Original Article

Unsteady flow of a Maxwell nanofluid over a stretching surface in the presence of magnetohydrodynamic and thermal radiation effects

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Abstract The problem of unsteady magnetohydrodynamic (MHD) boundary layer flow of a non-Newtonian Maxwell nanofluid over a stretching surface with thermal radiation is considered. The Maxwell model is used to characterize the non-Newtonian fluid behaviour. An appropriate similarity transformation is employed to transform the governing partial differential equations of mass, momentum, energy and nanoparticle concentration into ordinary differential equations. The coupled non-linear ordinary differential equations are solved by using the variational finite element method. The flow features and the heat transfer characteristics and nanoparticle volume fraction are analyzed and discussed in detail for several sets of values of the governing flow parameters. The results for the skin-friction coefficient, local Nusselt number and the local Sherwood number are presented in tables for various values of the flow controlling parameters.

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1. Introduction

Interest of the researchers in the flows of non-Newtonian fluids is on the leading edge during the last few decades because of their practical applications. Such interest in fact
is accelerated because of a broad range of applications of non-Newtonian fluids in the various disciplines, for instance in biological sciences, geophysics, chemical and petroleum industries. A number of industrially important fluids including molten plastics, polymers, pulps, food and fossil fuels display non-Newtonian behavior. The Navier–Stokes equations cannot adequately describe the flow of non-Newtonian fluids. The constitutive equations are able to predict the rheological characteristics. The flow characteristics of non-Newtonian fluids are quite different in comparison with Newtonian fluids. To get a clear idea about these fluids and their various applications, it is necessary to study the flow behavior of non-Newtonian fluids. Due to complexity, not a single constitutive equation exhibiting all properties of such non-Newtonian fluids is available. In the literature, the majority of non-Newtonian fluid models concern simple models like the power law and grade two or three [1–7]. The main drawbacks of these simple fluid models are that the results provided by them do not agree with fluid flows in reality. The power-law model is widely used in modeling fluids with shear-dependent viscosity. But it is unable to predict the effects of elasticity. The effects of elasticity can be obtained by the fluids of grade two or three. But the viscosity in these models is not shear dependent. Moreover, they are unable to predict the effects of stress relaxation. A subclass of rate type fluids, viz., the Maxwell model, can predict the stress relaxation and, therefore, have become more popular. Aliaikbar et al. [8] studied the influence of thermal radiation on magnetohydrodynamic (MHD) flow of Maxwell fluid over a stretching sheet. Hayat and Qasim [9] investigated the influence of thermal radiation and joule heating on MHD flow of Maxwell fluid over a stretching sheet. Mukhopadhyay [10] studied heat transfer analysis of the unsteady flow of a Maxwell fluid over a stretching sheet. Mukhopadhyay and Bhattacharyya [11] discussed the mass transfer effect on the Maxwell fluid flow passing through an unsteady stretching sheet.

Nanotechnology is considered by many to be one of the significant forces that drive the next major industrial revolution of this century. It represents the most relevant technological cutting edge currently being explored. Nanofluid heat transfer is an innovative technology which can be used to enhance heat transfer. Nanofluid is a suspension of solid nanoparticles (1–100 nm diameters) in conventional liquids like water, oil, and ethylene glycol. Depending on shape, size, and thermal properties of the solid nanoparticles, the thermal conductivity can be increased by about 40% with low concentration (1%–5% by volume) of solid nanoparticles in the mixture [12–20].

The term nanofluid is defined as a solid–liquid mixture consisting of nanoparticles and a base liquid. Choi is the first to use the term nanofluids to refer to fluids with suspended nanoparticles. Studies have shown that adding nanoparticles to a base fluid can effectively improve the thermal conductivity of the base fluid and enhance heat transfer performance of the liquid. This is why nanofluids have found such a wide range of applications in so many fields: energy, power, aerospace, aviation, vehicles, electronics, etc.

The thermal conductivity of the nanofluids is higher than that of base fluids. Further, the novel properties of Brownian motion and thermophoresis of such fluids make them potentially useful. Nanoparticles are used to enhance the thermal characteristics of ordinary base fluids such as water, ethylene glycol or oil. In addition the magneto-nanofluid is a unique material that has both liquid and magnetic properties. Such nanofluid has superficial role in

<table>
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blood analysis and cancer therapy. Buongiorno [21] provided a mathematical model of nanofluid which has the characteristics of thermophoresis and Brownian motion. Later on, Makinde and Aziz [22] investigated the boundary layer flow of viscous nanofluid with convective thermal boundary condition. Ul Haq et al. [23] examined the two dimensional boundary layer flow of natural convective micropolar nanofluid along a vertically stretching sheet. Noor et al. [24] investigated the mixed convection boundary layer flow past a stretching sheet with convective heating. Ramesh and Gireesha [26] studied the in

\[
\frac{\partial u}{\partial x} + u \frac{\partial u}{\partial y} + \frac{v}{\partial y} = 0
\]

(1)

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \lambda \left( u^2 \frac{\partial^2 u}{\partial x^2} + v^2 \frac{\partial^2 u}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho_f} u
\]

(2)

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial \rho_f}{\partial y} + \tau \left[ D_h \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 \right]
\]

(3)

\[
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_h \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}
\]

(4)

The principle aim of the present work is to study the unsteady laminar boundary layer two-dimensional flow, heat and mass transfer of a viscous incompressible electrically-conducting non-Newtonian Maxwell nanofluid over an unsteady permeable stretching sheet. The flow is subject to a non-uniform magnetic field and non-uniform velocity. To the best of authors’ knowledge, this problem has not been studied before. In this regard, the proper similarity transformations have been utilized for the reduction of the governing partial differential equations into ordinary differential equations. Graphical results for various flow parameters are presented to gain a thorough insight toward the physics of the problem.

2. Mathematical formulation

Consider a laminar boundary-layer two-dimensional flow, heat and mass transfer of an incompressible, electrically conducting non-Newtonian upper convected Maxwell fluid (in the presence of a transverse magnetic field) over an unsteady permeable stretching sheet. The flow is subjected to a non-uniform magnetic field of strength \( B = \frac{B_0}{\sqrt{1 - \alpha}} \) applied normal to the surface; \( B_0 \) is the initial strength of the magnetic field. The unsteady fluid, heat and mass flows start at \( t = 0 \). The sheet emerges out of a slit at \( x = 0 \), \( y = 0 \) and moves with non-uniform velocity \( U(x, t) = \frac{c t}{\sqrt{1 - \alpha}} \) where \( c, \alpha \) are positive constants with dimensions (time) \(^{-1} \), \( c \) is the initial stretching rate, and \( \sqrt{1 - \alpha} \) is the effective increasing stretching rate with time. In case of polymer extrusion, the material properties of the extruded sheet may vary with time. It is assumed that the magnetic Reynolds number is very small, and the electric field due to polarization of charges is neglected as there is no electric field. It is also assumed that the stretching surface is subjected to such amount of tension, which does not alter the structure of the permeable material.

The governing equations of such a type of flow are, in the usual notations, expressed as

\[
q_r = -\frac{4\sigma^* \partial T^4}{3k^* \frac{\partial T}{\partial y}},
\]

(6)

where \( \sigma^* \) and \( k^* \) are the Stefan–Boltzmann constant and the mean absorption coefficient, respectively.

It is assumed that the temperature differences within the flow, such as the term \( T^4 \), may be expressed as a linear function of temperature. Taylor series expanding for \( T^4 \) about a free stream temperature \( T_\infty \) after neglecting higher-order terms:

\[
T^4 = 4T^4 - 3T^2 \]

(7)

Using Eqs. (6) and (7), we obtain

\[
\frac{\partial \rho_f}{\partial y} = -\frac{16\sigma^* T^3}{3k^*} \frac{\partial T}{\partial y}.
\]

(8)

We now introduce the following relations for \( u, v, \theta \) and \( \phi \) as

\[
u = -\frac{\rho_f}{\partial x}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_\infty}
\]

(9)
where $\psi$ is the stream function. Let us introduce the following similarity transformations:

$$
\eta = \sqrt{\frac{c}{\nu (1-\alpha t) \partial_x}}, \quad \psi = \sqrt{\frac{\nu c}{1-\alpha t}} \phi(\eta),
$$

$$
T = T_\infty + \frac{cx}{(1-\alpha t)} \theta(\eta), \quad C = C_\infty + \frac{cx}{(1-\alpha t)} \phi(\eta) \quad (10)
$$

With the help of the above relations Eqs. (9) and (10), the governing equations finally reduce to

$$
A \left( \frac{\eta^2 f'' + f'}{2} \right) + f'' - ff'' + \beta \left( f'' f'' - 2f f'' \right) = f'' - M f' \quad (11)
$$

$$
A \left( \frac{\eta^2 \phi' + \phi}{2} + f \partial \phi - \partial f \phi' \right) = \frac{1}{Pr} \left( 1 + \frac{4R_d}{3} \right) \phi'' + N \phi' + N t \phi'' \quad (12)
$$

$$
\phi'' - Le \left( A \left( \frac{\eta^2 \phi' + \phi}{2} + f \partial \phi - \partial f \phi' \right) \right) + \frac{N t}{Nb} \phi'^2 = 0 \quad (13)
$$

The dimensionless boundary conditions become:

$$
f' = 1, \quad f = S, \quad \theta = 1, \quad N b \phi' + N t \phi'' = 0 \quad \text{at} \quad \eta = 0 \quad (14a)
$$

$$
f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty. \quad (14b)
$$

where $A = \frac{a_c}{L}$ is the unsteadiness parameter, $\beta = c a_0$ is the Maxwell parameter, $M = \frac{a_r}{R_c \nu}$ is the magnetic parameter, $Pr = \frac{\nu}{a_0}$ is Prandtl number, $R_d = \frac{\sqrt{\nu}}{a_0}$ is the radiation parameter, $Nb = \frac{a_d (C_m)}{\nu}$ is the Brownian motion, $N t = \frac{a_d (T_\infty - T_\infty)}{\nu}$ is the thermophoresis parameter and $Le = \frac{\nu}{D_b}$ is the Lewis number. Here $S = \frac{N a_0}{\nu}$ is the transpiration parameter, $S$ corresponds to suction or blowing according as $S > 0$ or $S < 0$.

For practical purposes, the functions $f(\eta)$, $\theta(\eta)$ and $\phi(\eta)$ allow us to determine the skin friction coefficient, local Nusselt number and the local Sherwood number, respectively. These are defined as follows:

$$
2C_f R e_s^{1/2} = f''(0), \quad Nu_s R e_s^{-1/2} = -\left( 1 + \frac{4R_d}{3} \right) \theta'(0),
$$

$$
S h_s R e_s^{-1/2} = -\phi'(0) \quad (15)
$$

where $Re_s = \frac{U_s}{v}$ is the local Reynolds number.

### 3. Method of solution

The finite element method is a powerful technique for solving ordinary or partial differential equations. The steps involved in the finite element analysis are as follows:

- Discretization of the domain into elements
- Derivation of element equations
- Assembly of element equations
- Imposition of boundary conditions
- Solution of assembled equations

The whole flow domain is divided into 1000 quadratic elements of equal size. Each element is three-noded and

<table>
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<th>A</th>
<th>Sharidan et al. [28]</th>
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Figure 1  Effect of unsteadiness parameter $A$ on velocity, temperature and nanoparticle volume fraction profiles.

Figure 2  Effect of magnetic parameter $M$ on velocity, temperature and nanoparticle volume fraction profiles.

Figure 3  Effect of Maxwell parameter $\beta$ on velocity, temperature and nanoparticle volume fraction profiles.
Figure 4  Effect of transpiration parameter $S$ on velocity, temperature and nanoparticle volume fraction profiles.

Figure 5  Effect of Prandtl number $Pr$ on temperature and nanoparticle volume fraction profiles.

Figure 6  Effect of $Nt$ on temperature and nanoparticle volume fraction profiles.
therefore the whole domain contains 2001 nodes. We obtain a system of equations contains 8004 equations. The obtained system is non-linear, therefore an iterative scheme is utilized in the solution. After imposing the boundary conditions the remaining system contains 7997 equations, which is solved by the Gauss elimination method while maintaining an accuracy of $10^{-5}$.

4. Results and discussion

In order to validate the variational finite element method used in the study and judge the present investigation, the results corresponding to the skin friction coefficient $f''(0)$ are compared with the available results of Chamkha et al. [29], shown in Table 1. Further more, the effect of the unsteady parameter $A$, magnetic field parameter $M$ and the Maxwell parameter $\beta$ on the skin friction coefficient, local Nusselt number and the local Sherwood number are presented in Table 2. The value of the skin friction coefficient $f''(0)$ decreases with the increase in the unsteady parameter $A$, magnetic field parameter $M$ and the Maxwell parameter $\beta$. It is clear from the table that the local Nusselt number increases with the increase in the unsteady parameter $A$, but decreases with the increase in the magnetic field parameter $M$ and the Maxwell parameter $\beta$. On the other hand, the local Sherwood number decreases with the increase in the unsteady parameter $A$ whereas it increases with the increase in the magnetic field parameter $M$ and the Maxwell parameter $\beta$.

Table 3 shows that the effect of the Prandtl number $Pr$, radiation parameter $R_d$, Brownian motion parameter $Nb$, thermophoresis parameter $Nt$ and the Lewis number $Le$ on the local Nusselt number and the local Sherwood number. One can see from the table that the influence of Prandtl number $Pr$ is to enhance the value of $-\theta'(0)$ and to reduce the value of $-\phi'(0)$. On the other hand, the effect of increasing the value of $R_d$ is to reduce the value of $-\theta'(0)$ whereas it increases the value of $-\phi'(0)$. However, increasing the value of the thermophoresis parameter $Nt$ leads to decreases in both the Nusselt and Sherwood numbers. The effect of increasing the Lewis number $Le$ is to reduce the local Nusselt number and to increase the local Sherwood number. However, the effect of increasing the Brownian motion parameter $Nb$ leads to a reduction in the local Sherwood number but causes no effect on the local Nusselt number.

The numerical computations are carried out for various values of the flow parameter which highlights the flow characteristics such as the unsteady parameter $A$, magnetic field parameter $M$, Maxwell parameter $\beta$, Prandtl number $Pr$, radiation parameter $R_d$, Brownian motion $Nb$, thermophoresis parameter $Nt$ and the Lewis number $Le$. In order to study the behaviour of velocity, temperature and nanoparticle volume fraction, various profiles are computed for different flow parameters and the results are presented graphically in Figures 1–9.
The effect of unsteadiness parameter \( A \) on velocity, temperature and nanoparticle volume fraction field is shown in Figure 1(a)–(c), respectively. It can be seen that the velocity along the sheet decreases with the increase in the unsteady parameter \( A \) and this implies an accompanying reduction of the thickness of the momentum boundary layer near the wall and away from the wall, the fluid velocity increases with the increasing in the unsteadiness parameter. From Figure 1(b), it is noticed that the temperature decreases significantly with the increase in the unsteadiness parameter and this implies that less heat is transferred from the fluid to the sheet and hence, the temperature \( \theta \) decreases. As the unsteadiness parameter \( A \) increases, the nanoparticle volume fraction decreases as well as observed from Figure 1(c). As the unsteadiness parameter \( A \) increases, less mass transfer from the fluid to the sheet takes place and hence, the volume fraction concentration decreases.

The effects of the magnetic field parameter \( M \) on the velocity, temperature and nanoparticle volume fraction profiles are presented in Figure 2(a)–(c), respectively. It can be seen that with the increase of the magnetic field parameter \( M \), the fluid velocity decreases whereas the temperature and nanoparticle volume fraction profiles increase. This is expected as application of a transverse magnetic field normal to the flow direction gives rise to a resistive type force called the Lorentz force. This force has the tendency to slow down the motion of the fluid.

The effect of the magnetic field parameter \( M \) is predicted to increase the wall temperature gradient. This is due to the fact that the thermal boundary layer thickness decreases with the increase of \( M \), which results in higher temperature gradient at the wall and hence, higher wall heat transfer. This is true through the magnetic field parameter is not directly appearing in the energy equation but it indirectly affecting on temperature field through changes in the velocity field.

Figure 3(a)–(c) depicts the effect of the Maxwell parameter \( \beta \) on the velocity, temperature and the nanoparticle volume fraction, respectively. It can be seen that the effect of increasing the values of \( \beta \) is to decrease the velocity field and hence, the boundary layer thickness decreases. This induces an increase in the absolute value of the temperature gradient at the surface. The trend of the temperature gradient at the surface with increasing values of \( \beta \) is shown in Figure 3(b). It can also be seen from Figure 3(c) that the nanoparticle volume fraction field increases with the increase in the Maxwell parameter \( \beta \).

The effect of the suction/injection parameter \( S \) on the velocity, temperature and the nanoparticle volume fraction profiles are shown in Figure 4(a)–(c), respectively. It is noticed from Figure 4(a) that the fluid velocity decreases with increasing values of the suction/injection parameter \( S \). The fluid velocity is found to decrease with the increase in suction \( (S>0) \) i.e., suction causes a decreasing fluid velocity in the boundary layer. However, increasing the injection effect \( (S<0) \) has the opposite effect. Figure 4(b) exhibits that the temperature in the boundary layer decreases with increasing values of suction \( (S>0) \). This causes a decrease in the rate of heat transfer (from the fluid to the surface) where as the temperature in the boundary layer increases in the case of injection \( (S<0) \). The thermal boundary layer is thinner in the case of suction and thicker in the case of injection when compared to the impermeability case \( (S=0) \). The result of increase in the suction effect \( (S>0) \) causes an increase in the nanoparticle volume fraction profile in the boundary layer region near the wall and the opposite effect is observed far away from the boundary. The effect of injection \( (S<0) \) is to increase the nanoparticle volume fraction profile near the boundary surface and the reverse phenomenon is observed far away from the boundary.

Figure 5(a) and (b) presents the temperature and the nanoparticle volume fraction profiles for different values of the Prandtl number \( Pr \). Figure 5(a) reveals that the effect of increasing the Prandtl number \( Pr \) is to decrease the temperature profile. This is due to the fact that increasing \( Pr \) leads to an increase in the kinematic viscosity and a decrease in the thermal conductivity. With the increase in the value of \( Pr \), the nanoparticle volume fraction decreases in the boundary region and the opposite effect is seen far away from the sheet surface. This is also related to the fact...
that as $Pr$ increases, the kinematic viscosity increases and the thermal conductivity decreases. When the viscosity increases, there is a decrease in the nanoparticle volume fraction in the vicinity of the boundary and far away from the surface, the effect is insignificant.

In order to understand the influence of $Nt$ on the heat and mass transfer, graphs for the temperature and nanoparticle volume fraction are plotted in Figure 6. It is clear from this figure that the effect of $Nt$ is to increase both of the temperature and nanoparticle volume fraction profiles. The effect of increasing the value of $Nt$ on the nanoparticle volume fraction is predicted to increase significantly in the boundary layer except close to the surface boundary where it shows significant decreases.

Figure 7 presents the effect of the Brownian motion parameter $Nb$ on the nanoparticle volume fraction profiles. From this figure, it is evident that increasing the value of $Nb$ has the tendency to decrease the nanoparticle volume fraction profiles. This is due to the fact that there would be a decrease in the nanoparticle volume fraction boundary layer thickness as the value of $Nb$ increases. No appreciable effect of $Nb$ on the temperature profiles is observed and hence, the temperature profiles are not presented for brevity.

Figure 8 illustrates the temperature profiles for different values of the radiation parameter $Rt$. From the figure, it can be seen that as the value of $Rt$ increases, the temperature profiles increase. Figure 9 shows the variation of the nanoparticle volume fraction profiles with the Lewis number $Le$. It can be seen that with the increase in the Lewis number $Le$, the nanoparticle volume decreases and hence, the concentration boundary layer thickness decreases.

Figure 10 shows the variation of local Nusselt number and local Sherwood number with the thermophoresis parameter $Nt$ for different values of Maxwell parameter $\beta$. It is evident from figure that the value of the local Nusselt number and local Sherwood number decreases with the increase of thermophoresis parameter $Nt$ for both Newtonian ($\beta=0$) and non-Newtonian fluid ($\beta \neq 0$). The figure reveals that as the increase of Maxwell parameter $\beta$, local nusselt number decreases where as the local Sherwood number increases.

5. Conclusion

This work was focused on the analysis of unsteady magnetohydrodynamic boundary layer flow of a non-Newtonian Maxwell nanofluid over a stretching surface with thermal radiation. The Maxwell model was used to characterize the non-Newtonian nanofluid behaviour. Appropriate similarity transformations were employed to transform the governing partial differential equations of mass, momentum, energy and nanoparticle concentration into ordinary differential equations. The coupled non-linear ordinary differential equations were solved by using the variational finite element method. The flow features and the heat transfer characteristics and nanoparticle volume fraction were presented graphically, analyzed and discussed in detail for several sets of values of the governing flow parameters. The results for the skin-friction coefficient, local Nusselt number and the local Sherwood number were presented in tabular for various values of the flow controlling parameters. It was found that the skin friction coefficient decreased with the increase in either of the unsteadiness parameter $A$, magnetic field parameter $M$ or the Maxwell parameter $\beta$. Also, the local Nusselt number increased with the increase in either of the unsteadiness parameter $A$ or the Prandtl number $Pr$ but decreased with the increase in either of the magnetic field parameter $M$, Maxwell parameter $\beta$, thermal radiation parameter $Rd$, thermophoresis parameter $Nt$ or the Lewis number $Le$. Moreover, the local Sherwood number increased with the increase in either of the magnetic field parameter $M$, the Maxwell parameter $\beta$, thermal radiation parameter $Rd$ or the Lewis number $Le$ whereas it decreased as either of the unsteadiness parameter $A$, Prandtl number $Pr$, thermophoresis parameter $Nt$, or the Brownian motion parameter $Nb$ increased.

References


