and heat transfer coefficients and (b) mass and nanoparticle transfer coefficients with the thermophoresis parameter

FIG. 9: Variation of the local (a) skin friction and heat transfer coefficients and (b) mass and nanoparticle transfer coefficients with the Soret parameter
The local skin friction, heat, mass, and nanoparticle transfer coefficients for various values of Brownian motion parameter $Nb$ are presented in Fig. 7. From Fig. 7(a), we notice that the local skin friction coefficient is increased for aiding flow (decreased for opposing flow) with an increase in $Nb$. From Fig. 7(a), it can be seen that the local heat transfer coefficient is diminished with a hike in the values of the Brownian motion parameter for both aiding and opposing flows. The higher values of the Brownian motion parameter results in higher local mass and nanoparticle transfer coefficients for both aiding and opposing flows, as depicted in Fig. 7(b).

The variation of the local skin friction, heat, mass, and nanoparticle transfer coefficients for various values of thermophoresis parameter $Nt$ are presented in Fig. 8. From Fig. 8(a), we notice that the local skin friction coefficient is enhanced for aiding flow (diminished for opposing flow) with the rise of $Nt$. From Fig. 8(a), we observe that the local heat transfer coefficient is diminished with escalating values of the thermophoresis parameter for both aiding and opposing flows. An increase in thermophoresis parameter significantly enhances the local mass transfer coefficient but reduces the local nanoparticle transfer coefficient for both aiding and opposing flows, as displayed in Fig. 8(b).

Figure 9 depicts the effect of the Soret parameter on the local skin friction, heat, mass, and nanoparticle transfer coefficients transfer rates for both aiding and opposing flows. For aiding flow, an increase in the values of the Soret parameter enhances (reduces for opposing flow) the local skin friction, heat, and mass transfer coefficients but reduces the local nanoparticle transfer coefficient for both aiding and opposing flows, as shown in Figs. 9(a) and 9(b), respectively.

The influence of the Forchheimer parameter on local skin friction, heat, mass, and nanoparticle transfer coefficients is depicted in Fig. 10. The local skin friction, heat, and mass transfer coefficients are increased for aiding flow (reduced for opposing flow) with the rising values of the Forchheimer parameter, as shown in Figs. 10(a) and 10(b), respectively. Higher $F_s$ values results in higher local nanoparticle transfer coefficients for both aiding and opposing flows, as shown in Fig. 10(b).

5. CONCLUSIONS

The boundary-layer stagnation-point flow past a stretching/shrinking sheet in a non-Darcy porous medium saturated with a nanofluid in the presence of the Soret effect is analyzed. The wall is subjected to uniform temperature, concentration, and nanoparticle volume fraction conditions. Using dimensionless variables, the governing equations are transformed into a set of nonlinear parabolic equations, where a numerical solution has been presented using the implicit, iterative finite-difference method discussed in Blottner (1970) for a wide range of parameters. Some important observations are given as follows:

- Higher values of the Soret parameter result in higher velocity distribution, skin friction, heat, and mass transfer rates for the aiding flow but a reverse trend is noticed for the opposing flow. Higher values of the Soret parameter result in a higher nanoparticle volume fraction distribution but a lower nanoparticle mass transfer rate in both aiding and opposing flows. Furthermore, the effect of the Soret parameter on the temperature and concentration distributions within the boundary layers is negligible.
In the aiding flow case, an increase in non-Darcy parameter $F_s$ decreases the velocity distribution and nanoparticle mass transfer rates, whereas it increases the temperature, concentration, nanoparticle volume fraction, skin friction, heat, and mass transfer rates. The opposite nature is found in the case of opposing flow.

The velocity distribution, heat, mass, and nanoparticle mass transfer rates are higher but the temperature, concentration, nanoparticle volume fraction, and skin friction are lower in the stretching parameter case when compared to the shrinking parameter case in both aiding and opposing flows.

The results also indicate that the presence of the Soret parameter on the stagnation-point flow past a stretching/shrinking sheet in a non-Darcy porous medium saturated with a nanofluid influences the flow, heat, mass, and nanoparticle volume fraction of the fluid flow.

ACKNOWLEDGMENT

The authors thank the anonymous reviewers for their valuable comments, which have improved the paper.

REFERENCES


Kameswaran, P. K., Sibanda, P., RamReddy, Ch., and Murthy, P. V. S. N., Dual solutions of stagnation-point flow of a nanofluid over a stretching surface, *Boundary Value Prob-


