Mixed convection in superposed nanofluid and porous layers in square enclosure with inner rotating cylinder

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\textbf{Abstract}

In this study, numerical simulation of mixed convection in a partitioned square cavity having CuO-Water nanofluid and superposed porous medium with an adiabatic rotating cylinder is performed. The bottom horizontal wall of the cavity is heated and the top horizontal wall is cooled while the remaining vertical walls are insulated. An adiabatic rotating cylinder is located at the center of the square cavity. Galerkin weighted residual finite element method is utilized to solve the governing equations of the system. The influence of Rayleigh number (between $10^3$ and $10^6$), angular rotational velocity of the cylinder (between 0 and 6000), solid volume fraction of the nanoparticle (between 0 % and 0.05 %), Darcy number (between $10^{-5}$ and $10^{-7}$) and three different vertical locations of the cylinder on the fluid flow and heat transfer characteristics are numerically investigated in detail for three different cylinder sizes. It is observed that the averaged heat transfer enhances as the value of Rayleigh number, angular rotational speed of the cylinder, nanoparticle volume fraction and Darcy number increase. The effect of the angular rotational speed of the cylinder on the averaged heat transfer enhancement is more pronounced for large cylinder size and 432.55% of averaged enhancement is achieved for $\Omega = 6000$ compared to motionless cylinder case at $\Omega = 0$ using cylinder sizes of $R=0.3$. The averaged heat transfer enhancement almost linearly with nanoparticle volume fraction for different cylinder sizes and adding solid nanoparticles to the base fluid is favorable for the locations when high values of local Nusselt number is observed. Local and averaged Nusselt number enhance as the cylinder approaches to the upper wall of the cavity.

1. Introduction

The insistent demands of convective heat transfer enhancement/control inside enclosures have produced various techniques such as; reforming the enclosure geometry, heating by discrete heaters with different positions, use of fins or baffles, inserting fixed or rotating objects, driving one or more of the enclosure walls, etc. Many engineering applications can be simplified to a cavity problem such as cooling of electronic equipments, thermal storage unit design, solar collectors, lubrication of journal bearings [1]. A vast amount of study related to natural convection in various geometries under different thermal boundary conditions can be found [2–4].

In order to control the natural convection, an adiabatic, stationary or rotated, bodies are inserted inside cavities. Many industrial applications use a thermally conductive rotating cylinders immersed in a fluid confined in an enclosures, such applications are: nuclear reactor fuel rods, drilling of oil wells, rotating-tube heat exchangers, rotating shafts and steel suspension bridge cables. Stationary cylinders inserted in square enclosures filled with fluids can be found in [5–8], vented enclosure in [9], lid driven enclosure in [10–13], and in a triangular enclosure [14,15]. However, rotating inner cylinders topic have received less investigations, this may refer to the complexity associated with numerical treatments in connecting curved rotating boundary with Cartesian coordinates. Lewis [16] obtained an early numerical solution for relatively low Rayleigh numbers of a circular cylinder rotating in a viscous fluid filled a square enclosure. Ghaddar and Thiele [17] reported the enhancement of heat transfer due to the rotation of an inner cylinder. Many other studies have demonstrated the gained heat transfer due to the presence of a rotating cylinder [18–25]. In the present decade, the problem has received noticeable attention as an attempt to enrich the state of the art of this field of investigation and hence improving the thermal performance of such practical applications. Misirlioglu [26] studied the cylinder rotation in a square enclosure saturated by a porous medium. His results showed that
Nomenclature

\begin{itemize}
\item \textbf{Cp} specific heat at constant pressure (J kg\(^{-1}\) K\(^{-1}\))
\item \textbf{Da} Darcy number, K/L\(^2\)
\item \textbf{g} gravitational acceleration (m s\(^{-2}\))
\item \textbf{Gr} Grashof number, \(Gr=\frac{g\beta_f(T_h-T_c)L^3}{\nu_f^2}\)
\item \textbf{L} side length of the enclosure (m)
\item \textbf{h} local heat transfer coefficient (W m\(^{-2}\) K\(^{-1}\))
\item \textbf{k} thermal conductivity (W m\(^{-1}\) K\(^{-1}\))
\item \textbf{K} permeability of the porous medium (m\(^2\))
\item \textbf{Nu} Nusselt number, \(Nu=hL/k_f\)
\item \textbf{p, P} pressure (N/m\(^2\)), dimensionless pressure
\item \textbf{Pr} Prandtl number, \(Pr=\frac{\nu_f}{\alpha_f}\)
\item \textbf{r} radius of the inner cylinder (m)
\item \textbf{R} dimensionless radius of the inner cylinder \(R=r/L\)
\item \textbf{Ra} Rayleigh number, \(Ra=\frac{g\beta(T_h-T_c)L}{\nu\alpha_f}\)
\item \textbf{Re} Reynolds number, \(Re=\frac{\omega rL}{\nu_f}\)
\item \textbf{Ri} Richardson number, \(Ri=\frac{Gr}{Re^2}\)
\item \textbf{T} temperature (K)
\item \textbf{u, v} velocity components (m s\(^{-1}\))
\item \textbf{U, V} dimensionless velocities components, \(U=uL/\alpha_f, V=vL/\alpha_f\)
\item \textbf{x, y} dimensional coordinates (m)
\item \textbf{X, Y} dimensionless coordinates, \(X=x/H, Y=y/H\)
\end{itemize}

Greek symbols

\begin{itemize}
\item \(\alpha\) thermal diffusivity (m\(^2\) s\(^{-1}\))
\item \(\beta\) thermal expansion coefficient (K\(^{-1}\))
\item \(\epsilon\) porosity of the porous layer
\item \(\nu\) kinematic viscosity (m\(^2\) s\(^{-1}\))
\item \(\theta\) dimensionless temperature, \(\theta=\frac{(T-T_c)}{(T_h-T_c)}\)
\item \(\phi\) nanoparticle volume fraction
\item \(\Psi\) dimensionless stream function
\item \(\omega\) rotational speed of the inner cylinder (rad s\(^{-1}\))
\item \(\Omega\) dimensionless rotational speed of the inner cylinder, \(\Omega=\frac{\omega rL}{\alpha_f}\)
\item \(\rho\) density (kg m\(^{-3}\))
\item \(\mu\) dynamic viscosity (N s m\(^{-2}\))
\end{itemize}

Subscripts

\begin{itemize}
\item \(c\) cold
\item \(f\) fluid
\item \(h\) hot
\item \(nf\) nanofluid
\item \(np\) nanoparticle
\item \(o\) cavity center, standard conditions
\item \(p\) porous
\end{itemize}

Table 1

Thermophysical properties of water and CuO nanoparticles.

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho) (kg/m(^3))</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>(c_p) (J/kg K)</td>
<td>4179</td>
<td>540</td>
</tr>
<tr>
<td>(k) (W/m K)</td>
<td>0.6</td>
<td>18</td>
</tr>
<tr>
<td>(\beta) (1/K)</td>
<td>(21\times10^{-5})</td>
<td>(0.85\times10^{-5})</td>
</tr>
<tr>
<td>(d_p) (nm)</td>
<td>0.384</td>
<td>29</td>
</tr>
</tbody>
</table>

Table 2

Grid independence test result for cylinder size of \(R=0.2\) (Ra = 10\(^8\), \(\Omega=6000, \phi = 0.05, Da = 10^{-3}\)).

<table>
<thead>
<tr>
<th>Grid name</th>
<th># Elements</th>
<th>Nu(_{nm})</th>
</tr>
</thead>
<tbody>
<tr>
<td>G1</td>
<td>2648</td>
<td>21.211</td>
</tr>
<tr>
<td>G2</td>
<td>10,592</td>
<td>18.836</td>
</tr>
<tr>
<td>G3</td>
<td>42,368</td>
<td>18.552</td>
</tr>
<tr>
<td>G4</td>
<td>62,848</td>
<td>18.515</td>
</tr>
</tbody>
</table>

Fig. 1. Schematic diagram of the physical model with boundary conditions and grid distribution.

Fig. 2. Code verification: local Nusselt number distribution computed in [13] and computed with present code for various Richardson numbers.
rotation is more effective in the forced convection regime than in mixed and natural convection regimes, and at high spin velocities the heat transfer is almost independent from the permeability of the porous medium. Hills [27] studied the two-roll mill composed of two disjoint, independently rotating circular cylinders in a rectangular box. Costa and Raimundo [28] used heatlines to visualize the heat transfer process in a differentially heated square enclosure with conductive rotating cylinder. Hussain and Hussain [29] included the effect of vertical position of rotating cylinder. Liao and Lin [30] investigated the heated rotating cylinder in a square cavity using an immersed-boundary method. Chatterjee et al. [31] conducted a numerical simulation for hydro-magnetic mixed convection transport in a square cavity subjected to an externally applied magnetic field and filled with electrically conducting fluid in the presence of thermally conductive rotating cylinder. Cavities of different shapes with inner isothermal rotating cylinder were studied by Shih and Cheng [32]. Their results revealed that the triangular cavity had the greatest ability to dissipate thermal energy while the circular cavity had the worst performance.

The geometry of a rotating cylinder inside an enclosure had been studied extensively in many early works. These early works (experimental and theoretical) had focused on examining the separation phenomena and the viscous cellular behavior [16]. These studies had used four or two rolls immersed in a bath of the test fluid [33].

![Image of flow patterns](image)

**Table 3**

Comparison results of averaged Nusselt number at the top wall of the lid driven cavity.

<table>
<thead>
<tr>
<th>Re = 400</th>
<th>Iwatsu et al. [64]</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gr = 100</td>
<td>3.84</td>
<td>3.81</td>
</tr>
<tr>
<td>Gr = 10^4</td>
<td>3.62</td>
<td>3.63</td>
</tr>
<tr>
<td>Gr = 10^6</td>
<td>1.22</td>
<td>1.26</td>
</tr>
</tbody>
</table>

![Fig. 3. Flow patterns for various Rayleigh numbers and cylinder sizes (Ω = 3000, φ = 0.02, Da = 5 × 10^5).](image)
Experimentally, these devices are used to study the micro-structural dynamics of complex fluids such as polymeric solutions, colloids, bubbles and drops [34].

However, the demands of heat transfer enhancement have resulted in many improvement techniques. One of these techniques is enclosure induced by gravity or vibration [35]. Besides, to the complexity of numerical and experimental efforts of this technique, device may damage when it used in engineering applications. As such, an alternative technique of inserting a rotary circular cylinder inside enclosures had been proposed and employed in clear fluid [36] and in porous media [26].

During the past and the present decades, a promising approach of heat transfer enhancement by dispersing nanometer-sized, metallic or non-metallic, particles of superior physical properties in liquids has received a progressively attention due to the favorable improvement of heat transfer. This topic is addressed as "Nanofluid", it began since the pioneer work of Choi and Eastman [37], which then supported by Pak and Cho [38]. The use of nanofluid led to increase the efficiency and compactness of many thermal devices like enhancement of heat dissipation from electronic components, improving the efficiency of thermal management system, industrial cooling systems, heating or cooling buildings, cooling nuclear system, energy storage system (solar absorber) and improvements of lubricants. However, the most important feature of nanofluids in industry is the energy saving and emissions reduction, hence, the environmental pollutants could be reduced [39]. However, since the pioneered works of [37] and [38] many studies, theoretical and experimental, were published dealing with both the development of the models of the thermophysical properties of nanofluid and with heat transfer enhancement aspects associated with the use of nanofluids. For instant, the works of Maxwell [40], Yu and Choi [41], Ho and Li [42], Chon and Kim [43] concentrated on the finding of the effective thermal conductivity models of the nanofluid, and
In porous media saturated enclosures, the nanofluids are also utilized as in Sun and Pop [52], Sheremet and Pop [53] and Chamkha and Ismael [54]. Nanofluids are also tested in composite enclosures partially composed of porous layer superimposed with a fluid layer as in Chamkha and Ismael [54] and Ismael and Chamkha [55]. A few published works concerned with natural convection in nanofluid confined in enclosure with stationary cylinder insert. Parvin et al. [56] investigated numerically the natural convection heat transfer from a heated cylinder contained in a square enclosure filled with water-Cu nanofluid. They presented a reduction of natural convection at larger cylinder diameter. Sheikholeslami et al. [57] studied the natural convection in a concentric annulus, filled with water-Cu nanofluid, between a cold outer square and heated inner circular cylinders in the presence of static radial magnetic field is investigated numerically using the lattice Boltzmann method. Sheikholeslami et al. [58] used the Control Volume-based Finite Element Method (CVFEM) in analyzing the effects of natural convection heat transfer in a cold outer circular enclosure filled with Cu-water nanofluid and containing a hot inner elliptic circular cylinder. Generally, the main findings of these three works is that the Cu-nanoparticles enhance the natural convection for relatively low Rayleigh numbers.

More recent works those concentrated with mixed convection in enclosures filled with nanofluids and containing a rotating cylinders. A critical recent survey made by the authors of the present paper have found very few papers regarding this subject. Roslan et al. [59] studied the effect of a rotating cylinder on heat transfer in a square enclosure filled with nanofluids using COMSOL. They showed that the thermal and fluid dynamics are highly influenced by the size of the cylinder, rotational speed and direction, nanoparticle concentration and its thermal conductivity values. Matin and Pop [60] presented a numerical study for mixed convection flow and heat transfer of Al2O3-Water nanofluid in an eccentric annulus with rotation on the inner cylinder. They reported that the average Nusselt number almost decreases with increase of Reynolds number. The effect of the position of the rotating cylinder (eccentricity) had a conditional behavior with both Reynolds and Rayleigh number. Wang et al. [61] investigated numerically the laminar mixed convective flow and heat transfer around a horizontal rotational cylinder in a concentric triangular enclosure filled with ethylene glycol-silicon carbide (SiC-EG) using CFD code (Fluent). Their main findings that the direction of inner cylinder rotation vigorous the heat transfer when it is as same as the flow of induced by natural convection while a deterioration occurs for opposite direction. They addressed also that the better heat transfer performance can be obtained with opposite rotation of smaller cylinder radius, and for motionless cylinder, an intermediate radius of cylinder must be chosen.

Some recent industrial and environmental applications such as; fibrous thermal insulation, solidification, fuel cell, cooling of nuclear fuel debris, solar collectors, and many others, have resulted in focusing many researches to simulate these applications. The problem can be simulated as a superposed fluid and porous layers confined in an enclosure (partly layered enclosure). To the best authors knowledge, this problem has never been simultaneously enhanced using inner rotated cylinder and nanofluids. This, therefore, guided us to investigate the aspects of the thermal and flow fields of the following problem. A superposed nanofluid and porous layers occupied in a square enclosure with inner rotated adiabatic cylinder. The porous layer is saturated with same nanofluid and the axis of cylinder rotation can be varied vertically. Adiabatic vertical walls, isothermally heated bottom wall, and isothermally cooled vertical wall are imposed as outer boundary conditions of the enclosure.

It is believed that the study of such problem will highly contribute in adding new aspects to the performance of the aforementioned industrial and environmental applications. The results of the present study can be utilized in several technological applications including controlling of natural convection heat transfer in solidification process, rotating tube-heat exchangers and oil extraction technology. In the oil
drilling operations, the flow behavior between the annular space of two circular cylinders which are stationary or rotating can be studied by extending it to include various effects such as the eccentricity and the fluid type. The results of this numerical study can also be used to optimize devices which are used for analyzing the micro-structural dynamics of complex fluids such as polymeric solutions, colloids, bubbles and drops [34]. Convection heat transfer can also be controlled by using an inner rotating cylinder for a variety of applications such as fibrous thermal insulation, solidification, fuel cell, cooling of nuclear fuel debris, solar collectors which can be modeled as superposed fluid and porous layers confined in an enclosure.

2. Mathematical formulation

Fig. 1 (a) shows schematic diagram of the problem under consideration. A superposed nanofluid and porous layers occupied in a square enclosure with side length \( L \) which contains a solid adiabatic inner rotated cylinder with radius \( r \) and angular speed \( \omega \). The cavity is partially divided into two layers, a nanofluid saturated porous layer occupying its lower half and a nanofluid occupying the remainder of the cavity. Nanofluid properties are assumed to be constant with temperature except the thermal conductivity and the density where it obeys Boussinesq approximation. The porous layer is saturated with the same nanofluid. The nanofluid is CuO-Water. Thermo-physical properties of water and CuO nanoparticles at the reference temperature are presented in Table 1. The axis of cylinder rotation \((\chi_1, \chi_2)\) is varied vertically \((\chi_2 = L/2)\) such that the cylinder is always in contact with both layers. The vertical walls are thermally insulated (adiabatic). The top wall is kept isothermally at low temperature \( T_c \), while the bottom wall is kept isothermally at higher temperature \( T_h \). All outer boundaries are impermeable except the interface between the porous and nanofluid layers is permeable. The pores within the porous layer are assumed to
be uniform and undeformable and in thermal equilibrium with nanofluid saturated it. Incompressible and laminar flow with ignored energy dissipation and radiation are assumed. The flow within the porous medium is adopted using Darcy-Brinkman model.

The dimensionless form of the governing equations is:

\[
\frac{\partial U'}{\partial X} + \frac{\partial V'}{\partial Y} = 0
\]

\[
U_{f,p}' \frac{\partial U_{f,p}'}{\partial X} + V_{f,p}' \frac{\partial U_{f,p}'}{\partial Y} = -\frac{\rho_e}{\rho_f} \nabla^2 U_{f,p}' + \frac{\rho_e}{\rho_f} \mu \left( \frac{\partial^2 U_{f,p}'}{\partial X^2} + \frac{\partial^2 U_{f,p}'}{\partial Y^2} \right) - \frac{\rho_e}{\rho_f} \mu_e \frac{\partial^2 U_{f,p}'}{\partial X^2} Pr
\]

\[
U_{f,p}' \frac{\partial U_{f,p}'}{\partial X} + V_{f,p}' \frac{\partial U_{f,p}'}{\partial Y} = \alpha_\theta \left( \frac{\partial^2 U_{f,p}'}{\partial X^2} + \frac{\partial^2 U_{f,p}'}{\partial Y^2} \right)
\]

where \( \epsilon \) is the porosity of the porous medium (equals to 0.398), \( \alpha \) is the thermal diffusivity, and \( U, V \) are the dimensionless velocity components in \( X, Y \) Cartesian coordinates. The above system can be written for fluid domain (subscript \( f \)) and for porous domain (subscript \( p \)) according to the following:

Fig. 8. Isotherms for various angular rotational velocities of the cylinder and cylinder sizes (\( \text{Ra} = 10^3, \phi = 0.02, \text{Da} = 5 \times 10^5 \)).
\[\epsilon = \begin{cases} 
1 & \text{for nanofluid layer} \\
\frac{1}{\epsilon} & \text{for porous layer} 
\end{cases}, \quad \delta = \begin{cases} 
0 & \text{for nanofluid layer} \\
1 & \text{for porous layer} 
\end{cases}, \]
\[\alpha_{nf} = \alpha_{f} \quad \text{for nanofluid layer} \quad \alpha_{p} = \alpha_{f} + \frac{k_{nf}}{(\rho C_{p})_{f}} \quad \alpha_{nf} = (1 - \epsilon)k_{s} + \epsilon k_{nf}, \]

where \(k_{s}\) is the thermal conductivity of solid matrix forming the porous layer (= 0.845 W/m K), \(k_{nf}\) is the thermal conductivity of nanofluid, and \((\rho C_{p})_{f}\) is the heat capacity of nanofluid.

The Non-dimensional quantities are:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha_{f}}, \quad V = \frac{vL}{\alpha_{f}}, \quad \theta = \frac{T - T_{1}}{(T_{3} - T_{1})},
\]
\[P = \frac{(\rho + \rho_{g} \gamma L^{2})}{\rho_{f} \alpha_{f}^{2}} R = \frac{R}{L} \quad (Xo, Yo) = \left(\frac{x}{L}, \frac{y}{L}\right)
\]
\[Pr = \frac{\nu}{\alpha_{f}} \quad \text{is the Prandtl number,} \quad Da = \frac{K}{L^{2}} \quad \text{is the Darcy number,}
\]
\[Ra = \frac{g\beta_{T}T_{3}L^{3}}{\nu \alpha_{f}} \quad \text{is the Rayleigh number and} \quad \Omega = \frac{w_{f}^{2}}{\rho_{f} \alpha_{f}} \quad \text{is the non-dimensional angular rotational speed of the cylinder.} \quad \text{Richardson number is important to be imposed in mixed convection study} \quad Ri = \frac{Ra}{\rho_{f} \Omega R^{2}} \quad \text{and Reynolds number is defined as} \quad Re = \frac{\nu R}{\nu}, \quad \text{hence,} \quad Ri = \frac{Ra}{Re^{2} Pr^{2}}
\]

2.1. Boundary conditions

1. On the left and right walls, \(\frac{\partial U}{\partial x} = 0, \quad U_{nf,p} = V_{nf,p} = 0\)
2. On the bottom wall, \(\theta_{x} = 1, \quad U_{y} = V_{y} = 0\)
3. On the top wall, \(\theta_{y} = 0, \quad U_{x} = V_{x} = 0\)
4. Along the fluid-porous interface (continuity condition),
   i. \[\frac{\partial \theta_{f}}{\partial y} = \frac{k_{nf}}{k_{f}} \frac{\partial \theta_{p}}{\partial y}, \quad \theta_{nf} = \theta_{p}\]
   within the fluid layer \quad within the porous layer
   ii. \[U_{nf} = U_{f}, \quad V_{nf} = V_{f} \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)_{nf} = \left(\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X}\right)_{p}
\]
5. On the solid surface of the rotating cylinder,
   i. \[U_{nf,p} = U_{o} \frac{(Y - Yo)}{R} = \Omega (Y - Yo) \quad \text{and} \quad V_{nf,p} = U_{o} \frac{(Xo - X)}{R} = \Omega (Xo - X).
\]
   ii. \[\frac{\partial \theta_{nf}}{\partial Y} = 0 \quad \text{and} \quad \frac{\partial \theta_{p}}{\partial Y} = 0
\]

within the fluid layer \quad within the porous layer

where \((Xo, Yo), \quad R\) are the cylinder position and its radius respectively, \(\Omega\) is the dimensionless rotational speed of the cylinder, and \(n\) is a vector normal to the cylinder. Once the dimensionless velocities \((U \quad \text{and} \quad V)\) and the dimensionless temperature \(\theta\) found, the interested quantities of the local \(Nu_{x}\) and average \(\overline{Nu_{m}}\) Nusselt numbers along the bottom hot wall can be calculated:
\[
Nu_{x} = -\frac{k_{nf} \partial \theta_{nf}}{k_{f} \partial Y} \bigg|_{Y=0}
\]
\[ \int N_u Y \]

Streamline contours are a comprehensive appliance to describe the flow fields within the cavity. They are calculated via the equation:

\[
\frac{\partial \psi_{nf,p}}{\partial x} + \frac{\partial \psi_{nf,p}}{\partial y} = \left( \frac{\partial U_{nf,p}}{\partial y} - \frac{\partial V_{nf,p}}{\partial x} \right)
\]

(7)

The boundary conditions for this equation are: \( \psi_{nf,p} = 0 \) on the four cavity walls. On the cylinder surface, the boundary condition can be set as:

\[
U_{\Omega} = -\frac{\partial \psi_{nf,p}}{\partial y}, \quad V_{\Omega} = -\frac{\partial \psi_{nf,p}}{\partial x}.
\]

2.2. Relations of nano fluid properties

Empirical correlation finding the role of temperature and particle size for nano fluid (Al2O3) thermal conductivity enhancement was given [43]:

\[
\frac{k_{nf}}{k_f} = 1 + 64.7d_{nf}^{0.746} \left( \frac{d_f}{d_{nf}} \right)^{0.369} \left( \frac{k_f}{k_{nf}} \right)^{0.7476} P_f^{0.9055} R_e_f^{1.2321} = \frac{\mu_f}{\partial \beta_f}.
\]

(8)

where \( k_o, d_{np}, d_f \) and \( f \) are the Boltzmann constant, diameter of the solid nanoparticles, size of the liquid molecules and mean path of the fluid particles, respectively. Although, this model was established for Al2O3, the validity of this model for CuO-water nano fluid was proved [62]. Thermal diffusivity and specific heat of the nano fluid can be given as:

\[
a_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}}
\]

(9)

\[
(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_{np}
\]

(10)

Dynamic viscosity of the nanofluid is defined by using the Brinkman model [45]:

\[
\mu_f = \frac{\rho_f}{(1 - \phi)^{0.5}}
\]

(11)

Recently, a modified expression for thermal expansion is given in Ref. [63] which contains the natural dependence of the density on temperature:

\[
(\rho \beta)_{nf} = (1 - \phi)(\rho \beta)_f + \phi(\rho \beta)_{np} - \phi(1 - \phi)(\rho_{np} - \rho_f) \beta_f - \beta_f
\]

(12)

3. Solution methodology and code validation

Eqs. (1)–(4) along with the boundary conditions are solved by using the finite element method. The finite element formulation is obtained by establishing the weak form of the governing equations by using the Galerkin procedure. The computational domain are divided into non-overlapping regions within each of the flow variables are approximated by using the interpolation functions. The field variables, velocity components, pressure and temperature within the computational domain are discretized by using Lagrange finite elements of different orders. When the relative error for each of the variables satisfy the following convergence criteria,
appropriate grid distribution with accurate results and minimal computational time grid independence study was performed. The averaged Nusselt number results for various grid sizes are shown in Table 2 for cylinder size of $R = 0.2$ ($Ra = 10^5$, $\Omega = 6000$, $\phi = 0.05$, $Da = 10^{-3}$). Grid type G3 with 42,368 number of elements was chosen in the subsequent computations for cylinder size of $R = 0.2$. The grid distribution of the computational domain is depicted in Fig. 1 (b). The present code is validated against the numerical results of [13] and [64]. Fig. 2 shows the comparison results of local Nusselt number distributions for a lid driven cavity for various Richardson numbers computed in [13]. Table 3 presents the comparison results of averaged Nusselt number at the top wall of the lid driven cavity. The results shown in Fig. 2 and Table 3 provide sufficient confidence for the present code.

4. Results and discussion

Numerical simulations are performed for a range of Rayleigh numbers (between $10^3$ and $10^6$), angular rotational speed of the cylinder (between 0 and 6000), nanoparticle volume fraction (between 0 and 0.05 %), Darcy number (between $10^{-5}$ and $10^{-3}$), cylinder sizes (between $R = 0.1$ and $R = 0.3$) and different vertical locations of the cylinder. The results are demonstrated in terms of streamlines, isotherms and Nusselt number distribution plots for various values of these parameters. The fluid is CuO-Water nanofluid.

Fig. 3 and 4 show the influence of varying the Rayleigh number on the streamlines and the isotherms for various cylinder sizes ($\Omega = 3000$, $\phi = 0.02$, $Da = 5 \times 10^{-5}$). The cylinder rotates in the counter-clockwise direction. Above the cylinder for cylinder size $R = 0.1$, a recirculating flow pattern is observed at $Ra = 10^3$. The flow velocity increases with $Ra$ number and this effect is more pronounced for the upper half of the cavity for cylinder size of $R = 0.1$. As the cylinder size increases more flow due to cylinder rotation interacts with the hot rising fluid. There is negligible change in the flow topology as the Rayleigh number enhances for cylinder size of $R = 0.3$. Isotherms are also affected by the variation of the Rayleigh number and cylinder size as shown in Fig. 4. As the Rayleigh number increases, isotherms are more (less) clustered towards the right (left) end of the bottom hot wall indicating enhanced heat transfer in these locations for cylinder size of $R = 0.1$. As the cylinder size increases, the location of the steep temperature gradient changes from right toward mid and left end of the bottom wall. The convection in the upper half of the cavity is more apparent with increasing Rayleigh numbers for small cylinder size as it can be seen from the plume like structure of the thermal patterns in the upper half. Nusselt number distributions along the bottom wall for various cylinder sizes and different values of Rayleigh numbers are shown in Figs. 5 and 6. Maximum heat transfer is seen adjacent to the mid of the bottom wall for cylinder sizes of $R = 0.2$ and $R = 0.3$, but for cylinder size of $R = 0.1$, peak value of the Nusselt number is seen at the right part of the bottom wall. Local heat transfer generally enhances for the large portion of the bottom wall for various cylinder sizes and the averaged heat transfer increases as the Rayleigh number increase. The discrepancy between the local and averaged heat transfer at $Ra = 10^5$ and $Ra = 10^6$ is high for small cylinder size.

Figs. 7 and 8 demonstrate the effects of varying angular rotational speed of the cylinder ($\Omega$) on the flow and thermal patterns for various cylinder sizes ($Ra = 10^3$, $\phi = 0.02$, $Da = 5 \times 10^{-5}$). The case $\Omega = 0$ corresponds to a stationary cylinder which is shown in Fig. 7(a), (d) and (g) and the cylinder rotates in the counter-clockwise direction. Two main recirculating regions are seen above the cylinder in the upper half of the cavity and some weak recirculation patterns in the lower half of the cavity for cylinder size of $R = 0.1$. The flow is symmetric with respect to the vertical axis passing through the center of the circular cylinder. As the size of the cylinder increases, the gap between the walls of the cavity and cylinder surface decreases and above the cylinder the recirculating vortices distorts in size and shape and below the cylinder two weak recirculating zones are seen (Fig. 7(d) and (g)). The upper two recirculating regions coalesce into a big one and the flow is
accelerated in the vicinity of the cylinder when the cylinder rotates in the counter clockwise direction for various cylinder sizes. The multi-cellular structure in the lower half of the cavity disappears with cylinder rotation and big cylinder sizes (Fig. 7(e), (f), (h) and (i)). More flow due to cylinder rotation interacts with the hot rising fluid from the bottom wall and the discrepancy between the flow topology is not noticeable for the lower half of the cavity with cylinder rotation for cylinder sizes of $R = 0.2$ and $R = 0.3$. Conduction is the dominant heat transfer mode for motionless cylinder case as it can be seen from the horizontally oriented isotherms in the lower half of the cavity and heat transfer along the hot bottom wall deteriorates as the size of the cylinder increases due to the blockage of the hot rising fluid from the bottom wall. Steep temperature gradient along the bottom wall is observed with cylinder rotation and this is more effective as the cylinder size increases. The effect of the angular rotational speed of the cylinder on the isotherms are more pronounced as the cylinder size increases since convection due to cylinder rotation assists natural convection of hot rising fluid along the bottom wall of the cavity especially in the vicinity of the left portion of the wall. The isotherms become less clustered indicating less heat transfer process for the right part of the bottom wall which is a small portion of the bottom wall as the angular rotational speed of the cylinder increases for cylinder sizes of $R = 0.2$ and $R = 0.3$. Figs. 9 and 10 show the local and averaged Nusselt number plots for various angular rotational velocities of the cylinder and cylinder sizes. The rotation of the cylinder acts in a way to increase the local Nusselt number for the large portion of the bottom wall for various cylinder sizes. As the angular rotational velocity of the cylinder increases the averaged heat transfer enhances and this effect is more pronounced for large cylinder size. Averaged Nusselt number enhancements of 42.56% and 432.55% are achieved for $\Omega = 6000$ compared to motionless cylinder case at $\Omega = 0$ for cylinder sizes of $R = 0.1$ and $R = 0.3$.

Streamlines and isotherms for various solid volume fractions of the nanoparticles $\phi$ are depicted in Fig. 11 for fixed values of $Ra = 10^5$, $\Omega = 3000$, $\phi = 0.02$. Small variation in the flow and thermal patterns are seen especially for the upper half of the cavity as the value of $\phi$ changes. Along the bottom wall, the shape of the isotherms does not change which indicates that the location of the maximum heat transfer does not change as the value of the solid volume fraction of the nanoparticle enhances. The increase in the convection due to an increase in nanoparticle volume fraction is also evident from the local Nusselt number plots (Fig. 12). This is due to the increased thermal conductivity of the nanofluid which results in better thermal transport within the cavity. The effect of adding nanoparticles to the base fluid at $\phi = 0$ is favorable in the mid of the cavity for cylinder sizes of $R = 0.2$ and $R = 0.3$ when high local heat transfer values are observed. The averaged heat transfer enhancements almost linearly with $\phi$ for various cylinder sizes as shown in Fig. 13. Averaged heat transfer enhancements of 7.35% and 7.45% are obtained for solid volume fraction of 0.05 compared to base fluid at $\phi = 0$ for cylinder sizes of $R = 0.3$ and $R = 0.1$, respectively.

Effects of Darcy numbers on the streamlines and isotherms are demonstrated in Fig. 14 for fixed values of $Ra = 10^5$, $\Omega = 3000$, $\phi = 0.02$ for cylinder size of $R = 0.2$. As the value of the Darcy number enhances, the strength of the convection inside the cavity increases and the cavity is filled with a single recirculation zone adjacent to the adiabatic cylinder and small vortices adjacent to the top corner are observed. The increase in the convection due to an increase in permeability is also evident from the isotherms and Nusselt number plots as shown in Figs. 14(d)-(f) and 15. Steep temperature gradients along the bottom wall toward the left end and plume like isotherms adjacent to the cylinder are seen as the value of the Darcy number enhances due to enhanced convection with an increase in permeability. The portion toward the right end of the bottom wall with inefficient heat transfer

![Flow and thermal patterns for various Darcy numbers and cylinder sizes (Ra = 10^5, $\Omega = 3000$, $\phi = 0.02$).](image-url)
process with increasing Darcy number diminishes in size as the cylinder size increases. The averaged Nusselt number and the discrepancy between the averaged heat transfer corresponding to different cylinder sizes increase as the value of Darcy number enhances as it is shown in Fig. 16. Averaged Nusselt number enhancements of 66.76% and 162.82% are achieved for $D_a = 10^{-2}$ compared to $D_a = 10^{-5}$ using cylinder sizes of $R = 0.1$ and $R = 0.3$, respectively due to the increased permeability of the porous medium.

Finally, the influence of vertical locations of the cylinder on the flow and thermal patterns are demonstrated in Fig. 17 for cylinder size of $R = 0.2$ and for fixed values of $Ra = 10^5$, $\Omega = 2500$, $\phi = 0.02$, $Da = 10^{-4}$. When the cylinder approaches to the bottom wall (L1), multi-cellular structure and a small recirculating vortex adjacent to the right top corner are seen within the cavity (Fig. 17(a)). As the cylinder approaches to the upper wall of the cavity (L3), fluid motion of the hot rising fluid toward the upper wall is less affected by the presence of the cylinder and the fluid flow is accelerated above the cylinder. Isotherms become more clustered along the bottom wall toward the left end for vertical location L3 and heat transfer process is inefficient for cylinder vertical location L1. The local enhancement of the heat transfer can be seen in Fig. 18 as the cylinder approaches to the upper wall. Averaged heat transfer enhancements (deteriorates) by 95.47% (69.34%) for location L3 (L1) compared to location where the cylinder is placed at the mid of the cavity (L2).

5. Conclusions

Numerical investigations of mixed convection in a partitioned cavity having upper half nano-fluid and lower half porous medium with an adiabatic rotating cylinder were performed. Some important conclusions from the numerical simulation results can be given as:

- Local Nusselt number increases for the large portion of the hot bottom wall for different cylinder sizes and the averaged Nusselt number enhances as the value of Rayleigh number increase. The convection above the cylinder in the upper half of the cavity is more pronounced with increasing Rayleigh numbers and the discrepancy between the local and averaged heat transfer at $Ra = 10^5$ and $Ra = 10^6$ is high when the cylinder size small.
- The flow patterns and isotherms within the cavity are affected with cylinder rotation. As the cylinder rotates, it acts in a way to increase the local heat transfer for the large part of the bottom wall for different cylinder sizes. The averaged Nusselt number enhances as the angular rotational speed of the cylinder increases and this is more effective when using a large cylinder size. 42.56% and 432.55% of averaged Nusselt number enhancements are obtained for $\Omega = 6000$ compared to motionless cylinder case at $\Omega = 0$ using cylinder sizes of $R = 0.1$ and $R = 0.3$. In the study of Costa and Raimundo [28], the cylinder size was also found to significantly affect the fluid flow and heat transfer characteristics due to the reduction of the gap between the cylinder surface and the wall of the cavity which enhances or deteriorates the local fluid motion. In the study by Roslan et al. [60], the size of the rotating cylinder was found to affect the heat transfer performance significantly in a vertically heated and cooled square cavity. An optimum value of the radius was found where the heat transfer attains its maximum average value.
- An increase in solid particle volume fraction of nanofluid results in an increase in the convection due to the increased thermal conductivity of the nanofluid which results in better thermal transport of the fluid within the partitioned cavity. Adding nanoparticles to the base fluid is favorable for the locations when high values of local heat transfer is seen. The averaged Nusselt number increases linearly with nanoparticle volume fraction for different cylinder sizes. 7.35% and 7.45% of enhancements in the averaged Nusselt number are achieved for solid volume fraction of 0.05 compared to base fluid at $\phi = 0$ using cylinder sizes of $R = 0.3$ and $R = 0.1$. In the previous study by Roslan et al. [60], heat transfer performance was found to enhance with higher nanoparticle concentration and higher thermal conductivity of the cylinder. The heat transfer enhancements are below 10% at the highest solid nanoparticle volume fraction as compared to base fluid.
- The local and averaged heat transfer enhance with increasing values...
of Darcy number due to the increased permeability of the porous medium. The discrepancy between the averaged Nusselt numbers corresponding to different cylinder sizes increase when the value of the Darcy number enhances. 66.76% and 162.82% enhancements of averaged Nusselt number are obtained for $Da=10^{-2}$ compared to $Da=10^{-5}$ using cylinder sizes of $R=0.1$ and $R=0.3$. Misirlioglu [26] reported that for higher cylinder rotational velocities, the average heat transfer does not change significantly with Darcy number.

- Local and averaged heat transfer enhance as the cylinder approaches to the upper wall of the cavity. Averaged heat transfer enhances by 95.47% for location L3 compared to vertical location L2. In the study by Hussain and Hussein [29], the cylinder upward and downward movement was also found to affect the average Nusselt number for a an isothermal hot cylinder in a cavity with all boundaries being isothermal at a constant low temperature.

The results of this numerical study can be used to optimize the devices which are used for analyzing the micro-structural dynamics of complex fluids such as polymeric solutions, colloids, bubbles and drops [34]. In the oil drilling operations, the flow behavior between the annular space of two circular cylinders which are stationary or rotating can be studied by extending it to include various effects such as the eccentricity and the fluid type which are in close relation with the present study [43]. The demand for increased convection in many applications such as in fibrous thermal insulation, solidification, fuel cell, cooling of nuclear fuel debris, solar collectors which can be modeled as superposed fluid and porous layers confined in an enclosure can be satisfied with an inner rotating cylinder placed between different media.

The study can be extended to include the effects of different nanofluid types, cylinder locations, thermal conductivity of the cylinder and unsteady flow effects which are not considered in this study.

References

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