MHD boundary layer flow, heat and mass transfer analysis over a rotating disk through porous medium saturated by Cu-water and Ag-water nanofluid with chemical reaction

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Abstract

In this article we have presented, magnetohydrodynamic (MHD) boundary layer flow, heat and mass transfer characteristics of Cu-water and Ag-water nanofluid (with volume fraction 1% and 4%) over a rotating disk through porous medium with thermal radiation, chemical reaction and partial slip. Using similarity variables the governing equations which represent the momentum, energy and diffusion are transformed into ordinary differential equations. The transformed conservation equations are solved numerically by using versatile, extensively validated, Finite element method. The sway of significant parameters such as nanoparticle volume fraction parameter (φ), Magnetic parameter (M), velocity slip parameter (λ), porous parameter (k), thermal radiation (R), space-dependent (A) and temperature dependent (B) heat source/sink parameters, temperature slip parameter (ξ), and chemical reaction parameter (Cr) on radial velocity, azimuthal velocity, temperature and concentration evaluations in the boundary layer region are examined in detail and the results are shown in graphically. Furthermore, the effect of these parameters on local skin friction coefficient (Cf), local Nusselt number (Nux) and local Sherwood number (Shx) is also investigated. The results are compared with previously published work and found to be admirable agreement. It is noted that the temperature profiles elevated with the increasing values of nanoparticle volume fraction parameter (φ).

Keywords:
Porous medium
Rotating disk
Cu-water and Ag-water nanofluid
Heat generation/absorption
Finite element method

List of symbols

\(C_p\) specific heat at constant pressure
\(C_w\) uniform constant concentration
K permeability parameter
\(\text{Nu}_x\) Nusselt number
Sc Schmidt number
\(q_r\) radiative heat flux
\(T_w\) uniform constant temperature
\(k_0\) rate of chemical reaction
\(\nu\) velocity in the \(\Phi\) - direction
\(K^*\) Mean absorption coefficient
R Radiation parameter
\(Re_c\) local rotational Reynolds number
\(q''''\) Non-uniform heat source/sink
\(k_s\) thermal conductivity of nanoparticle

\(C_w\) free stream concentration
M Magnetic parameter
Pr Prandtl number
\(\text{Sh}_x\) Sherwood number
T temperature of the fluid
\(T_w\) free stream temperature
u velocity in the \(r\) – direction
w velocity in the \(z\) – direction
\((r, \Phi, z)\) Cylindrical polar coordinates
\(\Phi\) nanoparticle volume fraction
Ω Angular velocity

Greek symbols

\(\theta\) non-dimensional temperature
\(\nu_f\) dynamic viscosity of the nanofluid
\(\nu_f\) kinematic viscosity of the base fluid
\(\rho_f\) density of the nanofluid
\((\rho C_p)_f\) heat capacitance of the nanofluid

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http://dx.doi.org/10.1016/j.powtec.2016.11.017
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\( \xi \) \hspace{1cm} \text{temperature slip parameter} \\
\( \mu_f \) \hspace{1cm} \text{dynamic viscosity of the base fluid} \\
\( \eta \) \hspace{1cm} \text{dimensionless similarity variable} \\
\( \rho_f \) \hspace{1cm} \text{density of the base fluid} \\
\( \kappa_f \) \hspace{1cm} \text{thermal conductivity of base fluid} \\
\( \varphi \) \hspace{1cm} \text{non-dimensional concentration} \\
\( \sigma^* \) \hspace{1cm} \text{stephan-Boltzmann constant} \\
\( \lambda \) \hspace{1cm} \text{velocity slip parameter} \\

\textbf{Subscripts} \\
\( f \) \hspace{1cm} \text{Base fluid} \\
\( nf \) \hspace{1cm} \text{nanofluid} \\

1. Introduction

Natural convection flow, heat and mass transfer through porous medium over curved bodies is an important area in recent years because of its wide range applications such as chemical engineering, thermal energy storage devices, heat exchangers, ground water systems, electronic cooling, boilers, heat loss from piping, nuclear process systems etc. Spherical geometries, cones, cylinders, ellipses, wavy channels, disks, torus geometries are some examples of curved bodies. In recent years, flow due to rotating disk has received much attention because of its wide range of applications in several industrial and engineering process such as spin coating, manufacturing, centrifugal pumps, pumping of liquid metals at high melting point, turbo-machinery etc. Good number of experimental and theoretical studies on transport phenomena over cylindrical bodies has been reported in literature which deals the process of polymer systems. Von Karman [1] was first studied and written the mathematical modeling of the problem of flow due to rotating disk. Attia [2] has analyzed the impact of temperature dependent viscosity on unsteady flow and heat transfer analysis due to rotating disk through porous medium. Asghar et al. [3] have reported unsteady flow due to non-coaxial rotations of disk and a fluid at infinity by taking slip effects into the account. All these studies are mainly focused on flow and heat transfer characteristics of the commonly used base fluids like water, ethylene glycol, oil etc. In recent times, the study of nanofluids has become the topic of extensive research because the presence of nanoparticles would appreciably increases the thermal conductivity of the fluids in heat transfer process. A nanofluid is a fluid containing small volumetric quantities of nanometer-sized (smaller than 100 nm) particles called nanoparticles. Nanofluids are the emerging composites consisting of nanometer size solid particles dispersed in the conventional heat transfer fluids like water, ethylene glycol, toluene, and engine oil. The very key characteristic of nanofluids is their high thermal conductivity relative to the base fluids. The thermal conductivity enhancement of nanofluids has become the most important phenomena than the limited heat transfer performance of available general fluids. Choi [4] was the first among all who introduced a new type of fluid called nanofluid while doing research on new coolants and cooling technologies. Eastman et al. [5] have also showed that the thermal conductivity has increased 40% when copper nanoparticles of volume fraction <1% are added to the ethylene glycol or oil. Buongiorno [6] has developed an analytical model for convective transport in nanofluids, in this study he concluded that there are seven possible mechanisms associating convection of nanofluids through moment of nanoparticles in the base fluid. These are nanoparticle size, inertia, particle agglomeration, Magnus effect, volume fraction of the nanoparticle, Brownian motion, thermophoresis. Among all the mechanisms Brownian motion and thermophoresis are found to be very important. The thermophoresis acts against temperature gradient, so that, the particles move from the region of higher temperature to the region of lower temperature. Also, Brownian motion tends to move the particles from higher concentration areas to lower concentration areas. Tiwari et al. [7] have presented heat transfer augmentation of nanofluids in a two-sided lid-driven heated square cavity. Santra et al. [8] have reported heat transfer augmentation of copper-water nanofluid in differentially heated square cavity. Abu-nada et al. [9] have analyzed natural convection applications of nanofluids over inclined two-dimensional enclosures filled with Cu-water nanofluid. Kuznetsov and Nield [10] studied the influence of Brownian motion and thermophoresis on natural convection boundary layer flow of a nanofluid past a vertical plate. Khan and pop [11] have discussed boundary layer flow, heat and mass transfer of a nanofluid past a stretching sheet. Ghasemi et al. [12] have discussed periodic natural convection flow and heat transfer over enclosure filled with nanofluid under the influence of oscillating heat flux. Bachok et al. [13] have presented boundary layer flow and heat transfer of nanofluid over a rotating disk embedded in porous medium. Niu et al. [14] have perceived the effect of partial slip on flow and heat transfer over a microtube filled with non-Newtonian nanofluid. Chamkha et al. [15] have discussed the mixed convection flow about vertical cone through porous medium saturated by a nanofluid with thermal radiation. Rashidi et al. [16] have discussed the nanofluid flow due to rotating porous disk with magnetic field and entropy generation. In addition, Gorla et al. [17] have studied nanofluid natural convection boundary layer flow through porous medium over a vertical cone. Chamkha et al. [18] were presented non-Darcy free convective nanofluid along a vertical plate with suction/injection and internal heat generation. Sheikholeslami et al. [19] have deliberated boundary layer flow and heat transfer analysis of nanofluid due to rotating disk. Türkyılmazoğlu et al. [20] discussed MHD nanofluid flow and heat transfer due to rotating disk. Chamkha et al. [21] have investigated Non-Newtonian nanofluid natural convection flow over a cone through porous medium with uniform heat and volume fraction fluxes. Zakari et al. [22] have presented the natural convection boundary layer flow, heat and mass transfer analysis of nanofluid influenced by various aspects like, size, shape, type of nanofluid, type of base fluid and working temperature of the base fluid. Ghalambaz et al. [23] have reported natural convection flow over a heated vertical plate through nanofluid saturated porous medium. Ghalambaz et al. [24] have analyzed natural convection of Al_{2}O_{3}–water nanofluid over a vertical cone with the influence of nanoparticles diameter, concentration and variable thermal conductivity. Noghrehabadi et al. [25] have noticed the boundary layer natural convection of nanofluid over a vertical plate. Sheremet et al. [26] presented Buongiorno’s mathematical model of nanofluid over a square cavity through porous medium. Sheremet et al. [27] have deliberated three-dimensional natural convection Buongiorno’s mathematical model of nanofluid over a porous enclosure. Zargartalebi et al. [28] have discussed Stagnation-point natural convection heat and mass transfer flow of nanofluid over a stretching sheet under the variable thermo-physical properties. Mliki et al. [29] presented MHD convective nanofluid flow over a linearly/sinusoidally heated cavity with heat generation/absorption. 

Magnezo nanofluids have specific applications in biomedicine, optical modulators, magnetic cell separation, magneto-optical wavelength filters, silk float separation, nonlinear optical materials, hyperthermia, optical switches, drug delivery, optical gratings etc. A magnetic nanofluid has both the liquid and magnetic properties. The used magnetic field influences the suspended particles and reorganizes their concentration in the fluid regime which powerfully influences the heat transfer analysis of the flow. Magneto nanofluids are useful to guide the particles up the blood stream to a tumor with magnets. This is due to the fact that the magnetic nanoparticles are regarded more adhesive to tumor cells than non-malignant cells. Such particles absorb more power than microparticles in alternating current magnetic fields tolerable in humans i.e. for cancer therapy. Several authors, Kefayati et al. [30], Chamkha et al. [31], Sudarsana Reddy et al. [32,33], Tasawar Hayat et al. [34] have discussed the MHD boundary layer flow, heat and mass transfer characteristics of nanofluids over different geometries.
The main aim of this article is to examine the impact of slip effects, heat generation/absorption, thermal radiation and chemical reaction on MHD boundary layer flow, heat and mass transfer analysis due to rotating disk embedded in porous medium saturated by Cu-water and Ag-water based nanofluids. Numerical solutions of radial velocity, azimuthal velocity, temperature and concentration distributions are obtained using Finite element method. To our knowledge, the problem is new and no such articles reported yet in the literature.

2. Mathematical model

Consider the steady, viscous incompressible, laminar, MHD boundary layer flow of Cu-water and Ag-water nanofluid due to rotating disk through porous medium with thermal radiation, chemical reaction and heat source/sink as depicted in Fig. 1. The disk is located in the plane at \( z = 0 \), and rotates with constant angular velocity \( \Omega \). The components of flow velocity are \((u,v,w)\) in the direction of increasing \((r,\phi,z)\) respectively. A constant magnetic field of strength \( B_0 \) is applied perpendicular to the surface of the disk. The field of strength \( B_0 \) is a water-based nanofluid containing two different types of nanoparticles Cu and Ag. The thermo-physical properties of the nanofluid are given in Table 1 [35]. Under the above assumptions, the governing equations describing the momentum, energy and concentration of nanoparticles in the presence of thermal radiation, chemical reaction and heat generation/absorption take the following form:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v \partial u}{\partial z} = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + \frac{u \partial u}{\partial r} + \frac{v \partial u}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{\mu_{nf}}{\rho_{nf}} u \frac{1}{k_{nf}} \alpha B_0^2 u \tag{2}
\]

\[
\frac{\partial v}{\partial t} + \frac{u \partial v}{\partial r} + \frac{v \partial v}{\partial z} = \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{\mu_{nf}}{\rho_{nf}} v \frac{1}{k_{nf}} \alpha B_0^2 v \tag{3}
\]

The associated boundary conditions are

\[
u = L \frac{\partial u}{\partial z}, v = r \Omega + L \frac{\partial v}{\partial z} = 0, T = T_w + k_{nf} \frac{\partial T}{\partial z} = C_w \text{ at } z = 0 \tag{7}
\]

\[
u \rightarrow 0, v \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } z \rightarrow \infty \tag{8}
\]

The stream function \( \psi \) can be defined as follows,

\[
u = \frac{\partial \psi}{\partial y} = -\frac{\partial \psi}{\partial x} \tag{4}
\]

The following similarity transformations are introduced to simplify the mathematical analysis of the problem

\[
u = r \Omega \Gamma' (\eta), v = r \Omega g, w = -\sqrt{2} \Omega \psi f (\eta), \eta = \sqrt{2} \psi f (\eta), \sigma = \frac{T - T_w}{T_w - T_m}, \beta = \frac{C_m - C_w}{C_w - C_m} \tag{5}
\]

The non-uniform heat source/sink, \( q^* \), is defined as

\[
q^* = f (\eta) \frac{k_f}{\psi f} [A1(T_w - T_m)f' + B1(T - T_m)] \tag{6}
\]

Table 1

<table>
<thead>
<tr>
<th>Fluid</th>
<th>( \rho_{nf}(1/T) )</th>
<th>( C_{pf}/(\rho_{nf} \Omega) )</th>
<th>( k_{nf}/(\rho_{nf} \Omega) )</th>
<th>( \beta \times 10^5 (K^{-1}) )</th>
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</thead>
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<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>21</td>
</tr>
<tr>
<td>Copper (Cu)</td>
<td>8933</td>
<td>385</td>
<td>401</td>
<td>1.67</td>
</tr>
<tr>
<td>Silver (Ag)</td>
<td>10,500</td>
<td>235</td>
<td>429</td>
<td>1.89</td>
</tr>
<tr>
<td>Alumina ((\text{Al}_2\text{O}_3))</td>
<td>3970</td>
<td>765</td>
<td>40</td>
<td>0.85</td>
</tr>
<tr>
<td>Titanium oxide ((\text{TiO}_2))</td>
<td>4250</td>
<td>668.2</td>
<td>8.9538</td>
<td>0.9</td>
</tr>
</tbody>
</table>

\[
\]
to internal heat source and the case $A_1 < 0, B_1 < 0$ corresponds to internal heat sink.

By using Rosseland approximation for radiation, the radiative heat flux $q_r$ is defined as

$$ q_r = -rac{4\sigma^*}{3K^*} \frac{\partial T^4}{\partial z} $$

where $\sigma^*$ is the Stephan-Boltzman constant, $K^*$ is the mean absorption coefficient. We assume that the temperature differences within the flow are such that the term $T^4$ may be expressed as a linear function of temperature. This is accomplished by expanding $T^4$ in a Taylor series about the free stream temperature $T_\infty$ as follows:

$$ T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \cdots $$

Neglecting higher-order terms in the above Eq. (12) beyond the first degree in $(T - T_\infty)$, we get

$$ T^4 \approx 4T_\infty^3 T - 3T_\infty^4 $$

Thus, substituting Eq. (13) into Eq. (11), we get

$$ q_r = -\frac{16T_\infty^3 \sigma^* \partial T}{3K^*} $$

Using Eqs. (9), (10) and (14), the governing non-linear partial differential Eqs. (1)–(6) together with the boundary conditions (7) and (8) reduce to

$$ f'' + \frac{A_1}{2} \left( f'' - 2f f'' - g'' \right) - k_1 f' - \frac{A_1}{A_2} M f' = 0 $$

$$ g'' + A_1(fg' - fg) - k_1 g - \frac{A_1}{A_2} M g = 0 $$

$$ (1 + R)\theta'' + Pr A_4 A_4 f' \theta' + A_4 \left( A f' + B \theta \right) = 0 $$

$$ S' + Sc f S' = Sc Cr S = 0 $$

$$ S'' + Sc f S'' = Sc Cr S = 0 $$

Fig. 2. Effect of $\phi$ on radial Velocity.

Fig. 3. Effect of $\phi$ on azimuthal Velocity.

Fig. 4. Effect of $\phi$ on temperature.

Fig. 5. Effect of $\phi$ on Concentration.
The transformed boundary conditions are

\[ \eta = 0, \quad f = 0, \quad f' = \lambda f'', \quad g = 1 + \lambda g', \quad \theta = 1 + \xi \theta', \quad \varphi = 1 \]

\[ \eta \to \infty, \quad f = 0, \quad g = 0, \quad \theta = 0, \quad \varphi = 0. \]  

(19)

where, prime indicates ordinary differentiation with respect to \( \eta \). In usual notations,

\[ \text{Pr} = \frac{\nu_f}{\sigma_f}, \quad M = \frac{Q_0 B_0^2}{2 \rho_f}, \quad k_1 = \frac{\nu_f}{2 K \Omega}, \quad \text{Sc} = \frac{\nu}{D_m}, \quad \text{Cr} = \frac{K_0}{2 \Omega}, \quad R = \frac{16 \Omega^2 \sigma_f}{3 K k_{nf}}, \]

\[ A = \frac{A_1}{2 \Omega}, \quad B = \frac{B_1}{2 \Omega}, \quad \lambda = L \sqrt{\frac{2 \Omega}{\nu_f}}, \quad \xi = K_1 \sqrt{\frac{2 \Omega}{\nu_f}} \]

and

\[ A_1 = (1 - \varphi)^{2.5} \left[ (1 - \varphi) + \varphi \left( \frac{\rho_c}{\rho_f} \right) \right], \quad A_2 = (1 - \varphi) + \varphi \left( \frac{\rho_c}{\rho_f} \right), \quad A_3 = \frac{k_{nf}}{k_f} \]

Quantities of practical interest in this problem are the local skin-friction coefficient \( C_f \) and \( C_g \) in radial and azimuthal directions, the local Nusselt number \( Nu_s \), and the local Sherwood number \( Sh_s \). These are defined, respectively, as

\[ C_f = \frac{\tau_{rz}}{\rho_f (r \Omega)^2}, \quad C_g = \frac{\tau_{\varphi z}}{\rho_f (r \Omega)^2}, \quad Nu_s = \frac{q_{w}}{\kappa (T_w - T_u)}, \quad Sh_s = \frac{J_{w}}{D_m (C_w - C_u)} \]

where \( \tau_{rz}, \tau_{\varphi z}, q_{w}, \) and \( J_{w} \) are the disk radial shear stress, azimuthal shear stress, surface heat flux and the mass flux, respectively. The
dimensionless versions of these key design quantities:

\[
C_f = \frac{f'(0)}{(1-\phi)^2 Re_\tau^\frac{1}{2}} \quad \text{and} \quad C_g = \frac{g'(0)}{(1-\phi)^2 Re_\tau^\frac{1}{2}}
\]

\[
Nu_k = -(1 + R) \frac{k_n}{k_f} \phi' (0) Re_\tau^\frac{1}{2} \quad \text{and} \quad Sh_k = -S' (0) Re_\tau^\frac{1}{2}
\]

Since the highly non-linear nature of ordinary differential Eqs. (15)–(18) together with boundary conditions (19), they cannot be solved analytically. So, the variational finite-element method [36,37,38,39] has been implemented.

3. Numerical method of solution

3.1. The finite-element method

The finite-element method (FEM) is such a powerful method for solving ordinary differential equations and partial differential equations. The basic idea of this method is dividing the whole domain into smaller elements of finite dimensions called finite elements. This method is such a good numerical method in modern engineering analysis, and it can be applied for solving integral equations including heat transfer, fluid mechanics, chemical processing, electrical systems, and many other fields. The steps involved in the finite-element are as follows.

i. Finite-element discretization

The whole domain is divided into a finite number of subdomains, which is called the discretization of the domain. Each subdomain is called an element. The collection of elements is called the finite-element mesh.

ii. Generation of the element equations

a. From the mesh, a typical element is isolated and the variational formulation of the given problem over the typical element is constructed.

b. An approximate solution of the variational problem is assumed, and the element equations are made by substituting this solution in the above system.

c. The element matrix, which is called stiffness matrix, is constructed by using the element interpolation functions.

Fig. 10. Effect of \( \lambda \) on radial Velocity.

Fig. 11. Effect of \( \lambda \) on azimuthal Velocity.

Fig. 12. Effect of \( \lambda \) on temperature.

Fig. 13. Effect of \( k_1 \) on radial Velocity.
iii. Assembly of element equations
The algebraic equations so obtained are assembled by imposing the interelement continuity conditions. This yields a large number of algebraic equations known as the global finite-element model, which governs the whole domain.

iv. Imposition of boundary conditions
The essential and natural boundary conditions are imposed on the assembled equations.
v. Solution of assembled equations
The assembled equations so obtained can be solved by any of the numerical techniques, namely, the Gauss elimination method, LU decomposition method, etc. An important consideration is that of the shape functions which are employed to approximate actual functions.

4. Results and discussion
Comprehensive numerical computations are calculated for different values of the parameters that describe the flow characteristics and the results are illustrated graphically. A representative set of computational results are presented in Figs. 2–19. In order to validate the present numerical procedure the results are compared with the results reported by Tasawar Hayat et al. [34] are shown in Table 2 and found to be excellent agreement.

The effect of nanoparticle volume fraction ($\phi$) on the radial velocity, azimuthal velocity, temperature and concentration profiles are depicted in Figs. 2–5 for the both Cu-water and Ag-water based nanofluids. It is noticed from Fig. 2 that the radial velocity profiles are decelerates and this deceleration is almost similar in both nanofluids as the nanoparticle volume fraction increases. This is due to the fact that increasing the nanoparticle volume fraction enhances the momentum boundary layer thickness in the flow regime. However, the azimuthal velocity profiles elevates with increasing values of ($\phi$) in the boundary layer regime (Fig. 3). The both temperature and concentration profiles heightens with increase in the values of nanoparticle volume fraction in both Cu-water and Ag-water nanofluids as shown in Figs. 4 and 5. This is because of the fact that the thermal boundary layer and the solutal boundary layer thickness enhances with higher values of ($\phi$). The enhancement in the both temperature and concentration profiles is higher in the Ag-water based nanofluid than the Cu-water nanofluid.

Figs. 6–9 depict the radial velocity ($f'$), azimuthal velocity ($g$), temperature ($\theta$) and concentration ($S$) distributions for different values of $A$, $R$, $B$, $k_1$, $M$, $Pr$, $Sc$, $Cr$. The results are shown for Cu-water and Ag-water nanofluids, respectively.
the magnetic parameter \((M)\). The both radial and azimuthal velocity profiles depreciates throughout the boundary layer regime as the strength of magnetic parameter is increases in both Cu-water and Ag-water based nanofluids. This is due to the fact that the presence of magnetic field in the flow creates a force known as the Lorentz force which acts as a retarding force and consequently, the momentum boundary layer thickness decelerates throughout the flow region. The deceleration in radial and azimuthal velocity profiles is more in Ag-water based nanofluids than the Cu-water nanofluids (Figs. 6, 7). We define the thermal energy as the additional force which drags the nanofluid from the influence of magnetic field. This additional force increases the thickness of the thermal boundary layer, so that the temperature profile enhances with the rise in \(M\) and this rise is more in the Ag-water nanofluid than the Cu-water nanofluids (Fig. 8). From Fig. 9, we notice that as the values of \(M\) increases the concentration distributions are increased in the both nanofluids.

The radial velocity, azimuthal velocity and temperature profiles of the Cu-water and Ag-water based nanofluids for different values of the velocity slip parameter \((\lambda)\) is depicted in Figs. 10–12. It is seen that the radial velocity \((f')\) and azimuthal velocity \((g)\) profiles deteriorates throughout the boundary layer regime as the values of velocity slip parameter \((\lambda)\) increases. This is due to the fact that as the values of velocity slip parameter \((\lambda)\) increases the stretching velocity is transferred to the fluid, which causes the deceleration in the both velocity profiles. However, the thermal boundary layer thickness is improved in the fluid regime with the higher values of velocity slip parameter \((\lambda)\).

The impact of porous parameter \((k_1)\) on radial velocity and azimuthal velocity profiles is shown in Figs. 13 and 14 for both Cu-water and Ag-water based nanofluids. It is noticed that the radial velocity and azimuthal velocity are both decelerates with the higher values of porous parameter \((k_1)\). This is because of the fact that, the porosity parameter in depending on the permeability parameter \((K)\), the values of porosity parameter increases means the values of permeability parameter decreases causes the depreciation in the both velocity profiles.

The effect of radiation parameter \((R)\) on temperature profiles is shown in Figs. 15 for both Cu-water and Ag-water based nanofluids. It is seen that as the values of thermal radiation parameter increases, the thermal boundary layer thickness is enhanced in both nanofluids. This is due to the fact that the presence of thermal radiation effect increases the temperature of the fluid in the entire flow region. In general, this is true because increasing the Rosseland diffusion approximation for radiation enhances the temperature of the fluid and this increase is higher in the Ag-water nanofluid than the Cu-water nanofluids (Fig. 15).

The temperature profiles of the Cu-water and Ag-water nanofluids for different values of the space-dependent \((A)\) and temperature-dependent \((B)\) coefficients for heat a source/sink is depicted in Figs. 16–17. It is observed that the temperature in the thermal boundary layer increases with the increasing values of \(A\) and \(B\) (positive values), whereas the thermal boundary layer thickness decelerates with decrease in the values of heat absorption parameters \(A\) and \(B\) (negative values). This is due to the fact that, with an increase in \(A>0, B>0\) (heat source), the boundary layer creates energy which causes the rise in the temperature profiles, whereas, with a decrease in \(A<0, B<0\) (heat absorption), the boundary layer absorbs the energy so that the thermal boundary layer thickness decreases in the fluid regime as shown in Figs. 16 and 17.

Impact of thermal slip parameter \((\xi)\) on the thermal boundary layer is shown in Fig. 18 for both Cu-water and Ag-water based nanofluids. It is observed from this figure that the temperature profiles deteriorate with the increasing values of thermal slip parameter \((\xi)\). This is because of the fact that as \((\xi)\) increases there is depreciation in the heat transfer from the disk surface to the adjacent fluid, which causes the thinner thermal boundary layer in the fluid regime. Fig. 19 illustrates the effect of chemical reaction parameter \((Cr)\) on the concentration distributions for both the Cu-water and Ag-water nanofluids. We see from this figure that the concentration profiles are highly influenced and are retards with the higher values of chemical reaction parameter in the flow region.

The values of the local skin-friction co-efficient \((C_f)\), local Nusselt number \((Nu_x)\), and the local Sherwood number \((Sh_x)\) for different values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>(\phi)</th>
<th>(M)</th>
<th>Hayat et al. [34]</th>
<th>Present study</th>
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<td>0.0</td>
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of the key parameters for the both Cu-water and Ag-water based nanofluids are presented in Table 3. It is noticed from this table that the rate of velocity rises in the fluid regime as the values of $(M)$ increases. However, the dimensionless heat transfer rates and mass transfer rates decrease in the both Cu-water and Ag-water based nanofluids with the increasing values of magnetic field parameter $(M)$. The skin-friction co-efficient heightens with the higher values of nanoparticle volume fraction $(\phi)$. It is found that the dimensionless heat transfer rates and the dimensionless mass transfer rates are both deteriorates in both Cu-water and Ag-water based nanofluids with the increasing values of $(\phi)$. It is evident that $\frac{f'(0)}{1}$ rises in the fluid regime as the values of velocity slip parameter $(\lambda)$ increases. However, $-\theta'(0)$ and $-\phi'(0)$ are both decreases in the both Cu-water and Ag-water based nanofluids with the increasing values of $(\lambda)$. The skin-friction co-efficient elevates whereas the Nusselt number and Sherwood number are decelerates with the improving values of radiation parameter $(R)$. It also evident from this table that the rate of velocity and heat transfer rates are both decelerates whereas the dimension less mass transfer rates improves with the increasing values of chemical reaction parameter $(Cr)$.

5. Conclusion

The MHD flow of Cu-water and Ag-water nanofluid over a rotating disk saturated by porous medium with heat generation/absorption, partial slip and first order chemical reaction is investigated numerically. The influence of various key parameters such as $\phi$, $M$, $\lambda$, $k_1$, $R$, $A$, $B$, $\xi$ and $Cr$ on hydrodynamic, thermal and concentration boundary layer is calculated and the results are shown in graphically. The summary of the significant results of the study are as follows:

1. As the values of nanoparticle volume fraction $(\phi)$ increases, the temperature and concentration profiles of the both fluids increase.

2. The radial and azimuthal velocity profiles decelerate whereas the thermal and concentration boundary layer thickness is heightens with the rising values of $(M)$.

3. The velocity slip parameter $(\lambda)$ deteriorates the both velocities $f'(\eta)$ and $g'(\eta)$ in the fluid regime.

4. The rate of fluid velocity $f'(0)$ rises and dimensionless heat transfer rates $-\theta(0)$ diminishes as the values of nanoparticle volume fraction $(\phi)$ increases.

References

17. A.J. Chamkha, S. Abasbandy, A.M. Rashad, Non-Darcy natural convection flow of non-Newtonian nanofluid over a cone saturated in a porous medium with uniform


