Influence of Lorentz forces on nanofluid forced convection considering Marangoni convection

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Abstract

Magnetohydrodynamic nanofluid forced convective heat transfer is investigated considering Marangoni convection. A two-phase model is selected for modeling of a nanofluid. The Runge-Kutta integration scheme is utilized to solve this problem. Influences of the Marangoni ratio, Schmidt number, Brownian motion parameter, magnetic number and thermophoretic parameter on the hydrothermal characteristics are presented. Results depict that the temperature augments with increases of the Schmidt number, Brownian motion, magnetic number and the thermophoretic parameters but it reduces with the rise of the Marangoni ratio. As the Marangoni ratio augments, the hydraulic boundary layer thickness enhances.

1. Introduction

Marangoni convection appears due to surface tension gradients. Liquid–liquid interfaces can generate Marangoni boundary layer. Newly, innovative kinds of fluids are required to reach more efficient performance. A nanofluid was proposed as innovative way to enhance heat transfer. Alsalawy et al. applied heatline analysis for the simulation of nanofluid conjugate free convection. They concluded that the Nusselt number augments with the rise of the wall thickness. Sheikholeslami and Ganji presented various applications of nanofluids in their review paper. Nasrin et al. analyzed nanofluid free convection heat transfer in a chamber. They showed that the uncertainties of heat transfer change with the variation of the nanofluid volume fraction.

Qing et al. examined the entropy production of a Casson nanofluid over a porous stretching plate in the presence of Lorentz forces. They utilized the Chebyshev spectral collocation method. Bhatti et al. investigated the fluid flow over a porous stretching plate using SLM. Slip motion impact on EMHD nanofluid flow over a sheet has been examined by Ayub et al. They presented impacts of active parameters on the Sherwood number. Bhatti and Rashidi presented the impact of thermo-diffusion on a Williamson nanofluid over a sheet. Parvin et al. studied Al2O3-water nanofluid free convection in a curved cavity.

Selimfendigil and Oztop examined nanofluid conjugate conduction-convection mechanism in a titled cavity. They proved that the temperature gradient augmented with the increase of the Grashof number.

Sheikholeslami et al. examined the non-uniform viscosity impact on natural convection of magnetic nanofluid. They illustrated that Nusselt number reduces with rise of viscosity parameter. Nanofluid mixed convection on a cone has been reported by Chamkha et al. They presented radiation impact in their article. Sheikholeslami et al. simulated about the influence of Lorentz forces on nanofluid radiative heat transfer. They depicted that temperature gradient decrease with augment of magnetic field. Influence of non-uniform Lorentz forces on nanofluid flow style has been studied by Sheikholeslami Kandelousi. He concluded that improvement in heat transfer reduces with the rise of the Kelvin forces. Reddy et al. utilized a two-phase model simulation of mixed convection over a plate. The problem of a wavy duct in existence of Brownian forces has been examined by Shehzad et al. They selected the Nelder-Mead method to find the solution. Nanofluid behaviors in various applications have been investigated by several researchers.

The aim of this article is to study the influence of magnetic field on a nanofluid with the existence of Marangoni convection. The Runge-Kutta integration scheme is chosen to simulate this problem. The influences of the active parameters on the hydrothermal treatment are examined.

2. Problem statement

Magnetohydrodynamic Marangoni boundary layer flow of a nanofluid is studied using a two-phase model. The influence of the
induced magnetic field and the Hall effects are neglected [30]. The interface temperature is considered as a function of x. Fig. 1 depicts the interface condition and the velocity components. The governing equations for this investigation are as follows:

\[
\frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} = 0
\]  

\[
\nu \frac{\partial u}{\partial y} + u \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial y^2} - \frac{B_0^2 \sigma_{el}}{\rho_f} u
\]  

\[
\nu \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} = \alpha \frac{\partial^2 T}{\partial y^2} + \left( \frac{\rho C_p}{\rho C_f} \right)_T \left( \frac{\partial C}{\partial y} \frac{\partial T}{\partial y} \right)_T + \left( \frac{D_r}{T_0} \right) \left( \frac{\partial^2 C}{\partial y^2} \right)
\]  

\[
\sigma = \sigma_0 [1 - \gamma_C (C - C_\infty) - \gamma_T (T - T_\infty)]
\]

\[
\gamma_C = - \frac{1}{\sigma_0} \frac{\partial \sigma}{\partial C} \bigg|_{T_\infty}, \quad \gamma_T = - \frac{1}{\sigma_0} \frac{\partial \sigma}{\partial T} \bigg|_{C_\infty}
\]

The corresponding boundary conditions are as follows [1]:

\[
C(x, \infty) = C_\infty, \quad u(x, \infty) = 0, \quad T(x, \infty) = T_\infty,
\]

\[
C(x, 0) = C_\infty + C_0 \xi, \quad X = \xi \frac{L}{x},
\]

\[
T(x, 0) = T_\infty + T_0 \xi^2,
\]

\[
\frac{\mu \partial u}{\partial y}|_{y=0} = - \frac{\partial \sigma}{\partial x} |_{y=0} = \sigma_0 \left( \gamma_C \frac{\partial C}{\partial x} |_{y=0} + \gamma_T \frac{\partial T}{\partial x} |_{y=0} \right)
\]

\[
\nu(x, 0) = 0.
\]

Fig. 1. Physical model of the problem.
In order to transform the above equations in self-similar form, the following equations are defined.

\[
C(x,y) = C_0 x^2 \phi(\eta) + C_w \\
T(x,y) = T_0 x^2 \theta(\eta) + T_w \\
\psi(x,y) = v(x)f(\eta), \quad \eta = y/L
\]

By using Eq. (8), the final dimensionless equations and conditions are given by:

\[
f'' + f'f - f^2 - Mf' = 0 \tag{9}
\]

\[
\theta'' + Pr(f'f - 2f'\theta) + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{10}
\]

\[
\phi'' + Sc(f'\phi - 2f\phi') + \frac{Nt}{Nb}\theta' = 0 \tag{11}
\]

\[
\theta(0) = 1, \quad \phi(0) = 1, \quad f(0) = 0, \quad f'(0) = -2(1 + r), \quad f(\infty) = 0, \quad \theta(\infty) = 0
\]

\[
r = \frac{C_0 \gamma C}{T_0 \gamma_T} \tag{13}
\]

and \( Pr = \frac{v}{\kappa}, \quad Nb = \frac{(\kappa_s \gamma_s)_{T=1}}{(\kappa_s \gamma_s)_{T=0}}, \quad Sc = \frac{(\kappa_s \gamma_s)_{T=1}}{(\kappa_s \gamma_s)_{T=0}}, \quad Nt = \frac{(\rho \gamma L)_{T=1}}{(\rho \gamma L)_{T=0}}, \quad Ha = B_0 L \sqrt{\frac{\omega \alpha}{\mu}} \)

\( M \leq Ha^2 \) are the Prandtl number, Brownian motion parameter, Schmidt number, thermophoretic parameter and the Hartmann number (magnetic number), respectively. \( L \) is a reference length and is defined as in [31]:

\[
L = -\frac{1}{\sigma_0 \gamma_T^2 \gamma_T} \tag{14}
\]

\( r \) can be introduced as \( r = Ma_C/Ma_T \) where \( Ma_C = \alpha_0 \gamma C_0 L/\alpha v \) and \( Ma_T = \alpha_0 \gamma T_0 L/\alpha v \).

3. Numerical Runge-Kutta method

In the Runge-Kutta method, at first, the following definitions are applied: \( x_2 = f, x_3 = \eta, x_4 = f', x_5 = \theta, x_6 = \theta', x_7 = \phi, x_8 = \phi' \). The

![Fig. 3](image-url) Effect of magnetic number on velocity, temperature and concentration profiles when \( r = 1, Sc = 1, Nb = 1, Nt = 0.001, Pr = 10 \).
Fig. 4. Effect of \( r \) on velocity, temperature and concentration profiles when \( M = 1, Sc = 1, Nb = 1, Nt = 0.001, Pr = 10. \)

Fig. 5. Effect of Schmidt number on temperature and concentration profiles when \( M = 1, r = 1, Nb = 1, Nt = 0.001, Pr = 10. \)
Fig. 6. Effect of Brownian motion parameter on temperature and concentration profiles when $M = 1, r = 1, Sc = 1, Nt = 0.0001, Pr = 10$.

Fig. 7. Effect of thermophoretic parameter on temperature and concentration profiles when $M = 1, r = 1, Sc = 1, Nb = 1, Pr = 10$.

Fig. 8. Effect of $M, r, Sc$ on Nusselt number.
Fig. 9. Contour plots for effect on active parameters on Nusselt number when Pr = 10.
final system and initial conditions are:

\[
\begin{align*}
\frac{dx_1}{dr} &= 0 \\
\frac{dx_2}{dr} &= 0 \\
\frac{dx_3}{dr} &= u_1 \\
\frac{dx_4}{dr} &= 1 \\
\frac{dx_5}{dr} &= u_2 \\
\frac{dx_6}{dr} &= 1 \\
\frac{dx_7}{dr} &= u_3 \\
\frac{dx_8}{dr} &= -2(1 + r)
\end{align*}
\]

Eqs. (15) and (16) are solved utilizing the 4th order Runge-Kutta method. According to \( f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \), unknown initial conditions can be obtained by the Newton’s method.

4. Results and discussion

Magnetohydrodynamic nanofluid hydrothermal behavior is examined considering Marangoni convection. Two-phase model is taken into account. The Runge-Kutta integration scheme is utilized to solve this problem. The MAPLE code has been validated by a comparison with the results of a previously published paper [1]. Fig. 2(a) indicates good accuracy of the present code. Fig. 2(b) shows the good agreement between the exact and numerical solutions. The concentration, temperature and the velocity profiles for several values of the Marangoni ratio, thermophoretic parameter, Brownian motion parameter, magnetic number and the Schmidt number are presented.

Fig. 3 depicts the impact of the Lorentz forces on the flow, mass and heat transfer. As the Lorentz forces augment, a back flow appears and in turn, both components of velocity reduce. The
hydraulic boundary layer thickness reduces in existence of the magnetic field. The influence of the Lorentz forces on the normal velocity is more pronounced than on the tangential velocity. Also, the temperature gradient near the hot surface reduces with augmentation of the Hartmann number. The concentration augments with the augmentation of the Lorentz forces. The influence of the Marangoni ratio on the temperature, concentration and the velocity distributions are depicted in Fig. 4. Both the tangential and the normal velocity enhance with the augmentation of the Marangoni ratio. The temperature reduces with the increase of this parameter. The influence of the Marangoni ratio on the concentration is similar to that obtained for the temperature.

Fig. 5 illustrates the impact of the Schmidt number on the concentration and the temperature distributions. The temperature reduces with the rise of the Schmidt number but the opposite trend is seen for the concentration. The influences of the thermophoretic and the Brownian motion parameters on the concentration and the temperature distributions are illustrated in Figs. 6 and 7. As these parameters rise, the temperature enhances. The concentration enhancements with the augmentation of the thermophoretic parameter but it reduces with increase of the Brownian motion.

The Nusselt number for various values of the active parameters is depicted in Figs. 8 and 9 and Tables 1 and 2. According to these data, a correlation is presented for the Nusselt number as follows:

\[
Nu = 6.22372 - 0.31371M + 1.4172r - 0.039731Sc - 0.858Nb - 4.333Nr - 0.0129Mr + 2.66 \times 10^{-3}Ms + 0.046096Nb + 0.233M + 0.079rSc + 0.0258rNr - 0.20721ScNb + 1.794M + 6.21 \times 10^{-7}M^{2} - 0.078r^{2} + 0.0183r^{2} + 0.0758Nb^{2} - 1.0896r^{2}
\]

(17)

It is predicted that the Nusselt number enhances with the augmentation of the Marangoni ratio. Enhancing the Lorentz forces causes the Nusselt number to augment due to the increment in the boundary layer thickness. Impacts of the thermophoretic parameter, Schmidt number and the Brownian motion parameter on Nu are similar to that of the Hartmann number.

5. Conclusion

MHD nanofluid hydrothermal flow in the presence of Marangoni convection is studied using a two-phase model. The Runge-Kutta integration method is selected to solve this problem. The impacts of the Marangoni ratio, thermophoretic parameter, Schmidt number, Brownian motion parameter and the magnetic number on hydrothermal characteristics are examined. The results indicate that the temperature gradient augments with the rise of the Marangoni ratio but it reduces with the increase of the other active parameters. As the Lorentz force increases, the hydraulic boundary layer thickness reduces but the opposite tendency is shown for the concentration and the temperature boundary layer thicknesses.

References