Analysis of mixed convection of nanofluid in a 3D lid-driven trapezoidal cavity with flexible side surfaces and inner cylinder

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\textbf{ARTICLE INFO}

\textbf{ABSTRACT}

Numerical study of mixed convection in a lid-driven 3D flexible walled trapezoidal cavity with nanofluids was performed by using Galerkin weighted residual finite element method. Effects of various pertinent parameters such as Richardson number (between 0.05 and 50), elastic modulus of the side surfaces (between 1000 and 10\textsuperscript{5}), side wall inclination angle (between 0° and 20°) and solid particle volume fraction (between 0 and 0.04) on the fluid flow and heat transfer characteristics in a 3D lid-driven-trapezoidal cavity were numerically examined. It was observed that these characteristics are influenced when the pertinent parameters change. Flexible side surface can be used as control element for heat transfer rate. Increment and reduction in the space which are provided by the flexible side walls result in heat transfer enhancement and deterioration for side wall inclination angle of 0° and 10°. Average Nusselt number enhances by about 9.80% when the value of the elastic modulus is increased from 1000 to 10\textsuperscript{5} for side wall inclination angles of \( \theta = 0° \). Adding nanoparticles to the base fluid results in linear increment of heat transfer and at the highest volume fraction, 25.30% of heat transfer enhancement is obtained. A polynomial type correlation for the average Nusselt number along the hot wall was proposed and it has a fourth order polynomial dependence upon the Richardson number and first order dependence upon the solid particle volume fraction.

\textbf{1. Introduction}

The interaction of natural convection effects and shear driven flow which is due to a moving surface has many important applications in practice and significant impacts on the heat transfer such as in cooling of nuclear reactors, electronic devices, some chemical processes and many others. Different geometrical shapes and various thermal boundary conditions may be encountered in practice. Studies for convection in trapezoidal shaped enclosures have received significant attention [1–10]. [11] examined the probable impacts on the heat transfer such as in cooling of nuclear reactors, electronic devices, some chemical processes and many others. [12] performed a simulation of lid-driven flow in a trapezoidal enclosure with Lattice Boltzmann method for Reynolds number in the range from 100 to 15,000 and the inclination angle from 50 to 90. A complex transitional of the flow in the trapezoidal cavity was observed as the value of Reynolds number is increased. [13]

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Nano-sized particles have been extensively used in various thermal engineering applications. Metallic or non-metallic fine solid particles such as Cu, CuO, Al₂O₃, TiO₂, SiO₂ with average particle size less than 100 nm are added to the heat transfer fluids such as water, ethylene glycol or oil.

Higher conductivity of the nanoparticle inclusion in the base fluid results in thermal conductivity enhancement of nanofluid even with a small amount of particle volume fraction. There are many factors that affect the thermal conductivity enhancement of nanofluids such as size, shape and type of the particles.

In heat transfer applications, a vast amount of research is dedicated to the nanofluids applications considering various geometries, thermal boundary conditions and different physical mechanisms [24–35]. [36] performed a mixed convection study of lid-driven trapezoidal cavity filled with nanofluid by using finite volume method. Different types of nanofluids were utilized and it was observed that SiO₂-water has the highest heat transfer rate among various nanoparticle types. In the study by [37], numerical simulation of mixed convection in a trapezoidal cavity filled with CuO-water nanofluid was performed by considering a new variable-property model. In the study by [38], a small amount of nanoparticle addition was found to enhance the heat transfer significantly for mixed convection in an inclined porous channel. [39] performed mixed convection in a lid-driven inclined square cavity with nanofluid. It was observed that heat transfer rate could be enhanced by the inclusion of nanoparticles and with inclination of the cavity for moderate and large values of Richardson numbers.

The main aim of the current study is to numerically examine the mixed convection in a nanofluid filled 3D trapezoidal cavity having flexible side walls with an inner stationary cylinder. This type of thermal configuration may be encountered in practice or convection in the trapezoidal cavity may be controlled by the use of some or all of the mentioned methods (flexible wall, inclusion of the nanoparticles). The mixed convection of fluid-structure interaction model with nanofluids has been performed for a three-dimensional trapezoidal cavity. The results of this present study could be utilized for the design and optimization of thermal systems in various engineering fields. A correlation for the Nusselt number along the hot wall of the trapezoidal cavity in polynomial form is also provided.

### 2. Mathematical formalism

Fig. 1 shows a schematic description of the physical model with boundary conditions and coordinates. A 3D trapezoidal cavity with side length H and inclination angle of θ was considered. The top and bottom surfaces are kept at constant temperatures of T_h and T_c, with T_h > T_c while the inclined side walls are assumed to be adiabatic. An inner stationary cylinder of diameter D was placed for the center location at (x_c, y_c, z_c). The inclined side surfaces are flexible with elastic modulus of E, density of ρ and Poisson’s ratio of ν. The top surface was moving with constant speed of u₀ in negative x direction. The 3D cavity was filled with CuO-water nanofluid at various volume fractions of solid particles (ϕ). The gravitational acceleration acts in negative y direction. The buoyancy term in the momentum equation was modeled by Boussinesq approximation and effects of thermal radiation and viscous dissipation were neglected. The Arbitrary Lagrangian-Eulerian method was used to describe the fluid motion for the flexible wall of the trapezoidal cavity in the fluid-structure interaction model. The fluid is Newtonian and the flow is accepted as 3D and laminar. Navier-Stokes and energy equations

![Fig. 1. Schematic representation of the physical problem with boundary conditions.](image-url)
Table 1
Thermophysical properties of base fluid and solid nanoparticle.

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$ (kg/m$^3$)</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>$c_p$ (J/kg K)</td>
<td>4179</td>
<td>540</td>
</tr>
<tr>
<td>$k$ (W/mK)</td>
<td>0.6</td>
<td>18</td>
</tr>
<tr>
<td>$\beta$ (1/K)</td>
<td>$21 \times 10^{-5}$</td>
<td>$0.85 \times 10^{-5}$</td>
</tr>
<tr>
<td>$d_p$ (nm)</td>
<td>0.384</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 2
Comparison of the average Nusselt number computed for a lid-driven cavity problem in ref. [44] and obtained with present solver.

<table>
<thead>
<tr>
<th>Re = 400</th>
<th>Ref. [44]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr = 100</td>
<td>3.84</td>
<td>3.81</td>
</tr>
<tr>
<td>Gr = $10^4$</td>
<td>3.62</td>
<td>3.63</td>
</tr>
<tr>
<td>Gr = $10^6$</td>
<td>1.22</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Fig. 2. Comparisons of the streamlines (a, b, c) and isotherms (d, e, f) computed in Basak et al. [9] and computed streamlines (g, h, i) and isotherms (j, k, l) with the current solver.
with effective thermo-physical properties (Table 1) of nano fluid can be written as [23]:

\[ \nabla \cdot \mathbf{u} = 0 \]  \hspace{1cm} (1)

\[ \rho_{nf}(\mathbf{u} - \mathbf{u}_b), \nabla \mathbf{u} = \nabla \sigma_{nf} + \rho_{nf} \mathbf{f}^b_j \] \hspace{1cm} (2)

\[ \mathbf{u} \nabla T = \sigma_{nf} \nabla^2 T \] \hspace{1cm} (3)

where \( f^b_j \), \( \sigma_{nf} \), \( \mathbf{u}_b \) denote the body force per unit volume, stress tensor and velocity of moving coordinate.

The equation for the solid-domain in the fluid-structure interaction model can be written as ([23]):

\[ \rho_s \mathbf{a}_s = \nabla \cdot \sigma_s + \mathbf{f}_s^b \] \hspace{1cm} (4)

where \( \sigma_s, \mathbf{f}_s^b \) denote the local acceleration, externally applied body force and solid stress tensor, respectively.

The relevant non-dimensional quantities are:

\[ Gr = \frac{g \beta (T_h - T_c) H^3}{\nu^2}, \quad Pr = \frac{\nu}{\alpha}, \quad Re = \frac{\omega H}{\nu}. \] \hspace{1cm} (5)

2.1. Boundary conditions and Nusselt number calculations

The appropriate forms of the dimensional boundary conditions can be given as:

- For the bottom surface:
  \( u = v = w = 0, T = T_b \)

- For the top surface:
  \( u = -u_b, v = w = 0, T = T_c \)

- For the side surfaces:
\[ \frac{\partial T}{\partial n} = 0 \]

Along the cylinder surface:
\[ u = v = w = 0, \quad \frac{\partial T}{\partial n} = 0 \]

At the fluid-structure interface displacement compatibility \((d_f = d_s)\)
and traction equilibrium \((\sigma_f = \sigma_s)\) should be satisfied \((\text{[23]})\).

Local and average Nusselt number along the hot surface can be calculated as:
\[ \int \int \theta \left( \partial_y \right) \sigma \left( \partial_y \right) dxdz = k \theta YH dxdz \]

\[ N_{u, Nu 1 Nu} = 0.2 \]

\[ xz_{nf}, ym_{HH}, 0 \]

2.2. Correlations for nanofluid effective thermo-physical properties

Effects of the temperature and particle size for Al2O3-water nanofluid on the thermal conductivity enhancement was given in ref. [40]:

\[ \frac{h_{nf}}{k_f} = 1 + 64.7\phi^{-0.64} \left( \frac{d}{d_f} \right) \left( \frac{d}{d_f} \right)^{0.369} \left( \frac{d}{d_f} \right)^{0.3476} \left( \frac{d}{d_f} \right)^{0.9955} \left( \frac{d}{d_f} \right)^{1.2321} \]

\[ Pr = \frac{\nu_f}{\nu}, \quad Re = \frac{\nu_f k_f T}{\nu f_{nf}} \]

where \( k_b \) is the Boltzmann constant. This model could also be used for CuO-water nanofluid ([41]). Specific heat of the nanofluid can be given as:

\[ (\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_f + \phi (\rho C_p)_m \phi \]

Dynamic viscosity of the nanofluid was according to Brinkman model [42]:

\[ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{1.3}} \]

Finally, thermal expansion of the nanofluid can be expressed as [43]:

\[ (\rho \beta)_n = (1 - \phi) (\rho \beta)_f + \phi (\rho \beta)_m - \phi (1 - \phi) (\rho \beta_f - \rho \beta_f) \]

3. Numerical solution method

Galerkin weighted residual finite element method was used to solve the governing system equations along with the boundary conditions. Each of the flow variables are approximated by using the interpolation functions within elements which are divided sub-domains of the computational domain as follows:

\[ u = \sum_{k=1}^{N} \psi_i \psi_{ik} U_k, \quad v = \sum_{k=1}^{N} \psi_i \psi_{ik} V_k, \quad w = \sum_{k=1}^{N} \psi_i \psi_{ik} W_k \]

\[ p = \sum_{k=1}^{N} \psi_i \psi_{ik} P_k, \quad T = \sum_{k=1}^{N} \psi_i \psi_{ik} T_k \]

where \( \psi_{ik} \) and \( \psi_{ik} \) denote the shape functions of velocity, pressure and temperature. \( U, V, P \) and \( T \) denote the values of the respective variables at the nodes of the element.

Flow variables within the computational domain were approximated by Lagrange finite elements. The approximate field variables when inserted into the governing equations results in residual \( R \) and the weighted average of this residual will be zeros over the computational domain:

\[ \int_{\Omega} w_0(x) R dx = 0 \]

where \( w_0 \) denotes the weight function. In the Galerkin method, the weight function is chosen from the same set of functions as of the trial functions. The resulting system of nonlinear ordinary differential equations were solved by Newton-Raphson method. Numerical experiments with different number of elements were performed to assure
a mesh independence of the solution. 250546, 232005 and 211963 number of tetrahedral elements were used for side inclination angles of $\theta = 0^\circ$, $\theta = 10^\circ$ and $\theta = 20^\circ$, respectively.

Present solver is validated with the existing results of ref. [9]. Fig. 2 presents the comparison results of streamlines and isotherms for various Rayleigh and Prandtl numbers computed in ref. [9] and obtained with the present solver. Distribution of flow and thermal patterns is similar.

A further validation of the present solver was made against the existing numerical results of [44]. Table 2 shows the average Nusselt number along the hot wall for various Grashof numbers at Reynolds number of 400 for a lid-driven cavity computed in [44] and obtained with the present solver. Fig. 3 shows another validation study that compares the results of Oztop and Abu-Nada [45] and present solver for Rayleigh number of $10^5$.

4. Results and discussion

In this numerical study, effects of Richardson number (between 0.05 and 50), elastic modulus of the side wall (between 1000 and $10^5$), side wall inclination angle (between $0^\circ$ and $20^\circ$) and solid particle volume fraction of the alumina (between 0 and 0.04) on the mixed convective heat transfer in a 3D lid-driven-trapezoidal cavity were numerically examined.

Effects of varying Richardson number on the fluid flow and thermal patterns are shown in Fig. 4 ($\theta = 10^\circ$, $E = 5000$, $\phi = 0.02$). Richardson number denotes the ratio of the natural convection to the forced convection due to the moving lid ($\frac{Gr}{Re^2}$). Grashof number was fixed to $10^5$ and the Richardson number was varied by changing the velocity of the moving lid. A lower value of Richardson number represents a higher value of the moving wall velocity and the center of the recirculating zone in the cavity is closer to the upper wall. As the Richardson number is increased to 10 and 50, natural convection effects become important and the center of the recirculation zones moves downward toward the mid of the cavity. The cylindrical obstacle slows down and slightly redirects the fluid motion. Average Nusselt number along the hot bottom surface decreases as the value of Richardson number enhances and by about 40.44% reduction in the average heat transfer is obtained when Ri is increased from 0.01 to 50 (Fig. 5). Variation of x and y component velocities at a plane $z = -0.5H$ and for two different x locations and for various values of Richardson number are shown in Fig. 6. An increment in the value of the Richardson number results in velocity enhancement which is due to the enhanced natural convection effects. The variation in the highest value of Richardson number is higher for y-velocity component for various x locations.

Flow and thermal patterns within the cavity for various elastic modulus of the side wall and for different inclination angles are demonstrated in Fig. 7 and Fig. 8 ($Ri = 0.5$, $\phi = 0.02$). The deflections of
the side walls are also shown in the figures. For the lowest value of elastic modulus, deformation of the surfaces are in the opposite and in the same direction of the surface normal for \( \theta = 0^\circ \) and for \( \theta = 10^\circ \) while it is negligible for side wall inclination angle of \( \theta = 20^\circ \). As the rigidity of the side wall increases, more (less) space is provided for the hot rising fluid from the bottom surface for \( \theta = 0^\circ \) (\( \theta = 10^\circ \)). Above the circular cylinder, a secondary weak recirculation zone is formed. Thermal patterns are more clustered adjacent to the hot bottom wall for small values of side wall inclination angles. There is negligible variation among different thermal patterns corresponding to different elastic modulus especially for higher values of \( \theta \).

Effects of elastic modulus of the side wall on the distribution of \( x \) and \( y \) velocity components along the \( y \)-axis for two different \( x \) locations in the plane \( z = -0.5 \, H \) are shown in Fig. 9. The magnitude of the \( y \)-velocity component increases slightly for lower values of elastic modulus of the side wall which is due to the additional space caused by surface deflection. Average Nusselt number variations (Fig. 10) along the hot bottom surface versus Richardson number increase, decrease and very slightly change for side wall inclination angles of \( \theta = 0^\circ \), \( \theta = 10^\circ \) and \( \theta = 20^\circ \). The additional space and reduction of the space obtained with flexible surface movement results in heat transfer enhancement and deterioration for \( \theta = 0^\circ \) and \( \theta = 10^\circ \). Average Nusselt number increases by about 9.80% and decreases 2.22% when the value of the elastic modulus is increased from \( E = 1000 \) to \( E = 10^5 \) for side wall inclination angles of \( \theta = 0^\circ \) and \( \theta = 10^\circ \).

Nanoparticle addition to the base fluid results in higher heat transfer rates due to the thermal conductivity enhancement even with small amount. Fig. 11 shows the \( x \) and \( y \) velocity along the \( y \)-axis at the plane \( z = -0.5 \, H \) (\( \theta = 10^\circ \), \( \text{Ri} = 2.5 \), \( E = 5000 \)). The shape of the velocity profiles along the \( y \)-axis does not change, but the magnitude of the \( y \)-velocity changes with nanoparticle addition especially in the location for \( x = 0.2 \). Average heat transfer versus solid particle volume fraction shows a linear trend. The enhancement is about 25.30% for nanofluid with the particle at the highest volume fraction as it is compared to the base fluid.

A polynomial type correlation was proposed for the average Nusselt number along the hot wall which is a function of Richardson number and solid particle volume fraction (\( E = 5000 \), \( \theta = 10^\circ \)). Richardson number is normalized with mean value of 10.46 and standard deviation of 16.07 while solid particle volume fraction is normalized with mean value of 0.02 and standard deviation of 0.0143 and the correlation is given as:
Fig. 9. Variation of the x and y velocity profiles along the y axis for various elastic modulus of the side walls and for two different x locations (z = -0.5H, θ = 10°, Ri = 0.5, φ = 0.02).

Fig. 10. Average Nusselt number along the hot surface versus elastic modulus of the side walls for various side wall inclination angles (Ri = 0.5, φ = 0.02).
The coefficients of the polynomial fit is given in Table 3. The dependence of the correlation on the solid particle volume fraction is first order which was already shown in Fig. 12. The complicated nature of mixed convection on the average Nusselt number is included by a fourth order polynomial dependence upon the Ri number. Surface fit data obtained with polynomial fit and data set computed with CFD are shown in Fig. 13 (a) while Fig. 13 (b) represents the residuals versus Ri number and \( \phi \) value. Table 4 shows the goodness of fit between polynomial fit and CFD data.

5. Conclusions

Mixed convection in a lid-driven 3D flexible sided trapezoidal cavity with nanofluids was numerically investigated. It was observed that the heat transfer and fluid flow characteristics are affected by the variations in the Richardson number, elastic modulus of the side wall and nanoparticle volume fraction for different side wall inclination angles of the trapezoidal cavity. Additional space is provided and reduction of the space is obtained with the elastic side surfaces which results in Nusselt number enhancement and deterioration for side wall inclination angle of \( \theta = 0^\circ \) and \( \theta = 10^\circ \). Average heat transfer increases by about 9.80% when the value of the elastic modulus is increased from 1000 to 10^5 for side wall inclination angles of \( \theta = 0^\circ \). Average Nusselt number increases linearly with solid particle volume fraction and heat transfer enhancement of 25.30% is obtained for nanofluid with the particle at the highest volume fraction as it is compared to the base fluid. Finally, a correlation for the average Nusselt number along the hot surface was provided which has a fourth order polynomial dependence upon the Ri number and first order dependence upon the solid particle volume fraction.

\[
\text{Nu}(R_i, \phi) = p_{00} + p_{10}R_i^0 + p_{01}R_i^1 + p_{20}R_i^2 + p_{11}R_i^1\phi + p_{30}R_i^3 + p_{21}R_i^2\phi + p_{40}R_i^4 + p_{31}R_i^3\phi
\]  

Table 3

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_{00} )</td>
<td>2.474</td>
</tr>
<tr>
<td>( p_{10} )</td>
<td>0.65</td>
</tr>
<tr>
<td>( p_{01} )</td>
<td>0.1841</td>
</tr>
<tr>
<td>( p_{20} )</td>
<td>0.7029</td>
</tr>
<tr>
<td>( p_{11} )</td>
<td>-0.05212</td>
</tr>
<tr>
<td>( p_{30} )</td>
<td>-2.828</td>
</tr>
<tr>
<td>( p_{21} )</td>
<td>0.07062</td>
</tr>
<tr>
<td>( p_{40} )</td>
<td>0.9781</td>
</tr>
<tr>
<td>( p_{31} )</td>
<td>-0.02138</td>
</tr>
</tbody>
</table>

Fig. 12. Average Nusselt number along the hot surface versus solid nanoparticle volume fractions (\( \theta = 10^\circ \), \( R_i = 2.5 \), \( E = 5000 \)).
Fig. 13. Surface fit plot versus Richardson number and solid particle volume fractions obtained with polynomial fit (a) and residuals between polynomial fit and CFD data (b).

Table 4
Goodness of fit between polynomial fit and CFD data.

<table>
<thead>
<tr>
<th>Goodness of fit</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>R-square</td>
<td>0.9616</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.1107</td>
</tr>
</tbody>
</table>

References

[23] K. Khanafar, Comparison of flow and heat transfer characteristics in a lid-driven cavity between flexible and modified geometry of a heated bottom wall, Int. J. Heat...


