Magnetohydrodynamics Mixed Convection in a Lid-Driven Cavity Having a Corrugated Bottom Wall and Filled With a Non-Newtonian Power-Law Fluid Under the Influence of an Inclined Magnetic Field

In this study, the problem of magnetohydrodynamics (MHD) mixed convection of lid-driven cavity with a triangular-wave shaped corrugated bottom wall filled with a non-Newtonian power-law fluid is numerically studied. The bottom corrugated wall of the cavity is heated and the top moving wall is kept at a constant lower temperature while the vertical walls of the enclosure are considered to be adiabatic. The governing equations are solved by the Galerkin weighted residual finite element formulation. The influence of the Richardson number (between 0.01 and 100), Hartmann number (between 0 and 50), inclination angle of the magnetic field (between 0 deg and 90 deg), and the power-law index (between 0.6 and 1.4) on the fluid flow and heat transfer characteristics are numerically investigated. It is observed that the effects of free convection are more pronounced for a shear-thinning fluid and the buoyancy force is weaker for the dilatant fluid flow compared to that of the Newtonian fluid. The averaged heat transfer decreases with increasing values of the Richardson number and enhancement is more effective for a shear-thickening fluid. At the highest value of the Hartmann number, the averaged heat transfer is the lowest for a pseudoplastic fluid. As the inclination angle of the magnetic field increases, the averaged Nusselt number generally enhances. [DOI: 10.1115/1.4032760]

1 Introduction

Due to its importance in many engineering applications such as cooling of electronic devices, food processing, coating, float glass production, solidification, and micro-electronic devices, a vast amount of literature is dedicated to the study of the interaction between the shear driven flow and natural convection in cavities. Recently, the magnetic field effect on the heat transfer and fluid flow have received some attention due to its importance in industrial applications such as micro-electronic devices, purification of molten metals, coolers of nuclear reactors, and many others [1]. Several studies have been conducted to use an external magnetic field for the control of heat transfer and fluid flow characteristics [2–12]. Mixed convection with a magnetic field in a top sided lid-driven cavity heated by a corner heater was studied in Ref. [13]. They showed that the magnetic field plays an important role to control the heat transfer and fluid flow. The effects of an external magnetic field on ferrofluid flow and heat transfer in a semi-annulus enclosure with a sinusoidal hot wall by using the Control Volume-based finite element method was investigated in Ref. [14]. They showed that, for low Rayleigh numbers, as the Hartmann number increases and the magnetic number decreases, the heat transfer enhances while the opposite trend was observed for high Rayleigh numbers.

Most of the studies for natural convection in enclosures are based on simple geometries such as square, triangular, or trapezoidal cavities. Heat transfer and pressure drop characteristics of a copper–water nanofluid flow for isothermally heated corrugated channel was studied in Ref. [15] by using the finite difference method. The effect of a corrugated side wall on the natural convection of a differentially heated cavity was investigated in Ref. [16] by using the finite element method. Their results showed that the heat transfer is greatly influenced by the variation of the Richardson number and enhancement is more effective for a shear-thickening fluid. At the highest value of the Hartmann number, the averaged heat transfer is the lowest for a pseudoplastic fluid. As the inclination angle of the magnetic field increases, the averaged Nusselt number generally enhances. [DOI: 10.1115/1.4032760]
the influence of a horizontally applied magnetic field for a range of Reynolds number, Stuart number, and power-law fluid index values was investigated in Ref. [21]. It was shown that as the power-law fluid index decreased, the effect of the magnetic field on the flow increased.

Based on the above literature survey and to the best of authors’ knowledge, the problem of MHD mixed convection of a lid-driven cavity with a corrugated bottom wall has never been reported in the literature. The present numerical study aims at investigating the effects of the Richardson number, Hartmann number, inclination angle of the magnetic field, the power-law fluid index on the fluid flow, and heat transfer characteristic in a cavity having a corrugated bottom wall.

2 Mathematical Formulation

The physical domain of a square cavity with a moving lid and a triangular wave-shaped bottom wall is depicted in Fig. 1 along with the boundary conditions. The height of the cavity is \( H \) and the length and height of the triangular waves are \( a = 0.25H \) and \( b = 0.1H \). The temperature of the bottom wall is maintained at a constant cold temperature of \( T_c \), while the vertical walls are assumed adiabatic. The Boussinesq approximation is assumed to be negligible. The buoyancy forces in the momentum equation are approximated by using the Boussinesq approximation.

The conservation equations of mass, momentum, and energy in a two-dimensional Cartesian coordinate system can be written in dimensional form as follows [22]:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \left( \frac{\partial \tau_{x\gamma}}{\partial x} + \frac{\partial \tau_{y\gamma}}{\partial y} \right) + \frac{\sigma B^2_x}{\rho} \left( u \sin(\gamma) \cos(\gamma) - v \sin^2(\gamma) \right) 
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left( \frac{\partial \tau_{x\gamma}}{\partial x} + \frac{\partial \tau_{y\gamma}}{\partial y} \right) + \beta_g \left( T - T_c \right) + \frac{\sigma B^2_x}{\rho} \left( u \sin(\gamma) \cos(\gamma) - v \cos^2(\gamma) \right) 
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\Pr} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) 
\]

For a power-law non-Newtonian fluid model, the shear stress tensor is given by [22]

\[
\tau_{ij} = m \left( 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right] + \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)^2 \right)^{(n-1)/2} \times \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) 
\]

where \( m \) and \( n \) denote the consistency coefficient and the power-law fluid index.

The relevant physical nondimensional numbers are the Prandtl number \( \text{Pr} = c_p \mu / k (u_0 / H)^2 \), Grashof number \( \text{Gr} = g \beta (T_h - T_c) H^3 \rho (H^3 / n (u_0 / H)^{1-\alpha}) \), Hartmann number \( \text{Ha} = B_0 (H^{n+1} / n (u_0 / H)^{1-\alpha})^{1/2} \) and the Reynolds number (Re = \( \rho H^2 u_0^{1-\alpha} / m \)). The Richardson number is defined as the ratio of the natural convection to the forced convection (RI = GR/Re^2).

The appropriate forms of the dimensional boundary conditions along the cavity walls are as follows:

For the vertical walls: \( u = v = 0, \partial T / \partial x = 0 \).

For the horizontal top wall: \( u = 1, v = 0, T = T_0 \).

For the triangular wave shaped bottom wall: \( u = v = 0, T = T_h \).

Local and averaged Nusselt number on the hot wall of the cavity is calculated as:

\[
\text{Nu}_s = \frac{n}{\partial \theta / \partial y_{\text{hot}}}, \quad \text{Nu}_{\text{avg}} = \frac{1}{S} \int_0^1 \text{Nu}_s \, ds 
\]

where \( \theta \) represents the nondimensional temperature, \( n \) denotes the coordinate direction normal to the surface. \( S \) represents the total length of the triangular wave.

3 Solution Methodology and Code Validation

The finite element formulation is utilized to solve Eqs. (1)–(4) using the appropriate boundary conditions described above. The finite element formulation is obtained by establishing the weak form of the governing equations with the Galerkin procedure. The computational domain is divided into nonoverlapping regions within each of the flow variables are approximated by using the interpolation functions. P2 – P1 Lagrange finite elements are used to discretize the velocity components and pressure, and the Lagrange–quadratic finite elements are chosen for the temperature. The convergence of the solution is assumed when the relative error for each of the variables satisfy a certain convergence criteria. In order to obtain an optimal grid distribution with accurate results and minimal computational time, different numerical studies with various grid sizes are tested. The averaged Nusselt number results for various grid sizes are shown in Table 1 for the
highest values of parameters of interest and for various power-law fluid indices and for three different values of Richardson number (Ha = 50, γ = 90 deg). Grid 4 (G4) with 40,512 number of elements was chosen in the subsequent computations. The grid distribution of the computational domain is depicted in Fig. 1(b). The present code is validated with the results of Refs. [23,24]. Figure 2 demonstrates the comparison results of streamlines and isotherms for (Ra = 7 × 10^3, Ha = 25) and (Ra = 7 × 10^5, Ha = 100). The numerical results of Ref. [15] for a corrugated channel flow at Re = 500 were also used to validate the code as shown in Fig. 3. The results shown in Figs. 2 and 3 provide sufficient confidence for the present code.

<table>
<thead>
<tr>
<th>Grid name</th>
<th># Elements</th>
<th>Nu_{m} (Ri = 100, n = 0.6, 1, 1.4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>2536</td>
<td>1.224, 1.523, 1.856</td>
</tr>
<tr>
<td>G2</td>
<td>10,128</td>
<td>1.039, 1.073, 1.150</td>
</tr>
<tr>
<td>G3</td>
<td>25,384</td>
<td>1.056, 1.092, 1.172</td>
</tr>
<tr>
<td>G4</td>
<td>40,512</td>
<td>1.049, 1.084, 1.163</td>
</tr>
<tr>
<td>G5</td>
<td>75,264</td>
<td>1.049, 1.086, 1.166</td>
</tr>
</tbody>
</table>

4 Results and Discussion
Numerical simulation results are presented in terms of streamlines, isotherms, and Nusselt number distributions for various values of the Richardson numbers (between 0.01 and 100), Hartmann number (between 0 and 50), and the inclination angle of the magnetic field (between 0 and 90). The Prandtl number of the fluid is set to 10. The power-law fluid index n is varied between 0.6 and 1.4.
1.45. The Newtonian fluid behavior is characterized with $n = 1$, for pseudoplastic fluids $n < 1$ and for dilatant fluids $n > 1$. The amplitude and the frequency of the triangular waveform of the bottom wall are set to $a = 0.25H$ and $b = 0.1H$, respectively.

The effects of varying the Richardson number on the streamlines and the isotherms are demonstrated in Figs. 4 and 5 for various power-law fluid indices ($Ha = 30, a = 0.25H, b = 0.1H, \gamma = 45\,\text{deg}$). The Richardson number denotes the ratio of natural convection to the forced convection due to the upper moving wall. For the Newtonian fluid case, two main recirculation zones are observed within the cavity. As the value of the Richardson number increases, the influence of the buoyant force increases. The shape and extent of the recirculation zone adjacent to the bottom wavy wall increases with increasing the $Ri$ values. The natural convection effect is more pronounced for the shear-thinning fluid ($n = 0.6$) as the extent of the recirculating zone adjacent to the bottom wall increases and occupies most of the cavity. For the shear-thickening fluid ($n = 1.4$), half of the cavity is occupied by the vortex due to the moving upper wall at $Ri = 100$. The buoyancy force is weaker for the dilatant fluid flow compared to that of the Newtonian fluid. The isotherms are less clustered along the bottom wall and are horizontally aligned, which indicates the dominance of heat conduction with increasing $Ri$ values for the Newtonian fluid. At $Ri = 100$, the gradient of the temperature on the bottom wall increases toward the left end as the power-law fluid index decreases, which indicates a locally enhanced heat transfer there. The local and the averaged Nusselt numbers plots are shown in Figs. 6 and 7 for various Richardson numbers and power-law fluid indices. The local enhancement of the heat transfer is seen for a Newtonian fluid as the value of $Ri$ enhances and this effect is more pronounced for a
shear-thickening fluid. The local Nusselt number is higher along the left part of the bottom wavy wall for pseudoplastic fluids at the highest Richardson number. The averaged heat transfer decreases with increasing Ri values and the enhancement is more effective for a shear-thickening fluid. Figure 7(a) presents the effect of varying the Richardson number on the \(v\)-velocity profiles for a horizontal plane located at \(y = 0.5H\) from the bottom wall for different power-law fluid indices. The magnitude of the velocity component are seen to increase with the power-law fluid index at \(\text{Ri} = 0.01\). Due to the different extensions of the bottom recirculation zone with the power-law fluid indices at \(\text{Ri} = 100\), positive and negative values of the \(v\)-velocity are obtained until \(x = 0.3H\).

The influence of varying the Hartmann number on the streamlines and the isotherms are demonstrated in Figs. 8 and 9 for different power-law fluid indices (\(\text{Ri} = 0.5\); \(a = 0.25H\); \(b = 0.1H\); \(\gamma = 45\) deg). The cavity is occupied with a single main cell, and small vortices are seen at the corners of the bottom wall in the absence of the magnetic field for the Newtonian fluid case. As the value of the Hartmann number increases, the main recirculating cell within the cavity breaks into two cells and the extent of the bottom vortex elongates diagonally. The flow topology looks similar for pseudoplastic fluids and shear-thickening fluids compared to a Newtonian fluid. The extent of the bottom recirculating zone diminishes in size as the power-law fluid index increases for \(\text{Ha} = 20\) and \(\text{Ha} = 50\). An increase in the Hartmann number results in causing the isotherms to become less dense for the corrugated heated wall. At the highest value of the Hartmann number, the thermal boundary layer is thicker along the bottom wavy wall for pseudoplastic fluids. The isotherms for Newtonian and shear-thickening fluids are similar. The variations of the local and averaged Nusselt numbers with respect to a change in the Hartmann number are depicted in Figs. 10 and 11 for various power-law fluid indices.

The averaged heat transfer rates decrease with increasing values of the Hartman number due to the damping of the fluid motion within the cavity, which is due to the Lorentz forces of the magnetic field. The reduction in the local Nusselt number with increasing Ha values is apparent for the right part of the bottom wavy hot wall for all power-law fluid indices. The averaged heat transfer rate is the same for Newtonian and shear-thickening fluids at the highest Hartmann number and attains lower values for a pseudoplastic fluid. The effects of varying the

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Hartmann number on the $v$-velocity profiles for a horizontal plane located at $y = 0.5H$ from the bottom wall are shown in Fig. 11(a) for various values of the power-law fluid index. The magnitude of the velocity component decreases with the Hartmann number.

The effects of the inclination angle of the magnetic field on the flow and thermal patterns are depicted in Figs. 12 and 13 for three inclination angles $\gamma = 0\deg, 45\deg, 90\deg$ and various power-law fluid indices. The cavity is occupied with two recirculation zones for the horizontally aligned magnetic field case. The core of the upper recirculating zone moves downward and lesser flow is induced into the cavity due to the upper moving wall with increasing values of the power-law fluid index. As the inclination angle increases from $\gamma = 0\deg$ to $\gamma = 45\deg$, the bottom recirculating zone diminishes in size, and at $\gamma = 90\deg$, it disappears while the cavity is filled with a single recirculating zone. The natural convection effect is more effective for the shear-thickening fluid. The thermal patterns show the conduction-dominated flow regime due to the horizontally aligned isotherms for the inclination angle of $\gamma = 0\deg$. As the value of $\gamma$ increases, locally enhanced heat transfer regions adjacent to the right part of the bottom wavy wall are seen for the Newtonian and shear-thickening fluids cases due to the Lorentz force resulted from the magnetic field. The augmentation of the local heat transfer along the right portion of the bottom wall is apparent for the power-law fluid indices $n = 1$ and $n = 1.4$ as shown in Fig. 14. The averaged heat transfer plots are shown in Figs. 15(a) and 15(c) for various inclination angle of the magnetic field and three different Hartmann number values. The averaged Nusselt number decreases with the power-law fluid index and generally enhances with the inclination angle of the magnetic field. The discrepancy between the averaged heat transfer values for various inclination angles decreases for high ($n = 1.4$) and low ($n = 0.6$) values of power law indices when the
Hartmann number are 20 and 50 (Figs. 15(b) and 15(c)). For low values of magnetic field strength, the heat transfer values are higher for the same power law index compared to the case in Figs. 15(b) and 15(c), which is due to the dampening of the flow motion with increasing magnetic field and the discrepancy between the averaged Nusselt numbers are higher for power law index of 0.6 and decreases with as power law index enhances which is due to the different contributions of the Lorentz force in the x and y momentum equations resulted from the magnetic field.

5 Conclusions

A numerical study of mixed convection lid-driven cavity under the influence of an inclined magnetic field with a triangular wave-shaped corrugated bottom wall filled with a non-Newtonian power-law fluid was performed. Some important conclusions can be drawn from the numerical simulation results as:

- The influence of natural convection is more effective for the shear-thinning fluid and the buoyancy force is weaker for the dilatant fluid flow compared to that of the Newtonian fluid.
- As the value of Ri increases, the heat transfer is locally enhanced for the Newtonian fluid and the shear-thickening fluid. The averaged Nusselt number decreases with...
increasing Ri values, and the enhancement is more effective for a shear-thickening fluid.  
- Heat transfer is locally reduced as the value of the Hartmann number increases for all power-law index fluids. At the highest value of the Hartmann number, the averaged heat transfer is the same for Newtonian and shear-thickening fluids and has the lowest value for the pseudoplastic fluid.

- As the inclination angle of the magnetic field increases, the bottom recirculating zone diminishes in size and disappears and the cavity is filled with a single recirculating zone for the vertically aligned magnetic field.

- As the value of the inclination angle of the magnetic field increases, the averaged heat transfer rate generally enhances. The difference between the averaged Nusselt number for different inclination angles is lower for a fluid having a higher power law index. The study can be extended to include the effects of corrugation amplitude, corrugation frequency, type of corrugation (rectangular, trapezoidal, sinusoidal, etc.), and unsteady flow effects, which are not considered in this study.

**Nomenclature**

- B = magnetic induction
- Gr = Grashof number
- h = local heat transfer coefficient
- H = magnetic field
- L = length of the enclosure
- k = thermal conductivity
- M = magnetic number
- n = power law index
- Nu = local Nusselt number
- p = pressure
- Pr = Prandtl number
- Re = Reynolds number
- Ri = Richardson number
- T = temperature
- u, v = x-y velocity components
- x, y = Cartesian coordinates
- z = thermal diffusivity
- β = thermal expansion coefficient
- θ = nondimensional temperature
- ν = kinematic viscosity
- ρ = density of the fluid
- τ = nondimensional time

**References**


