Heat and mass transfer analysis in natural convection flow of nanofluid over a vertical cone with chemical reaction
P. Sudarsana Reddy A. Chamkha

Article information:
To cite this document:
Permanent link to this document: http://dx.doi.org/10.1108/HFF-10-2015-0412

Downloaded on: 07 March 2017, At: 02:30 (PT)
References: this document contains references to 40 other documents.
To copy this document: permissions@emeraldinsight.com

Users who downloaded this article also downloaded:


Access to this document was granted through an Emerald subscription provided by emerald-srm:557711 []

For Authors
If you would like to write for this, or any other Emerald publication, then please use our Emerald for Authors service information about how to choose which publication to write for and submission guidelines are available for all. Please visit www.emeraldinsight.com/authors for more information.

About Emerald www.emeraldinsight.com
Emerald is a global publisher linking research and practice to the benefit of society. The company manages a portfolio of more than 290 journals and over 2,350 books and book series volumes, as well as providing an extensive range of online products and additional customer resources and services.

Emerald is both COUNTER 4 and TRANSFER compliant. The organization is a partner of the Committee on Publication Ethics (COPE) and also works with Portico and the LOCKSS initiative for digital archive preservation.

*Related content and download information correct at time of download.
Heat and mass transfer analysis in natural convection flow of nanofluid over a vertical cone with chemical reaction

P. Sudarsana Reddy
Department of Mathematics, RGM College of Eng. & Tech, Nandyal, India, and
A. Chamkha
Department of Mechanical Engineering, Prince Mohammad Bin Fahd University, Al-Khobar, Saudi Arabia

Abstract
Purpose – In recent years, nanofluids are being widely used in many thermal systems because of their higher thermal conductivity and heat transfer rate. The higher thermal conductivity depends on many parameters such as size, shape and volume and the Brownian motion and thermophoresis of added nanoparticles. The purpose of this paper is to analyze the influence of the Brownian motion and thermophoresis on natural convection heat and mass transfer boundary layer flow of nanofluids over a vertical cone with radiation.

Design/methodology/approach – Using similarity variables, the non-linear partial differential equations, which represent momentum, energy and diffusion, are transformed into ordinary differential equations. The transformed conservation equations are solved numerically subject to the boundary conditions by using versatile, extensively validated, variational finite-element method.

Findings – The sway of significant parameters such as magnetic field (M), buoyancy ratio parameter (Nr), Brownian motion parameter (Nb), thermophoresis parameter (Nt), thermal radiation (R), Lewis number (Le) and chemical reaction parameter (Cr) on velocity, temperature and concentration evaluation in the boundary layer region is examined in detail. The results are compared with previously published work and are found to be in agreement. The velocity distributions are reduced, while temperature and concentration profiles elevate with a higher (M). With the improving values of (R), the velocity and temperature sketches improve, while concentration distributions are lowered in the boundary layer region. The temperature and concentration profiles are elevated in the boundary layer region for higher values of (Nt). With the increasing values of (Nb), temperature profiles are enhanced, whereas concentration profiles get depreciated in the flow region.

Social implications – In recent years, it has been found that magneto-nanofluids are significant in many areas of science and technology. It has applications in optical modulators, magnetooptical wavelength filters, tunable optical fiber filters and optical switches. Magnetic nanoparticles are especially useful in biomedicine, sink float separation, cancer therapy, etc. Specific biomedical applications involving nanofluids include hyperthermia, magnetic cell separation, drug delivery and contrast enhancement in magnetic resonance imaging.

Originality/value – To the best of the authors’ knowledge, no studies have assessed the impact of the two slip effects, namely, Brownian motion and thermophoresis, on the natural convection of electrically conducted heat and mass transfer to the nanofluid boundary layer flow over a vertical cone in the presence of radiation and chemical reaction; therefore, this problem has been addressed in this study.

The authors are thankful to the reviewers for their decent suggestions and observations to improve the quality of the manuscript.
Comparison of the results of this study’s with those of previously published work was found to be in good agreement.

**Keywords** Nanofluid, Thermophoresis, Brownian motion, Radiation, Chemical reaction, Vertical cone

**Paper type** Research paper

**Nomenclature**

- $g$ = Gravitational acceleration vector ($m/s^2$)
- $K_m$ = Thermal conductivity ($W m^{-1} K^{-1}$)
- $C$ = Nanoparticle volume fraction
- $C_w$ = Ambient nanoparticle volume fraction
- $T_w$ = Temperature at the cone surface
- $T$ = Fluid temperature (K)
- $q_w$ = Wall heat flux
- $D_B$ = Brownian diffusion coefficient ($m^2/s$)
- $f(\eta)$ = Dimensionless stream function
- $N_t$ = Thermophoresis parameter
- $P$ = Pressure (Pa)
- $K^*$ = Mean absorption coefficient
- $M$ = Magnetic parameter
- $C_r$ = Scaled chemical reaction parameter
- $B_0$ = Magnetic field strength
- $N_r$ = Buoyancy ratio parameter
- $(u,v)$ = Velocity components in x- and y-axis (m/s)
- $C_i$ = Skin-friction coefficient
- $c_p$ = Specific heat ($J/kg K$)

**Greek symbols**

- $\alpha$ = Thermal diffusivity of base fluid ($m^2/s$)
- $\rho_f$ = Fluid density ($kg m^{-3}$)
- $\psi$ = Stream function
- $\tau$ = Parameter defined by $e \left( (\rho c)_{f} / (\rho c)_{w} \right)$
- $\phi (\eta)$ = Dimensionless nanoparticle volume fraction Similarity variable
- $\theta (\eta)$ = Dimensionless temperature
- $\beta$ = Thermal expansion coefficient ($1/K$)

**Subscripts**

- $w$ = Condition at cone surface
- $\eta$ = Similarity variable
- $Ra_x$ = Convention parameter
- $Nu_x$ = Nusselt number
- $C_{w}$ = Nanoparticle volume fraction on the cone
- $(x, y)$ = Cartesian coordinates
- $T_{w}$ = Ambient temperature attained
- $K_{r}$ = Chemical reaction parameter
- $j_w$ = Wall mass flux
- $D_T$ = Thermophoretic diffusion coefficient ($m^2/s$)
1. Introduction

The theory of nanofluids is an old concept, and it was first introduced by Choi and Eastman (1995) when they were researching on new coolants and cooling technologies, and it became popular because of its numerous applications in heat exchangers, nuclear reactor systems, boilers, electronic cooling and energy storage devices (Ostrach, 1988). The thermal conductivity and ultra-small particle size are the valuable thermophysical properties of nanofluids, and because of this, nanofluids show significantly better performance than the normal single- and multi-phase fluids (Li et al., 2009; Buongiorno and Venerus, 2009; Lazarus Godson et al., 2010; Ghadimi et al., 2011). Many theoretical and experimental studies have suggested that thermal conductivity and dynamic viscosity depend on the shape, size and constructive materials of nanoparticles and the type and working temperature of the base fluid (Saidur et al., 2011; Kleinstreuer and Feng, 2011; Abu-Nada, 2009; Sundar et al., 2013; Li and Peterson, 2007). There are some other important mechanisms, methods of synthesis of nanofluid and sonication time, that influence the thermophysical properties and the heat transfer enhancement of nanofluids. In addition, Brownian motion and thermophoresis are the mass transfer mechanisms that affect the convective heat transfer performance of nanofluids (Kuznetsov et al., 2010; Zaraki et al., 2015). Many experimental and numerical studies in the literature have mentioned about the importance of nanofluids’ natural convection heat transfer property (Muthtamiliselvan et al., 2010; Parvin et al., 2012; Kamyar et al., 2012). However, we can witness diverse conclusions in those experimental and numerical investigations (Haddad et al., 2012). In an experimental investigation of nanofluids, deterioration in the natural convection heat transfer is usually noticed, whereas in numerical investigation, enhancement is reported. In his benchmark study, Buongiorno (2006) reported seven possible mechanisms associating the natural convection of nanofluids with the movement of nanoparticles in the base fluid by using scale analysis. These mechanisms include nanoparticle size, inertia, particle agglomeration, Magnus effect, volume fraction of the nanoparticle, Brownian motion, particle size, and thermophoresis. Among these mechanisms, the Brownian motion and thermophoresis are found to be of importance. The thermophoresis acts against a temperature gradient, aiding the movement of the particles from higher- to lower-temperature regions. In addition, the Brownian motion aids the movement of particles from higher- to lower-concentration areas. Kuznetsov et al.
(2010) discussed the influence of the Brownian motion and thermophoresis on nanofluids’ natural convection boundary layer flow over a vertical plate, and because of the importance of the Brownian motion and thermophoresis effects, they studied the concentration boundary layer of nanoparticles. Aziz and Khan (2012) presented nanofluids’ natural convection boundary layer flow over a vertical plate subject to the convective boundary conditions.

In recent years, it has been found that magneto-nanofluids are significant in many areas of science and technology, with applications in optical modulators, magneto-optical wavelength filters, tunable optical fiber filters and optical switches. Magnetic nanoparticles are especially useful in biomedicine, sink float separation, cancer therapy, etc. Specific biomedical applications involving nanofluids include hyperthermia, magnetic cell separation, drug delivery and contrast enhancement in magnetic resonance imaging. In view of the aforementioned applications, Chamkha et al. (2011) studied the mixed-convection magneto-hydrodynamic flow of a nanofluid past a stretching permeable surface in the presence of the Brownian motion and thermophoresis effects. Rashidi et al. (2014a, 2014b) discussed the dynamics of nanofluids from a non-linearly stretching sheet with transpiration using Homotopy simulation. Rashidi et al. (2014a, 2014b) studied single- and double-phase models of nanofluid heat transfer in Wavy Channel. Noghrehbadi and Behseresht (2013) analyzed how flow and heat transfer are affected by variable properties of nanofluids over a vertical cone saturated in a porous medium. Noghrehbadi et al. (2013a, 2013b, 2013c) analyzed the natural convection of nanofluids under different geometries of the stretching sheet and a vertical plate. Behseresht et al. (2013) presented natural convection heat and mass transfer of nanofluid over a vertical cone by considering the practical range of nanofluids’ thermo-physical properties. Chamkha et al. (2013) discussed the mixed-convection flow over a vertical cone through a porous medium saturated by a nanofluid with thermal radiation. In addition, Gorla et al. (2014) studied nanofluids’ natural convection boundary layer flow through a porous medium over a vertical cone. Chamkha et al. (2014) presented non-Darcy free convective nanofluids along a vertical plate with suction/injection and internal heat generation. Recently, Chamkha et al. (2015) investigated non-Newtonian nanofluid natural convection flow over a cone through a porous medium with uniform heat and volume fraction fluxes. Very recently, Garoosi et al. (2015a, 2015b) presented natural convection of nanofluids in a square cavity and heat exchangers.

To the best of the authors’ knowledge, no studies have assessed the impact of the two slip effects, namely, Brownian motion and thermophoresis, on the natural convection of electrically conducted heat and mass transfer to the nanofluid over a vertical cone in the presence of radiation and chemical reaction; therefore, this problem has been addressed in this study. The transformed boundary layer equations, which represent the flow, temperature and concentration, are solved numerically using finite-element method (FEM). The problem reported in the present study is of immediate interest in the upcoming generation of heat exchange technology, boilers, solar film collectors and geothermal energy storage systems.

2. Mathematical analysis

Figure 1 shows a two-dimensional, study of an electrically conducting heat and mass transfer boundary layer flow of a nanofluid over a vertical cone. The coordinate system is chosen, as the x-axis is coincident with the flow direction over the cone surface. It is assumed that $T_w$ and $C_w$ are the temperature and nanoparticle volume fraction at the surface of the cone ($y = 0$) and $T_w$ and $C_w$ are the temperature and nanoparticle volume fraction of the ambient fluid, respectively. In the present analysis, the nanoparticles are persuaded in the base fluid according to the Brownian motion and thermophoresis.
The nanoparticles tend to propel from a hot to cold surface because of thermophoresis. In contrast, the Brownian motion drive nanoparticles to move from higher- to lower-concentration surface. Accordingly, because of the hot surface, nanoparticles move away from the surface of the cone. Because of the Brownian motion force, the concentration of nanoparticles becomes uniform, and the concentration boundary layer of nanoparticles exists over the surface of the cone. Similarly, because of the thermophoresis force, the temperature of nanoparticles becomes uniform; so, a thermal boundary layer of nanoparticles exists at the cone surface.

An external magnetic field of strength $B_0$ is applied in the direction of the $y$-axis. By considering the works of Kuznetsov et al. (2010) and by using the Oberbeck–Boussinesq approximation, the governing equations describing the steady-state conservation of mass, momentum and energy, as well as conservation of nanoparticles for nanofluids in the presence of thermal radiation and other important parameters, take the following form:

\[
\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\left[1 - C_w\right] \beta(T - T_w) - (\rho_p - \rho) (C - C_w) \cos \gamma - \frac{\alpha B_0^2}{\rho_f} u
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \left[ D_h \frac{\partial C}{\partial y} \cdot \frac{\partial T}{\partial y} + \left( \frac{D_T}{T_w} \right) \frac{\partial T}{\partial y} \right] - \frac{1}{\rho c_p} \frac{\partial q_r}{\partial y}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_h \frac{\partial^2 C}{\partial y^2} + \left( \frac{D_T}{T_w} \right) \frac{\partial^2 T}{\partial y^2} - K_s (C - C_w)
\]

The associated boundary conditions are as follows:
\[ u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w \quad \text{at} \quad y = 0 \quad (5) \]

\[ u \to 0, \quad T \to T_w, \quad C \to C_w \quad \text{at} \quad y \to \infty \quad (6) \]

The radiative heat flux \( q_r \) (using Rosseland Approximation) is defined as follows:

\[ q_r = -\frac{4\sigma^* \partial T^4}{3K^*} \frac{\partial y}{\partial y}, \quad (7) \]

We assume that the temperature variances inside the flow are such that the term \( T^4 \) can be represented as a linear function of temperature, and so, it has the Taylor series expansion. After neglecting higher-order terms from the Taylor series expansion of \( T^4 \) about \( T_w \), we get the following equation:

\[ T^4 \equiv 4T_w^3 T - 3T_w^4. \quad (8) \]

Thus, substituting equation (8) in equation (7), we get the following equation:

\[ q_r = -\frac{16T_w^3\sigma^* \partial T}{3K^*} \frac{\partial y}{\partial y}. \quad (9) \]

We now introduce the following similarity variables to transform the governing equations into a system of ordinary differential equations:

\[ u = axf(\eta), \quad v = -\sqrt{\nu}yf(\eta), \quad \eta = \sqrt{\frac{a}{\nu}} y, \]

\[ \theta(\eta) = \frac{T - T_w}{T_w - T}, \quad \phi(\eta) = \frac{C - C_w}{C_w - C}. \quad (10) \]

Here, \( r \) can be approximated by the local radius of the cone, if the thermal boundary layer is thin, and is related to the \( x \) coordinate by \( r = x \sin \gamma \).

Substituting equations (9) and (10) into equations (1)-(4), we get the following system of non-linear ordinary differential equations:

\[ f'''' - (f')^2 + ff''' + Ra_1(\theta - Nr\phi) - Mf' = 0 \quad (11) \]

\[ (1 + R)\theta'' + Prf\theta' + Nb\theta'\varphi' + Ni(\theta')^2 = 0 \quad (12) \]

\[ \varphi'' + Le\varphi' - Le \cdot Cr \cdot \phi + \frac{Nt}{Nb} \theta'' = 0 \quad (13) \]

The transformed boundary conditions are as follows:

\[ \eta = 0, \quad f = 0, \quad f'(0) = 1, \quad \theta = 1, \quad \phi = 1. \quad (14) \]

\[ \eta \to \infty, \quad f' = 0, \quad \theta = 0, \quad \phi = 0. \]

where the prime (') denotes differentiation with respect to \( \eta \); the significant thermophysical parameters dictating the flow dynamics are defined as follows:
Quantities of practical interest in this problem are the skin-friction coefficient, local Nusselt number $Nux$, and the local Sherwood number $Shx$, which are defined as follows:

$$Cf = \frac{2\tau \rho}{\rho}, \quad Nux = \frac{xq_T}{k(T_w - T_x)}, \quad Shx = \frac{xJ_T}{D_B(C_w - C_x)}$$

The set of ordinary differential equations (11)–(13) is highly non-linear, and therefore, it cannot be solved analytically. The FEM (Bhargava et al., 2009; Bég et al., 2008; Reddy, 1985; Rana and Bhargava, 2012; Rana et al., 2012; Goyal and Bhargava, 2013) has been implemented to solve these non-linear equations.

3. Numerical method of solution
3.1 The finite-element method
The FEM is such a powerful method for solving ordinary and partial differential equations. The basic idea of this method is dividing the whole domain into smaller elements of finite dimensions called finite elements. This method is a good numerical method in modern engineering analysis, and it can be applied for solving integral equations including heat transfer, fluid mechanics, chemical processing, electrical systems and many others. The steps involved in the FEM are as follows.

1. **Finite-element discretization**: In finite element discretization, the entire interval is divided into a finite number of subintervals and this subinterval is called an element. The set of all these elements is called the finite-element mesh.

2. **Generation of the element equations**:
   - A variational formulation of the mathematical model over the typical element (an element from the mesh) is performed.
   - An approximate solution of the variational problem is assumed, and the element equations are made by substituting this solution in the above system.
   - Using interpolating polynomials, the stiffness matrix is constructed.

3. **Assembly of element equations**: By imposing the inter-element continuity conditions, all the algebraic equations are assembled. This result in a large number of algebraic equations called global FEM, and it represents the whole domain.

4. **Imposition of boundary conditions**: The boundary conditions which represent the flow model are imposed on the assembled equations.

5. **Solution of assembled equations**: The assembled equations so obtained can be solved by any of the numerical techniques, namely, Gauss elimination method, LU decomposition method, etc. An important consideration is that of the shape functions which are used to approximate actual functions.
For the solution of system of non-linear ordinary differential equations (11)-(13) together with boundary conditions (14), we first assume that:

$$\frac{df}{d\eta} = h$$  \hspace{1cm} (17)

Equations (11)-(13) are then reduced to the following equations:

$$h'' - h^2 + fh' + Ra_x(\theta - Nr\phi) - Mh = 0$$  \hspace{1cm} (18)

$$(1 + R)\theta'' + Prf\theta' + Nb \theta' \phi' + Nt(\theta')^2 = 0$$  \hspace{1cm} (19)

$$\varphi'' + Le \phi' - Le . Cr . \phi + \frac{Nt}{Nb} \theta'' = 0$$  \hspace{1cm} (20)

The boundary conditions take the following form:

$$f = 0, \quad h = 1, \quad \theta = 1, \quad \varphi = 1, \quad \text{at} \quad \eta = 0,$$

$$h = 0, \quad \theta = 0, \quad \varphi = 0, \quad \text{at} \quad \eta \to \infty.$$  \hspace{1cm} (21)

3.2 Variational formulation

The variational form associated with equations (17)-(20) over a typical linear element \((\eta, \eta_{+1})\) is given by the following equations:

$$\int_{\eta}^{\eta_{+1}} w_1 \left( \frac{df}{d\eta} - h \right) d\eta = 0$$  \hspace{1cm} (22)

$$\int_{\eta}^{\eta_{+1}} w_2 (h'' - h^2 + fh' + Ra_x(\theta - Nr\phi) - Mh) d\eta = 0$$  \hspace{1cm} (23)

$$\int_{\eta}^{\eta_{+1}} w_3 (1 + R)\theta'' + Prf\theta' + Nb \theta' \phi' + Nt(\theta')^2 d\eta = 0$$  \hspace{1cm} (24)

$$\int_{\eta}^{\eta_{+1}} w_4 (\varphi'' + Le \phi' - Le . Cr . \phi + \frac{Nt}{Nb} \theta'') d\eta = 0$$  \hspace{1cm} (25)

Where \(w_1, w_2, w_3,\) and \(w_4\) are arbitrary test functions and may be viewed as the variations in \(f, h, \theta,\) and \(\varphi,\) respectively.

3.3 Finite-element formulation

The FEM may be obtained from the aforementioned equations by substituting finite-element approximations of the following form:

$$f = \sum_{j=0}^{2} f_j \Psi_j, \quad h = \sum_{j=0}^{2} h_j \Psi_j, \quad \theta = \sum_{j=0}^{2} \theta_j \Psi_j, \quad \phi = \sum_{j=0}^{2} \phi_j \Psi_j$$  \hspace{1cm} (26)

With \(w_1 = w_2 = w_3 = w_4 = \Psi_i \quad (i = 1, 2).\)
Where $\Psi_i$ is the shape function for a typical element $(\eta_e, \eta_{e+1})$ and is defined as follows:

$$
\Psi_i^1 = \frac{(\eta_{e+1} - \eta)}{(\eta_{e+1} - \eta_e)}, \quad \Psi_i^2 = \frac{(\eta - \eta_e)}{(\eta_{e+1} - \eta_e)}, \quad \eta_e \leq \eta \leq \eta_{e+1}.
$$

(27)

The FEM of the equations thus formed is given by the following equation:

$$
\begin{bmatrix}
[K^{11}] & [K^{12}] & [K^{13}] & [K^{14}] \\
[K^{21}] & [K^{22}] & [K^{23}] & [K^{24}] \\
[K^{31}] & [K^{32}] & [K^{33}] & [K^{34}] \\
[K^{41}] & [K^{42}] & [K^{43}] & [K^{44}]
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
 f \\
h \\
\theta \\
\phi
\end{bmatrix} \\
\begin{bmatrix}
 r^1 \\
r^2 \\
r^3 \\
r^4
\end{bmatrix}
\end{bmatrix} = 0.
$$

Where $[K^{mn}]$ and $[r^m]$ $(m, n = 1, 2, 3, 4)$ are defined as follows:

$$
K_{ij}^{11} = \int_{\eta_e}^{\eta_{e+1}} \psi_j \frac{\partial \Psi_i}{\partial \eta} d\eta, \quad K_{ij}^{12} = -\int_{\eta_e}^{\eta_{e+1}} \psi_i \frac{\partial \Psi_j}{\partial \eta} d\eta, \quad K_{ij}^{13} = K_{ij}^{14} = 0.
$$

$$
K_{ij}^{21} = \int_{\eta_e}^{\eta_{e+1}} \psi_j \psi_i d\eta, \quad K_{ij}^{22} = -\int_{\eta_e}^{\eta_{e+1}} \frac{\partial \psi_j}{\partial \eta} \frac{\partial \psi_i}{\partial \eta} d\eta + \int_{\eta_e}^{\eta_{e+1}} \psi_j \psi_i^2 d\eta + M \int_{\eta_e}^{\eta_{e+1}} \psi_j d\eta.
$$

$$
K_{ij}^{31} = Pr \int_{\eta_e}^{\eta_{e+1}} \psi_j \theta^i d\eta, \quad K_{ij}^{32} = 0.
$$

$$
K_{ij}^{33} = (1 + R) \int_{\eta_e}^{\eta_{e+1}} \frac{\partial \psi_j}{\partial \eta} \frac{\partial \psi_i}{\partial \eta} d\eta + Nt \int_{\eta_e}^{\eta_{e+1}} \psi_j \theta^i \frac{\partial \psi_i}{\partial \eta} d\eta.
$$

$$
K_{ij}^{34} = Nb \int_{\eta_e}^{\eta_{e+1}} \psi_j \frac{\partial \psi_i}{\partial \eta} d\eta, \quad K_{ij}^{41} = \frac{1}{2} Le \int_{\eta_e}^{\eta_{e+1}} \psi_i \theta^j d\eta, \quad K_{ij}^{42} = 0.
$$

$$
K_{ij}^{43} = \frac{-Nt}{Nb} \int_{\eta_e}^{\eta_{e+1}} \frac{\partial \psi_j}{\partial \eta} \frac{\partial \psi_i}{\partial \eta} d\eta, \quad K_{ij}^{44} = \int_{\eta_e}^{\eta_{e+1}} \frac{\partial \psi_j}{\partial \eta} \frac{\partial \psi_i}{\partial \eta} d\eta - Le \cdot Cr \int_{\eta_e}^{\eta_{e+1}} \psi_j \psi_i d\eta.
$$

$$
\begin{align*}
r^2_i &= 0, \\
r^3_i &= -\left(\frac{d\psi_i}{d\eta}\right)_{\eta_e}^{\eta_{e+1}}, \\
r^4_i &= -\left(\frac{d\psi_i}{d\psi_j}\right)_{\eta_e}^{\eta_{e+1}} - \left(\frac{d\psi_i}{d\psi_j}\right)_{\eta_e}^{\eta_{e+1}}.
\end{align*}
$$

Where:

$$
\begin{align*}
f^j &= \sum_{j=0}^{2} f_i \frac{\partial \psi_i}{\partial \eta}, \\
h^i &= \sum_{j=0}^{2} h_i \frac{\partial \psi_i}{\partial \eta}, \\
\theta^i &= \sum_{j=0}^{2} \theta_i \frac{\partial \psi_i}{\partial \eta}, \\
\phi^i &= \sum_{j=0}^{2} \phi_i \frac{\partial \psi_i}{\partial \eta}.
\end{align*}
$$
The important aspect in this numerical procedure is to select an approximate finite value of \( \eta \). So, to estimate the relevant value of \( \eta \), the solution process has been started with an initial value of \( \eta = 4 \), and then equations (22)-(25) are solved with associated boundary conditions. We have updated the value of \( \eta \) and the solution process is continued until the results are not affected with further values of \( \eta \). The choice of \( \eta_{\text{max}} = 5 \) and \( \eta = 8 \) for velocity and temperature and concentration has confirmed that all the numerical solutions approach the asymptotic values at the free stream conditions. To investigate the sensitivity of the solutions to mesh density, we have performed the grid invariance test for velocity, temperature and concentration distributions, and the results of the test are shown in Table I. It is observed from this table that in the same domain, the accuracy is not affected, even though the number of elements increases, by decreasing the size of the elements.

4. Results and discussion

Comprehensive numerical computations are conducted for different values of the parameters that describe the flow characteristics, and the results are illustrated graphically from Figures 2-19. In most practical situations, heat should be detached from the hot surface into the ambient space; so, it is worth mentioning that a cone with a hot surface is more practical than that with a cold surface. However, there are some cases in which there is heat reaction or heat absorbing process inside the cone in which the cone should be heated from the ambient space. Hence, the main aim of the present study is to discuss the heat and mass transfer characteristics over a cone with hot surface. The results of this study are compared with those of previously published work and are shown in Table II.

The influence of magnetic field parameter \( M \) on velocity, temperature and concentration profiles in the boundary layer is depicted in Figures 2-4. It is noticed from these figures that...
Figure 2. Effect of M on velocity profile

Figure 3. Effect of M on temperature profile

Figure 4. Effect of M on concentration profile

M = 0.1, 0.4, 0.7, 1.0, 1.5.

$Ra_x = 0.5$, $Nt = 0.5$, $Nr = 0.5$
$Pr = 1.0$, $R = 0.1$, $Le = 2.0$,
$Cr = 0.2$, $Nb = 0.5$
the hydrodynamic boundary layer thickness decelerates, whereas thermal boundary layer thickness and solutal boundary layer thickness heighten with enhances in the values of (M). This is because of the fact that the presence of a magnetic field in an electrically conducting fluid produces a force called Lorentz force; this force acts against the flow direction and causes depreciation in velocity profiles (Figure 2), and, at the same time, to overcome the drag force imposed by the Lorentzian retardation, the fluid has to perform extra work. This supplementary work can be converted into thermal energy which increases the temperature of the fluid (Figure 3) and also increases the concentration profiles (Figure 4).

Figures 5-7 depict the effect of convection parameter (Ra_x) on velocity, temperature and concentration distributions. An increase in (Ra_x) elevates the velocity of the fluid because of the enhancement of convection currents (Figure 5). It is analyzed that both temperature and concentration profiles decelerate with increasing values of the convection parameter Ra_x. This is because of the fact that the convection parameter is more dominant as compared to the buoyancy ratio parameter, so that, there is retardation in the thickness of thermal and solutal boundary layers. Furthermore, the temperature and concentration profiles increase when Ra_x = 0 (forced convection) because of no buoyancy forces, and both profiles retard with the increasing values of (Ra_x).

Figures 8-10 illustrate the effect of buoyancy ratio parameter (Nr) on velocity, temperature and concentration distributions through the boundary layer regime. It can be seen from Figure 8 that the thickness of hydrodynamic boundary layer is reduced with enhancing values (Nr). The temperature profiles of the fluid increases with increasing values of buoyancy ratio parameter (Nr). This is from the reality that higher values of buoyancy ratio parameter enhance the fluids temperature, so that thermal boundary layer thickness increases (Figure 9). The concentration profiles enhance throughout the fluid region for different improving values of the buoyancy ratio parameter (Nr). This is because of the fact that solutal boundary layer thickness elevates with increasing values of Nr (Figure 10).

The effect of thermal radiation parameter (R) on velocity, temperature and concentration profiles is shown in Figures 11-13. It is noticed from Figures 11 and 12 that the hydrodynamic and thermal boundary layer thickness is enhanced with the higher values of (R) in the entire flow region. This is because of the fact that imposing thermal radiation into the flow, will increase the temperature of the fluid, causing an increment in the velocity of the fluid and a further increase in the temperature of the fluid. However, there is deceleration in the concentration boundary layer thickness with increasing values of R (Figure 13).
Variation of non-dimensional temperature and concentration distributions for different values of thermophoretic parameter (Nt) is depicted in Figures 14 and 15. The thermophoresis acts against the temperature gradient, so that, the particles move from higher- to lower-temperature regions. It is noticed from these figures that both temperature and concentration profiles get elevated in the boundary layer region for the higher values of thermophoretic parameter (Nt). This is from the reality that particles near the hot surface...
create a thermophoretic force; this force enhances the temperature and concentration of the fluid in the boundary layer region.

The effect of the Brownian motion parameter (Nb) on temperature and concentration profiles is illustrated in Figures 16 and 17. The Brownian motion is the random motion of suspended nanoparticles in the base fluid and is more influenced by its fast-moving atoms or molecules in the base fluid. It is worth mentioning that Brownian motion is related to the size of nanoparticles that are often in the form of agglomerates and/or aggregates. It is noticed that, with the increasing values of Brownian motion parameter (Nb), the temperature profiles also enhance as shown in Figure 16, whereas concentration profiles get depreciated (Figure 17). Clearly, we noticed that the Brownian motion parameter has a significant influence on both temperature and concentration profiles.

The impact of the Lewis number (Le) on concentration profiles is plotted in Figure 18. It is observed that concentration distributions decelerate with the increasing values of the Lewis number in the entire boundary layer region. By definition, the Lewis number represents the ratio of the thermal diffusivity to the mass diffusivity. Increasing the Lewis number means a higher thermal diffusivity and a lower mass diffusivity, and this produces a thinner concentration boundary layer. Figure 19 depicts the variations in concentration distributions in the boundary layer region for different values of the chemical reaction parameter (Cr). We
Figure 11. Effect of $R$ on velocity profile

$R = 0.1, 0.6, 1.2, 1.6, 2.0$

$Ra_x = 0.5$, $Nt = 0.5$, $Nr = 0.5$, $Pr = 1.0$, $M = 0.5$, $Le = 2.0$, $Cr = 0.2$, $Nb = 0.5$

Figure 12. Effect of $R$ on temperature profile

$R = 0.1, 0.6, 1.2, 1.6, 2.0$

$Ra_x = 0.5$, $Nt = 0.5$, $Nr = 0.5$, $Pr = 1.0$, $M = 0.5$, $Le = 2.0$, $Cr = 0.2$, $Nb = 0.5$

Figure 13. Effect of $R$ on concentration profile

$R = 0.1, 0.6, 1.2, 1.6, 2.0$

$Ra_x = 0.5$, $Nt = 0.5$, $Nr = 0.5$, $Pr = 1.0$, $M = 0.5$, $Le = 2.0$, $Cr = 0.2$, $Nb = 0.5$
Heat and mass transfer analysis

Figure 14. Effect of $Nt$ on temperature profile

Figure 15. Effect of $Nt$ on concentration profile

Figure 16. Effect of $Nb$ on temperature profile
Figure 17. Effect of Nb on concentration profile

Figure 18. Effect of Le on concentration profile

Figure 19. Effect of Cr on concentration profile
see from this figure that the concentration profiles are highly influenced and are decelerated with the higher values of the chemical reaction parameter.

The values of skin-friction coefficient $-f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ are calculated for diverse values of the parameters that entered into the problem when the cone surface is hot and are shown in Table III. It is evident that skin-friction coefficient enhances, whereas the Nusselt number and Sherwood number decelerates with the increasing values of both parameters (M and Nr). It is also seen that the skin-friction coefficient and Nusselt number decreases, whereas Sherwood number elevates with the higher values of (R). We have noticed depreciation in skin-friction coefficient, Nusselt number and Sherwood number with an increment in the values of (Nt) in the entire boundary layer region. With the higher values of the Brownian motion parameter (Nb), the rate of change of velocity and heat transfer rates decelerates, whereas mass transfer rates enhance in the boundary layer regime. It is observed from this table that both skin-friction coefficient and Nusselt number diminish, while the Sherwood number enhances with the improving values of chemical reaction parameter (Cr).

5. Conclusion
In this present study, we have numerically examined the electrically conducting natural convection nanofluid boundary layer flow of heat and mass transfer along a vertical cone with thermal radiation and chemical reaction. The slip of the flow in this problem is because of the Brownian motion and thermophoresis. The powerful mathematical tool similarity variables approach is applied to convert the governing equations into the set of ordinary differential equations. These equations are solved numerically using FEM. The important results of the present study can be summarized as follows.

<table>
<thead>
<tr>
<th>M</th>
<th>Nr</th>
<th>R</th>
<th>Nt</th>
<th>Nb</th>
<th>Cr</th>
<th>$-f''(0)$</th>
<th>$-\theta'(0)$</th>
<th>$-\phi'(0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>0.976224</td>
<td>0.517279</td>
<td>0.738589</td>
</tr>
<tr>
<td>0.4</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.110412</td>
<td>0.497377</td>
<td>0.726902</td>
</tr>
<tr>
<td>0.7</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.231896</td>
<td>0.479657</td>
<td>0.717882</td>
</tr>
<tr>
<td>1.0</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.343567</td>
<td>0.463737</td>
<td>0.710818</td>
</tr>
<tr>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.096252</td>
<td>0.498656</td>
<td>0.772449</td>
</tr>
<tr>
<td>0.5</td>
<td>0.6</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.177676</td>
<td>0.484109</td>
<td>0.763785</td>
</tr>
<tr>
<td>0.5</td>
<td>1.2</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.316139</td>
<td>0.466640</td>
<td>0.754750</td>
</tr>
<tr>
<td>0.5</td>
<td>1.6</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.304647</td>
<td>0.456324</td>
<td>0.750177</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.120620</td>
<td>0.535439</td>
<td>0.777955</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.106437</td>
<td>0.481675</td>
<td>0.820010</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.2</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.093811</td>
<td>0.429649</td>
<td>0.856753</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.6</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.088752</td>
<td>0.395820</td>
<td>0.870576</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.1</td>
<td>0.5</td>
<td>0.2</td>
<td>1.109380</td>
<td>0.545613</td>
<td>0.954760</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.6</td>
<td>0.5</td>
<td>0.2</td>
<td>1.108952</td>
<td>0.492900</td>
<td>0.812529</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>1.2</td>
<td>0.5</td>
<td>0.2</td>
<td>1.107875</td>
<td>0.438150</td>
<td>0.703462</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>1.6</td>
<td>0.5</td>
<td>0.2</td>
<td>1.107291</td>
<td>0.417332</td>
<td>0.677069</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.2</td>
<td>1.132646</td>
<td>0.487364</td>
<td>0.809149</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.2</td>
<td>1.139752</td>
<td>0.431585</td>
<td>0.909090</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>1.2</td>
<td>1.134029</td>
<td>0.380205</td>
<td>0.952647</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.6</td>
<td>0.2</td>
<td>1.129435</td>
<td>0.339861</td>
<td>0.976201</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>1.147894</td>
<td>0.487007</td>
<td>0.784499</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.2</td>
<td>1.144011</td>
<td>0.482534</td>
<td>0.925675</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.3</td>
<td>1.140938</td>
<td>0.478600</td>
<td>1.047727</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.1</td>
<td>0.5</td>
<td>0.5</td>
<td>0.4</td>
<td>1.138416</td>
<td>0.476007</td>
<td>1.156171</td>
</tr>
</tbody>
</table>

Table III. The values of skin-friction coefficient $-f''(0)$, Nusselt number $-\theta'(0)$ and Sherwood number $-\phi'(0)$ for different values of M, Nr, R, Nt, Nb, Cr.
• The velocity distributions are reduced, while temperature and concentration profiles elevate with a higher (M).
• With the improving values of (R), the velocity and temperature sketches improve, while concentration distributions are lowered in the boundary layer region.
• The temperature and concentration profiles are elevated in the boundary layer region for higher values of (Nt).
• With the increasing values of (Nb), temperature profiles are enhanced, whereas concentration profiles get depreciated in the flow region.
• Skin-friction coefficient, Nusselt number and Sherwood number diminish with an increase in the values of (Nb) in the entire boundary layer region.
• The skin-friction coefficient and Nusselt number decrease, although Sherwood number increases with the higher values of (R).

References


**Corresponding author**
P. Sudarsan Reddy can be contacted at: suda1983@gmail.com

For instructions on how to order reprints of this article, please visit our website: [www.emeraldgrouppublishing.com/licensing/reprints.htm](http://www.emeraldgrouppublishing.com/licensing/reprints.htm)
Or contact us for further details: permissions@emeraldinsight.com