Radiative non linear heat transfer analysis on wire coating from a bath of third-grade fluid

M.K. Nayak\textsuperscript{a,}\textsuperscript{*}, Sachin Shaw\textsuperscript{b}, Ali J. Chamkha\textsuperscript{c,}\textsuperscript{d}

\textsuperscript{a} Department of Physics, Radhakrishna Institute of Technology and Engineering, BijuPatnaik University of Technology, Odisha, India
\textsuperscript{b} Department of Mathematics and Statistical Sciences, Botswana International University of Science and Technology, Private Bag 16, Palapye, Botswana
\textsuperscript{c} Department of Mechanical Engineering, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia
\textsuperscript{d} Prince Sultan Endowment for Energy and Environment, Prince Mohammad Bin Fahd University, Al-Khobar 31952, Saudi Arabia

\section*{A R T I C L E   I N F O}

\textbf{Keywords:}
Wire coating  
Third-grade fluid  
Heat transfer  
Non-linear thermal radiation  
Joule heating  
Pressure-type die

\section*{A B S T R A C T}

Wire coating as an industrial process coats bare conducting wires for primary insulation so as to accomplish mechanical strength and provide protection for aggressive environments. In the present study, we have investigated and discussed the influence of radiative linear as well as non-linear heat transfer on wire coating with melt polymer as a coating fluid in response to a third-grade fluid model subject to Joule heating. In our analysis, we deal with (i) Reynolds model and (ii) Vogel’s model to implement the temperature-dependent viscosity. The governing equations characterizing the flow and heat transfer are solved numerically by the fourth-order Runge-Kutta method. It is heartening to note that the temperature parameter $\theta_k$ is an indicator of the small/large temperature difference between the surface and the ambient fluid, which has a remarkable effect on the heat transfer characteristics and the temperature distributions in the flow region within the die. It is visualized that an increase in $\theta_k$ and the radiation parameter $R$ decrease the fluid temperature of the coating fluid, thereby enhancing the rate of heat transfer associated with a thinner thermal boundary layer.

\section{1. Introduction}

At the beginning of the 20th century, due to enormous applications in industries and technological processes such as polymeric extrusion, drawing of wires, petroleum drilling, manufacturing of food and papers, etc., many investigators have been motivated in the study of viscoelastic fluid flow and heat transfer in the wire coating process. In fact, the most efficient process used for wire coating is the coaxial extrusion process.

The co-extrusion process is an operation in which either molten polymer is extruded continuously on an axially moving wire or the bare preheated wire is dragged inside a die filled with molten polymer. In this process of coating, the velocity of continuum as well as the melt polymer (third grade fluid) develops high pressure in a specific region. This high pressure generates strong bonding between the melt polymer and the wire and hence provides fast coating. Wire-coating in pressure-type die for Newtonian as well as non-Newtonian fluids were carried out by pioneer researchers, namely Bernhardt [1], Bagley and Storey [2], Han and Rao [3], Carley et al. [4], and Wagner and Mitsoulis [5] in the beginning. The analysis of wire coating for pressure-type die for Newtonian and non-Newtonian fluids is presented in the books of Middleman [6], Tadmor and Gogos [7], and Mckelvey [8].

A series of studies were undertaken by many researchers, notably Fata et al. [9], Hayat et al. [10] and Nayak and Dash [11], where they analyzed wire coating using third grade fluid in various situations. Regarding wire coating, third-grade fluid was considered as it itself a viscoelastic fluid of industrial importance. A third-grade fluid was used because it exhibits features such as shear thickening and shear thinning. In addition, researchers viz. Ali and Javed [12], Ali et al. [13], Javed et al. [14] performed studies investigating wire coating associated with different kinds of fluids such as FENE-P fluid, Giesekus viscoelastic fluid and Phan-Thien-Tanner fluid.

Nayak et al. [15] in their study explored the influence of transverse magnetic field in wire coating using third grade fluid as coating material. This is one of the major 20th century contributions, regarding flow as well as heat transfer of third-grade fluid on wire coating, to the development of a better quality final product (coated wire), due to better controlled rate of cooling. However, they did not investigate the influence of linear as well as non-linear thermal radiation in their study. The objective of the present study is to analyze the influence of linear as well as non-linear thermal radiation in the wire coating process, wherein a coating material is modeled as third grade fluid viz. melt...
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_w$</td>
<td>radius of the wire (m)</td>
</tr>
<tr>
<td>$u_w$</td>
<td>dragging velocity (m/s)</td>
</tr>
<tr>
<td>$\theta_u$</td>
<td>wire temperature (K)</td>
</tr>
<tr>
<td>$L$</td>
<td>length of die (m)</td>
</tr>
<tr>
<td>$R_d$</td>
<td>radius of die (m)</td>
</tr>
<tr>
<td>$q_r$</td>
<td>radiative heat flux (W m⁻²)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>dissipation function (W m⁻²)</td>
</tr>
<tr>
<td>$q$</td>
<td>velocity of fluid (m/s)</td>
</tr>
<tr>
<td>$S$</td>
<td>extra stress tensor</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>fluid temperature (K)</td>
</tr>
<tr>
<td>$B_i$</td>
<td>Brinkman number</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>reference viscosity (N s m⁻²)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density of the fluid (kg m⁻³)</td>
</tr>
<tr>
<td>$\kappa_i, \alpha_i, \beta_i, \sigma_i$</td>
<td>material constants</td>
</tr>
<tr>
<td>$\rho, p, \theta$</td>
<td>substantial derivative</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure (N m⁻²)</td>
</tr>
<tr>
<td>$F$</td>
<td>viscous force per unit volume (N m⁻³)</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity (W m⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure (J kg⁻¹ K⁻¹)</td>
</tr>
<tr>
<td>$\Theta_d$</td>
<td>die temperature (K)</td>
</tr>
<tr>
<td>$\theta_R$</td>
<td>temperature parameter</td>
</tr>
<tr>
<td>$R$</td>
<td>radiation parameter</td>
</tr>
<tr>
<td>$\delta$</td>
<td>wire coating aspect ratio</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>non-Newtonian parameter</td>
</tr>
<tr>
<td>$k_i$</td>
<td>mean absorption coefficient (m⁻¹)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>dynamic viscosity (N s m⁻²)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant (W m⁻² K⁴)</td>
</tr>
<tr>
<td>$m$</td>
<td>Reynolds model viscosity parameter</td>
</tr>
</tbody>
</table>

polymer, and includes constant viscosity as well as temperature dependent viscosity in response to Reynolds and Vogel’s models. In cases where the temperature difference between the continuum and ambient fluid is large, the thermal radiation effect is vital as it alters the structure of the thermal boundary layer and the rate of heat transfer. This implies the relevance of radiative heat transfer in industrial processes including nuclear power plants for power generation, gas turbines, nuclear reactor cooling, satellites, etc. [16–22]. Usually the linearized Rosseland approximation is accomplished by assuming sufficiently small temperature differences between the surface and ambient fluid. However, non-linear Rosseland approximation provides results for both small and large temperature differences between $\theta_u$ (constant wire surface temperature) and $\theta_d$ (constant ambient temperature of die). Introduction of non-linear radiative effects in the energy equation develops a high non-linearity in the governing equations [Cortel[23], Hayat et al. [24]]. Since then, the problem of steady or unsteady flow with linear or non-linear thermal radiation effects has been observed and investigated in [25–30].

The novelty of the present study is to explore the effects of linear and non-linear thermal radiation in wire coating of third grade fluid inspired by temperature dependent viscosity in response to Reynolds and Vogel’s models in presence of Joule heating.

### 2. Formulation of the problem

Consider the boundary layer flow of an incompressible third grade fluid such as molten polymer like polyvinyl chloride (PVC) inside a stationary pressure type die of finite length $L$ having radius $R_d$ and temperature $\theta_d$. Suppose a wire of radius $R_w$ is extruded along the axis of the die with velocity $u_w$ and temperature $\theta_u$ as shown in Fig. 1. Let us make the following assumptions: (1) the flow is steady (2) the polymer melt flows through a suitably long cylindrical die in which a wire moves axially at a constant speed (3) the flow is laminar (4) the velocity in the radial direction is negligibly small compared to that in the axial direction (5) the inertial effect is negligibly small compared to viscous effect that is reasonably due to the very high viscosity of melt polymer (7) excessive wall shear stress is avoided as it may lead to elongation or frequent breakup of the wire when coating operation, and also uneven and rough extruded coating (8) heat conduction in the flow direction is negligibly small compared to that in the radial direction (9) the melt density, specific heat, and thermal conductivity are independent of temperature, whilst the viscosity depends on temperature (10) no-slip boundary conditions are subjected to the moving wire as well as the stationary die wall (11) the gravitational effect is negligible (12) the fluid is acted upon by a constant pressure gradient $\frac{\partial p}{\partial z}$ in the axial direction. The wire and die are concentric and a cylindrical co-ordinate system $(r,z)$ is chosen at the center of the wire to analyze the flow situation where $z$ and $r$-axes are taken along and perpendicular to the direction of flow respectively.

The design of the wire coating die is of primary importance since it greatly affects the quality of the final product. The pressure type die is considered because within this die the melt polymer meets the wire where a complex flow field exists, and its understanding is vital for the better design of dies with optimum performance. Taking into account the above mentioned frame of reference and assumptions, consider the fluid velocity, extra stress tensor and temperature field as

$$V = [0,0,w(r)], S = S(r)$$ and $\Theta = \Theta(r)$ (1)

The equations of continuity, momentum and energy governing the flow of an incompressible fluid are

$$\nabla \cdot V = 0$$

$$\frac{\partial D}{\partial t} = -\nabla p + F + J \times B$$

$$\rho C_p \frac{\partial \Theta}{\partial t} = k \nabla^2 \Theta + \phi - q' + J_h$$

where $F = V \cdot S$ is the viscous force, $\phi = S \cdot VV$ is the viscous dissipation, $q_i$ is the radiative heat flux so that $q_i'$ is the derivative of $q_i$ with respect to $r, J_h$ is the Joule heating term, $\frac{\partial p}{\partial r}$ is the material derivative.

The relevant boundary conditions are:

$$w = u_w, \Theta = \theta_u \text{ at } r = R_w$$

$$w = 0, \Theta = \theta_d \text{ at } r = R_d$$

(5)

For third grade fluid, the extra stress tensor $S$ is defined as

$$S = -pI + \mu A_i + \alpha_1 A_2 + \alpha_2 A_3 + \beta_1 A_1 + \beta_2 (A_1 A_2 + A_1 A_3) + \beta_3 (tr A_i^2) A_i$$

(6)

where $p$ is the pressure, $I$ is the identity tensor, $\mu = \mu(\Theta)$ is the coefficient of viscosity (kg m⁻¹ s⁻¹). Here $\alpha_1$ and $\alpha_2$ are the second order material constants (kg m⁻¹), symbols $\beta_1, \beta_2$, and $\beta_3$ are the third order material constants (kg m⁻¹ s⁻¹) and $tr$ is the trace operator. The quantities $A_i (i = 1, 2, 3)$ are the Rivlin-Ericksen tensors which are defined by

![Fig. 1. Wire coating process in a pressure type die.](image-url)
the recursive relation as follows:
\[ A_n = I A_1 = L^n + L \text{ and } A_n = A_{n-1} L + LA_{n-1} + \frac{DA_{n-1}}{Dt}, \quad n = 2,3, \]
where \( T \) denotes the transpose of the matrix and \( L = \text{grad} V \). Because of interaction of conducting fluid with magnetic field, a body force of retarding nature i.e., \( \nabla \times \mathbf{J} \times \mathbf{B} \) is attained. This drag force acting along the \( z \)-axis is given by
\[ J \times B = (0,0,-\sigma B_0^2 w) \]
where \( B_0 \) is the uniform magnetic field applied along the positive radial direction.

Taking into account Eq. (1), (2) is satisfied indicating that the fluid flow is possible. The non-zero components of extra tensor \( S \) are
\[ S_{rr} = (2\alpha_1 + \alpha_2) \left( \frac{d\phi}{dr} \right)^2, \quad S_{rr} = \alpha_2 \left( \frac{d\phi}{dr} \right)^2, \quad S_{rr} = S_{\theta \theta} \]
\[ = 2(\beta_2 + \beta_3) \left( \frac{d\phi}{dr} \right)^3 + \mu \Omega \left( \frac{d\phi}{dr} \right) \]
Making substitution of Eqs. (7) and (8), equation of balance of momentum (3) becomes
\[ -\frac{\delta p}{\delta r} = \frac{1}{r} \left( 2\alpha_1 + \alpha_2 \right) r \left( \frac{d\phi}{dr} \right)^2 \]
(9)
\[ \frac{\delta p}{\delta \theta} = 0 \]
(10)
\[ \frac{\delta p}{\delta \zeta} = \frac{1}{r} \left( \mu \Omega \left( \frac{d\phi}{dr} \right) + \frac{1}{r} \frac{d}{dr} \left( 2(\beta_2 + \beta_3) r \left( \frac{d\phi}{dr} \right) \right) - \sigma B_0^2 w \right) \]
(11)
Eq. (11) describes the flow due to pressure gradient. As drag of the wire prevails outside the die, the pressure gradient is assumed to be zero i.e., \( \frac{\delta p}{\delta r} = 0 \). So the Eq. (11) takes the form
\[ 2\beta_2 \frac{1}{r} \frac{d}{dr} \left( r \left( \frac{d\phi}{dr} \right) \right) + \frac{1}{r} \frac{d}{dr} \left( \mu \Omega \left( \frac{d\phi}{dr} \right) \right) - \sigma B_0^2 w = 0 \]
(12)
where \( \beta_2 = \beta_3 + \beta_3 \).

The viscous dissipation term
\[ \phi = S : \mathbf{V} \mathbf{V} = \mu \left( \frac{d\phi}{dr} \right)^2 + 2\mu \left( \frac{d\phi}{dr} \right)^4 \]
(13)
Using the Rosseland approximation for thermal radiation [31] the radiative heat flux is modeled as
\[ q_r = \frac{4\sigma_0 \theta^4}{3k_0} \]
(14)
Following Pantokratoras and Fang [32], the Eq. (14) can be written as
\[ q_r = \frac{16\sigma_0 \theta^4 \Omega \frac{d\theta}{dr}}{3k_0} \]
(15)
Using Eqs. (13), (14) and (15), the energy equation (4) reads
\[ k \left[ \frac{d\theta}{dr} \right] + \frac{1}{r} \frac{d}{dr} \left( \mu \left( \frac{d\phi}{dr} \right)^2 + 2\mu \left( \frac{d\phi}{dr} \right)^4 \right) + \frac{16\sigma_0 \theta^4 \Omega \frac{d\theta}{dr}}{3k_0} \]
\[ + \sigma B_0^2 w = 0 \]
(16)
Let us introduce the dimensionless parameters as
\[ r = \frac{r}{R_0}, \quad \Omega = \frac{w}{U_{in}}, \quad \phi = \frac{\Theta - \Theta_0}{\Theta_0 - \Theta_m} \]
(17)
From Eq. (17),
\[ \Theta = \Theta_m [1 + \Theta_0 (\Theta_m - 1)] \]
where \( \Theta_m = \frac{\Theta_m}{\Theta_0} \) is the temperature parameter.

We introduce the non-dimensional constants as
\[ M_0 = \frac{\sigma B_0^2 R_0^2}{\mu_0}, \quad \rho_0 = \frac{u_{in} R_0}{\mu_0 R_{in}}, \quad B_0 = \frac{\mu_0 u_{in}^2}{k \Theta_m}, \quad \delta = \frac{R_d}{R_{in}}, \quad \Gamma = \frac{16\sigma_0 \theta^4 \Omega}{3k_0} \]
(18)
2.1. Constant viscosity

Let \( \mu(\Theta) = \mu_0 \) Using Eqs. (17) and (18) in the Eqs. (12) and (16) and dropping the bar for simplicity we get the non-dimensional momentum, and energy equations along with reduced boundary conditions as

\[
\frac{d^2 w}{dr^2} + \frac{dw}{dr} + 2\beta_0 \left[ 3r \left( \frac{dw}{dr} \right)^2 + \left( \frac{dw}{dr} \right)^3 \right] - M^2 \Phi = 0
\]

(19)

\[w(1) = 1\ and\ w(\delta) = 0\]  

(20)

\[\mu(\Theta) = \mu_0 e^{-\beta_0 \Theta} \]

(23)

where \( m \) is a non-dimensional viscosity parameter associated with Reynolds model.

Using Eqs. (17), (18) and (23) in the Eqs. (12) and (16) and

\[
-(1 + R\beta_0^2)\Phi(1)\]

\[
\begin{array}{cccc}
\text{Br} & M = 0 & M = 0.5 & M = 1 & M = 1.2 \\
0 & -23.72763208 & -23.72763208 & -23.72763208 & -23.72763208 \\
0.8 & -26.12965935 & -26.39125090 & -27.13958248 & -27.5568944 \\
2.4 & -30.92958133 & -31.71319858 & -33.95649625 & -35.26501824 \\
3.2 & -33.32724587 & -34.37172779 & -37.36176272 & -39.02585948 \\
4 & -35.72354804 & -37.02874683 & -40.7658019 & -42.84450407 \\
\end{array}
\]

Table 2

Local Nusselt Number for different M and Br for \( \beta_0 = 0.1, R = 1, \delta = 1 \) (Constant Viscosity model).

\[
-(1 + R\beta_0^2)\Phi(1)
\]

\[
\begin{array}{cccc}
\Theta_R & M = 0 & M = 0.5 & M = 1 & M = 1.2 \\
0 & -23.72763208 & -23.72763208 & -23.72763208 & -23.72763208 \\
0.8 & -26.12965935 & -26.39125090 & -27.13958248 & -27.5568944 \\
2.4 & -30.92958133 & -31.71319858 & -33.95649625 & -35.26501824 \\
3.2 & -33.32724587 & -34.37172779 & -37.36176272 & -39.02585948 \\
4 & -35.72354804 & -37.02874683 & -40.7658019 & -42.84450407 \\
\end{array}
\]

Table 1

Local Nusselt Number for different R and \( \Theta_R \) for \( M = 0.1, \beta_0 = 0.1, R = 1, \delta = 1 \) (Constant Viscosity model).
Fig. 8. Influence of $R$ on velocity for $M = 1, \beta_0 = 0.01, m = 5, \beta_r = 10, \theta_R = 1.1$ (Reynolds model).

Fig. 9. Influence of $\theta_R$ on temperature for $M = 1, \beta_0 = 0.01, m = 5, \beta_r = 10, R = 1$ with $R = 1$ (Reynolds model).

Fig. 10. Influence of $R$ on temperature for $M = 1, \beta_0 = 0.01, m = 5, \beta_r = 10, R = 1$ with $\theta_R = 1.1$ (Reynolds model).

Fig. 11. Influence of $R$ on temperature for $M = 1, \beta_0 = 0.01, m = 5, \beta_r = 10, \theta_R = 1.5$ with $R = 1$ (Reynolds model).

Fig. 12. Influence of $R$ on temperature for $M = 1, \beta_0 = 0.01, m = 5, \beta_r = 10, \theta_R = 1.5$ with $R = 2.5$ (Reynolds model).

Fig. 13. Variation of local Nusselt number with $R$ associated with different $\theta_R$ for $M = 1, \beta_0 = 0.01, \beta_r = 10, m = 5$ (Reynolds model).
dropping the bar for simplicity we get the non-dimensional momentum, and energy equations along with reduced boundary conditions as

\[ e^{-\tilde{\nu}_{0}(\frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr})} + 2\tilde{\nu}_{0} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 = 0 \]  

\[ w(1) = 1 \text{ and } w(\delta) = 0 \]  

\[ \Theta(1) = 0 \text{ and } \Theta(\delta) = 1 \]  

2.2.2. Vogel model

It is a model that accounts for temperature dependent viscosity. In this model, the expression for temperature dependent viscosity is

\[ \mu(\Theta) = \mu_0 e^{-\beta_{\Theta}(\Theta)} \]  

where \( D \) and \( B_1 \) are viscosity parameters affiliated with Vogel’s model and \( \beta_{\Theta} = \mu_0 e^{-\Theta} \). Here it is remarkable to note that the previous authors had considered the first order approximation of the Taylor’s series expansion in (23) and (28). However, we have considered the higher order approximations in (23) and (28) so as to accomplish the characteristic behavior of higher order terms involving the parameters \( \beta_{\Theta}, \mu_0, D, B_1, \Theta_0 \). Using Eqs. (17), (18) and (28) in the Eqs. (12) and (16) and dropping the bar for simplicity we get the non-dimensional momentum, and energy equations along with reduced boundary conditions as

\[ e^{-\tilde{\nu}_{0}(\frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr})} + \frac{2\tilde{\nu}_{0}}{D} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 = 0 \]  

\[ w(1) = 1 \text{ and } w(\delta) = 0 \]  

\[ \Theta(1) = 0 \text{ and } \Theta(\delta) = 1 \]  

\[ \frac{D}{\Theta_{0} + 1} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right) + rB_1 e^{-\frac{D}{\Theta_{0} + 1}} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 + 2\tilde{\nu}_{0} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 \]  

\[ + rB_{1} \tilde{\nu}_{0} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 + 2\tilde{\nu}_{0} \left( \frac{d^2 w}{dr^2} + \frac{dw}{dr} \frac{d\Theta}{dr} \right)^2 = 0 \]  

\[ \Theta(1) = 0 \text{ and } \Theta(\delta) = 1 \]  

The relevant quantity of practical interest here is the local Nusselt number \( N_u \) defined as

\[ N_u = \frac{q_w}{k(\Theta_{0} - \Theta_{0})} \]  

where \( q_w = -k \frac{d\Theta}{dr} = q_w \) is the heat flux at the surface of the wire.

| Table 3 |
| Local Nusselt number for different R and \( \Theta_0 \) for \( \beta_{\Theta} = 0.1, R = 1, m = 10, \Theta_{0} = 1 \) (Reynolds model).
<p>|</p>
<table>
<thead>
<tr>
<th>R</th>
<th>( \Theta_0 = 0.8 )</th>
<th>( \Theta_0 = 1.2 )</th>
<th>( \Theta_0 = 1.5 )</th>
<th>( \Theta_0 = 1.8 )</th>
<th>( \Theta_0 = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2.97290158</td>
<td>-5.5478846</td>
<td>-3.11989444</td>
<td>-2.49660255</td>
<td>-2.28734220</td>
</tr>
<tr>
<td>1</td>
<td>1.37187191</td>
<td>-8.76575604</td>
<td>-9.41229471</td>
<td>-13.59316847</td>
<td>-17.95981467</td>
</tr>
<tr>
<td>2</td>
<td>0.57892917</td>
<td>-11.15661082</td>
<td>-14.53359363</td>
<td>-23.55915406</td>
<td>-32.91105591</td>
</tr>
<tr>
<td>3</td>
<td>-0.03979814</td>
<td>-13.32689463</td>
<td>-19.26358513</td>
<td>-33.25004650</td>
<td>-47.69536281</td>
</tr>
<tr>
<td>4</td>
<td>-0.59176179</td>
<td>-15.40717892</td>
<td>-24.07627070</td>
<td>-42.83179095</td>
<td>-62.41297214</td>
</tr>
<tr>
<td>5</td>
<td>-1.11131430</td>
<td>-17.44197999</td>
<td>-28.73056897</td>
<td>-52.35790225</td>
<td>-79.09717038</td>
</tr>
</tbody>
</table>
The local Nusselt number in non-dimensional form can be obtained as

\[ Nu_r = -(1 + R \Theta R) \Theta'(1) \]  

(34)

where \( \Theta' \) is the differentiation of \( \Theta \) with respect to \( r \).

\[ Nu_r = -(1 + R \Theta R) \Theta'(1) \]  

(34)

where \( \Theta' \) is the differentiation of \( \Theta \) with respect to \( r \).
3. Results and discussion

The influence of radiative linear as well as non-linear heat transfer on wire coating of third grade fluid inspired by temperature dependent viscosity using Reynolds and Vogel's models in presence of Joule heating has been analyzed. The non-linear Rosseland approximation is adopted to develop the coupled non-linear energy equation. The fourth order Runge-Kutta method is employed to obtain the solutions of the modified governing boundary layer equations along with the reduced boundary conditions. The numerical results explored the effects of thermal radiation (linear as well as non-linear) on velocity, temperature and heat transfer of coating fluid in the wire coating process in presence of magnetic field through requisite graphs and hence discussion. In order to ensure the accuracy of our results, the results obtained and presented in the present study are compared quantitatively with the noteworthy researchers viz. Cortel [21], Hayat et al. [22]. This comparison confirms to us that our numerical results are found to be in excellent agreement for the considered values of the parameters and therefore we are confident about the accuracy and generality of our results.

### 3.1. Constant viscosity case

We will now discuss the heat transfer variation due to the influence of linear as well as non-linear thermal radiation in detail, as shown in Figs. 2–6, by saying that temperature of the coating fluid gets reduced with increasing values of temperature parameter \( \Theta_0 \) in presence of low magnetic field strength and moderate viscous heating as is visualized in Fig. 2. However, with a low value of \( \Theta_0 \) (i.e., when the wire temperature is 0.8 times greater than the dye temperature), the temperature of the coating fluid reduces significantly, leading to a better heat transfer rate and hence considerable cooling, yielding better quality of coating. It is convenient to express the variations in fluid temperature in the presence of thermal radiation for different values of temperature parameter as shown in Figs. 3 and 4. The observations based on these figures have revealed that the temperature of coating fluid diminishes with increasing values of thermal radiation parameter \( R \) for fixed value of \( \Theta_0 \) (\( \Theta_0 = 0.01 \text{ and } 1.1 \Delta = 1 \text{ (Vogel's Model)} \)). This is well in agreement with the results reported earlier by Cortel [21] and Hayat [22]. We will now describe why the fluid temperature decreases due to the enhancement of \( R \). Physically this is because of the fact that with the increase in \( R \), the mean absorption coefficient decreases. As a consequence, the rate of radiative heat transfer to the coating fluid gets enhanced. An important point is to be mentioned here that the temperature profiles for \( \Theta_0 = 0.01 \) (for greater wire temperature) exhibits opposite trend against \( \Theta_0 = 1.5 \) (for lower wire temperature). Fig. 5 reveals the variation of heat transfer rate with thermal radiation for different values of \( \Theta_0 \). While Fig. 6 yields the variation of heat transfer rate with \( Br \) for different \( M \). There are some observations suggesting that increasing values of \( \Theta_0 \) enhances the heat transfer rate (absolute value) in response to different values of radiation parameter \( R \) (Fig. 5). Further, as the magnetic field strength goes on increasing, the rate of heat transfer (absolute value) produced follows the same trend (i.e., diverging trend) that acts as a source of temperature in association with different values of \( Br \). An important result regarding heat transfer rate is reported in Tables 1 and 2. Nevertheless, the value of temperature parameter, the Nusselt number and hence heat transfer rate higher increases due to an increase in \( R \) at fixed higher value of \( \Theta_0 \) indicating more cooling and hence better cooling. However, for lower value of \( \Theta_0 \) (\( \Theta_0 = 0.8 \)) a reverse trend is attained. Furthermore, at fixed non-zero \( R \), the absolute value of Nusselt number and hence heat transfer rate gets enhanced due to increase in temperature parameter contributing to more cooling (Table 1). It is too significant to be given attention in the context that with increasing value of \( Br \) (i.e., with more viscous heating compared to heat conducted), the rate of heat transfer gets reduced for any strength of magnetic field (even in the absence or presence of magnetic field) [15] as illustrated in Table 2. It is from Table 2 that at fixed value of \( Br \), an increase in magnetic field strength reduces the rate of heat transfer [15]. Another interesting point to be noted is that the heat transfer rate shows no deviation in the absence of \( Br \), irrespective of magnitude (absence/presence) of magnetic field strength.

#### 3.2. Variable viscosity

##### 3.2.1. Reynolds model

We will have to keep on observing the variation of fluid velocity for different values of temperature parameter \( \Theta_0 \) and radiation parameter \( R \) as shown in Figs. 7 and 8. We can now say that the velocity of the coating fluid gets accelerated near the surface of the wire (\( r < 1.3 \)) with an increase in \( \Theta_0 \) whereas a reverse trend is attained towards the die surface as is seen in Fig. 7. Further, Fig. 8 conveys to us the fluid velocity behavior under the influence of thermal radiation. It is understood from this figure that the velocity of coating fluid increases though insignificantly due to increasing values of radiation parameter \( R \) in association with lower magnetic field strength and moderate viscous heating. However, it is hard to track the velocity behavior under the

### Table 5

Local Nusselt number for different \( R \) and \( \Theta_0 \) for \( M = 1, \beta_e = 0.01, B_r = 1, \Omega = 0.01 \text{ (Vogel's Model)} \).

<table>
<thead>
<tr>
<th>( R )</th>
<th>( \Theta_0 = 0.8 )</th>
<th>( \Theta_0 = 1.2 )</th>
<th>( \Theta_0 = 1.5 )</th>
<th>( \Theta_0 = 1.8 )</th>
<th>( \Theta_0 = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1.32649919</td>
<td>-1.55879443</td>
<td>-1.46914638</td>
<td>-1.47172894</td>
<td>-1.46592264</td>
</tr>
<tr>
<td>1</td>
<td>-1.52929952</td>
<td>-3.93705688</td>
<td>-7.75274721</td>
<td>-15.36278233</td>
<td>-23.82256985</td>
</tr>
<tr>
<td>2</td>
<td>-1.85171139</td>
<td>-6.27478409</td>
<td>-14.41607947</td>
<td>-31.55268167</td>
<td>-51.30872064</td>
</tr>
<tr>
<td>4</td>
<td>-2.5642243</td>
<td>-10.92742207</td>
<td>-27.94757929</td>
<td>-65.27981066</td>
<td>-109.30275191</td>
</tr>
<tr>
<td>5</td>
<td>-2.93173535</td>
<td>-13.25002807</td>
<td>-34.79207664</td>
<td>-82.3664313</td>
<td>-138.80180879</td>
</tr>
</tbody>
</table>
influence of R near the surface of the wire.

We must now explore the effects of fluid temperature for different values of \( \Theta_0 \) and R, as are observed in Figs. 9–12. It may be stated here that the fluid temperature decreases with increasing values of \( \Theta_0 \), however, that shows an opposite trend for lower value of \( \Theta_0 \) (\( \Theta_0 = 0.8 \)) (Fig. 9). It is preferentially observed that the decreasing trend of temperature of coating fluid is more significant in the Reynolds model case compared to that in the constant viscosity case. Figs. 10–12 present a result showing that increase in radiation parameter R decreases the temperature of the coating fluid \([16,17]\). It should be mentioned that the decreasing trend appears to be prominent for \( \Theta_0 \) = 1.1 compared to \( \Theta_0 = 1.5 \) and \( \Theta_0 = 2.5 \). It is clear that the heat transfer rate is comparatively more in the former (\( \Theta_0 = 1.1 \)) than the latter (\( \Theta_0 = 1.5 \) and \( \Theta_0 = 2.5 \)).

It must be noted from Figs. 13 and 14 that the heat transfer rates show similar behavior under the influence of the temperature parameter and the magnetic field strength in response to different values of \( R \) and \( Br \), as in the constant viscosity case. However, this trend (enhancement of absolute value of the heat transfer rate) continues faster in this case compared to the constant viscosity case.

Irrespective of \( \Theta_0 \) (except at \( \Theta_0 = 0.8 \)), an increase in R enhances the heat transfer rate (absolute value) while at fixed moderate R, an increase in \( \Theta_0 \) shows the same trend on rate of heat transfer in presence of magnetic field strength and viscous heating (Table 3). It is evident from Table 4 that the Nusselt number (absolute value) and hence the heat transfer undergoes a similar fashion to that of the constant viscosity case but with a greater magnitude.

3.2.2. Vogel’s model

What has been discussed so far indicates that the fluid velocity, somehow, varies in response to linear as well as non-linear thermal radiation in the Reynolds model of variable viscosity. Figs. 15 and 16 indicate that the variation of coating fluid is insignificant (there is no trace of any change in fluid velocity) for increasing values of the radiation parameter as well as the temperature ratio parameter.

Confirmed evidence for the influence of non-linear thermal radiation on temperature profiles can be seen in Fig. 17. Here we explore whether the fluid temperature decreases due to an increase in temperature ratio parameter \( \Theta_0 \). An important point to be kept in mind in this regard is that the temperature profiles for \( \Theta_0 = 0.8 \) exhibits a diametrically opposite response compared to that for \( \Theta_0 = 1.1,1.2,1.5,2 \).

Figs. 18–20 represent the picture of well-designed temperature profiles for different values of radiation parameter R. It is found from this representation that fluid temperature decreases in response to rising values of R affiliated with Vogel’s viscosity model \([16,17]\). However, it must be noted that the decreasing trend of fluid temperature is prominent and symmetric (at \( r = 1.5 \)) for \( \Theta_0 = 1.1 \) compared to \( \Theta_0 = 1.5 \) and \( \Theta_0 = 2.5 \) as is illustrated in Figs. 18–20. Figs. 21 and 22 demonstrate the influence of temperature parameter \( \Theta_0 \) and magnetic field strength on heat transfer rate respectively. It is again interesting to note from Figs. 21 and 22 that heat transfer rates exhibit the exact behavior under the influence of temperature parameter and magnetic field strength in response to different values of \( R \) and \( Br \) as in the constant viscosity case, as well as the Reynolds model case. However, this discussion confirms to us that this trend (enhancement of absolute value of heat transfer rate) continues at the fastest rate among all three cases considered in the present study.

Let us turn our attention to the data incorporated in Table 5, where we observed similar behavior of heat transfer rate (absolute value) in interaction with \( \Theta_0 \) and R influenced by magnetic field strength and viscous heating.

In sum, we can now see how different the temperature behavior of coating fluid is in Vogel’s model compared to that in Reynolds model as well as the constant viscosity model. There are some observations indicating that the decreasing trend of fluid temperature is in the fashion \( \Theta_{Vogel} < \Theta_{Reynolds} < \Theta_{Constantviscosity} \) in the flow domain. This suggests that more heat transfer rate and hence more cooling is achieved in Vogel’s model rather in Reynolds model and the constant viscosity case.

4. Conclusion

The effects of linear as well as non-linear thermal radiation associated with pertinent governing parameters on the flow of a third-grade fluid in wire coating under the influence of Joule heating have been discussed in some detail with the help of graphs and tables. In the present study, the fourth order Runge-Kutta method is employed to solve the non-dimensional momentum and energy equations in association with reduced boundary conditions. The numerical results achieved in the present study agree quantitatively with the previously published results. The major contributions of the present study are as follows:

1. The velocity of the coating fluid gets accelerated near the surface of the wire (\( \kappa < 1.3 \)) with an increase in \( \Theta_0 \) whereas the reverse trend is attained towards the die surface.
2. The temperature behavior (a decreasing trend) of the coating fluid in Vogel’s model compared to that in the Reynolds model as well as the constant viscosity model is in the fashion \( \Theta_{Vogel} < \Theta_{Reynolds} < \Theta_{Constantviscosity} \) in the entire flow domain.
3. The heat transfer rate exhibits a similar behavior (increasing trend of rate of heat transfer) under the influence of the temperature parameter and the magnetic field strength in response to different values of \( R \) and \( Br \) in the constant viscosity, Reynolds model and Vogel’s model cases. However, this trend continues to be at the fastest rate in Vogel’s model case compared to the constant viscosity and Reynolds model case.

References


