1 Introduction

Natural convection heat transfer is an important phenomenon in engineering systems because of its wide range of applications in electronic cooling, heat exchangers, and double pane windows [1]. Natural convection inside closed cavities filled with a porous medium has received considerable attention over the past few years and has seen an expansion in applications. Such applications include the design of solar collectors, crystal growth, solar thermal receivers, packed sphere beds, geothermal energy systems, uncovered solar flat plates, and the migration of moisture. There is a wide interest in natural convection in a porous medium in scientific, engineering, and technology fields. The work on natural convection in porous cavities is exhaustive and a good number of excellent books currently available are testimonies to this wave of interest. An excellent survey was made on the fundamental topic of flows in porous media with an important issue from diverse fields such as energy, civil, biotechnology, chemical, and environmental engineering by Bejan et al. [2]. The book by Ingham and Pop [3] presents an inclusive account of available information in the field of transport phenomena in porous media and the area of fluid mechanics and heat transfer which is closely related to energy utilization and conservation. A recent comprehensive literature survey concerning convection in porous media was given by Nield and Bejan [4]. The authors of this book provided an excellent background in the field of convection in porous media, and they also presented a comprehensive survey on the natural convection and heat transfer in porous media. A very useful acquaintance on the natural convection heat transfer of a porous medium in the local thermal nonequilibrium (LTNE) model is also given in Ref. [4].

Convection due to thermal interaction between solids and fluids is termed conjugate heat transfer. Conjugate natural convection heat transfer in cavities has received much attention because of its importance in many engineering systems, such as solar energy collectors, material processing, heat preservation of thermal transport circuits, building energy components, and the cooling of electrical units. Kaminski and Prakash [5] numerically studied the effect of conjugate natural convection heat transfer in a square cavity with a finite-thickness vertical wall and compared different models of wall heat conduction. Baytas et al. [6] numerically studied the conjugate natural convection heat transfer in a square cavity filled with a porous medium and having two finite-thickness horizontal plates. Saed [7] numerically investigated the conjugate natural convection in a porous square cavity with the effect of conduction in one of the vertical walls. He found that the heat transfer rate increased with an increase in the thermal conductivity ratio. Saleh et al. [8] investigated the effect of conjugate natural convection at the bottom wall on Darcy–Bénard convection in a square porous cavity. Varol [9] first considered the free convection in a porous cavity with a centered conducting body. Chamkha and Ismael [10] studied the effect of conjugate natural convection heat transfer in a square porous cavity filled with nanofluids and heated by a thick triangular wall. Sheremet and Pop [11] used the finite difference method to investigate the conjugate natural convection heat transfer in a square porous cavity filled with a nanofluid. Recently, Rees and Nield [12] investigated the effect of an embedded solid block on unsteady Darcy–Bénard convection in a square porous cavity. The horizontal walls were maintained at different constant temperatures and the vertical walls were kept adiabatic.

Most of the previous studies considered the natural convection heat transfer of a porous medium in local thermal equilibrium (LTE), where the fluid temperature is equal to the solid temperature. The case when the fluid temperature is different from the solid temperature is called LTNE. Haddad et al. [13] studied the validity of the LTE assumption for natural convection in a porous medium along a vertical flat plate. Their study was based on the two-phase model, and they used the Brinkman term (no-slip condition) to solve the problem. Zhang and Liu [14] proposed a criterion for the LTE of forced convection flows in porous media using...
a numerical method based on the Brinkman–Forchheimer equation. On the other hand, several studies have considered the LTNE for a porous medium. Baytaş and Pop [15] investigated the effect of using the LTNE on natural convection in a porous cavity. Anjum Badruddin et al. [16] studied natural convection and heat transfer through a vertical annulus with a porous medium using the LTNE model. Rees et al. [17] elaborated on the nuances of the effect of LTNE model on heat transfer. The study of Kuznetsov and Nield [18] analyzed the effect of natural convection on a horizontal layer of a porous medium filled with a nanofluid using the LTNE model. Very recently, Alsabery et al. [19] numerically considered the unsteady natural convection in a nanofluid-saturated porous cavity by using the LTNE model.

Recently, the problem of natural convection in closed cavities with various thermal boundary conditions has been given considerable attention by several authors. Sarris et al. [20] considered natural convection in a 2D cavity with a sinusoidal temperature profile on the upper wall while the other walls were adiabatic by using the finite volume method. Saeid and Yaacob [21] numerically considered natural convection in a square cavity filled with a pure fluid with a nonuniform hot-wall temperature and a uniform cold-wall temperature. Deng and Chang [22] numerically studied the natural convection in a rectangular cavity filled with a pure fluid and heated by two sinusoidal temperature distributions on the vertical left and right sidewalls. Khandelwal et al. [23] numerically studied the effect of periodicity of a sinusoidal boundary condition on natural convection in a fluid-saturated porous cavity. They found that the local heat transfer rate decreased with the increasing of the height of the porous cavity. Recently, Wu et al. [24] numerically considered the non-Darcy natural convection in a square porous cavity with sidewalls heated partially by a sinusoidal temperature profile under the LTNE condition. However, to the best of our knowledge, the study of natural convection in a porous cavity with a conjugate bottom wall based on the LTNE has not been undertaken yet. Thus, the authors of the present study believe that this work is valuable. The aim of this study is to investigate the effects of nonuniform heating and a finite wall thickness on the natural convection in a square porous cavity based on the LTNE model. The conjugate heat transfer in porous media when the thermal nonequilibrium conditions taken into consideration is a very important phenomenon in the applications of the high power heat transfer. In the present work, the distribution of the heat that splits from the conjugate wall into the fluid and the porous matrix phases are investigated.

2 Mathematical Formulation

The steady two-dimensional natural convection problem in a square porous cavity with side \( L \) is illustrated in Fig. 1. The left and right vertical surfaces are maintained at a constant temperature \( T_e \), and the bottom solid wall of the porous cavity is heated either uniformly or nonuniformly via the function \( T = (T_b - T_e) \sin(2\pi x/L) + T_e \). The top horizontal wall is adiabatic. The following assumptions are made:

(a) The Darcy model and the Boussinesq approximation are applicable.

(b) The convective fluid and the solid matrix are not in local thermodynamic equilibrium.

(c) The properties of the fluid and the porous medium are homogeneous and isotropic.

(d) The fluid physical properties are constant except for its density.

(e) The boundaries of the cavity are assumed to be impermeable.

Under these assumptions, the governing equations can be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{gK(p\vec{F}_p)}{p_f} \frac{\partial T_f}{\partial x} \quad (2)
\]

\[
(pC_p)\left( \frac{\partial T_f}{\partial x} + v \frac{\partial T_f}{\partial y} \right) = \frac{q}{\rho_f} \left( \frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} \right) + h [T_p - T_f] \quad (3)
\]

\[
(1 - \phi)K \left( \frac{\partial^2 T_p}{\partial x^2} + \frac{\partial^2 T_p}{\partial y^2} \right) = h [T_f - T_p] \quad (4)
\]

where \( K \) is the permeability of the porous medium, \( \phi \) is the porosity of the medium, and \( g \) is the acceleration due to gravity. The energy equation for the impermeable solid wall is

\[
\frac{\partial^2 T_w}{\partial x^2} + \frac{\partial^2 T_w}{\partial y^2} = 0 \quad (5)
\]
We now define the stream function as follows:

\[ u = \frac{\partial \phi}{\partial y}, \quad v = -\frac{\partial \phi}{\partial x} \]  

and also introduce the following nondimensional variables:

\[ X = \frac{y}{L}, \quad Y = \frac{x}{L}, \quad \Psi = \frac{\Psi}{\phi \sigma \eta}, \quad \theta_j = \frac{T_j - T_c}{T_h - T_c}, \quad \theta_p = \frac{T_p - T_c}{T_h - T_c}, \quad \theta_n = \frac{T_n - T_c}{T_h - T_c}, \quad \gamma = \frac{\phi \sigma f}{\rho C_p} \]

Equations (1)–(5), on using Eqs. (6) and (7), now become

\[ \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -Ra \frac{\partial \theta_j}{\partial X} \]

(8)

\[ \frac{\partial \Psi}{\partial Y} - \frac{\partial \Psi}{\partial Y} = \frac{\partial^2 \theta_j}{\partial X^2} + \frac{\partial^2 \theta_j}{\partial Y^2} + H[\theta_n - \theta_j] \]

(9)

\[ \frac{\partial^2 \theta_j}{\partial X^2} + \alpha \frac{\partial^2 \theta_j}{\partial Y^2} = \gamma H[\theta_n - \theta_j] \]

(10)

\[ \frac{\partial^2 \theta_n}{\partial X^2} + \frac{\partial^2 \theta_n}{\partial Y^2} = 0 \]

(11)

where \( Ra = g \beta \rho f (T_h - T_c) L / (\rho \sigma \alpha) \) is the Darcy–Rayleigh number of the porous medium. The dimensionless boundary conditions for solving Eqs. (8)–(11) are [25]

\[ \Psi = 0, \quad \text{on all solid boundaries} \]

(12)

\[ \theta_n(X, 0) = 1 \quad \text{or} \quad \sin(2\pi X) \]

(13)

\[ \theta_j(0, Y) = \theta_j(0, Y) = \theta_n(0, Y) = 0 \]

(14)

\[ \theta_j(1, Y) = \theta_j(1, Y) = \theta_n(1, Y) = 0 \]

(15)

\[ \frac{\partial \theta_j(X, 1)}{\partial Y} = \frac{\partial \theta_n(X, 1)}{\partial Y} = 0 \]

(16)

\[ \frac{\partial \theta_j(X, S)}{\partial Y} = \frac{\partial \theta_n(X, S)}{\partial Y} = \frac{\partial \theta_n(X, S)}{\partial Y} \]

(17)

\[ \frac{\partial \theta_j(X, S)}{\partial Y} = K_s \frac{\partial \theta_n(X, S)}{\partial Y} = \gamma \frac{\partial \theta_n(X, S)}{\partial Y} \]

(18)

where \( S = W/L \) is the solid wall thickness and \( K_s = k_n / \phi \sigma f \) is the thermal conductivity ratio.

The local Nusselt number along the interface between the solid wall and the porous region is defined as

\[ Nu = -\left[ \frac{\partial \theta}{\partial Y} \right]_{Y=S} \]

(19)

The average fluid and porous Nusselt numbers evaluated at the wall interface are defined, respectively, as

\[ \bar{Nu}_f = \int_0^1 \left[ -\frac{\partial \theta}{\partial Y} \right]_{Y=S} \, dX \]

(20)

\[ \bar{Nu}_p = \int_0^1 \left[ -\frac{\partial \theta}{\partial Y} \right]_{Y=S} \, dX \]

(21)

We now define a weighted-average Nusselt number, \( Nu_{wa} \), in the following way:

\[ Nu_{wa} = \varphi \bar{Nu}_f + (1 - \varphi) \bar{Nu}_p \]

(22)

### 3 Numerical Method and Validation

An iterative finite difference method (FDM) is employed to solve the governing equations (8)–(11) subject to the boundary conditions (12)–(18). The FDM solution begins with the finite difference equation (FDE) of the energy function and the stream function equation. The FDEs of Eqs. (10) and (20) are written in the Gaussian successive over-relaxation formulation, respectively, as

\[ \theta_j^{i+1} = \theta_j^i + \frac{\lambda_s}{2 (1 + B^2)} \left( \theta_j^{i+1} + \theta_j^{i+1} \right) 
+ B^2 \left( \theta_j^{i+1} + \theta_j^{i+1} + \theta_j^{i+1} - 2(1 + B^2) \theta_j^{i+1} \right)
- \left( \Delta X \right)^2 \left( S_{ij} \right) \]

(23)

\[ \bar{Nu}_f = \frac{\Delta Y}{2} \left[ -\frac{\partial \theta_j}{\partial Y} \right]_{1,1} + 2 \left[ -\frac{\partial \theta_j}{\partial Y} \right]_{1,2} + \left( -\frac{\partial \theta_j}{\partial Y} \right)_{1,3}
+ \ldots + \left( -\frac{\partial \theta_j}{\partial Y} \right)_{1,ND} + \left( -\frac{\partial \theta_j}{\partial Y} \right)_{1,ND+1} \]

(24)

where

\[ \left( S_{ij} \right)_{ij} = H \left[ \theta_j^{i+1} - \left( \theta_j^{i+1} \right)_{ij} \right] \]

(25)

The FDE of the other equations can be treated in the same way, where \( B = \Delta X / \Delta Y \) and \( \lambda_s \) is the relaxation parameter.

In this paper, several grid tests are performed: 20 × 20, 40 × 40, 60 × 60, 80 × 80, 100 × 100, 120 × 120, 140 × 140, and 160 × 160. Table 1 shows the calculated Nusselt number at different mesh numbers for \( Ra = 10^3 \), \( \gamma = 1 \), \( H = 1 \), \( K_s = 1 \), and \( S = 0.2 \). The results show insignificant differences for the 140 × 140 grids and above. Therefore, for all computations in this paper for similar problems to this subsection, the 140 × 140 uniform grid is employed.

For the purpose of validating the data, the present figures are compared with the ones provided by Saeid [7] for \( Ra = 10^3 \), \( K_s = 0.1 \), and \( S = 0.2 \) as depicted in Fig. 2. In addition, a comparison is made between the resulting figures and the one provided by Baytas and Pop [15] as shown in Fig. 3. Figure 3 demonstrates the comparison between the results of this study and the results presented by Baytas and Pop [15] for \( Ra = 10^3 \), \( \gamma = 10 \), \( H = 1 \), and \( S = 0 \). A comparison is made between the resulting figures and the one provided by Varol et al. [26] where the bottom wall was heated by a nonuniform function of \( T_s = T_c + 0.5 \sin(2\pi x) \) as shown in Fig. 4. Figure 4 demonstrates the comparison between the results of this study and the results presented by Varol et al. [26] for \( Ra = 10^3 \) and \( S = 0 \). As observed from these comparisons, a very good agreement between the results occurs.

### 4 Results and Discussion

This section presents numerical results for the streamlines, isotherms of the fluid phase and the isotherms of the porous phase with various values of the Rayleigh number (10^2 ≤ Ra ≤ 10^3).

<table>
<thead>
<tr>
<th>Grid size</th>
<th>( Nu_f )</th>
<th>( Nu_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>20 × 20</td>
<td>-4.65621</td>
<td>1.21029</td>
</tr>
<tr>
<td>40 × 40</td>
<td>-4.69997</td>
<td>1.59403</td>
</tr>
<tr>
<td>60 × 60</td>
<td>-4.74504</td>
<td>1.77536</td>
</tr>
<tr>
<td>80 × 80</td>
<td>-4.72563</td>
<td>1.85767</td>
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<tr>
<td>100 × 100</td>
<td>-4.73266</td>
<td>1.91738</td>
</tr>
<tr>
<td>120 × 120</td>
<td>-4.73755</td>
<td>1.99852</td>
</tr>
<tr>
<td>140 × 140</td>
<td>-4.74073</td>
<td>2.09152</td>
</tr>
<tr>
<td>160 × 160</td>
<td>-4.74175</td>
<td>2.09255</td>
</tr>
</tbody>
</table>

**Table 1 Grid tests Nusselt numbers at different grid sizes for the case Ra = 10^3, \( \gamma = 1 \), \( H = 1 \), \( K_s = 1 \), and \( S = 0.2 \).**

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thermal conductivity ratio \( (K_r = 0.44, 1, 2.40, 9.90, \text{ and } 23.8) \) (epoxy–water: 0.44, brickwork–water: 1, glass–water: 2.40, epoxy–air: 9.90, stainless steel–water: 23.8), the ratio of the wall thickness to its height \((0 \leq S \leq 0.7)\), the modified conductivity ratio \((10^{-1} \leq \gamma \leq 10^{1})\), and the interphase heat transfer coefficient \((10^{-1} \leq H \leq 10^{1})\). The weighted-average Nusselt number \((\text{Nu}_{wa})\) is calculated for various values of \(Ra, S, \text{ and } \gamma\).

Figure 5 presents the effects of the modified conductivity ratio \((\gamma)\) and the Rayleigh number \((Ra)\) on the streamlines (left), isotherms of the global fluid phase (middle), and the isotherms of the global porous phase (right) when the bottom wall is heated uniformly \((h = 1)\) for \(H = 1, K_r = 1, \text{ and } S = 0.2\). The isotherms of the global fluid phase is the contour result of Eqs. (9) and (11) and the isotherms of the global porous phase is the contour result of Eqs. (10) and (11). Figure 5(a) compares the local thermal nonequilibrium and local thermal equilibrium cases with a low Rayleigh number \((Ra = 10^{3})\). In general, changing the flow motion with the uniform boundary conditions set causes the streamlines to create two similar rotating cells in different directions within the porous cavity. Contour level labels define the direction of the fluid flow (clockwise or anticlockwise direction) and also the strength of the flow. Positive \(\Psi\) denotes anticlockwise fluid flow, whereas negative \(\Psi\) designates clockwise fluid flow. \(\Psi_{\text{min}}\) and \(\Psi_{\text{max}}\) represent the extreme values of the stream function. These values are important to show the minimum and maximum changes of the flow. The streamlines tend to appear with high intensity next to the vertical walls of the cavity. The isotherms pattern within the fluid phase tend to appear with almost vertical lines close to the left and the right cold vertical walls of the porous cavity, while in the middle of the cavity, the isotherm patterns appear with curve lines due to the convective heat transfer effects, whereas the isotherms pattern within the porous phase occurs with smooth curve lines and less distorted due to the low modified conductivity ratio. The isotherms pattern appears smooth with less curve lines within the solid wall affected by the high resistance to the conductive heat transfer in the solid region. Increasing the value of the modified conductivity ratio to 1000 causes the flow and temperature distributions to appear like the local thermal equilibrium case. The strength of the flow circulation increases with the increasing of the modified conductivity ratio. Because of the high modified conductivity ratio \((\gamma = 1000)\), we achieved the thermal equilibrium case, and the isotherms of the global fluid phase became similar to the isotherms of the global porous phase. The streamlines clockwise and anticlockwise cells slightly expand. As the Rayleigh number increases, the intensity of the streamlines increases together with the cells sizes due to the strong buoyancy forces compared to the viscous forces (see Fig. 5(b)). The strength of the flow circulation is clearly increased with the increasing of the Rayleigh number affected by the rising of the convection intensity. The isotherms pattern of the fluid phase is highly distorted especially next to the interface wall, whereas we observe less change in the isotherms pattern of the porous phase. At a higher Rayleigh number value, the strength of the flow circulation decreases with an increase in the modified conductivity ratio (see \(\Psi_{\text{min}}\) and \(\Psi_{\text{max}}\) values).

Fig. 2 Streamlines (a)—Saeid [7] (left) and present study (right) and isotherms (b)—Saeid [7] (left) and present study (right) for \(Ra = 10^{3}, K_r = 0.1, \text{ and } S = 0.2\)
Figure 6 depicts the effects of the modified conductivity ratio ($\gamma$) and the Rayleigh number (Ra) on the streamlines (left), isotherms of the global fluid phase (middle), and the isotherms of the global porous phase (right) when the bottom wall is heated nonuniformly ($\theta = \sin(2\pi x)$) for $H = 1$, $K_r = 1$, and $S = 0.2$. Figure 6(a) presents the flow behavior and the temperature distribution within the porous cavity for the local thermal nonequilibrium case ($\gamma = 0.1$) and low Rayleigh number (Ra = $10^2$). Applying a nonuniform heating on the bottom horizontal wall significantly changes the flow behavior in which three streamlines rotating cells appear within the porous cavity. This is due to the fact that the lower heating provided by the nonuniform bottom boundary which leads to a less buoyancy effect and thermal gradient. We observe the reduction on the strength of the flow circulation with the nonuniform heating (see $\Psi_{\text{min}}$ and $\Psi_{\text{max}}$ values). The isotherm patterns of the fluid and porous phases appear as curved lines near the interface wall, while in the middle of the porous cavity, the isotherms patterns show less density. This is due to the greater thermal boundary layer thickness with a nonuniform heating which leads to the inhibition of the temperature gradient. The intensity of the streamlines is significantly increased, and the streamlines occupy the entire porous cavity with the increasing of the Rayleigh number. The streamlines rotating cells are clearly affected with the Rayleigh number rising; the streamlines clockwise cell expands diagonally within the porous cavity, the streamlines anticlockwise cells near the vertical walls, the big (primary) one expends vertically, while the secondary small cell tends to shrink close to the right boundary (see Fig. 6(b)). The isotherms pattern density is slightly increased within the fluid phase with the increasing of the Rayleigh number. This is due to the fact that the conductive heat transfer increases with the Rayleigh number. It may also be observed that the isotherms pattern within the porous phase remains unchanged. The minimum strength of the flow circulation decreases with the increasing of the modified conductivity ratio, while the maximum strength increases (see $\Psi_{\text{min}}$ and $\Psi_{\text{max}}$ values). A higher modified conductivity ratio ($\gamma = 1000$) causes the system to achieve the thermal equilibrium case which leads to the similarity between the isotherms of the global fluid phase and the isotherms of the global porous phase.

Figure 7 shows the effects of various thermal conductivity ratios ($K_r$) on the streamlines (left), isotherms of the global fluid phase (middle), and the isotherms of the global porous phase (right) when the bottom wall is heated nonuniformly ($\theta = \sin(2\pi x)$) for $Ra = 10^4$, $\gamma = 10$, $H = 1$, and $S = 0.2$. Changing the flow motion with the boundary conditions set causes the streamlines to create three different streamlines rotating cells. The clockwise cell occurs in the center of the porous cavity, while the two anticlockwise cells appear next to the cold vertical boundaries; the primary cell takes place next to the left wall and the secondary cell tends to appear close to the bottom corner of the right boundary. The isotherms pattern of the fluid and porous phases appear with low density, while it occurs with high density within the solid wall due to the lower thermal conductivity ratio ($K_r = 0.44$) (see Fig. 7(a)). The intensity of the streamlines increases within the middle of the porous cavity with the increase of the thermal conductivity ratio, and as a result, the strength of the flow circulation increases (see $\Psi_{\text{min}}$ and $\Psi_{\text{max}}$ values). It is also noted that the clockwise and the primary anticlockwise cells expand with this increase in the thermal conductivity ratio. This happens due to the reduction of the thermal resistance as the thermal conductivity ratio increases. The isotherms pattern within the solid wall shows less density and more heat moves to the porous cavity. This is due to the reduction of thermal resistance of the solid wall which leads to a strong temperature gradient with the increase in the thermal conductivity ratio. At the higher thermal conductivity ratio, the streamlines clockwise cell expands diagonally within the porous cavity. The primary streamlines anticlockwise cell near the vertical left wall tends to expand vertically, while the secondary cell close to the right cold wall tends to vanish. It may also be observed that the isotherms pattern appears with a high density and a strong temperature gradient within the porous cavity, especially within the isotherms of the fluid phase, as depicted in Figs. 7(c) and 7(d).

Figure 8 illustrates the effects of various solid wall thicknesses ($S$) on the streamlines (left), isotherms of the global fluid phase...
(middle), and the isotherms of the global porous phase (right) when the bottom wall is heated nonuniformly ($\theta = \sin(2\pi X)$) for $Ra = 10^3$, $\gamma = 10$, $H = 1$, and $Kr = 1$. Figure 8(a) presents the flow behavior and the temperature distribution within the porous cavity in the absence of the solid wall ($S = 0$). Due to nonuniform heating of the bottom horizontal wall and uniform cooling of the vertical walls, the flow within the porous cavity produces a streamlines cell in the clockwise direction located within the middle of the porous cavity and a primary streamlines cell in the anticlockwise direction next the left vertical boundary. It can be noted that the isotherms pattern of the fluid and the porous phases occurs with high density near the bottom boundary of the porous cavity. However, a higher density of the isotherms pattern appears within the fluid phase compared to the density of the isotherms pattern within the porous phase. Inserting a thin solid wall ($S = 0.1$) clearly affects the flow behavior, producing a third streamlines cell in the anticlockwise direction at the bottom right corner of the porous cavity. We may also observe that the strength of the flow circulation decreases with the insertion of the solid wall due to the resistance of the solid wall which decreases the buoyancy effect (see $\Psi_{\text{min}}$ and $\Psi_{\text{max}}$ values). The isotherms pattern of the fluid phase shows lesser density, while we observe no change in the isotherms pattern of the porous phase. This is due to the thermal property of the solid insert which decreases the temperature gradient resulting in less heating effect. Increasing the thickness of the solid wall to 0.5 clearly affects the distribution of flow behavior and the temperature profiles. Two streamlines cells in the anticlockwise direction occur next to the cold vertical walls of the porous cavity, while the main cell in the clockwise direction shrinks horizontally within the middle of the porous cavity. The density and the amount of the isotherms pattern of the global fluid phase are clearly decreased and become similar to that of the global porous phase. This is due to the fact that the solid wall offers high resistance, which leads to a weak temperature gradient, and as a result, the isotherms pattern of the global fluid phase becomes similar to the isotherms of the global porous phase which leads to the thermal equilibrium case, as shown in Fig. 8(d).

Figure 9(a) presents the effect of various values of the modified conductivity ratio on the weighted-average Nusselt number with the Rayleigh number for $H = 10$, $Kr = 1$, and $S = 0.2$. The convection heat transfer is increased with the increase of the Rayleigh number in which at high Rayleigh number values ($Ra > 10^5$), a significant enhancement of the convection heat transfer can be noted. This is due to the strong buoyancy effects compared to the viscous forces. Moreover, a higher modified conductivity ratio ($\gamma = 1000$) leads to a higher enhancement of the heat transfer rate, which has the maximum weighted-average Nusselt number. The effect of various values of the solid wall thickness on the weighted-average Nusselt number with the Rayleigh number is depicted in Fig. 9(b) for $\gamma = 10$, $H = 10$, $Kr = 1$, and $S = 0.2$. As with the previous case, the best enhancement of the overall heat transfer occurs with increasing the Rayleigh number, which tends to increase the average Nusselt number. It can be observed that the convection heat transfer is strongly enhanced in the absence of
Fig. 5 Variation of the streamlines (left), isotherms of the global fluid phase (middle), and isotherms of the global porous phase (right) with modified conductivity ratio ($\gamma$) and Rayleigh number (Ra) when the bottom wall heated uniformly ($\theta = 1$) for $H = 1$, $K_r = 1$, and $S = 0.2$: (a) $Ra = 10^2$ and (b) $Ra = 10^3$
the solid wall ($S = 0$), this is due to the fact that fluids transfer more heat than solids. In addition, it can be seen that the heat transfer rate shows no change with an increase in the thickness of the solid wall due to the resistance of the solid wall in which there is no heating effects. It may also be observed that the convection heat transfer increases with an increase in the thermal conductivity ratio. Due to the high thermal conductivity ratio which tends to decrease the resistance of the solid wall, the heat transfer rate increases (Fig. 9(c)).

Figure 10(a) shows the effect of various values of the Rayleigh number on the weighted-average Nusselt number with the solid wall thickness for $\gamma = 10$, $H = 10$, and $K_r = 1$. We clearly observe that by increasing the thickness of the solid wall, the overall heat transfer is decreases affected by the resistance of the solid wall which obviously leads to the reduction of the heat transfer rate. It may also be observed that the heat transfer rate is highly enhanced with higher Rayleigh numbers ($Ra = 10^3$). This due to the strong buoyancy effects which lead to a high temperature gradient, and as a result, the convection heat transfer increases. However, there is no change in the heat transfer rate for all Rayleigh numbers with the high solid wall thicknesses ($S \leq 0.5$), which indicates that with a high solid wall thickness there is no effect of changing the

![Figure 6](http://heattransfer.asmedigitalcollection.asme.org/pdfaccess.ashx?url=/data/journals/jhtrao/936353/ on 08/03/2017 Terms of Use: http://www.asme.org/about-asme/terms-of-use)
flow buoyancy. We also note that the best enhancement in the convection heat transfer is obtained with lower $S$ values ($S < 0.4$) due to the fact that fluids transfer heat more than solids. Furthermore, a higher modified conductivity ratio ($\gamma = 1000$) leads to a strong enhancement of the convection heat transfer, which has a maximum weighted-average Nusselt number (Fig. 10(b)). Figure 10(c) illustrates the effects of various values of the thermal conductivity ratio on the weighted-average Nusselt number with the solid wall thickness for $\gamma = 10$ and $H = 10$. In this figure, we may note different behaviors on the heat transfer rate for which increasing the thickness of the solid wall tends to increase the overall heat transfer for all thermal conductivity ratios. This is due to the fact that fluids transfer heat more than solids.
to the fact that increasing the thermal conductivity ratio tends to lower the resistance of the solid wall which allows the cavity to transfer more heat, particularly with the high thermal conductivity ratio ($K_r = 23.8$). Furthermore, for a fixed high value of $K_r$, the heat transfer is drastically increased for the case of relatively small wall thickness. This could be due to the fact that a small thickness wall with a high conductivity shows there is almost no resistance to the heat to transfer to the porous region.

Figure 11(a) demonstrates the effect of various values of the interphase heat transfer coefficient ($H$) on the weighted-average Nusselt number with the modified conductivity ratio for $Ra = 10^4$, $K_r = 1$, and $S = 0.2$. The heat transfer rate increases as the

![Streamlines, Isotherms](image)

**Fig. 8 Variation of the streamlines (left), isotherms of the global fluid phase (middle), and isotherms of the global porous phase (right) with the solid wall thickness ($S$) when the bottom wall heated nonuniformly ($\theta = \sin(2\pi x)$) for $Ra = 10^4$, $\gamma = 10$, $H = 1$, and $K_r = 1$: (a) $S = 0$, (b) $S = 0.1$, (c) $S = 0.3$, and (d) $S = 0.5$**
modified thermal conductivity ratio increases due to the enhancing of the thermal conductivity which clearly increases the values of the weighted-average Nusselt number. We also observe that a significant enhancement of the overall heat transfer is obtained with the higher interphase heat transfer coefficients affected by the higher convective heat transfer. It may also be noted from this figure that the heat transfer rate is noticeably increased in the absence of the solid wall, particularly with the high modified conductivity ratio ($\gamma > 100$), as shown in Fig. 11(b). The effect of various values of the thermal conductivity ratio on the weighted-average Nusselt number with the modified conductivity ratio is depicted in Fig. 11(c) for $Ra = 10^3$, $H = 1$, and $S = 0.2$. As in the

\[ ...\]
previous figure, the overall heat transfer enhances with an increase of the modified thermal conductivity ratio. Furthermore, the convection heat transfer enhances with the increase in the thermal conductivity ratio due to the low resistance of the solid wall which leads to a stronger temperature gradient, and as a result, the maximum weighted-average Nusselt number is obtained.

5 Conclusions

Some important conclusions from the study are given as follows:

(1) Increasing the solid wall thickness up to a higher value clearly enhances the flow behavior and the temperature distribution. Due to the fact that the solid wall offers a high resistance which leads to a weak temperature gradient, the isotherms pattern of the global fluid phase becomes similar to the isotherms of the global porous phase which lead to the thermal equilibrium case.

(2) We clearly observe a high resistance due to the conductive heat transfer in the solid wall with the thermal nonequilibrium case (low modified conductivity ratio) compared to that of the thermal equilibrium case. In other words, a thicker wall or a low thermal conductivity wall tends to inhibit heat transport in the porous medium.

(3) It is clearly observed that increasing the thickness of the solid wall tends to decrease the overall heat transfer affected by the resistance of the solid wall, particularly with high Rayleigh numbers.

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Nomenclature

- \( C_p \) = specific heat capacity
- \( g \) = gravitational acceleration
- \( h \) = convective heat transfer coefficient
- \( H \) = interphase heat transfer coefficient
- \( k \) = thermal conductivity
- \( K \) = permeability of the porous medium
- \( K_r \) = square wall to base fluid thermal conductivity ratio \( K_r = k_w/k_f \)
- \( L \) = length of the cavity
- \( N \) = periodicity parameter
- \( Nu \) = Nusselt number
- \( Ra \) = Rayleigh number
- \( S \) = dimensionless solid wall thickness \( (W/L) \)
- \( T \) = temperature
- \( u, v \) = velocity components in the \( x \)- and \( y \)-directions, respectively
- \( U, V \) = dimensionless velocity components in the \( X \)- and \( Y \)-directions, respectively
- \( x, y \) and \( X, Y \) = space coordinates and dimensionless space coordinates, respectively

Greek Symbols

- \( \alpha \) = thermal diffusivity
- \( \beta \) = thermal expansion coefficient
- \( \gamma \) = modified conductivity ratio
- \( \theta \) = dimensionless temperature
- \( \mu \) = dynamic viscosity
- \( \nu \) = kinematic viscosity
- \( \rho \) = density
- \( \phi \) = porosity of the medium
- \( \psi \) and \( \Psi \) = stream function and dimensionless stream function

Subscripts

- \( c \) = cold
- \( f \) = fluid phase
- \( h \) = hot
- \( p \) = solid of the porous medium
- \( w \) = solid wall