Mixed convection in a square cavity filled with CuO-water nanofluid heated by corner heater

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A R T I C L E   I N F O

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A B S T R A C T

This paper investigates the heat transfer and fluid flow in a lid-driven cavity filled with CuO-water nanofluid and heated by a corner heater. The left vertical wall is cooled isothermally and moves upward. The corner heater is configured to be acting in the horizontal and vertical right walls. The remainder walls are adiabatic. Temperature dependent models for thermal conductivity and dynamic viscosity have been invoked. The governing equations have been solved using finite difference method. The governing parameters are nanofluid volume fraction \( \varphi = 0.0 - 0.07 \), Richardson number \( Ri = 0.01 - 100 \), Reynolds number \( (Re = 100 - 300) \) and five configurations of corner heater governed by the distance of its lower edge \( \Delta = 0 - 1 \). The results show that for low and intermediate values of Richardson number (i.e., \( Ri = 0.01 \) and \( Ri = 1 \)) the effects of the nanoparticles on heat transfer enhancement is not greatly pronounced. However, for high values of Richardson number (i.e., \( Ri = 10 \) and \( Ri = 100 \)) the Nusselt number enhances due to the addition of nanoparticles. A single case of heat transfer deterioration, due to the presence of nanoparticles, is observed for the case of \( Ri = 0.01 \) and \( \Delta = 0 \). For all studied Richardson numbers, the case of \( \Delta = 0 \) gives the best scenario for heat transfer when compared to other heater’s location.

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1. Introduction

Plentiful publications deal with the mixed convection in cavities of moving walls have been found in the open literature, this because the vital relevance of such geometry in industrial applications and mathematical sciences. Forming of sheet glass, continuous casting and galvanizing of metals, cooling of electronics, lubrication, and food processing are examples to such applications. Purely mathematical attention are also interested in this field of investigation [1–5].

Torrance et al. [6] studied different aspect ratio cavities with moving walls. Marked influence of buoyancy force at high aspect ratio was noticed in their study. Iwatsu et al. [7,8] studied the three-dimensional cavity with moving surface. In a certain Richardson number range, Aydin [9] reported a dominance of the opposing-buoyancy over the aiding-buoyancy. Chamkha [10] developed the problem of Aydin [9] by including the impact of heat source/sink with an externally applied magnetic force. He reported an adverse tendency between the mean Nusselt number and Hartman number. Al-Amiri et al. [11] investigated the double-diffusive mixed convection in a lid- driven square cavity. Sharif [12] studied the mixed convection in an inclined shallow enclosure. His results confirmed the increase of average Nusselt number with the tilted angle of the enclosure for the forced convection mode. Waheed [13] has investigated two different cases of lid-driven cavity, namely either the top or the bottom wall was lid. Ismael [14] has introduced the mixed convection in a cavity using an arc-shaped moving wall. He demonstrated diverse effects of the radius and rotation of the arc-shaped moving wall on the heat transfer.

Few experimental works [15,16] regarding the lid driven cavity have found in the open literature, this may be because the practical difficulty attending with the problem of sealing the meeting lines of moving and fixed walls.

The convective heat transfer can be augmented if some thermophysical properties of working fluid are promoted. This can be achieved by dispersing nano-sized solid particles to form what is called nanofluid. Tiwari and Das [17] have studied different categories of lid–driven walls to examine the effect of aiding and opposing mechanisms. Spot studies concerning the lid-driven cavities filled with nanofluids can be referred to Talebi et al. [18], Alinia et al. [19], Salari et al. [20], Chamkha and Abu Nada [21], Abbassian Arani et al. [22], Nasrin et al. [23], and Cho et al. [24]. However, Sivasankaran et al. [25] reported the effect of im-
posing an external magnetic field onto driven cavities. Studies concerning the characteristics of nanofluids under the effect of magnetic field in lid-driven cavities can be referred to Ghasemi et al. [26], Kadri et al. [27], and Chatterjee et al. [28].

Rather than rectangular lid-driven cavities, different geometries have been investigated also as an extension to their industrial applications (Migeon et al. [29], Mercan and Atlik [30], Rahman et al. [31], Ghasemi and Aminosadati [32], and Battachharya et al. [33]). Abu Nada and Chamkha [34] modeled a steady laminar mixed convection flow inside a lid-driven cavity with a cold bottom wavy wall with the top wall was hot and being moved at a constant speed. A CuO-water nanofluid fills the cavity. Sheremet and Pop [35] studied the double-diffusive mixed convection in a lid-driven square cavity filled with water-based nanofluid using Buongiorno’s model. The top and bottom walls were isothermal and being moved in a constant speed. Recently, Chamkha and Ismael [36] have studied the mixed convection inside a sidewall lid–driven trapezoidal cavity filled with Cu-water nanofluid under the effect of a transverse magnetic field. More recently, Malleswaran and Sivasankaran [37] have discussed the effects of corner heating on mixed convection under the influence of a uniform magnetic field in a lid-driven square cavity. They observed that the heater length in the horizontal direction is more effective than that of in the vertical direction on the heat transfer and on the flow pattern.

In accordance with the literature survey above, we ascertain that a mixed convection in a lid-driven cavity filled with nanofluid and heated by a corner heater has not been investigated yet. The driven left wall is to be kept isothermal with low temperature while the heater is to be varied between the horizontal and the right vertical walls. Temperature dependent models for the dynamic viscosity and the thermal conductivity are used.

2. Mathematical modeling

Fig. 1 shows the problem under study, a square lid-driven cavity (side length = W) filled with CuO-water nanofluid. A corner heater coincides along the left lower corner of the cavity and adds heat to the nanofluid at constant temperature $T_H$. The leading edge of the heater varies from $\delta = 0$ to H. The case of $\delta = 0$ refers to a heater completely located on the bottom wall of the cavity and the case of $\delta = H$, is when the heater coincide along the right vertical wall of the cavity, i.e. the total heater length is taken equal to the cavity side length. The left vertical wall is kept cold at constant temperature $T_C$ and being lid upward at speed $U_p$. The temperature difference between the walls and the nanofluid are considered moderate, thus, no slip boundary condition is assumed between the nanofluid and the solid walls, even at the moving solid wall. The base fluid (water) and the nanoparticles (CuO) are assumed to be in thermal equilibrium. The flow is assumed Steady and the parameters ranges are taken within the laminar range. The density varies with temperature according to Boussinesq approximation. The followings issues are neglected; thermal dissipation, Joule, Soret and Dufour effects. The dimensional governing equations are as follow:

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(1)

X–direction momentum equation:

$$\rho_{nf}\left(\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu_{nf}\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right)$$

(2)
Y–direction momentum equation:

\[
\rho_{nf} \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \frac{\partial p}{\partial y} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho_{nf} \beta_{nf} k (T - T_c) \tag{3}
\]

Energy equation:

\[
u \frac{\partial T}{\partial x} + u \frac{\partial T}{\partial y} = \alpha_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

Where \( \beta \) is the thermal expansion coefficient, \( \rho \) is the density, \( \mu \) is the dynamic viscosity, and \( \alpha \) is the thermal diffusivity. The subscript \( nf \) stands for nanofluid.

For the given problem, the appropriate boundary conditions at the system boundaries are,

1. On \( x = 0, u = 0, v = Up = T = T_c \).
2. On \( x = W, u = v = 0, T = T_H \) for \( 0 < y \leq \delta, \frac{\partial T}{\partial x} = 0 \) for \( \delta < y \leq W \).
3. On \( y = 0, u = v = 0, \text{and} \frac{\partial T}{\partial y} = 0 \) for \( 0 < x \leq \delta, T = T_H \) for \( \delta < x \leq W \).
4. On \( y = W, u = v = 0, \) and \( \frac{\partial T}{\partial y} = 0 \).

It is planning to solve the problem using finite difference method. As such, it is necessary to attenuate the pressure gradient terms of momentum equations. This task is achieved using the stream function–vorticity formulation strategy.

\[
\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} = -\Omega
\]

Continuity:

\[
\frac{\partial \Psi}{\partial x} + \frac{\partial \Omega}{\partial y} = 0
\]

Momentum:

\[
\frac{\partial \Psi}{\partial y} - \frac{\partial \Omega}{\partial x} = \frac{\nu_f}{\nu} \frac{1}{Re} \left[ \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right] + \frac{(\rho \beta)_{nf} k}{\rho_f \beta_{nf} \nu_f} \frac{\partial T}{\partial x} \tag{5}
\]

Energy:

\[
\frac{\partial \Psi}{\partial y} - \frac{\partial \Omega}{\partial x} = \frac{\alpha_f}{\alpha} \frac{1}{Pr Re} \left[ \frac{\partial^2 \Omega}{\partial x^2} + \frac{\partial^2 \Omega}{\partial y^2} \right] \tag{6}
\]

Where \( Pr = \frac{\nu}{\alpha} \) is the Prandtl number, \( Gr = \frac{\delta^3 \alpha \Delta T W^3}{\gamma} \) is the Grashof number, \( Ri = \frac{\alpha_f \Delta T W^2}{\nu_f} \) is the Richardson number, and \( Re = \frac{U_L W}{\nu_f} \) is the Reynolds number.

1. On \( x = 0, \frac{\partial \Psi}{\partial x} = 0, \theta = 0 \).
2. On \( x = 1, \frac{\partial \Psi}{\partial x} = 0, \) and \( \theta = 1 \) for \( 0 < Y \leq \Delta, \frac{\partial \Psi}{\partial x} = 0 \) for \( Y < \Delta \).
3. On \( Y = 0, \frac{\partial \Psi}{\partial y} = 0, \) and \( \frac{\partial \Omega}{\partial x} = 0 \) for \( 0 < X \leq \Delta, \) \( \theta = 1 \) for \( X < \Delta \).
4. On \( Y = 1, \frac{\partial \Psi}{\partial y} = 0, \) and \( \frac{\partial \Omega}{\partial x} = 0 \).

Jensen’s formula is used for the boundary condition of the vorticity on the solid boundary is given by [38]:

\[
\Omega_{n_f} = \frac{\partial \Psi}{\partial n} \text{ for moving wall} = \frac{3 \Psi}{2 \Delta n} \text{ for fixed wall}
\]

where \( \gamma = \left( \frac{\partial \Psi}{\partial n} \right) \) for moving wall, \( = \) for fixed wall, \( n \) is a normal vector that is \( n = X \) for horizontal wall and \( n = Y \) for vertical wall and subscripts 0 refers to points on the wall, 1 refers to points adjacent to the wall and 2 refers to the second line of points adjacent to the wall.

2.1. Nanofluid relations

The suggested nanofluid is CuO-Water with its properties shown in Table 1 [39,40] and its thermophysical properties are as follow:

<table>
<thead>
<tr>
<th>Property</th>
<th>Fluid (Water)</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cp (J/kg.K)</td>
<td>4179</td>
<td>540</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>k (W/m.K)</td>
<td>0.613</td>
<td>18</td>
</tr>
<tr>
<td>( \beta ) (K⁻¹)</td>
<td>21 × 10⁻⁵</td>
<td>5.1 × 10⁻⁵</td>
</tr>
</tbody>
</table>

The thermal conductivity is according to Chon et al. [41]

\[
k_{nf} = k_f + \frac{1}{259} \frac{36.44 \varphi_0}{\varphi_0 + 1}(0.259 \varphi_0^2 + 0.369 \varphi_0 + 0.746) \tag{7}
\]

\[
Pr = \frac{\mu_f}{\rho_f \beta_{nf}} \text{, } Re_f = \frac{\rho_f \beta_{nf} T}{5 \pi \mu_f} \tag{8}
\]

Where \( k_b \) is the Boltzmann constant \((1.3807 × 10^{-23} J/K)\) and \( \varphi \) is the mean path of fluid particles given as 0.07 nm. \( T \) in Eq. (9) is in Centigrade. This model was also confirmed by Mintsa et al. [42] for CuO and adopted afterwards in many studies, such as by Popa et al. [43].

The base fluid viscosity \( \mu_f \) is calculated from the following empirical relation [44]:

\[
\mu_f = 2.414 \times 10^{-5} \times 10^{247.8/(T-f(140))}
\]

Where \( T \) here in Kelvin and \( \mu_f \) in Pa.s

Dynamic viscosity:

\[
\mu_{nf}(10^3) = -0.6967 + \left( \frac{15.937}{T} \right) + 1.238 \varphi + \left( \frac{1356.14}{T^3} \right) \tag{9}
\]

The interest quantity of convective heat transfer is the Nusselt number, which is calculated on the heater locally using:

\[
Nu = \frac{k_{nf}}{\left( \rho C_p \right)_{nf}} \tag{10}
\]

3. Numerical solution

The dimensionless governing Eqs. (5–7) are discretized uniformly \((\Delta X = \Delta Y)\) over the square domain and along with the mentioned boundary conditions, we solved the discretized equations using the finite-difference method based on an in-house computer code written in FORTRAN. The instability of solution resulting from convective terms has
been treated by up-wind scheme procedure. Under relaxation factors were introduced to accelerate the accurate solution which is terminated by verification the following criterion:

\[
\max \left| \frac{X_{neu}(i,j) - X_{old}(i,j)}{X_{old}(i,j)} \right| \leq 10^{-6}
\]

(16)

3.1. Grid dependency test

To set the suitable grid size, we did a grid test independency on the case of Ri = 100, Re = 100, \( \varphi = 0 \), and \( \Delta = 0.5 \). The results regarding the variation of the average Nusselt number with mesh size are presented in Table 2. A grid size of 81 x 81 has been chosen as a tradeoff between the accuracy and the time required for the FORTRAN code.

3.2. Comparisons with others

Comparing the results of the present code with the open literature have been achieved with the average Nusselt number over the top moving wall for wide (laminar) ranges of Ri and Re. Several researchers as in Abu Nada and Chamkha [34], Waheed [40], Tiwari and Das [17] pointed out this case. As can be seen in Table 3, the maximum absolute error is about 4.45% attending with \( Re = 1000 \). However, this discrepancy is acceptable when considering both the flow and heat transfer fields. Another case of square cavity with moving vertical walls conducted by Tiwari and Das [17] has also checked. The results of comparison are tabulated in Table 4. Globally, the corresponding errors recorded in Tables 3 and 4 demonstrate a reasonable confidence in the outcome of our present code.

4. Results and discussion

In the present study, the range of Richardson number investigated is 0.01 \( \leq \text{Ri} \leq 100 \). The value of the Reynolds number is varied as \( Re = 100, 200 \) and 300 while the value of Grashof varies in order to adjust the desired value of Richardson number. The location of the heater and the cold moving wall are set such that the currents of natural and forced convection interact in adverse directions. This scenario generates multicellular convection and this in turn provides a suitable condition to investigate the enhancement of nanofluids in mixed convection applications. Hence, the current results present the effects of the heater’s location, volume fraction of nanoparticles, and the Richardson number on the heat transfer in the cavity. Unless stated, most results are presented for \( Re = 100 \).

Starting with the influence of the heater’s location on the flow and thermal patterns, Fig. 2 presents the contours of isotherms and streamlines using different locations of the corner heater for \( \text{Ri} = 10 \) and \( \varphi = 3\% \). Five different values of heater’s location are presented, namely \( \Delta = 0, 0.25, 0.50, 0.75, \) and 1.0, respectively as portrayed in Fig. 2(a)–(e). For \( \text{Ri} = 10 \), natural convection heat transfer dominates over forced convection. As shown in Fig. 2, two recirculation cells are formed within the cavity where a main cell spreads over most of the cavity domain and another slim cell is formed adjacent to the left cold wall of the cavity. Due to the predominant natural convection, \( \text{Ri} = 10 \), a main recirculation cell is generated whereas a slim cell is formed on the left of the cavity, close to the driven left wall. This pattern of recirculation is repeated in all other locations of the corner heater. In addition, it can be seen from this figure that the heater location affects the core of the primary recirculation. Moreover, Fig. 2 shows that the isotherms are closely clustered at the cold wall and adjacent to the heater. For the case of \( \Delta = 1 \), the isotherms become horizontal in the middle of the cavity which

---

Table 2
Grid independency for \( \text{Ri} = 100, \text{Re} = 100, \varphi = 0, \Delta = 0.5 \).

<table>
<thead>
<tr>
<th>Grid Size</th>
<th>( 21 \times 21 )</th>
<th>( 41 \times 41 )</th>
<th>( 61 \times 61 )</th>
<th>( 81 \times 81 )</th>
<th>( 101 \times 101 )</th>
<th>( 121 \times 121 )</th>
<th>( 141 \times 141 )</th>
</tr>
</thead>
</table>

Table 3
Mean Nusselt number on the top moving wall of a square cavity for \( Pr = 0.71 \).

<table>
<thead>
<tr>
<th>( Re )</th>
<th>( Ri )</th>
<th>Current code</th>
<th>Chamkha and Abu Nada [34]</th>
<th>Waheed [40]</th>
<th>Tiwari and Das [17]</th>
<th>Khanafar and Chamkha [41]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>100</td>
<td>1.0197</td>
<td>1.010134</td>
<td>1.00033</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>100</td>
<td>0.01</td>
<td>2.08</td>
<td>2.090837</td>
<td>2.03116</td>
<td>2.10</td>
<td>2.01</td>
</tr>
<tr>
<td>400</td>
<td>0.000625</td>
<td>4.036</td>
<td>4.162057</td>
<td>4.02462</td>
<td>3.85</td>
<td>3.91</td>
</tr>
<tr>
<td>500</td>
<td>0.0004</td>
<td>4.548</td>
<td>4.663689</td>
<td>4.52671</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>1000</td>
<td>0.0001</td>
<td>6.2599</td>
<td>6.351615</td>
<td>6.48423</td>
<td>6.33</td>
<td>6.33</td>
</tr>
</tbody>
</table>

Table 4
Comparison of \( \text{Nu}_{\text{av}} \) with [17] for Prandtl number of 6.2.

<table>
<thead>
<tr>
<th>( \text{Ri(Re)} )</th>
<th>Current code</th>
<th>% error</th>
<th>( \text{Ri(Re)} )</th>
<th>Current code</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1(316.23)</td>
<td>30.67</td>
<td>–4.7</td>
<td>11.54</td>
<td>11.15</td>
<td>–3.38</td>
</tr>
<tr>
<td>1(100)</td>
<td>17.96</td>
<td>–2.95</td>
<td>6.65</td>
<td>6.61</td>
<td>–0.6</td>
</tr>
<tr>
<td>10(316.23)</td>
<td>10.19</td>
<td>–2.3</td>
<td>4.25</td>
<td>4.22</td>
<td>–0.7</td>
</tr>
</tbody>
</table>
For other values of $\Delta$, the isotherms are not as horizontal as the case of $\Delta = 1$, which indicates that the middle of the cavity encounters some degree of varying temperature distribution.

Fig. 3 presents the effect of $\Delta$ on the local Nusselt number along the heater. The Nusselt number is presented along the heater surface for $Ri = 100$, $\varphi = 3\%$ and for the five heater locations $\Delta = 0, 0.25, 0.75$, and 1.0. The heater situations of $\Delta = 0.25, 0.5$, and 0.75 have a common behavior, that is the value of the Nusselt number is highest at the horizontal edge of the corner heater and it decays to a very low value exactly at the corner of the heater (i.e., right bottom corner of the cavity). Then, it increases to a higher value afterward the corner and finally it decreases again toward the trailing edge of the heater. This decrease, near the trailing edge, is due to the direct contact of the insulated portion of the right wall with the trailing edge of the heater. Different patterns were recorded with the case of $\Delta = 0$ and 1.0, that is the local Nusselt number increases along a substantial length of the heater up to a peak value and then drops. The recorded minimum values at the edge of meeting the lower and the moving up left walls for $\Delta = 0$ and 1 refer to the formation of the secondary clockwise recirculation at this edge. This recirculation resists the convective current and leading to drop the local Nusselt number there.

Fig. 4 presents the diversity of the mean Nusselt number with the location of the corner heater. The figure demonstrates that at $\Delta = 0$ and $Ri < 100$, the Nusselt number is maximum while the $\Delta$ value at which the Nusselt number is minimum varies with $Ri$ value. For instance, at the case of $Ri = 0.01$ (i.e., a case of forced convection domination) the minimum Nusselt number occurs at $\Delta = 0.75$, whereas when $Ri = 1$ (i.e., equivalent forced and natural convolutions) the worst location is found at $\Delta = 0.50$. However, for $Ri = 10$ and 100, (i.e., cases of natural convection domination) the worst location is at $\Delta = 0.25$. The maximum Nusselt value at $\Delta = 0$ is attributed to the direct point contact of the leading edge of the heater with the cold wall, which makes the heat transfer for such configuration very high.

Fig. 5 presents the effects of nanoparticles concentration, using two concentrations $\varphi = 0\%$ and $\varphi = 7\%$, on the streamlines and isotherms within the cavity. For the case of $Ri = 0.01$, the effects of the nanoparticles on heat transfer enhancement is not greatly pronounced. For this Richardson value, the heat is transferred mainly by forced convection and the addition of nanoparticles will increase the viscosity of the nanofluids. For example, according to Eq. (10), the viscosity of the
nanofluid, at \( \varphi = 7\% \), is almost 2.5 times of a clear fluid. This substantial increase in the nanofluid’s viscosity will weaken the forced convection currents within the cavity and causes a decrease in the heat transfer. However, the presence of highly conductive nanoparticles will balance the deterioration caused by viscosity and in total, the addition of nanoparticles on the heat transfer will be minimal. Also, for the case \( Ri = 1 \), Fig. 5 shows that the edges of the right and left circulation cells meet. Consequently, the enhancement brought by the high conductive nanoparticles will slightly overcome the adverse effect of viscosity. Thus, the convective heat transfer becomes stronger which in turn thins the thermal boundary layer as portrayed by the high clustering of isotherms. For the case of dominant Natural convection, \( Ri = 10 \), the expanding of the natural convection (clockwise) circulation results in thinner thermal boundary layers and mostly horizontal isotherms.

Fig. 6(a) shows the local Nusselt number along the heater for \( Ri = 100 \) using various concentration of nanoparticles and \( \Delta = 0.5 \). The maximum local Nusselt number occurs at the leading edge of the cavity and the minimum local Nusselt number occurs exactly at the corner of the heater as was discussed in Fig. 3. The figure shows the local Nusselt number enhances due to the addition of nanoparticles due to the increased level of mixing brought by the addition of nanoparticles, as was discussed in Fig. 5.

Fig. 6(b) completes the perception of the nanoparticles effects, where it demonstrates that the average Nusselt number, at low values of Richardson number, is less sensitive to the volume concentration of nanoparticles when compared to high values of Richardson number. For the case of \( Ri = 100 \), the natural convection becomes predominant and buoyant forces will enhance the mixing within the cavity and the role of nanoparticles in heat transfer enhancement becomes more pronounced.

Fig. 7 presents the effect of Richardson number on the flow and the thermal patterns within the cavity for the case of \( \varphi = 5\% \) and \( \Delta = 0.5 \). When \( Ri = 0.01 \), a single recirculation cell extends over the whole cavity. The rotation of this cell is clockwise due to the movement of the lid in the positive y-direction. As the Richardson number increases to \( Ri = 1 \), the natural convection increases and another cell at the right hand side of the cavity takes place. The rotation of the right circulation cell is in the counter clockwise direction. Further increasing in Richardson number will cause the right circulation cell to further increase in size and spreads over a larger area in the cavity and accordingly the left cell size decreases.

The effect of Richardson number on the local Nusselt number for the case of \( \varphi = 3\% \), and \( \Delta = 0.5 \) is portrayed in Fig. 8. The maximum heat transfer occurs at the leading edge of the heater for the case of \( Ri = 1 \) and \( Ri = 10 \). However, for \( Ri = 0.01 \), the maximum Nusselt number occurs at the trailing edge of the heater. The minimum Nusselt number occurs at the corner of the heater, which coincides, with lower right corner of the cavity and it has the same trend that was discussed in Fig. 2.

The effect of nanoparticle concentration on the average Nusselt number is shown in Figs. 9 and 10 for the cases of \( Ri = 0.01 \) and \( Ri = 10 \), respectively. At \( Ri = 0.01 \) and \( \Delta = 0 \), the increased viscosity resulting from the addition of nanoparticles acts as a drag to the dominance forced convection and overcome the positive effect of the increased thermal conductivity, thus the Nusselt number decreases as shown in Fig. 9. However, for other values of \( \Delta \) the effect of nanoparticles on heat transfer enhancement is not much pronounced as discussed in Fig. 5. The minimum average Nusselt number occurs at \( \Delta = 0.75 \). The case of \( Ri = 10 \) demonstrates a better enhancement role of the nanoparticles on heat transfer due to the release of the thermal conductivity action which enhance the dominant natural convection, as was discussed previously for Figs. 5 and 6. The minimum average Nusselt number occurs at \( \Delta = 0.25 \). Similar to the observation mentioned in Fig. 4, the maximum value of average Nusselt number at \( \Delta = 0 \) is attributed to the direct point contact of the leading edge of the heater with the cold wall.

Finally, it is interested to study further values of Reynolds number and check how it affects the thermal and fluid fields. Fig. 11 shows that for \( Ri = 1 \) and \( \Delta = 0.5 \) as \( Re \) increases, the clock-wise recirculation slims while the counter clock-wise one expands, but both recirculation weaken with \( Re \). The corresponding isotherms show overcrowding at the zone where both recirculation meet. Fig. 12 shows the variation of the average Nusselt number with \( Ri \) for three values of \( Re \) when \( \Delta = 0.5 \) and \( \varphi = 0.03 \). At \( Ri = 100 \), the Nusselt number increases by about 50% and 30% when \( Re \) increases from 100 to 300 by a step of 100. However, at \( Ri = 0.1 \), Nusselt number deteriorates when \( Re \) is increased from 200 to 300. This deterioration can be referred to the up growth of the adverse action of the friction forces generated by the transcendent motion of the left wall.
5. Conclusions

For low and intermediate values of $Ri = 0.01$ and $Ri = 1$, the role of nanoparticles on heat transfer enhancement is not significantly pronounced. However, for high values of Richardson number ($Ri = 10$ and $Ri = 100$) the Nusselt number enhances due to the addition of nanoparticles. A single case of adverse effect of nanoparticles on heat transfer is detected for the case of $Ri = 0.01$ and $\Delta = 0$. For all studied Richardson numbers, when the heater stands completely along the bottom wall ($\Delta = 0$), is shown to have the best situation of heat transfer when compared to other heater locations (i.e., $\Delta = 0.25$, 0.5, 0.75, and 1.0). The local distribution of Nusselt number along the heater surface shows that
Fig. 8. Distribution of local Nusselt number along the corner heater for different CuO volume fractions at $\varphi = 0.03$ and $\Delta = 0.5$.

Fig. 9. Average Nusselt number with volume fraction for different corner heater configurations at $Ri = 0.01$.

Fig. 10. Average Nusselt number with volume fraction for different corner heater configurations at $Ri = 10$.

Fig. 11. Streamlines and isotherms for different Reynolds numbers at $Ri = 1$, $\Delta = 0.5$, and $\varphi = 0.03$.

Fig. 12. Variation of Nusselt number with $Ri$ for different values of Reynolds number at $\Delta = 0.5$ and $\varphi = 0.03$. 

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Figures clearly depict the distribution and variation of Nusselt numbers with various parameters, including volume fraction, corner heater configuration, and Reynolds number, showcasing the effects on local and average heat transfer characteristics.
the minimum Nusselt number occurs at the bottom right corner of the
heater for $\Delta = 0.25, 0.50$ and 0.75 and occurs at the leading edge of the
heater for the case of $\Delta = 0$ and 1.0. The increase of Reynolds number
is useful at higher values of Richardson numbers.

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