MIXED CONVECTIVE HEAT TRANSFER OF IMMISCIBLE FLUIDS IN A VERTICAL CHANNEL WITH BOUNDARY CONDITIONS OF THE THIRD KIND

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The effect of viscous dissipation and boundary conditions of the third kind on fully developed mixed convection for the laminar flow in a parallel plate vertical channel filled with two immiscible viscous fluids is studied analytically. The plate exchanges heat with an external fluid. Both conditions of equal and different reference temperatures of external fluid are considered. Separate solutions are matched at the interface using suitable matching conditions. First, the simple cases of the negligible Brinkman or negligible Grashof numbers are solved. Then, the combined effects of buoyancy forces and viscous dissipation are analyzed by the perturbation series method (PM), valid for small values of the perturbation parameter, and by the differential transform method (DTM), valid for all values of perturbation parameter. Numerical results are presented graphically for the distribution of velocity and temperature fields for varying physical parameters, such as the mixed convection parameter, perturbation parameter, viscosity ratio, width ratio, conductivity ratio, and Biot numbers. The effect of these parameters on the Nusselt number at the walls is also presented graphically. It is found that the mixed convection parameter and perturbation parameter enhance the flow field; whereas, the viscosity ratio, width ratio, and conductivity ratio suppress the flow field. It is also found that both PM and DTM solutions agree very well for small values of the perturbation parameter.

KEY WORDS: mixed convection, immiscible fluids, differential transform method, boundary condition of third kind

1. INTRODUCTION

Mixed convection flow in a vertical channel has been the subject of many previous investigations due to its possible applications in industrial and engineering processes. These include nuclear reactors, fire research, thermal insulation, thermal storage system, electric transmission cables, and break housing of an aircraft (Desai and Vafai, 1992), which can be modeled by various extensions of this type of geometry. Tao (1960) analyzed laminar fully developed mixed convection flow in a vertical parallel-plate channel with uniform wall temperatures. Aung and Worku (1986a,b) discussed the theory of combined free and forced convection, respectively, in a vertical channel with flow reversal conditions for both developing and fully developed flows. Aung and Worku (1987) assumed that the walls of the
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NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$b$</td>
<td>ratio of thermal expansion coefficient ($\beta_2/\beta_1$)</td>
</tr>
<tr>
<td>$Br$</td>
<td>Brinkman number ($\mu_1 U_0^{(1)} / K_1 \Delta T$)</td>
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<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
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<tr>
<td>$g$</td>
<td>acceleration due to gravity</td>
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<td>$Gr$</td>
<td>Grashof number ($g \beta_1 D_1^2 \Delta T / \nu_1^2$)</td>
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<tr>
<td>$h$</td>
<td>width ratio ($h_2/h_1$)</td>
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<td>$h_1$</td>
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</tr>
<tr>
<td>$h_2$</td>
<td>width of region II</td>
</tr>
<tr>
<td>$k$</td>
<td>ratio of thermal conductivities ($k_1/k_2$)</td>
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<td>$k_1$</td>
<td>thermal conductivity of the fluid in region I</td>
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<tr>
<td>$k_2$</td>
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<td>$m$</td>
<td>ratio of viscosities ($\mu_1/\mu_2$)</td>
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<td>$p$</td>
<td>dimensional pressure</td>
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<tr>
<td>$Re$</td>
<td>Reynolds number ($D_1 U_0^{(1)} / \nu_1$)</td>
</tr>
<tr>
<td>$T$</td>
<td>temperature</td>
</tr>
<tr>
<td>$T_{q_1}, T_{q_2}$</td>
<td>temperature at the boundaries</td>
</tr>
<tr>
<td>$U_0^{(1)}$</td>
<td>reference velocity</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity</td>
</tr>
<tr>
<td>$Y$</td>
<td>space coordinate</td>
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Greek Symbols

<table>
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<tr>
<td>$\alpha_1$</td>
<td>thermal diffusivity of the fluid in region I</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>thermal diffusivity of the fluid in region II</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>coefficient of thermal expansion of the fluid I region I</td>
</tr>
<tr>
<td>$\beta_2$</td>
<td>coefficient of thermal expansion of the fluid I region II</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>difference in temperature ($T_2 - T_1$)</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>dimensionless parameter</td>
</tr>
<tr>
<td>$\Lambda$</td>
<td>mixed convection parameter ($Gr/Re$)</td>
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<td>$\epsilon$</td>
<td>perturbation parameter ($\Lambda Br$)</td>
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<td>$\theta_1$</td>
<td>temperature in region I</td>
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<td>$\mu_1$</td>
<td>viscosity of the fluid in region I</td>
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<td>$\nu_1$</td>
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<td>kinematics viscosity of the fluid in region II</td>
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<td>$\rho_1$</td>
<td>density of fluid in region I</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>density of fluid in region II</td>
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</table>

channel were having asymmetric temperatures. A comprehensive review of the literature dealing with mixed convection in internal flow was reported by Aung (1987). Ingham et al. (1988) also reported on the flow reversal situation for mixed convection in a vertical channel for different wall heating conditions. The laminar mixed convection in a parallel plate vertical channel with viscous dissipation in the fully developed region has been studied by Barletta (1998). Following the work of Aung and Worku (1986a,b) and Barletta (1998), Umavathi and Malashetty (2005), Umavathi et al. (2006), and Prathap Kumar et al. (2009, 2011a,b) analyzed the mixed convection in a vertical channel.

It is well known that buoyancy plays an important role on the forced fluid flow and heat transfer in a heated vertical channel. For an aiding flow with a sufficiently high $Gr/Re^2$, where $Gr$ is the Grashof number and $Re$ is Reynolds number, the fluid near the heated walls is accelerated to a very high speed, causing the flow reversal in the central portion of the channel in order to maintain mass conservation. On the other hand, in general, a recirculation flow is observed near the heated walls when the opposing buoyancy force is strong enough to reverse the forced flow locally. Consequently, an understanding of mixed convection heat transfer becomes important and necessary. Habchi and Acharya (1986) numerically investigated the aiding mixed convection of air. Their results show that the air temperature increases with $Gr/Re^2$ and the Nusselt number decreases monotonically. A similar study was performed by Aung and Worku (1986b) indicating that buoyancy force can cause a severe distortion in the velocity profile especially under asymmetric heat condition. Aung and Worku (1986a) and Lavine (1988) proposed the criteria for the presence of reverse flow in vertical and inclined ducts, respectively. Actually, heat transfer may be greatly enhanced over the section containing a strong reverse flow. Umavathi et al. (2005a,b) presented an analytical study of mixed convection in a vertical channel including the effects of viscous dissipation.

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In the past, the laminar forced convection heat transfer in the thermal entrance region of a rectangular channel has been analyzed by Lyczkowski et al. (1969) and Javeri (1976) either for the temperature boundary condition of the first kind, characterized by prescribed wall temperature, or for the boundary condition of the second kind, expressed by the prescribed wall heat flux by Hicken (1968) and Sparrow and Siegal (1960). A more realistic condition in many applications, however, will be the temperature boundary condition of the third kind: the local wall heat flux is a linear function of the local wall temperature. Heat transfer in the laminar region of a flat channel for the temperature boundary condition of the third kind was explored by Javeri (1977). Javeri (1978) also investigated the influence of the temperature boundary condition of the third kind on the laminar heat transfer in the thermal entrance region of a rectangular channel. Later, Zanchini et al. (1998) analyzed the effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of the third kind.

Most existing research has been principally devoted to the case of a single fluid filling the entire enclosure. In realistic situations however, the fluid system oftentimes consists of two (and possibly more) separate, immiscible liquids—a layer of one liquid overlying a layer of another liquid—in the described side-heated cavity. The problem formulation now contains additional dynamical ingredients, such as the interfacial stress and the deformation of the interface shape. The behavior of a two-layer liquid is of interest in the course of design and operation of fluid experiments in a low gravity space environment (Ostrach, 1982). Also, a multilayered liquid arrangement provides an improved model for a buoyancy-driven convection process in growing high-quality crystals. In general, multiphase flows are driven by gravitational and viscous flows. Malashetty and Leela (1992) performed theoretical and experimental work on the laminar flow of two immiscible fluids in a horizontal pipe. Two-fluid flow and heat transfer in various geometries were studied by Malashetty et al. (2004), Umavathi and Malashetty (2005), and Umavathi et al. (2005a,b, 2008).

The concept of the differential transform method was first introduced by Zhou (1986) and used to solve both linear and nonlinear initial value problems in electric circuit analysis. The main advantage of this method is that it can be applied directly to nonlinear differential equations without requiring linearization, discretization, or perturbation. It is a semi-analytical/semi-numerical technique that formulizes the Taylor series in a very different manner. This method constructs an analytical solution in the form of a polynomial. Not like the traditional high-order Taylor series method, which requires symbolic computation, the DTM is an iterative procedure for obtaining Taylor series solutions. Another important advantage is that this method reduces the size of computational work; whereas, the Taylor series method computationally takes a long time for large orders. This method is well-addressed in Chen and Ho (1999), Ravi Kanth and Aruna (2008), and Villafuerte and Chen-Charpentier (2012).

Keeping in view the practical applications of mixed convection flow as mentioned above, it is the aim of the present work to extend studies available in the literature—and especially the work of Zanchini et al. (1998) on mixed convection in a vertical channel with boundary conditions of the third kind—with viscous dissipation by considering two immiscible viscous fluids. The solutions are obtained by regular perturbation method and by the semi-analytical method known as DTM. When the viscosity ratio, conductivity ratio, and width ratio are taken as one, the solutions obtained in this paper coincide with those of Zanchini et al. (1998).

2. MATHEMATICAL FORMULATION

The geometry under consideration illustrated in Fig. 1 consists of two infinite parallel plates extending in the X and Z directions. The region \(0 \leq Y \leq -h, Y \leq \sqrt{2} \) is occupied by the viscous fluid of density \(\rho_1\), viscosity \(\mu_1\), thermal conductivity \(k_1\) and thermal expansion coefficient \(\beta_1\) and the region is occupied by a different (immiscible) viscous fluid of density \(\rho_2\), viscosity \(\mu_2\), thermal conductivity \(k_2\), and thermal expansion coefficient \(\beta_2\). The fluids are assumed to have constant properties, except the density in the buoyancy term in the momentum equation \(\rho_1 = \rho_0 [1 - \beta_1 (T_1 - T_0)]\) and \(\rho_2 = \rho_0 [1 - \beta_2 (T_2 - T_0)]\). A fluid rises in the channel driven by buoyancy forces. The transport properties of both fluids are assumed to be constant. We consider the fluid to be incompressible and the flow is steady, laminar, and fully developed. It is assumed that the only nonzero components of the velocity \(\vec{q}\) are the X-component \(U_i\) \((i = 1, 2)\). Thus, as a consequence of the mass balance equation, one obtains

\[
\frac{\partial U_i}{\partial X} = 0
\]

(1)
so that $U_i$ depends only on $Y$. The streamwise and transverse momentum balance equations yield the following:

**Region I**

\[
g \beta_1 (T_1 - T_0) - \frac{1}{\rho_1} \frac{\partial P}{\partial X} + \nu_1 \frac{d^2 U_1}{dY^2} = 0
\]

(2)

and $Y$-momentum balance equation can be expressed as follows:

\[
\frac{\partial P}{\partial Y} = 0
\]

(3)

**Region II**

\[
g \beta_2 (T_2 - T_0) - \frac{1}{\rho_2} \frac{\partial P}{\partial X} + \nu_2 \frac{d^2 U_2}{dY^2} = 0
\]

(4)

\[
\frac{\partial P}{\partial Y} = 0
\]

(5)

where $P = p + \rho_0 g x$ (assuming $p_1 = p_2 = p$) is the difference between the pressure and hydrostatic pressure. Because of Eqs. (3) and (5), $p$ depends only on $X$ so that Eqs. (2) and (4) can be rewritten as follows

**Region I**

\[
T_1 - T_0 = \frac{1}{g \beta_1 \rho_1} \frac{dP}{dX} - \nu_1 \frac{d^2 U_1}{dY^2}
\]

(6)

**Region II**

\[
T_2 - T_0 = \frac{1}{g \beta_2 \rho_2} \frac{dP}{dX} - \nu_2 \frac{d^2 U_2}{dY^2}
\]

(7)

From Eqs. (6) and (7), one obtains the following:
Region I

\[
\frac{\partial T_1}{\partial X} = \frac{1}{g \beta_1 \rho_1} \frac{d^2 P}{dX^2} \tag{8}
\]

\[
\frac{\partial T_1}{\partial Y} = -\frac{\nu_1}{g \beta_1} \frac{dU_1}{dY^3} \tag{9}
\]

\[
\frac{\partial^2 T_1}{\partial Y^2} = -\frac{\nu_1}{g \beta_1} \frac{d^2 U_1}{dY^4} \tag{10}
\]

Region II

\[
\frac{\partial T_2}{\partial X} = \frac{1}{g \beta_2 \rho_2} \frac{d^2 P}{dX^2} \tag{11}
\]

\[
\frac{\partial T_2}{\partial Y} = -\frac{\nu_2}{g \beta_2} \frac{d^3 U_2}{dY^3} \tag{12}
\]

\[
\frac{\partial^2 T_2}{\partial Y^2} = -\frac{\nu_2}{g \beta_2} \frac{d^4 U_2}{dY^4} \tag{13}
\]

Both walls of the channel will be assumed to have a negligible thickness and exchange heat by convection with an external fluid. In particular, at \( Y = -h_1/2 \) the external convection coefficient will be considered as uniform with the value \( q_1 \) and the fluid in the region \( 0 \leq Y \leq -h_1/2 \) will be assumed to have a uniform reference temperature \( T_{q_1} \).

At \( Y = h_2/2 \) the external convection coefficient will be considered as uniform with the value \( q_2 \) and the fluid in the region \( 0 \geq Y \geq h_2/2 \) will be supposed to have a uniform reference temperature \( T_{q_2} \geq T_{q_1} \). Therefore, the boundary conditions on the temperature field can be expressed as follows:

\[
-k_1 \frac{\partial T_1}{\partial Y} \bigg|_{Y = -(h_1/2)} = q_1 \left[ T_{q_1} - T_1 \left( X, \frac{-h_1}{2} \right) \right] \tag{14}
\]

\[
-k_2 \frac{\partial T_2}{\partial Y} \bigg|_{Y = h_2/2} = q_2 \left[ T_2 \left( X, \frac{h_2}{2} \right) - T_{q_2} \right] \tag{15}
\]

Because of Eqs. (9) and (12), Eqs. (14) and (15) can be rewritten as follows:

\[
\frac{d^3 U_1}{dY^3} \bigg|_{Y = -(h_1/2)} = \frac{g \beta_1 q_1}{k_1 \nu_1} \left[ T_{q_1} - T_1 \left( X, \frac{-h_1}{2} \right) \right] \tag{16}
\]

\[
\frac{d^3 U_2}{dY^3} \bigg|_{Y = h_1/2} = \frac{g \beta_2 q_2}{k_2 \nu_2} \left[ T_2 \left( X, \frac{h_2}{2} \right) - T_{q_2} \right] \tag{17}
\]

Because of Eqs. (6) and (7), there exists a constant \( A \) such that

\[
\frac{dP}{dX} = A \tag{18}
\]

For the problem under examination, the energy balance equation in the presence of viscous dissipation can be written as follows:

Region I

\[
\frac{d^2 T_1}{dY^2} = -\frac{\nu_1}{\alpha_1 c_p} \left( \frac{dU_1}{dY} \right)^2 \tag{19}
\]

Region II

\[
\frac{d^2 T_2}{dY^2} = -\frac{\nu_2}{\alpha_2 c_p} \left( \frac{dU_2}{dY} \right)^2 \tag{20}
\]
From Eqs. (10) and (19), Eqs. (13) and (20) allow one to obtain differential equations for $U_1$, namely,
Region I
\[
\frac{d^4 U_1}{dY^4} = \frac{g\beta_1}{\alpha_1 c_p} \left( \frac{dU_1}{dY} \right)^2
\]  
(21)
Region II
\[
\frac{d^4 U_2}{dY^4} = \frac{g\beta_2}{\alpha_2 c_p} \left( \frac{dU_2}{dY} \right)^2
\]  
(22)
The boundary conditions on velocity are no-slip conditions and those induced by boundary conditions on temperature. In addition, the continuity of velocity, shear stress, temperature, and heat flux at the interface between the two layers are assumed as follows:
\[
U_1 \left( \frac{-h_1}{2} \right) = U_2 \left( \frac{h_2}{2} \right) = 0
\]  
(23)
together with Eqs. (16) and (17), which due to Eqs. (6) and (7), can be rewritten as follows:
\[
\left. \frac{d^3 U_1}{dY^3} \right|_{Y = -(h_1/2)} - \frac{q_1}{k_1} \left. \frac{d^2 U_1}{dY^2} \right|_{Y = -(h_1/2)} = -\frac{Aq_1}{\mu_1 k_1} - \frac{g\beta_1 q_1}{k_1 \nu_1} [T_0 - T_{q_1}]
\]  
(24)
\[
\left. \frac{d^3 U_2}{dY^3} \right|_{Y = h_2/2} + \frac{q_2}{k_2} \left. \frac{d^2 U_2}{dY^2} \right|_{Y = h_2/2} = \frac{Aq_2}{\mu_2 k_2} - \frac{g\beta_2 q_2}{\nu_2 k_2} [T_{q_2} - T_0]
\]
(25)
Equations (21)–(25) determine the velocity distribution. They can be written in a dimensionless form by means of the following dimensionless parameters:
\[
U_1 = \frac{U_1}{U_0}, \quad U_2 = \frac{U_2}{U_0}, \quad \nu_1 = \frac{Y_1}{D_1}, \quad \nu_2 = \frac{Y_2}{D_2}, \quad \text{Gr} = \frac{g\beta_1 \Delta T D_1^3}{\nu_1^2}, \quad \text{Re} = \frac{U_0^2 D_1}{\nu_1}, \quad \text{Br} = \frac{U_0^2 \Omega_1}{K_1 \Delta T}, \quad \Lambda = \frac{\text{Gr}}{\text{Re}}
\]  
(26)
\[
\theta_1 = \frac{T_1 - T_0}{\Delta T}, \quad \theta_2 = \frac{T_2 - T_0}{\Delta T}, \quad R_T = \frac{T_2 - T_0}{\Delta T}, \quad s = \frac{h_1 D}{k_1}, \quad \text{Bi}_1 = \frac{h_2 D}{k_2}
\]
where $D = 2h$ is the hydraulic diameter. The reference velocity and the reference temperature are given by
\[
U_0^{(1)} = -\frac{AD_1^2}{48\mu_1}, \quad U_0^{(2)} = -\frac{AD_2^2}{48\mu_2}, \quad T_0 = \frac{T_{q_1} + T_{q_2}}{2} + s \left( \frac{1}{\text{Bi}_1} - \frac{1}{\text{Bi}_2} \right) (T_{q_2} - T_{q_1})
\]  
(27)
Moreover, the temperature difference $\Delta T$ is given by $\Delta T = T_{q_2} - T_{q_1}$ if $T_{q_1} < T_{q_2}$. As a consequence, the dimensionless parameter $R_T$ can only take the values 0 or 1. More precisely, the temperature difference ratio $R_T$ is equal to 1 for asymmetric heating, i.e., $T_{q_1} < T_{q_2}$, while $R_T = 0$ for symmetric heating, i.e., $T_{q_1} = T_{q_2}$, respectively. Equation (18) implies that $\Delta$ can be either positive or negative. If $A < 0$, then $U_0^{(1)}$, $\text{Re}$ and $\Lambda$ are negative, i.e., the flow is downward. On the other hand, if $A > 0$, the flow is upward, so that $U_0^{(1)}$, $\text{Re}$ and $\Lambda$ are positive. Using Eqs. (26) and (27), Eqs. (21)–(25) become
Region I
\[
\frac{d^4 u_1}{dy^4} = \Lambda \text{Br} \left( \frac{du_1}{dy} \right)^2
\]  
(28)
Region II

\[ \frac{d^4 u_2}{dy^4} = \Lambda Br bh^4 knm \left( \frac{du_2}{dy} \right)^2 \]  

(29)

The boundary and interface conditions become

\[ u_1 = 0 \quad \text{at} \quad y = -\frac{1}{4}; \quad u_2 = 0 \quad \text{at} \quad y = \frac{1}{4} \]

\[ \frac{d^2 u_1}{dy^2} - \frac{1}{B_i_1} \frac{d^3 u_1}{dy^3} = -48 + R_{T S} \Lambda \left( 1 + \frac{4}{B_i_1} \right) \quad \text{at} \quad y = -\frac{1}{4} \]

\[ \frac{d^2 u_2}{dy^2} + \frac{1}{B_i_2} \frac{d^3 u_2}{dy^3} = -48 - \Lambda R_{T S} \Theta \left( 1 + \frac{4}{B_i_2} \right) \quad \text{at} \quad y = \frac{1}{4} \]

\[ u_1 = mh^2 u_2; \quad \frac{du_1}{dy} = \frac{1}{h} \frac{du_2}{dy}; \quad \frac{d^2 u_1}{dy^2} = \frac{1}{nb} \left[ \frac{d^2 u_2}{dy^2} + 48 (1 - nb) \right]; \quad \frac{d^3 u_1}{dy^3} = \frac{1}{nbh} \frac{d^3 u_2}{dy^3} \quad \text{at} \quad y = 0 \]  

(30)

2.1 Basic Idea of Differential Transform Method

The basic definitions and fundamental operations of differential transform are defined in Zhou (1986). The differential transform of the function \( u(y) \) is defined as the following

\[ U(j) = \frac{1}{j!} \left[ \frac{d^j u(y)}{dy^j} \right]_{y=0} \]  

(31)

where \( u(y) \) is the original function and \( U(k) \) is the transformed function.

The differential inverse transform of \( U(k) \) is defined as follows:

\[ u(y) = \sum_{j=0}^{\infty} U(j) y^j \]  

(32)

In real applications, the function \( u(y) \) is considered as a finite series. Therefore, Eq. (32) can be written as follows:

\[ u(y) = \sum_{j=0}^{n} U(j) y^j \]  

(33)

and, therefore, \( u(y) = \sum_{j=n+1}^{\infty} U(j) y^j \) is neglected as it is small. Usually, the values of \( n \) are decided by a convergence of the series coefficients.

In order to assess the accuracy of DTM, the solutions obtained from DTM are compared to perturbation method solutions. The fundamental mathematical operations performed by the DTM are listed in Table 1.

3. SOLUTIONS

3.1 Separated Effects of Buoyancy Forces and Viscous Dissipation

The solution of Eqs. (28) and (29) using boundary and interface conditions as in Eq. (30) in the absence of the viscous dissipation term \( (Br = 0) \) is given by

Region I

\[ u_1 = \frac{d_1 y^3}{6} + \frac{d_2 y^2}{2} + d_3 y + d_4 \]  

(34)
TABLE 1: Operations for the one-dimensional differential transform method

<table>
<thead>
<tr>
<th>Original function</th>
<th>Transformed function</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y(x) = g(x) \pm h(x) )</td>
<td>( Y(j) = G(j) \pm H(j) )</td>
</tr>
<tr>
<td>( y(x) = \alpha g(x) )</td>
<td>( Y(j) = \alpha G(j) )</td>
</tr>
<tr>
<td>( y(x) = \frac{dg(x)}{dx} )</td>
<td>( Y(j) = (j + 1)G(j + 1) )</td>
</tr>
<tr>
<td>( y(x) = \frac{d^2g(x)}{dx^2} )</td>
<td>( Y(j) = (j + 1)(j + 2)G(j + 2) )</td>
</tr>
<tr>
<td>( y(x) = g(x)h(x) )</td>
<td>( Y(j) = \sum_{l=0}^{k} G(l)H(j - l) )</td>
</tr>
<tr>
<td>( y(x) = x^m )</td>
<td>( Y(j) = \delta(j - m) = \begin{cases} \ 1 , &amp; \text{if } j = m \ 0 , &amp; \text{if } j \neq m \end{cases} )</td>
</tr>
</tbody>
</table>

Region II

\[
u_2 = \frac{d_3 y^3}{6} + \frac{d_6 y^2}{2} + d_7 y + d_8 \tag{35}\]

Using Eq. (30) in Eqs. (6) and (7), the energy balance equations become

Region I

\[
\theta_1 = -\frac{1}{\Lambda} \left( 48 + \frac{d^2 u_1}{dy^2} \right) \tag{36}\]

Region II

\[
\theta_2 = -\frac{1}{\Lambda \beta n} \left( 48 + \frac{d^2 u_2}{dy^2} \right) \tag{37}\]

Using the expressions obtained in Eqs. (34) and (35), the energy balance equations, (36) and (37), become

Region I

\[
\theta_1 = -\frac{1}{\Lambda} \left( 48 + d_1 y + d_2 \right) \tag{38}\]

Region II

\[
\theta_2 = -\frac{1}{\Lambda \beta n} \left( 48 + d_3 y + d_6 \right) \tag{39}\]

The solution of Eqs. (28) and (29) can be obtained when buoyancy forces are negligible (\( \Lambda = 0 \)) and viscous dissipation is dominating (\( \beta \neq 0 \)), so that purely forced convection occurs. For this case [solutions of Eqs. (28) and (29)], using the boundary and interface conditions given by (30), the velocities are given by

Region I

\[
u_1 = \frac{B_1 y^3}{6} + \frac{B_2 y^2}{2} + B_3 y + B_4 \tag{40}\]

Region II

\[
u_2 = \frac{B_5 y^3}{6} + \frac{B_6 y^2}{2} + B_7 y + B_8 \tag{41}\]

The energy balance equations, (19) and (20), in nondimensional form can also be written as follows:

Region I

\[
\frac{d^2 \theta_1}{dy^2} = -\beta r \left( \frac{du_1}{dy} \right)^2 \tag{42}\]

Region II

\[
\frac{d^2 \theta_2}{dy^2} = -\beta r k h^4 \left( \frac{du_2}{dy} \right)^2 \tag{43}\]
The boundary and interface conditions for temperature are

\[
\frac{d\theta_1}{dy} = B_i \left[ \theta_1 + \frac{R_T s}{2} \left( 1 + \frac{4}{B_i} \right) \right] \quad \text{at} \quad y = -\frac{1}{4}
\]

\[
\frac{d\theta_2}{dy} = B_i \left[ -\theta_2 + \frac{R_T s}{2} \left( 1 + \frac{4}{B_i} \right) \right] \quad \text{at} \quad y = \frac{1}{4}
\]

\[
\theta_1 = \theta_2 \quad \text{at} \quad y = 0; \quad \frac{d\theta_1}{dy} = \frac{1}{kh} \frac{d\theta_2}{dy} \quad \text{at} \quad y = 0
\]  

Using Eqs. (40) and (41), solving Eqs. (42) and (43), we obtain

Region I

\[
\theta_1 = g_1 y^6 + g_2 y^5 + g_3 y^4 + g_4 y^3 + g_5 y^2 + c_1 y + c_2
\]

Region II

\[
\theta_2 = g_6 y^6 + g_7 y^5 + g_8 y^4 + g_9 y^3 + g_{10} y^2 + c_3 y + c_4
\]

### 3.2 Combined Effects of Buoyancy Forces and Viscous Dissipation

In this section, both buoyancy forces and viscous dissipation are considered as non-negligible. We solve Eqs. (28) and (29) using the perturbation method with a dimensionless parameter \( \varepsilon \) defined as follows:

\[
\varepsilon = \Delta Br
\]

and does not depend on the reference temperature difference \( \Delta T \). To this end, the solutions are assumed in the form

\[
u (y) = u_0 (y) + \varepsilon u_1 (y) + \varepsilon^2 u_2 (y) + \ldots = \sum_{n=0}^{\infty} \varepsilon^n u_n (y)
\]

Substituting Eq. (48) into Eq. (28) and (29) and equating the coefficients of like powers of \( \varepsilon \) to 0, we obtain the zeroth- and first-order equations as follows:

Region I

Zeroth-order equations

\[
\frac{d^4 u_{10}}{dy^4} = 0
\]

First-order equations

\[
\frac{d^4 u_{11}}{dy^4} = \left( \frac{du_{10}}{dy} \right)^2
\]

Region II

Zeroth-order equations

\[
\frac{d^4 u_{20}}{dy^4} = 0
\]

First-order equations

\[
\frac{d^4 u_{21}}{dy^4} = m n b k h^4 \left( \frac{du_{20}}{dy} \right)^2
\]

The corresponding boundary and interface conditions given by Eq. (30) for the zeroth and first order reduces to

Zeroth-order equation

\[
u_{10} = 0 \quad \text{at} \quad y = -\frac{1}{4}, \quad u_{20} = 0 \quad \text{at} \quad y = \frac{1}{4}
\]
First-order equations

\[
\frac{d^2 u_{10}}{dy^2} - \frac{1}{B_1} \frac{d^3 u_{10}}{dy^3} = -48 + \frac{R_T \Lambda}{2} \left( 1 + \frac{4}{B_1} \right) \quad \text{at} \quad y = -\frac{1}{4}
\]

\[
\frac{d^2 u_{20}}{dy^2} + \frac{1}{B_2} \frac{d^3 u_{20}}{dy^3} = -48 - \frac{R_T \text{sh} \alpha}{2} \left( 1 + \frac{4}{B_2} \right) \quad \text{at} \quad y = \frac{1}{4}
\]

\[
u_1 = \mu h^2 u_{20} \quad \text{at} \quad y = 0, \quad \frac{du_{10}}{dy} = \mu \frac{du_{20}}{dy}
\]

\[
\frac{d^2 u_{10}}{dy^2} = \frac{1}{n_b} \left( \frac{d^2 u_{20}}{dy^2} + 48 (1 - n_b) \right) \quad \text{at} \quad y = 0
\]

\[
\frac{d^3 u_{10}}{dy^3} = \frac{1}{n_b h} \frac{d^3 u_{20}}{dy^3} \quad \text{at} \quad y = 0
\] (53)

First-order equations

\[
u_{11} = 0 \quad \text{at} \quad y = -\frac{1}{4}, \quad \nu_{21} = 0 \quad \text{at} \quad y = \frac{1}{4}
\]

\[
\frac{d^2 u_{11}}{dy^2} - \frac{1}{B_1} \frac{d^3 u_{11}}{dy^3} = 0 \quad \text{at} \quad y = -\frac{1}{4}
\]

\[
\frac{d^2 u_{21}}{dy^2} + \frac{1}{B_2} \frac{d^3 u_{21}}{dy^3} = 0 \quad \text{at} \quad y = \frac{1}{4}
\]

\[
u_{11} = \mu h^2 u_{21}, \quad \frac{du_{11}}{dy} = \mu \frac{du_{21}}{dy}
\]

\[
\frac{d^2 u_{11}}{dy^2} = \frac{1}{n_b} \left( \frac{d^2 u_{21}}{dy^2} + 48 (1 - n_b) \right) \quad \text{at} \quad y = 0
\] (54)

\[
\frac{d^3 u_{11}}{dy^3} = \frac{1}{n_b h} \frac{d^3 u_{21}}{dy^3} \quad \text{at} \quad y = 0
\]

Solutions of zeroth-order Eqs. (49) and (51) using boundary and interface conditions (53) are

\[
u_{10} = \frac{a_1 y^3}{6} + \frac{a_2 y^2}{2} + a_3 y + a_4
\] (55)

\[
u_{20} = \frac{a_5 y^3}{6} + \frac{a_6 y^2}{2} + a_7 y + a_8
\] (56)

Solutions of first-order Eqs. (50) and (52), using boundary and interface conditions (54) are

\[
u_{11} = f_1 y^8 + f_2 y^7 + f_3 y^6 + f_4 y^5 + f_5 y^4 + \frac{e_{11} y^3}{6} + \frac{e_{21} y^2}{2} + e_3 y + e_4
\] (57)

\[
u_{21} = f_6 y^8 + f_7 y^7 + f_8 y^6 + f_9 y^5 + f_{10} y^4 + \frac{e_{11} y^3}{6} + \frac{e_{6} y^2}{2} + e_7 y + e_8
\] (58)

Using velocities given by relations (55)–(58), the expressions for energy balance equations, (36) and (37), become Region I

\[
\theta_1 = -\frac{1}{\Lambda} \left[ 48 + d_1 y + d_2 + \varepsilon \left( 56 f_1 y^6 + 42 f_2 y^5 + 30 f_3 y^4 + 20 f_4 y^3 + 12 f_5 y^2 + e_{11} y + e_2 \right) \right]
\] (59)

Region II

\[
\theta_2 = -\frac{1}{\Lambda n} \left[ 48 + d_3 y + d_6 + \varepsilon \left( 56 f_6 y^6 + 42 f_7 y^5 + 30 f_8 y^4 + 20 f_9 y^3 + 12 f_{10} y^2 + e_{11} y + e_6 \right) \right]
\] (60)
3.3 Solutions using DTM

Now, we apply the differential transformation method into Eqs. (28) and (29). Taking the differential transform of Eqs. (28) and (29) with respect to \( r \), according to Table 1, gives

\[
U(r + 4) = \frac{1}{(r + 1)(r + 2)(r + 3)(r + 4)} \times \left( \Lambda Br \sum_{s=0}^{r} (r - s + 1)(s + 1) U(r - s + 1) U(s + 1) \right)
\]

(61)

\[
V(r + 4) = \frac{1}{(r + 1)(r + 2)(r + 3)(r + 4)} \times \left( \Lambda Br A_1 \sum_{s=0}^{r} (r - s + 1)(s + 1) V(r - s + 1) V(s + 1) \right)
\]

(62)

where \( U(r) \) and \( V(r) \) are transformed functions of \( u_1(y) \) and \( u_2(y) \) respectively.

The differential transform of the initial conditions are as follows:

\[
U(0) = c_1, \quad U(1) = c_2, \quad U(2) = c_3, \quad U(3) = c_4
\]

\[
V(0) = \frac{c_1}{mh^2}, \quad V(1) = \frac{c_2}{h}, \quad V(2) = c_3 n b + 48(n b - 1), \quad V(3) = c_4 A_2,
\]

(63)

Using the conditions as given in Eq. (63), one can evaluate the unknowns \( c_1, c_2, c_3, \) and \( c_4 \). By using the DTM and the transformed boundary conditions, the above equations finally lead to the solution of a system of algebraic equations.

A Nusselt number can be defined at each boundary, as follows:

\[
Nu_1 = \frac{2(h_1 + h_2)}{R_T \{ T_2 (h_2/2) - T_1 (-h_1/2) \}} \frac{dT_1}{dY} \bigg|_{Y = -(h_1/2)}
\]

\[
Nu_2 = \frac{2(h_1 + h_2)}{R_T \{ T_2 (h_2/2) - T_1 (-h_1/2) \}} \frac{dT_2}{dY} \bigg|_{Y = h_2/2}
\]

(64)

By employing Eq. (26), Eq. (64) can be written as follows

\[
Nu_1 = \frac{(1 + h)}{R_T [\theta_2 (1/4) - \theta_1 (-1/4) + (1 - R_T) \theta_1 (-1/4)]} \frac{d\theta_1}{dy} \bigg|_{y = -(1/4)}
\]

\[
Nu_2 = \frac{1 + 1/h}{R_T [\theta_2 (1/4) - \theta_1 (-1/4) + (1 - R_T) \theta_1 (-1/4)]} \frac{d\theta_2}{dy} \bigg|_{y = 1/4}
\]

(65)

The constants that appeared in all the above equations are not presented to save space.

4. RESULTS AND DISCUSSION

Analytical and semi-analytical methods, such as the regular perturbation method (PM) and differential transform method (DTM) for the steady mixed convection of two immiscible viscous fluids in a vertical channel, are analyzed with boundary conditions of the third kind. The solutions are obtained using the regular PM with the product of \( \Lambda (Gr/Re) \), and Brinkman number \( Br \) as the perturbation parameter valid for small values of the perturbation parameter. The restriction on the perturbation parameter to be small is relaxed by finding the solutions for the basic equations using DTM, which is a semi-analytical method. The flow field in the case of asymmetric heating \( (R_T = 1) \) and symmetric heating \( (R_T = 0) \) are obtained and depicted in Figs. 2–10.

Tables 2–4 present the velocity and temperature solutions obtained by PM and DTM for symmetric and asymmetric wall heating conditions, varying the perturbation parameter \( \epsilon \) for equal and unequal Biot numbers. In Table 2,
FIG. 2: Velocity profiles for different values of $\Lambda$

FIG. 3: (a) Temperature profiles for different values of $Br$ for equal Biot numbers; (b) Temperature profiles for different values of $Br$ for unequal Biot numbers

FIG. 4: (a) Velocity profiles for different values of $\varepsilon$; (b) Temperature profile for different values of $\varepsilon$
Mixed Convective Heat Transfer of Immiscible Fluids

FIG. 5: (a) Velocity profile for different values of \( m \); (b) Temperature profile for different values of \( m \)

FIG. 6: (a) Velocity profiles for different values of \( h \); (b) Temperature profiles for different values of \( h \)

FIG. 7: (a) Velocity profiles for different values of \( k \); (b) Temperature profiles for different values of \( k \)
FIG. 8: (a) Velocity profile for different values of $\varepsilon$ and $\Lambda$; (b) Temperature profiles for different values of $\varepsilon$ and $\Lambda$

FIG. 9: (a) Velocity profile for different values of $\varepsilon$ for equal Biot numbers; (b) Temperature profile for different values of $\varepsilon$ for equal Biot numbers

FIG. 10: (a) Velocity profile for different values of $\varepsilon$ for unequal Biot numbers; (b) Temperature profile for different values of $\varepsilon$ for unequal Biot numbers
TABLE 2: Values of velocity for $\Lambda = 500$ and $R_T = 1$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\varepsilon = 0$, $B_{i_1} = B_{i_2} = 10$</th>
<th>$\varepsilon = 2$, $B_{i_1} = B_{i_2} = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PM</td>
<td>DTM</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.250$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$-0.150$</td>
<td>0.245710</td>
<td>0.245710</td>
</tr>
<tr>
<td>$-0.050$</td>
<td>1.082860</td>
<td>1.082860</td>
</tr>
<tr>
<td>$0.000$</td>
<td>1.500000</td>
<td>1.500000</td>
</tr>
<tr>
<td>$0.050$</td>
<td>1.797140</td>
<td>1.797140</td>
</tr>
<tr>
<td>$0.150$</td>
<td>1.674290</td>
<td>1.674290</td>
</tr>
<tr>
<td>$0.250$</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

TABLE 3: Values of velocity for $\Lambda = 500$ and $R_T = 1$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\varepsilon = 0$, $B_{i_1} = B_{i_2} = 10$</th>
<th>$\varepsilon = 2$, $B_{i_1} = B_{i_2} = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PM</td>
<td>DTM</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.250$</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>$-0.150$</td>
<td>0.647500</td>
<td>0.647500</td>
</tr>
<tr>
<td>$-0.050$</td>
<td>1.283750</td>
<td>1.283750</td>
</tr>
<tr>
<td>$0.000$</td>
<td>1.500000</td>
<td>1.500000</td>
</tr>
<tr>
<td>$0.050$</td>
<td>1.596250</td>
<td>1.596250</td>
</tr>
<tr>
<td>$0.150$</td>
<td>1.272500</td>
<td>1.272500</td>
</tr>
<tr>
<td>$0.250$</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

it is seen that, in the absence of perturbation parameter, the PM and DTM solutions are equal for both the velocity and temperature fields. When the perturbation parameter $\varepsilon$ is increased ($\varepsilon = 2$), it is seen that the PM and DTM solutions do not agree. A similar nature is also observed in Tables 3 and 4 for PM and DTM solutions.
TABLE 4: Values of velocity for $\Lambda = 500$ and $R_T = 0$

<table>
<thead>
<tr>
<th>$y$</th>
<th>$\varepsilon = 0$, $\text{Bi}_1 = \text{Bi}_2 = 10$</th>
<th>$\varepsilon = 2$, $\text{Bi}_1 = \text{Bi}_2 = 10$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PM</td>
<td>DTM</td>
</tr>
<tr>
<td>Velocity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.250$</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
<tr>
<td>$-0.150$</td>
<td>0.960000</td>
<td>0.960000</td>
</tr>
<tr>
<td>$-0.050$</td>
<td>1.440000</td>
<td>1.440000</td>
</tr>
<tr>
<td>0.000</td>
<td>1.500000</td>
<td>1.500000</td>
</tr>
<tr>
<td>0.050</td>
<td>1.440000</td>
<td>1.440000</td>
</tr>
<tr>
<td>0.150</td>
<td>0.960000</td>
<td>0.960000</td>
</tr>
<tr>
<td>0.250</td>
<td>0.000000</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.250$</td>
</tr>
<tr>
<td>$-0.150$</td>
</tr>
<tr>
<td>$-0.050$</td>
</tr>
<tr>
<td>0.000</td>
</tr>
<tr>
<td>0.050</td>
</tr>
<tr>
<td>0.150</td>
</tr>
</tbody>
</table>

Tables 2 and 3 are the solutions of velocity and temperature for asymmetric wall heating conditions for equal and unequal Biot numbers, respectively. Tables 2 and 3 also reveal that the percentage of error is large at the interface for velocity when compared to the error at the boundaries. Furthermore, the percentage of error between the PM and DTM is large for unequal Biot numbers when compared to equal Biot numbers. Table 4 displays the solutions of symmetric wall heating conditions for equal Biot numbers. The percentage of error is less for symmetric wall heating conditions for equal Biot numbers when compared to asymmetric wall heating conditions.

Plots of $u$ versus $y$ for asymmetric heating ($R_T = 1$) with $\Lambda = 0$, 200, 400 is shown in Fig. 2 for equal Biot numbers. It is seen from Fig. 2 that for large values of $\Lambda$ (400), there is flow reversal near the cold wall ($y = -1/4$), which is the similar result obtained by Zanchini (1998) for boundary conditions of the third kind and by Barletta (1998) for isoflux boundary conditions.

Let us now consider the case of negligible buoyancy force with a relevant viscous dissipation, which corresponds to $\Lambda = 0$. Plots of temperature versus $y$ are shown in Figs. 3(a) and 3(b), for some values of $\text{Br}$ for equal and unequal Biot number, respectively. As Brinkman number increases, temperature also increases in both regions for equal and unequal Biot numbers. However, the nature of profiles for equal and unequal Biot numbers is different at the cold plate. This nature is because when $\text{Bi}_1 = 1$, $\text{Bi}_2 = 10$, one obtains $T(-L/2) > T(L/2)$ [following Zanchini (1998)]. A similar result was also obtained by Zanchini and Barletta for $\Lambda = 0$. In Figs. 2 and 3, the ratios of physical parameters are fixed as one so that two fluid models represent that of one fluid model for which we can compare the result with one fluid.

Figures 4(a) and 4(b) show the variation of nondimensional velocity and temperature profile $u$ and $\theta$ for $\Lambda = \pm 500$ (both assisting and opposing flows) for different values of $\varepsilon$. It is observed for upward flow, velocity and temperature at each position are increasing functions of $\varepsilon$. However, the effect of $\varepsilon$ on $u$ is stronger for $\Lambda$, while that on $\theta$ is weaker. It is also seen from Figs. 4(a) and 4(b) that for downward flow at each position $u$ is a decreasing function for $\varepsilon < 0$; whereas, $\theta$ is an increasing function for $\varepsilon < 0$. Furthermore, one can infer from this graph that flow reversal occurs at both the plates for $\Lambda = \pm 500$. This is because enhancement of viscous dissipation results in a higher value of temperature, which in turn enhances the values of buoyancy force. A positive value of $\Lambda$ increases the...
fluid velocity; whereas, negative values of $\Lambda$ decrease the fluid velocity. The effect of $\Lambda$ and $\varepsilon$ show a similar result obtained by Prathap Kumar et al. (2009) for isoflux boundary conditions for immiscible fluids. Figures 4(a) and 4(b) also pictorially depict the difference between PM and DTM solutions. It is noted that the agreement between and PM and DTM is good for small values of perturbation parameter and the difference increases substantially as $\varepsilon$ increases. Furthermore, the effect of $\varepsilon$ for two fluid flows is similar in nature observed by Zanchini (1998) for one fluid flow.

The effect of viscosity ratio $m$ on the flow is shown in Figs. 5(a) and 5(b). As $m$ increases, velocity increases in region I; whereas, it decreases in region II. There is no effect of viscosity ratio on temperature. One can interpret this nature as, when $m = 1$, both fluids have the same viscosity. When $m = 2$, the viscosity of the fluid in region II is twice the viscosity of region I, and when $m = 3$, the viscosity of the fluid in region II is three times the viscosity of region I. It is well known that as the viscosity of the fluid increases, the velocity decreases. Therefore, the velocity of the fluid in region II shows the opposite effect when compared to region I. Furthermore, one can also observe from Fig. 5(a) that, at the interface, the profile for $m = 1$ shows the continuity; whereas for $m = 2$ and 3, the profile at $y = 0$ jumps suddenly, region I to region II. This nature is due to the condition imposed at the interface.

The effect of width ratio $h(h_2/h_1)$ on velocity and temperature fields is shown in Figs. 6(a) and 6(b), respectively. As the width ratio increases, both the velocity and temperature decrease in both regions; that is, the larger the width of the fluid in region I, the lower the velocity and temperature field are. It is observed from Fig. 6 that the ratio of width ratio has a significant effect on flow.

The effect of conductivity ratio $k(k_1/k_2)$ on velocity and temperature is shown in Figs. 7(a) and 7(b), respectively, for equal Biot numbers. As the conductivity ratio $k$ increases, the velocity decreases at any given point; that is, the larger the conductivity of the fluid in region II, the smaller the flow rate is. The flow reversal occurs at the left wall as $k$ increases. The effect of viscosity ratio $m$, width ratio $h$, and conductivity ratio $k$ is similar to the result obtained by Prathap Kumar et al. (2011a) for isoflux boundary condition.

In Figs. 8(a) and 8(b), the dimensionless velocity $u$ and temperature $\theta$ for $\Lambda = \pm 500$ and for different values of $\varepsilon$ for unequal Biot numbers are depicted. It is noted that there is no flow reversal for large values of mixed convection parameter $\Lambda$ when compared to equal Biot numbers for both buoyancy-assisting and buoyancy-opposing flow. Figure 8(b) shows that temperature increases more at the cold wall when compared to the hot wall; that is, temperature increases more at the wall that has the smaller external convection coefficient. A comparison between equal Biot numbers [Figs. 4(a) and 4(b)] and unequal Biot numbers [Figs. 8(a) and 8(b)] shows that the effect of $\Lambda$ on $u$ and $\theta$ becomes stronger if either $B_i$ or $B_2$ becomes smaller.

Figures 9(a), 9(b), 10(a), and 10(b) show the velocity and $\Lambda\theta$ for symmetric wall heating conditions for equal and unequal Biot numbers. It is observed from these figures that both $u$ and $\Lambda\theta$ are increasing functions of $\varepsilon$. Figures 9(a), 9(b), 10(a), and 10(b) also reveal that the temperature profiles are symmetric for equal Biot numbers and is more significant in the case of upward flow than in the case of downward flow. Furthermore, one can also come to the conclusion the effect of $\Lambda$ on $u$ and $\theta$ become stronger if either $B_1$ or $B_2$ becomes smaller for symmetric heating (a similar nature was also observed for asymmetric heating). The effect of $B_1$ and $B_2$ for asymmetric and symmetric heating on the flow show a similar nature to that obtained by Zanchini (1998).

Figures 8(a), 8(b), 9(a), 9(b), 10(a), and 10(b) suggest that the solutions obtained by PM and DTM agree very well for small values of $\varepsilon$, and the difference becomes very large as $\varepsilon$ increases for both assisting and opposing flow.

The rate of heat transfer at both walls for variation of $\varepsilon$ is shown in Fig. 11. The Nusselt numbers at the cold wall is a decreasing function of mixed convection parameter $\Lambda$ for upward flow and increasing function of $\Lambda$ for downward flow. The rate of heat transfer is more for smaller values of $\Lambda$ at the left wall. The Nusselt number is an increasing function of mixed convection parameter $\Lambda$ for downward flow at the hot wall and decreasing function of $\Lambda$ for upward flow. The flow nature of Nusselt number on $\varepsilon$ is a similar result to that obtained by Zanchini (1998).

5. CONCLUSION

The problem of steady laminar mixed convective flow in a vertical channel filled with immiscible viscous fluid was analyzed using boundary conditions of the third kind. The governing equations were solved analytically using PM valid for small values of perturbation parameter and using the DTM valid for all values of perturbation parameter. The numerical values evaluated by the PM and DTM are tabulated and displayed graphically for all the governing
parameters. It was observed that the flow at each position was increasing function of $\varepsilon$ for upward flow and decreasing function of $\varepsilon$ for downward flow. Flow reversal was observed for asymmetric wall heating for equal Biot numbers, and there is no flow reversal for unequal Biot numbers. The viscosity ratio increases the fluid flow in region I and decreases in region II. The width ratio and conductivity ratio reduces the flow in both the regions for equal Biot numbers and asymmetric wall heating conditions. The Nusselt number at the cold wall was an increasing function of $|\varepsilon|$ and decreasing function of $|\varepsilon|$ at the hot wall. Considering equal values for viscosity, width and conductivity for fluids in both the regions, we get back the results of Zanchini (1998) for one fluid model. The percentage of error between PM and DTM was found to be large for large values of perturbation parameter.

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