1 Introduction

Colloidal suspensions of nanometer-sized particles (size range of 1–100 nm) in base fluids are famed as nanofluids and have a greater thermal conductivity than the base fluid. This substantial thermal conductivity can promote the rate of heat transfer in manufacturing applications such as solar heating, thermal storage systems, cooling of electronic components, and heat exchangers. Several applications in manufacturing employ natural convection as the principle heat transfer mechanism. Subsequently, it is important to comprehend the thermal pattern by natural convection of such systems.

Various numerical and experimental studies have been reported to examine the several aspects of nanofluids [1]. Thermophysical properties of nanofluids such as thermal diffusivity, thermal conductivity, viscosity, and the density of several nanofluids have been analyzed by Kang et al. [2], Velagapudi et al. [3], Turgut et al. [4], Rudyak et al. [5], Murugesan and Sivan [6], and Nayak et al. [7]. Natural convection of nanofluids in simple geometries such as enclosures with different kinds (square, rectangular, etc.) have been discussed in detail [8–22].

Investigation of natural, forced, and mixed convection mechanism in an annular space is among the most important heat transfer research, by reason of its presence in several applications in industrial heat exchangers. Detailed survey investigations are introduced by Togun et al. [23] containing numerical and experimental explorations dealing distinct kinds of fluids and several boundary conditions for vertical and horizontal annular spaces. The influence of heating conditions, as well as the dimensionless numbers (Rayleigh, Grashof, Prandtl, Reynolds, and Darcy), on heat transfer in concentric and eccentric annular passages is investigated. The influence of nanoparticles on the increase in heat transfer in an annular channel is studied.

Heat transfer and flow in a nanofluid flowing in a horizontal annulus is a specially interesting problem (the annulus has several applications in the manufacture of heat exchangers). Hence, many works have been accomplished for the case of concentric and eccentric horizontal circular annuli [23–33].

Many investigators have adopted an analysis to examine heat transfer and fluid flow with different boundary conditions in a horizontal eccentric annular passage. Lee and Lee [34] considered free convection mechanism for symmetrical cases of elliptical annuli and reported experiments for this geometry with a few cases.

Schreiber and Singh [35] have analyzed convection heat transfer in horizontal oriented confocal elliptical cylinders in terms of elliptical coordinates. Elshamy et al. [36] reported a numerical investigation of laminar natural convection in a horizontal confocal elliptical annulus and developed some feasible correlations for the average Nusselt number. Cheng and Chao [37] considered the heat transfer and fluid flow in a horizontal eccentric elliptical annular space. Considerable rise in heat transfer rate is noted due to the buoyancy strength created by fluid motion. Mota et al. [38] investigated the natural heat transfer in a horizontal eccentric elliptic annulus filled by saturated porous medium utilizing the Darcy–Boussinesq equations. Sufficient increment in heat transfer was demonstrated for the eccentric elliptical annular passage in comparison to the concentric case.

Hirose et al. [39] examined numerically and experimentally the heat transfer and fluid flow in a horizontal eccentric annular space with heated outer cylinder and cooled inner circular elliptical cylinder. The influence of oriented angle and eccentric configuration on heat transfer enhancement is studied. Mixed and forced convection in the annulus passage between two horizontal confocal elliptical cylinders was considered numerically by Zerari et al. [40]. It was noted that the heat transfer in natural convection rises.
by boosting the Grashof number. Recently, Bouras et al. [41] have reported a numerical investigation for double-diffusive natural convection and fluid flow in an annular passage between confocal elliptic cylinders. Constant temperatures and concentrations are imposed along walls of the annular passage. Controlling equations were formulated by applying the dimensionless form in terms of elliptical coordinates for laminar two-dimensional incompressible flow using stream function-vorticity formulation. The differential equations were discretized by the control volume method. They showed that both the heat and mass transfer enhance with prompting the Rayleigh number. At large Rayleigh numbers, the isocenters show a plume as the isotherms. However, the plume becomes robust and diffuses through the annular passage, since the Lewis number is larger than 1.

Heat transfer of nanofluids in a horizontal elliptic annulus has found to have limited literature. Izadi et al. [42] have numerically studied the laminar forced convection in water-based Al₂O₃ nanofluid in a two-dimensional annulus with single-phase approach. Three-dimensional numerical simulations for laminar mixed convective heat transfer for many nanofluids flow in an elliptic annulus with constant heat flux were considered by Dawood et al. [43]. A numerical analysis was carried out by solving the basic equations (continuity, momentum, and energy) applying the finite-volume method with the assistance of SIMPLE algorithm. Four various kinds of nanofluids (water–Al₂O₃, water–CuO, water–SiO₂, and water–ZnO) with different nanoparticles size 20, 40, 60, and 80 nm, and various volume fractions ranged from 0% to 12%, and the eccentricity of the inner ellipse, $e_1 = 0.7, 0.8, \text{and } 0.9$. The eccentricity of outer ellipse and the angle of orientation are fixed at 0.6 deg and 0 deg, respectively. The governing equations are in elliptical coordinates system [44] as the physical boundaries are identified with constant value coordinates. The dimensional transformation from elliptical $(\eta, \theta)$ to Cartesian coordinates $(x, y)$ is

$$
\begin{align*}
  x &= a \, c h(\eta) \cos(\theta) \\
  y &= a \, s h(\eta) \sin(\theta)
\end{align*}
$$

(1)

where $a$ is the half-elliptical focal distance

$$
\frac{A_1}{c h(\eta_1)} = \frac{A_2}{c h(\eta_2)}
$$

(2)

Based on the Boussinesq approximation, the governing equation for the problem is given in elliptical coordinates as follows:

- **Continuity equation**

$$
\frac{\partial}{\partial \eta} (hV_{\eta}) + \frac{\partial}{\partial \theta} (hV_{\theta}) = 0
$$

(3)

- **Momentum (vorticity) equation**

$$
\begin{align*}
V_{\eta} \frac{\partial \omega}{\partial \eta} + V_{\theta} \frac{\partial \omega}{\partial \theta} &= \left( \frac{\rho_0}{\rho_{nf}} \right) \frac{1}{h^2} \left( F(\eta, \theta) \cos(\theta) - G(\eta, \theta) \sin(\theta) \right) \frac{\partial T}{\partial \eta} \\
&+ G(\eta, \theta) \cos(\theta) \frac{\partial T}{\partial \theta} \\
&+ \frac{\mu_{nf}}{\rho_{nf}} \frac{1}{h^2} \left( \frac{\partial^2 \omega}{\partial \eta^2} + \frac{\partial^2 \omega}{\partial \theta^2} \right)
\end{align*}
$$

(4)

- **Energy equation**

$$
\frac{\partial T}{\partial \eta} V_{\eta} + V_{\theta} \frac{\partial T}{\partial \theta} = \frac{K_{nf}}{(\rho c_p)_{nf}} \frac{1}{h} \left( \frac{\partial^2 T}{\partial \eta^2} + \frac{\partial^2 T}{\partial \theta^2} \right)
$$

(5)

The vorticity is defined by

$$
\omega_\ell = - \frac{1}{h^2} \left( \frac{\partial \psi}{\partial \eta^2} + \frac{\partial \psi}{\partial \theta^2} \right)
$$

(6)

where $h$ is the dimensional metric coefficient of the transformation from the Cartesian to the elliptical coordinates.

2 Mathematical Modeling

Figure 1 displays the considered enclosure. It is an annulus formed by two confocal elliptic cylinders filled with water-based nanofluid including Cu nanoparticles. The natural convection is modeled by solving the equation of continuity, momentum, and the energy equation. The imposed boundary conditions are no-slip and isothermal on both cylinder walls. The flow is assumed to be two-dimensional (the cylinders are long enough). The pure fluid (water) and copper nanoparticles are in thermal equilibrium. The thermophysical properties of the nanofluid are considered constant with the exception of the density which changes according to the Boussingues approximation.

The problem is formulated using elliptical coordinates that are natural for this problem since the physical boundaries are
When the grid performed in the current work, constant \( \eta \) values are associated with elliptic curves, whereas constant \( \theta \) values result in the lines joining the two walls (see Fig. 2).

The corresponding eccentricities of the inner and outer cylinders are given as

\[
\begin{align*}
h &= a \sqrt{sh^2(\eta) \sin^2(\theta)} \\
F(\eta, \theta) &= \frac{sh(\eta) \cos(\theta)}{\sqrt{sh^2(\eta) \sin^2(\theta)}} \\
G(\eta, \theta) &= \frac{ch(\eta) \sin(\theta)}{\sqrt{sh^2(\eta) \sin^2(\theta)}}
\end{align*}
\] (7)

The effective density of the nanofluid is

\[
(\rho)_{ad} = \phi \rho_p + (1 - \phi) \rho_f
\] (9)

The thermal diffusivity of the nanofluid is

\[
(\alpha)_{ad} = \frac{K_{nf}}{\rho C_p}_{nf}
\] (10)

The effective dynamic viscosity of the nanofluid given by Brinkman [47] is

\[
(\mu)_{ad} = \frac{\mu_f}{(1 - \phi)^{\frac{1}{3}}}
\] (13)

In Eq. (10), \( K_{nf} \) is the effective thermal conductivity of the nanofluid, which for low dense mixtures with microsized spherical particles, according to Maxwell [48], is

\[
K_{nf} = \frac{K_f (K_p + 2K_f) - 2\phi(K_f - K_p)}{(K_p + 2K_f) + \phi(K_f - K_p)}
\] (14)

These models (Eqs. (13) and (14)) have been used recently in literature for estimation of effective dynamic viscosity and thermal conductivity of nanofluids in numerical simulation of free convection [49].

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**Fig. 2** (a) Physical domain and (b) computational domain

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**Table 1** Effect of the grid density on the mean Nusselt numbers of inner wall for \( Ra = 10^4 \), \( \varepsilon_1 = 0.8 \), \( \varepsilon_2 = 0.6 \), and \( \phi = 0 \)

<table>
<thead>
<tr>
<th>Grid size</th>
<th>11 \times 11</th>
<th>11 \times 21</th>
<th>21 \times 41</th>
<th>31 \times 61</th>
<th>41 \times 81</th>
<th>51 \times 101</th>
<th>61 \times 121</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu\text{avg}</td>
<td>2.360</td>
<td>2.521</td>
<td>2.633</td>
<td>2.676</td>
<td>2.696</td>
<td>2.707</td>
<td>2.708</td>
</tr>
</tbody>
</table>

**Table 2** Comparisons of the present results for the average Nusselt number with the literature

<table>
<thead>
<tr>
<th>( \varepsilon_1 )</th>
<th>( \varepsilon_2 )</th>
<th>( \alpha ) (deg)</th>
<th>( Ra )</th>
<th>Present results</th>
<th>Ref. [36]</th>
<th>Present results</th>
<th>Ref. [36]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9</td>
<td>0.4</td>
<td>0</td>
<td>( 10^4 )</td>
<td>3.5382</td>
<td>3.53</td>
<td>1.1469</td>
<td>1.19</td>
</tr>
<tr>
<td>0.86</td>
<td>0.4</td>
<td>90</td>
<td>( 10^5 )</td>
<td>3.7084</td>
<td>3.68</td>
<td>1.3903</td>
<td>1.35</td>
</tr>
<tr>
<td>0.86</td>
<td>0.4</td>
<td>90</td>
<td>( 4 \times 10^4 )</td>
<td>5.2702</td>
<td>5.34</td>
<td>1.8752</td>
<td>1.93</td>
</tr>
<tr>
<td>0.68</td>
<td>0.4</td>
<td>90</td>
<td>( 10^5 )</td>
<td>2.8711</td>
<td>2.66</td>
<td>1.4122</td>
<td>1.38</td>
</tr>
</tbody>
</table>

**Table 3** Thermophysical properties of the base fluid and the nanoparticles [50]

<table>
<thead>
<tr>
<th></th>
<th>( \rho ) (kg ( m^3 ))</th>
<th>( C_p ) (J kg(^{-1}) K(^{-1}))</th>
<th>( K ) (W m(^{-1}) K(^{-1}))</th>
<th>( \beta ) (K(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>( 21 \times 10^{-5} )</td>
</tr>
<tr>
<td>Copper</td>
<td>8933</td>
<td>385</td>
<td>401</td>
<td>1.67 \times 10^{-5}</td>
</tr>
</tbody>
</table>
The following dimensionless variables are introduced to convert the governing equations into dimensionless form:

\[
D_h = \frac{a}{D_h}, \quad H = \frac{h}{D_h}, \quad V_\eta = \frac{D_h V_\psi}{x_f}, \quad V_\theta = \frac{D_h V_\phi}{x_f}, \quad \psi^* = \frac{\psi}{x_f}, \quad \phi^* = \frac{\phi}{x_f},
\]

\[
\omega^* = \frac{D_h^2}{x_f}, \quad T^* = \frac{T - T_c}{T_h - T_c}
\]

The governing equations are written in the following dimensionless form:

\[
\frac{\partial}{\partial \eta} \left( HV_\eta^* \right) \frac{\partial}{\partial \theta} \left( HV_\theta^* \right) = 0
\]

\[
HV_\eta^* \frac{\partial \phi^*}{\partial \eta} + HV_\theta^* \frac{\partial \phi^*}{\partial \theta} = Pr Ra \left( \frac{1}{(1 - \phi) \rho_f + \phi \rho_p} + \frac{1}{(1 - \phi) \rho_f + 1} \right)
\]

\[
\times \left\{ \left[ F(\eta, \theta) \cos(\alpha) - G(\eta, \theta) \sin(\alpha) \right] \frac{\partial T^*}{\partial \eta} 
\right. 
\]

\[
- \left[ F(\eta, \theta) \sin(\alpha) + G(\eta, \theta) \cos(\alpha) \right] \frac{\partial T^*}{\partial \theta} 
\]

\[
+ \frac{Pr}{(1 - \phi)^2} \left[ \frac{1}{(1 - \phi) + \phi \left( \rho C_p \right) p} \right] \left( \frac{\partial^2 \phi^*}{\partial \eta^2} + \frac{\partial^2 \phi^*}{\partial \theta^2} \right)
\]

\[
\frac{\partial T^*}{\partial \eta} HV_\eta^* + HV_\theta^* \frac{\partial T^*}{\partial \theta} = \frac{K_{ef}}{K_f} \left[ \frac{1}{(1 - \phi) + \phi \left( \rho C_p \right) p} \right] \left( \frac{\partial^2 T^*}{\partial \eta^2} + \frac{\partial^2 T^*}{\partial \theta^2} \right)
\]

\[
\omega = \frac{1}{H^2} \left( \frac{\partial^2 \phi^*}{\partial \eta^2} + \frac{\partial^2 \phi^*}{\partial \theta^2} \right)
\]

In the above equations, the Prandtl number (Pr) and the Rayleigh number (Ra) are defined as

\[
Pr = \frac{\nu_f}{x_f}, \quad Ra = \frac{g \beta_f D_h (T_h - T_c)}{\nu_f x_f}
\]

The dimensionless boundary conditions are as follows:

- Inner cylinder wall (\( \eta = \eta_1 = \text{constant} \))

\[
V_\eta^* = V_\theta^* = \frac{\partial \psi^*}{\partial \eta} = \frac{\partial \phi^*}{\partial \eta} = 0
\]

\[
\omega^* = - \frac{1}{H^2} \left( \frac{\partial^2 \psi^*}{\partial \eta^2} + \frac{\partial^2 \phi^*}{\partial \theta^2} \right)
\]

\[
T^* = 1
\]

- Outer cylinder wall (\( \eta = \eta_2 = \text{constant} \))

\[
V_\eta^* = V_\theta^* = \frac{\partial \psi^*}{\partial \theta} = \frac{\partial \phi^*}{\partial \eta} = 0
\]

\[
\omega^* = - \frac{1}{H^2} \left( \frac{\partial^2 \psi^*}{\partial \eta^2} + \frac{\partial^2 \phi^*}{\partial \theta^2} \right)
\]

\[
T^* = 0
\]
Calculation of the local Nusselt number for the cylinder walls is performed by

$$\text{Nu} = \left( \frac{K_{al}}{K_f} \right) \frac{1}{H} \frac{\partial T}{\partial n} \bigg|_{n=2}$$  \hspace{1cm} (20)

The mean Nusselt number is calculated

$$\text{Nu}_{avg} = \frac{1}{\theta_{NN} - \theta_1} \int_{\theta_1}^{\theta_{NN}} \text{Nu} \, d\theta$$ \hspace{1cm} (21)

The integral in this equation is approximated with the aid of Simpson’s method.

3 Numerical Implementation

The physical domain of the annulus is complex, and therefore transformed to a rectangular domain with a mapping, which enables the equations to be discretized on an orthogonal uniform mesh.

The system of equations (16) and (17) subject to the appropriate boundary conditions is numerically solved using finite-volume method. Equation (18) is solved using centered differences method. The iterative method used for the numerical solution of algebraic system of equations (matrix) is the Gauss–Seidel, with an under-relaxation process. As convergence criteria, $10^{-6}$ is chosen for all dependent variables, and the value of 0.75 is taken for under-relaxation parameter.

The following condition is used to obtain the convergence

$$\frac{\max \Phi_{i,j}^{n+1} - \max \Phi_{i,j}^n}{\max \Phi_{i,j}^{n+1}} \leq 10^{-6}$$ \hspace{1cm} (22)

where $n + 1$ denotes the current iteration and $n$ previous iteration; $\Phi$ stands for $\psi$ or $T$; and $i, j$ refer to space coordinates. The 2D generated mesh grid including physical and computational grids is shown in Fig. 2.

To check for a mesh-independent solution, an extensive mesh testing procedure was performed. The mesh size is increased from $11 \times 11$ to $61 \times 121$. The variations in the mean Nusselt numbers of inner surface with the grid number are given in Table 1 at $\text{Ra} = 10^4$, $\phi = 0$, the inclination angle $\alpha = 0 \text{ deg}$, and the eccentricity of the inner and outer cylinders is $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.6$, respectively. Thus, it is decided to choose $51 \times 101$.

First, the governing equations have been solved for the natural convection flow between two concentric elliptic cylinders filled by pure fluid (air), in order to compare the results of the present code with those obtained by Elshamy et al. [36]. The comparison of the present results with the literature is found in a good agreement (see Table 2). These comparisons are encouraging and thereby validate the accuracy of the proposed numerical method.

4 Results and Discussion

In this study, natural convection heat transfer in a horizontal annuli formed by two confocal elliptic cylinders filled with nanofluid is investigated numerically using the finite-volume method (FVM). The fluid in the annuli is Cu–water nanofluid. The thermophysical properties of cooper nanoparticles and base fluid are given in Table 3. Calculations are made for Rayleigh number ($10^3 \leq \text{Ra} \leq 3 \times 10^5$), volume fraction of nanoparticles $0 \leq \phi \leq 0.12$, and eccentricity of the inner ellipse, $0.7 \leq \epsilon_1 \leq 0.9$. The eccentricity of outer ellipse and the angle of orientation are fixed at $\epsilon_2 = 0.6$ and $\alpha = 0 \text{ deg}$, respectively. The Prandtl number water is 6.2. Results are presented in the form of streamlines, isotherm plots, and local and average Nusselt numbers.
Figure 3 illustrates the variation in the mean Nusselt number with the Rayleigh number for different values of volume fraction of nanoparticles and different eccentricities $e_1$ of the inner ellipse. For $e_1 = 0.7$, it is observed that for a fixed value of $\phi$, the mean Nusselt number increases slowly for Ra less than $10^4$, when the dominant mode of heat transfer is conduction, and then increases significantly as the Rayleigh increases. When the values of eccentricity $e_1$ increase, the increase in the average Nusselt number according to Rayleigh is very significant; the reason is that as the gap between the two cylinders becomes very much larger, and consequently, the convection becomes more intense. We also note that, for fixed values of Ra, the Nusselt number is affected much by addition of cooper nanoparticles to the base fluid when the parameter $e_1$ decreases.

For better comparison, the distribution of the mean Nusselt numbers with volume fraction of the nanoparticles for each eccentricities of the inner ellipse ($e_1$) separately is shown in Fig. 4. It is observed that for fixed value of the Rayleigh number, the Nusselt number increases linearly by increasing $\phi$, but this increase became less considerable (the line slope is decreased) when we increase the gap between the ellipses. For a fixed value of $\phi$, when the parameter $e_1$ increases, the gap between the ellipses becomes larger so it leads to raise the fluid convection inside the annulus, and the heat transfer rate depends strongly on the value of Rayleigh number.

The distribution of local Nusselt number on the inner and outer surfaces of the annulus is illustrated in Fig. 5 for different values of the nanoparticle volume fraction when Ra = $10^1$, and $e_1 = 0.8$. It is observed that the outer and inner Nusselt numbers present an opposite distribution. The inner wall exhibits a maximum, while the outer exhibits a minimum. These two extremes are achieved at an angle close to $\theta = 90$ deg (at the plume region). This finding is compatible with the thermal contours shown in Fig. 6. We can also note that the maximum values (whether the inner surface or the outer one) are affected by the presence of nanoparticles, while the minimum values are not affected and the plume region does not move about the angular position with increasing volume fraction, but only narrowed.

The isotherms and streamlines for different values of Rayleigh number and various values of eccentricity of the inner elliptic cylinder when $\phi = 0$ are shown in Figs. 7–9. Figure 7 illustrates the stream function and isotherm contours for Ra = $10^1$, $10^2$, $10^3$, and $3 \times 10^3$ when $e_1 = 0.7$. At Ra = $10^3$, the isotherms are almost parallel and concentric curves which follow well the active wall’s profiles and the distribution of temperatures by simply decreasing the hot wall to the cold wall. As for streamlines, it is observed that the structure of flow is formed by two symmetrical cells with clockwise (in the right side) and anticlockwise (in the left side) rotations. The magnitudes of stream functions are small signifying conduction dominant heat transfer within the cavity. For Ra = 10$^3$, increasing the Rayleigh number reflects an intensification of natural convection, allowed the appearance of a bifurcation giving birth to four additional cells turning in the opposite direction of neighboring cells and three plumes in the isotherms. When the Rayleigh number achieves at $3 \times 10^3$, the cells and the plumes regions join together forming one plume over the hot elliptic cylinder and two main symmetrical cells.

Figure 8 shows that for Ra = $10^1$, the conduction is the mode of heat transfer dominant, where the isotherms are almost parallel curves and adopt well enough the wall profiles, and the flow is organized in two cells which turn very slowly in opposite directions. Figure 8 indicates that the density of the isotherms near the cylinders increases by increasing the Rayleigh number. That

![Fig. 6 Isotherms (left) and streamlines (right) for different values of volume fraction when $e_1 = 0.8$ and Ra = $10^2$](image)

![Fig. 7 Isotherms (left) and streamlines (right) for different values of the Rayleigh number Ra at $e_1 = 0.7$ and $\phi = 0$](image)
means that the temperature gradients at the walls of the cylinders increase, and consequently, the mean Nusselt number also increases. In addition, it is observable that the streamlines become thicker near the left and right sides of the inner cylinder by increasing the Rayleigh number; this indicates that the flow is accelerated in those areas. The main vortex deforms from an oval shape to a stretched shape, and the center of inner vortex moves upward under the buoyancy effect.

However, when the gap between the two cylinders becomes more and more larger, a disturbance of isothermal lines can be observed and a plume appears over the hot elliptic cylinder at the beginning of a Rayleigh number equal to $10^4$ (Fig. 9), and the natural convection is dominant. It is clear from Fig. 9 that with an increase in the Rayleigh number, the vortices move upward. As a result of Figs. 7–9, it can be concluded that the convection is amplified with increasing the Rayleigh number and the gap between the inner and outer elliptic cylinders. As the Rayleigh number increases, the plumes of the isotherms ascend toward the outer elliptic cylinder, and the decrease in the thickness of the thermal boundary layer causes the increase in the Nusselt number.

It should be mentioned to this end that all results depicted in Figs. 5 and 7–9 are in agreement with those reported by Elshamy et al. [36] and recently by Bouras et al. [41].

The effects of Cu nanoparticles volume fractions on the flow structure are presented in Fig. 6, for $e_1 = 0.8$ and $Ra = 10^5$. As can be seen from the figures, with the increase in the solid volume fraction of nanofluid, the magnitude of the stream functions increases, and the vortices move upward. The intensity of the plume is not increase much with the addition of nanoparticles. Instead, in the sides of the annulus in the narrow gap, a local region of conduction-dominated heat transfer is produced, a phenomenon which is usually found at the bottom portion of the annulus where the flow is inert and stably stratified for all situations.

## 5 Conclusion

The effects of the eccentricity $e_1$ of inner surface, the nanoparticles volume fraction parameter $\phi$, and the Rayleigh number $Ra$ on local and mean Nusselt numbers, streamlines, and isotherms are investigated numerically for a natural convective flow in an annular space between confocal horizontal elliptical cylinders with isothermal surfaces filled with a copper–water-based nanofluid. We can draw the following conclusions from this study:

- The contours of stream functions and isotherms are symmetric about the vertical line for all situations due to the imposed boundary conditions.
- The plume region does not move about the angular position by increasing both Rayleigh number and volume fraction of nanoparticles for all eccentricity $e_1$ considered.
- The plume region presents the low values of Nusselt numbers for the inner cylinder and the maximum values for that of the outer.
- Results indicated also that the addition of copper nanoparticles has produced a significant enhancement of heat transfer.

![Fig. 8 Isotherms (left) and streamlines (right) for different values of the Rayleigh number Ra at $e_1 = 0.8$ and $\phi = 0$](image)

![Fig. 9 Isotherms (left) and streamlines (right) for different values of the Rayleigh number Ra at $e_1 = 0.9$ and $\phi = 0$](image)
with respect to that of the base fluid, but the effect of nanoparticles concentration on the mean Nusselt number is more pronounced when we considered a small gap between the elliptical cylinders (low eccentricity $\varepsilon$ of the inner cylinder).

- The heat transfer enhances with increasing the Rayleigh number, but its effect is more pronounced for largest gap between the cylinders.

The study throws light into this area and indicates the need for more investigations in the future by browsing effects of the other main controlling parameters, namely, the inclination of the enclosure ($\varphi$), Prandtl number (Pr), and other type of nanofluid, on the flow structure and the heat transfer.

Nomenclature

\[ A_1, A_2 = \text{major axes of the inner and outer elliptical cylinders (m)} \]
\[ B_1, B_2 = \text{minor axes of the inner and outer elliptical cylinders (m)} \]
\[ C_p = \text{specific heat at constant pressure (J kg}^{-1} \text{K}^{-1}) \]
\[ g = \text{gravitational acceleration (m s}^{-2}) \]
\[ h = \text{metric coefficient (m)} \]
\[ H = \text{dimensionless} \]
\[ k = \text{thermal conductivity (W m}^{-1} \text{K}^{-1}) \]
\[ Nu = \text{Nusselt number} \]
\[ Pr = \text{Prandtl number} \]
\[ Ra = \text{Rayleigh number} \]
\[ T = \text{dimension temperature (K)} \]
\[ u, v = \text{axial and radial velocities (m s}^{-1}) \]
\[ V_{\rho}, V_{\theta} = \text{velocity components in } \eta, \theta \text{ directions (m s}^{-1}) \]
\[ x, y = \text{Cartesian coordinates (m)} \]

Greek Symbols

\[ \alpha = \text{thermal diffusivity (m}^{2} \text{s}^{-1}) \]
\[ \beta = \text{thermal expansion coefficient (K}^{-1}) \]
\[ \varepsilon_1, \varepsilon_2 = \text{eccentricities of ellipses} \]
\[ \eta, \theta = \text{elliptic coordinates (m)} \]
\[ \mu = \text{dynamic viscosity, kg/m s} \]
\[ \rho = \text{density (kg m}^{-3}) \]
\[ \nu = \text{kinematic viscosity (m}^{2} \text{s}^{-1}) \]
\[ \phi = \text{volume fraction of the nanoparticles} \]
\[ \psi = \text{stream function (m}^{2} \text{s}^{-1}) \]
\[ \omega = \text{vorticity (s}^{-1}) \]

Subscripts

\[ c = \text{cold} \]
\[ f = \text{fluid} \]
\[ h = \text{hot} \]
\[ nf = \text{nanofluid} \]
\[ p = \text{solid particles} \]
\[ 1 = \text{inner cylinder} \]
\[ 2 = \text{outer cylinder} \]

Superscript

\[ * = \text{dimensionless parameters} \]

References


