Magneto-hydrodynamics heat and mass transfer analysis of single and multi-wall carbon nanotubes over vertical cone with convective boundary condition

P. Sreedevi a, P. Sudarsana Reddy b,∗, Ali. J. Chamkha b,c

a Department of Mathematics, RGM College of Eng. & Tech., Nandyal, AP, India
b Mechanical Engineering Department, P Prince Mohammad Bin Fahd University, Al-Khebar, Saudi Arabia
c RAK Research and Innovation Center, American University of Ras Al Khaimah, United Arab Emirates

A R T I C L E   I N F O

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A B S T R A C T

This paper investigate the numerical study of MHD boundary layer flow, heat and mass transfer analysis of water based nanofluids containing single and multi-walled CNTs over a vertical cone embedded in porous medium with convective boundary condition under the influence of chemical reaction and suction/injection. The similarity transformation technique is used for converting the governing non-linear partial differential equations, which represents the momentum, temperature and concentration of nanofluid, into the system of coupled ordinary differential equations. The transformed conservation equations together with boundary conditions are solved by using Finite element method. The sway of various pertinent parameters on hydrodynamic, thermal and solutal boundary layers is investigated and the results are displayed graphically. Furthermore, the values of local skin-friction coefficient, rate of temperature and concentration is also inspected for various values of non-dimensional parameters and the results are shown in tabular form. The numerical data results are compared with available data for special cases and found in good agreement. It is found that the skin-friction coefficient, Nusselt number and Sherwood number enhancements with rising values of Biot number (B1) in both SWCNTs-water and MWCNTs-water based nanofluids.

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1. Introduction

The thermal properties of materials have become the rapid growth of interest in recent years because of its extensive range of applications in scientific and engineering communities. Conventional heat transfer fluids like water, kerosene, ethylene glycol, oils etc., have low heat transfer capabilities due to their limited thermal conductivity. By suspending nanometer sized (1–100 nm) particles, called nanoparticles, into the base fluids we increase its thermal conductivity. Though we have different varieties of nanoparticles like, metals, oxides, carbides, nitrides or nonmetals (graphite, carbon nanotubes), in which carbon nanotubes are the most promising materials in terms of their capability to conduct heat. The thermal conductivity of carbon materials at room temperature is almost over five orders of magnitude compare to the general materials. Higher thermal conductivity is not only the unique quality of the carbon nanotubes but they have exceptional mechanical, electrical properties and specific applications in nano-sensors and atomic transportation. The exceptional mechanical properties of CNTs are Ultra-high Young’s modulus around 1 TPa and tensile strengths varying from 11–63 GPa. In electronic industry the removal of heat has become a crucial issue for continuing progress to increased levels of degenerate power. So, it is necessary to identify the materials that have more heat transfer capabilities and are used in the design of next generations of integrated circuits (ICs) and three-dimensional (3D) electronics. Carbon nanotube is hypothetically different as a cylinder fabricated of rolled up graphene sheet and is classified as single wall or multi walls. Single-wall carbon nanotubes (SWCNTs) can be described as the nanotubes with single well having the diameter 0.4 nm–3 nm, whereas, nanotubes with multi walls can be described as the multi-wall carbon nanotubes (MWCNTs) with diameter ranging from 0.4 nm to 30 nm. Due to numerous applications, carbon nanotubes are used in purification processes, medicine, nanotechnology, batteries, super conductors, electronics, waste salvaging, tissue engineering, semiconductors, transistors, solar storage and biosensors.

In view of the above applications of nanofluids, the first endeavor made by Choi et al. [1] while doing research on new cooling technologies and they have noticed up to 150% enhancement in thermal conductivity when carbon nanotubes are suspended into the ethylene glycol or oil. Eastman et al. [2] have revealed that the thermal conductivity of the...
ethylene glycol or oil has increased 40% when copper nanoparticles of volume fraction less than 1% are added. Xue [3] suggested a theoretical model for thermal conductivity by taking rotational elliptical nanotubes with extensive axial ratio based on Maxwell theory and this proposed model also designates the properties of space dispersion of the CNTs on thermal conductivity. Ding et al. [4] reported that the thermal conductivity of CNTs based nanofluid is depending on the temperature of the base fluid and found that up to 30% enhancement in thermal conductivity when the temperature of the fluid is 25 °C, however, 79% enhancement is noticed at 40 °C. Buongiorno [5] has reported in his bench mark study about the natural convection heat transfer enhancement of nanofluids using scale analysis. Wen et al. [6] have discussed the effective thermal conductivity of aqueous suspension of multi-walled carbon nanotubes. They found that as the concentration of carbon nanotubes enhances, the effective thermal conductivity increases. Assael et al. [7] presented the thermal conductivity enhancement of MWCNTs-water based nanofluid and noticed 34% enhancement in thermal conductivity when 0.6% volume fraction of carbon nanotubes are suspended into the water. Garg et al. [8] have examined the influence of dispersing energy on viscos-ity and heat transfer enhancement of multi-walled carbon nanotubes and found 20% enhancement in thermal conductivity. Amrollahi et al. [9] have studied experimentally about the heat transfer coefficients of multi-walled carbon nanotubes dispersed in distilled water over the uniformly heated horizontal tube in entrance region. They noticed up to 33– 40% convective heat transfer coefficient enhancement when the volume fraction of nanotubes is 0.25%. Rashmi et al. [10] deliberated the stability and thermal conductivity of carbon nanotubes-water based nanofluid experimentally and noticed 100–250% enhancement in thermal conductivity. Phuoc et al. [11] have discussed the thermophysical properties of nanofluids and reported up to 2.3–16% enhancement in thermal conductivity of nanofluids prepared by carbon nanotubes stabilized by chitosan. Harish et al. [12] inspected the heat transfer augmentation of SWCNTs-ethylene glycol based nanofluid and found 14.8% enhancement in thermal conductivity when 0% volume fraction of SNTs is added to the ethylene glycol. Kumaresan et al. [13] have studied thermal conductivity of CNTs-water and ethylene glycol mixture nanofluid experimentally based on the temperature. They noticed that the specific heat of the nanofluid enhances remarkably and maximum is 19.8% when the volume fraction of added MWCNTs if 0.45%. Pakdaman et al. [14] reported experimentally about the thermophysical properties and overall performance of MWCNTs-oil based nanofluid flow over helically coiled tubes; furthermore, they have observed the thermal conductivity enhancement by taking the volume fraction of added CNTs is 0.1%, 0.2% and 0.4%. Wang et al. [15] have discussed the heat transfer and pressure drop of carbon nanotubes based nanofluid over horizontal circular tube by taking distilled water as the base fluid. Furthermore, they have reported 70% - 190% enhancement in convective heat transfer when the volume fraction of CNTs is 0.05%–0.24%. The experimental investiga-tion of thermophysical characteristics of single and multi-walled carbon nanotubes based nanofluid was discussed by several authors [16–19]. Khan et al. [20] have analyzed the impact of Navier slip on flow and heat transfer analysis of carbon nanotubes over a flat plate. Halefald et al. [21] have studied the efficiency and heat transfer properties of carbon nanotubes-water based nanofluid over coaxial heat exchanger under laminar regime. Ul Haq et al. [22] examined the thermophysical properties of MHD boundary layer flow and heat transfer analysis over a stretching surface filled with carbon nanotubes-water based nanofluid. Hayat et al. [23] discussed the stagnation point flow of carbon nanotubes with the influence of homogeneous-heterogeneous reactions and Newtonian heating. Ellahi et al. [24] have studied the ther-mal conductivity enhancement of single and multi-walled CNTs-salt water based nanofluid. Imtiaz et al. [25] presented the convective flow over between rotating stretchable disks filled with carbon nanotubes-water based nanofluid. Hayat et al. [26] have investigated the stagnation point flow and melting heat transfer of SWCNTs-water nanofluid and MWCNTs-water based nanofluid over permeable stretching sheet.
with suction/injection and convective boundary conditions. Gavili et al. [27] have analyzed the heat transfer properties of carbon nanotubes over two-sided lid-driven differentially heated square cavity depending on fluid temperature. Khan et al. [28] have studied the influence of nanoparticle volume fraction on MHD boundary layer CNTs-water based nanofluid over static/moving wedge. Karami et al. [29] have deliberated the various applications of carbon nanotubes in solar collectors. Magneto nanofluids have specific applications in biomedicine, optical modulators, magnetic cell separation, magneto-optical wavelength filters, silk float separation, nonlinear optical materials, hyperthermia, optical switches, drug delivery, optical gratings etc. A magnetic nanofluid has both the liquid and magnetic properties. Keeping above applications of magnetonanofluids several authors [30–35] have discussed the flow, heat and mass transfer inspection of nanofluids by taking various parameters into the account. Oztop et al. [36] presented headline analysis to study heat transport path in an inclined non-uniformly heated enclosure filled with CuO-water based based nanofluid. Selimfendigil et al. [37] studied conjugate natural convection-conduction heat transfer in an inclined partitioned heated cavity filled with Al2O3-water and CuO-water based nanofluid by using Finite element analysis and found the rate of heat transfer elevates as the volume fraction of nanoparticles raises. Selimfendigil et al. [38] discussed MHD mixed convection heat transfer enhancement of CuO-water based nanofluid over lid-driven cavity and found absolute value of averaged heat transfer enhances as the values of Richardson number increases from 1 to 100. Miroshnichenko et al. [39] have studied natural convection in an open trapezoidal cavity filled with CuO nanofluid and found reduction in heat transfer as the values of Hartman number increases. Oztop et al. [40] have perceived mixed convection of MHD flow in nanofluid filled and partially heated wavy walled lid-driven enclosure.

The above literature review reveals that there are no studies that investigate the influence of magnetic field and radiation on heat and mass transfer characteristics of SWCNTs-water and MWCNTs-water based nanofluid over a vertical cone through porous media under the presence of suction/injection and convective boundary condition. The problem discussed in this study has immediate applications in solar thermal system, purification system, health care materials, thermal conductors etc. The solution of governing equations is executed in Mathematica 10.0.

2. Mathematical analysis

We consider two dimensional, study, laminar flow over a vertical cone saturated by porous media filled with single and multi-wall CNTs-water based nanofluid under the convective boundary condition with suction/injection as illustrated in Fig. 1. The thermophysical properties of water and both CNTs are depicted in Table 1. The coordinate system is chosen as the x-axis is coincident with the flow direction over the cone surface. It is assumed that T∞ to be determined, is the result of convective heating process which is characterized by a temperature T0 and heat transfer coefficient h0, and φω is the nanoparticle volume fraction at the surface of the cone and Tω and φω are the temperature and nanoparticle volume fraction of the ambient fluid, respectively. An external magnetic field of strength B0 is applied in the direction of the y-axis. By considering the Oberbeck–Boussinesq approximations the governing equations describing the steady-state conservation of mass, momentum, energy as well as conservation of nanoparticles for nanofluids in the presence of thermal radiation and other important parameters take the following form:

\[ \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \]  

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_{nf} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{1}{\rho_{nf}} \frac{\partial \left( \rho(T - T_\infty) \right)}{\partial y} + g \left[ \rho(T - T_\infty) - \rho(T - T_\infty) \right] \cos y \]  

\[ \frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho c_p} \frac{\partial \rho}{\partial y} \]  

\[ \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = D_{nf} \frac{\partial^2 \phi}{\partial y^2} - K \left( \phi - \phi_\infty \right) \]  

The associated boundary conditions are

\[ u = 0, v = V_0(x), -k_{nf} \frac{\partial T}{\partial y} = h_0 \left( T_j - T_\infty \right), \phi = \phi_\infty \text{ at } y = 0 \]  

\[ u \rightarrow 0, T \rightarrow T_\infty, \phi \rightarrow \phi_\infty \text{ at } y \rightarrow \infty \]

In the above equations x and y represents coordinate axis along the cone surface in the direction of motion and perpendicular to it, u and v are the velocity components along x and y directions, respectively. The term \( V_0 = \frac{\partial}{\partial x} \) represents the mass transfer at the surface with \( V_0 < 0 \) for suction and \( V_0 > 0 \) for injection.

The radiative heat flux \( q_r \) (using Rosseland approximation) is defined as

\[ q_r = -\frac{4k_{nf} \partial T}{\partial y} \]  

We assume that the temperature variances inside the flow are such that the term \( T^4 \) can be represented as linear function of temperature. This is accomplished by expanding \( T^4 \) in a Taylor series about a free stream temperature \( T_\infty \) as follows:

\[ T^4 = T_\infty^4 + 4T_\infty^3 \left( T - T_\infty \right) + 6T_\infty^2 \left( T - T_\infty \right)^2 + \ldots \]

After neglecting higher-order terms in the above equation beyond the first degree term in(\( T - T_\infty \)), we get

\[ T^4 \approx 4T_\infty^3 \left( T - T_\infty \right) \]
Thus substituting Eq. (8) in Eq. (7), we get
\[
q_r = \frac{16T_0 \alpha^3 \sigma^* \partial T}{3K^*} \frac{\partial T}{\partial y} \tag{9}
\]

The dynamic viscosity \( \mu_{df} \), density \( \rho_{df} \), thermal diffusivity \( \alpha_{df} \), heat capacitance \( (\rho C_p)_{df} \) and thermal conductivity \( k_{df} \) of the nanofluid are defined as follows:
\[
\mu_{df} = \frac{\mu_f}{(1 - \phi)^2}, \quad \rho_{df} = (1 - \phi)\rho_f + \rho_{CNT} \phi_f, \quad \alpha_{df} = \frac{k_{df}}{(\rho C_p)_{df}}.
\]
\[
(\rho C_p)_{df} = (1 - \phi)(\rho C_p)_{f} + \phi(\rho C_p)_{CNT}.
\]
\[
k_{df} = k_f \left( \frac{(1 - \phi) + 2\phi \left( \frac{k_{CNT} + k_f}{k_{CNT} - k_f} \right) \ln \left( \frac{k_{CNT} + k_f}{k_{CNT} - k_f} \right)}{(1 - \phi) + 2\phi \left( \frac{k_{CNT} + k_f}{k_{CNT} - k_f} \right) \ln \left( \frac{k_{CNT} + k_f}{k_{CNT} - k_f} \right)} \right)
\]
where \( \phi \) is the nanoparticles volume fraction. The subscript \( df \) signifies the thermophysical properties of nanofluid, \( f \) indicates the properties of base fluid and \( CNT \) represents carbon nanotubes.

The similarity transformations are as follows:
\[
\eta = \frac{2}{R \alpha} \frac{1}{x}, \quad f(\eta) = \frac{\psi}{\alpha \sqrt{R \alpha}}, \quad \theta(\eta) = \frac{T - T_w}{T_w - T_{\infty}}, \quad S(\eta) = \frac{\phi - \phi_{\infty}}{\phi_{\infty} - \phi_{\infty}} \tag{10}
\]
Where \( R \alpha \) is the local Rayleigh number and is defined as
\[
R \alpha = \frac{g \beta f \beta f (T - T_{\infty})^3 \cos \gamma}{\mu_f \alpha_f}
\]
and \( r \) can be approximated by the local radius of the cone, if the thermal boundary layer is thin, and is related to the \( x \) coordinate by \( r = x \sin \gamma \).

Substituting Eq. (10) into Eqs. (1)–(4), we get the following system of non-linear ordinary differential equations
\[
f'''' + \frac{A_1}{(1 - \phi)^2} f''' + \frac{1}{2} \left( f' \right)^2 - k_1 f'' - \frac{A_1}{A_2} f' + A_1 [\theta - \varphi S] S = 0 \tag{11}
\]
\[
(1 + R)\theta'' + \frac{3}{4} A_1 A_4 f \theta' = 0 \tag{12}
\]
\[
S'''' + \frac{3}{4} Sc f S' - Cr S = 0 \tag{13}
\]

The transformed boundary conditions are
\[
\eta = 0, \quad f = V_0, \quad f' = 0, \quad \theta(0) = -A_1 B_1 (1 - \theta(0)), \quad S = 1.
\]
\[
\eta \to \infty, \quad f' = 0, \quad \theta = 0, \quad S = 0. \tag{14}
\]
where prime denotes differentiation with respect to \( \eta \), and the significant thermophysical parameters dictating the flow dynamics are defined by
\[
N_r = \frac{\beta^* (\phi_f - \phi_{\infty})}{\beta(T_0 - T_{\infty})}, \quad k_1 = \frac{\chi^2}{KR_d \alpha_f}, \quad C_r = \frac{K_s x^2}{D_n R_d \alpha_f^2}, \quad Pr = \frac{\mu_f}{\alpha f}, \quad R = \frac{16T_0 \alpha^3 \sigma^*}{3K^* k_{df}}, \quad M = \frac{\sigma f \chi^2}{\mu_f R_d \alpha_f^2},
\]
\[
S_c = \frac{\alpha}{D_n}, \quad A_1 = (1 - \phi)^3 \left[ (1 - \phi) + \phi \left( \frac{\rho_{CNT}}{\rho_f} \right) \right], \quad B_1 = \frac{h_f x}{k_f R_d \alpha_f^2}, \quad A_2 = (1 - \phi) + \phi \left( \frac{\rho_{CNT}}{\rho_f} \right), \quad A_3 = (1 - \phi) + \phi \left( \frac{(\rho C_p)_{CNT}}{(\rho C_p)_f} \right), \quad A_4 = \frac{k_f}{k_{df}}.
\]

Quantities of practical interest in this problem are skin-friction coefficient, local Nusselt number \( Nu_x \), and the local Sherwood number \( Sh_x \), which are defined as
\[
C_f = \frac{r_m}{\rho U_m}, \quad Nu_x = \frac{x q_{df}}{k(T_w - T_{\infty})}, \quad Sh_x = \frac{x J}{D_b (\phi_f - \phi_{\infty})}. \tag{15}
\]

3. Numerical method

The set of ordinary differential Eqs. (11)–(13) are highly non-linear, and therefore cannot be solved analytically. The Finite-element method [41–44] has been employed to solve these non-linear equations. The procedure of Finite element method is as follows.

3.1. Finite element method

For the solution of system of non-linear ordinary differential Eqs. (11)–(13) together with boundary conditions (14), first we assume that
\[
\frac{df}{dn} = h \tag{16}
\]

The Eqs. (11)–(13) then reduces to
\[
h'''' - \frac{A_1}{Pr} \left[ \frac{3}{4} f h'' - \frac{1}{2} (h')^2 \right] - k_1 h - \frac{A_1}{A_2} M h + A_1 [\theta - \varphi S] = 0 \tag{17}
\]
\[
(1 + R)\theta'' + \frac{3}{4} A_1 A_4 f \theta' = 0 \tag{18}
\]
\[
S'''' + \frac{3}{4} Sc f S' - Cr S = 0 \tag{19}
\]

The boundary conditions take the form
\[
\eta = 0, \quad f = V_0, \quad h = 0, \quad \theta'(0) = -A_1 B_1 (1 - \theta(0)), \quad S = 1.
\]
\[
\eta \to \infty, \quad h = 0, \quad \theta = 0, \quad S = 0. \tag{20}
\]

3.2. Variational formulation

The variational form associated with Eqs. (17)–(19) over a typical linear element \((\eta_x, \eta_x+1)\) is given by
\[
\int_{\eta_x}^{\eta_x+1} \psi_1 \left( \frac{df}{dn} - h \right) d\eta = 0 \tag{21}
\]
\[
\int_{\eta_x}^{\eta_x+1} \psi_2 \left( h'''' - \frac{A_1}{Pr} \left[ \frac{3}{4} f h'' - \frac{1}{2} (h')^2 \right] - k_1 h - \frac{A_1}{A_2} M h + A_1 [\theta - \varphi S] \right) d\eta = 0 \tag{22}
\]
\[
\int_{\eta_x}^{\eta_x+1} \psi_3 \left( (1 + R)\theta'' + \frac{3}{4} A_1 A_4 f \theta' \right) d\eta = 0 \tag{23}
\]
\[
\int_{\eta_x}^{\eta_x+1} \psi_4 \left( S'''' + \frac{3}{4} Sc f S' - Cr S \right) d\eta = 0 \tag{24}
\]

Where \( \psi_1, \psi_2, \psi_3, \) and \( \psi_4 \) are arbitrary test functions and may be viewed as the variations in \( f, h, \theta, \) and \( S, \) respectively.

3.3. Finite- element formulation

The finite-element model may be obtained from above equations by substituting finite-element approximations of the form
\[
f = \sum_{j=1}^{3} f_j \psi_j, \quad h = \sum_{j=1}^{3} h_j \psi_j, \quad \theta = \sum_{j=1}^{3} \theta_j \psi_j, \quad S = \sum_{j=1}^{3} S_j \psi_j. \tag{25}
\]

With, \( w_1 = w_2 = w_3 = w_4 = \psi_1, \) \( (i = 1, 2, 3). \)

Where \( \psi_i \) are the shape functions for a typical element \((\eta_x, \eta_x+1)\) and are defined as
\[
\psi_1 = \frac{(\eta_x+1 + \eta - 2 \eta)}{(\eta_x+1 - \eta)} \left( \frac{\eta_x+1 - \eta}{(\eta_x+1 - \eta)^2} \right), \quad \psi_2 = \frac{4(\eta - \eta_x)(\eta_x+1 - \eta)}{(\eta_x+1 - \eta)^2}, \quad \psi_3 = \frac{(\eta_x+1 + \eta - 2 \eta)}{(\eta_x+1 - \eta)^2}, \quad \psi_4 = \eta_x \leq \eta \leq \eta_x+1. \tag{26}
\]
Table 2
Comparison of \( f'(0) \) and \(-\phi'(0)\) with previously published work (water as the base fluid) for fixed values of \( Nr=0, Pr=6.2, V_0=0, B_1=0, \gamma=0, R=0, Cr=0, M=0, K_1=0 \).

<table>
<thead>
<tr>
<th>( \psi )</th>
<th>Khan et al. [20]</th>
<th>Present results</th>
</tr>
</thead>
<tbody>
<tr>
<td>SWCNTs</td>
<td>MWNTs</td>
<td>SWCNTs</td>
</tr>
<tr>
<td>( f'(0) )</td>
<td>( f'(0) )</td>
<td>( f'(0) )</td>
</tr>
<tr>
<td>0.01</td>
<td>0.33984</td>
<td>0.33927</td>
</tr>
<tr>
<td>0.1</td>
<td>0.40811</td>
<td>0.39608</td>
</tr>
<tr>
<td>0.2</td>
<td>0.50452</td>
<td>0.46466</td>
</tr>
</tbody>
</table>

The finite element model of the equations thus formed is given by

\[
\begin{bmatrix}
K_{11} & K_{12} & K_{13} & K_{14} \\
K_{21} & K_{22} & K_{23} & K_{24} \\
K_{31} & K_{32} & K_{33} & K_{34} \\
K_{41} & K_{42} & K_{43} & K_{44}
\end{bmatrix}
\begin{bmatrix}
\eta \\
\phi \\
\theta \\
\psi
\end{bmatrix}
= \begin{bmatrix}
\{ f' \} \\
\{ f' \} \\
\{ f' \} \\
\{ f' \}
\end{bmatrix}
\]

Where \( [K_{mn}] \) and \( \{r_{ni}\} \) \((m, n = 1, 2, 3, 4)\) are defined as

\[
K_{11} = \int_{\eta_0}^{\eta_1} \psi \frac{\partial \psi}{\partial \eta} d\eta, \quad K_{12} = -\int_{\eta_0}^{\eta_1} \psi \frac{\partial \xi}{\partial \eta} d\eta, \quad K_{13} = 3 \int_{\eta_0}^{\eta_1} \psi \frac{\partial \xi}{\partial \eta} d\eta, \quad K_{14} = 0.
\]

The very important aspect in this numerical procedure is to select an approximate finite value of \( \eta_\text{max} \). So, in order to estimate the relevant value of \( \eta_\text{max} \), the solution process has been started with an initial value of \( \eta_\text{max} = 5 \), and then the Eqs. (11)–(13) are solved together with boundary conditions (14). We have updated the value of \( \eta_\text{max} \) and the solution process is continued until the results are not affected with further values of \( \eta_\text{max} \). The choice of \( \eta_\text{max} = 10 \) and \( \eta_\text{max} = 8 \) for velocity, temperature and concentration have confirmed that all the numerical solutions approach to the asymptotic values at the free stream conditions. The step size has taken as \( \psi = 0.001 \) and the error is less than 1.E-05.

4. Results & discussion

Numerical solution to the problem of MHD boundary layer nanofluid flow is presented and velocity, temperature and concentration profiles for different pertinent parameters are displayed graphically from Figs. 2–21. Comparison of present numerical code with existing results, Khan et al. [20], is made for different values of nanoparticle volume fraction parameter \( \phi \) by fixing other variables as zero and found good agreement which is shown in Table 2.
The impact of volume fraction parameter ($\phi$) on velocity, temperature and concentration profiles of the water based nanofluid containing single and multi-walled CNTs are displayed in Figs. 2–4. The velocity distributions declined with higher values of volume fraction parameter ($\phi$) in both nanofluids. Remarkable deterioration in fluid velocity is noticed in MWCNTs-water based nanofluid than the SWCNTs-water nanofluid. Furthermore, the temperature and concentration distributions are both decelerate with increasing values of ($\phi$) in the both nanofluids. It is observed that the retardation in thickness of the thermal boundary layer of the entire fluid regime is higher in MWCNTs-water nanofluid than the SWCNTs-water nanofluid. However, the depreciation in the thickness of solutal boundary layer is slightly more in SWCNTs-water nanofluid than the MWCNTs-water nanofluid.

Figs. 5–7 are sketched for various values of magnetic parameter ($M$) on velocity, temperature and concentration profiles. Fig. 5 reveals that velocity profiles are impedes throughout the boundary layer with higher values of magnetic field parameter ($M$) in the both SWCNTs-water and MWCNTs-water nanofluid. This is due to the fact that the presence of magnetic field in the flow creates a force known as the Lorentz force which acts as a retarding force and consequently, the momentum boundary layer thickness decelerates throughout the flow region. Deterioration in the thickness of hydrodynamic boundary layer is more in MWCNTs-water nanofluid than the SWCNTs-water nanofluids with increasing values of ($M$). The temperature profiles of the both fluids
rises as the values of \((M)\) increases and this rise is more in SWCNTs-water nanofluid than the MWCNTs-water nanofluid (Fig. 6). We define the thermal energy as the additional force which drags the nanofluid from the influence of magnetic field. This additional force increases the thickness of the thermal boundary layer. From Fig. 7, we noticed exact reverse trend in concentration profiles comparing with temperature profiles with the rising values of \((M)\).

The sway of Buoyancy ratio parameter \((Nr)\) on velocity, temperature and concentration distributions is depicted in Figs. 8–10. Noticeable velocity deceleration is observed in the both nanofluids with the enhancing values of \((Nr)\) and is shown in Fig. 8. It is noticed that temperature profiles of the both nanofluids enriches with higher values of \((Nr)\). This is from the reality that higher the values of \((Nr)\) enhance the fluids temperature, so that thermal boundary layer thickness is elevated (Fig. 9). Furthermore, the concentration profiles also increases with \((Nr)\). It is observed that elevation in concentration profiles is more in MWCNTs-water than the SWCNTs-water nanofluids (Fig. 10).

The temperature and concentration distributions for various values of radiation parameter \((R)\) are illustrated in Figs. 11 and 12 in the both single and multi-walled CNTs-water created nanofluids. It is seen that as the values of thermal radiation parameter \((R)\) increases, the thermal boundary layer thickness is enhanced in both nanofluids. This is because the Rosseland radiative absorptive \(k^*\) decreases with increasing radiation parameter. As a result, the divergence of radiative heat flux \(\frac{\partial q_r}{\partial y}\) and
then rate of radiative heat transfer into the fluid increase. The larger radiative heat transfer is favourable for thermal boundary layer growth. However, the concentration profiles weaken in the fluid region as the values of (R) increases. It is perceived that this deceleration in solutal boundary layer thickness is more in MWCNTs-water than the SWCNTs-water nanofluids.

Figs. 13 and 14 reveals, the influence of the Prandtl number (Pr) on velocity and temperature profiles. The velocity profiles reduce throughout the boundary layer regime as the strength of (Pr) increases and this decrement is more in MWCNTs-water nanofluid than the SWCNTs-water nanofluid (Fig. 13). Furthermore, the temperature profiles also decreases with improved values of (Pr) in the both nanofluids. This is because of the reality that a fluid with large Prandtl number has low thermal diffusivity which causes low heat penetration. Consequently, the temperature distributions as well as thermal boundary layer thickness decreases and this deceleration is higher in MWCNTs-water than the SWCNTs-water nanofluids (Fig. 14).

The variations in non-dimensional velocity, temperature and concentration scatterings for diverse values of suction/injection parameter (Vo) are plotted in Figs. 15–17. It is perceived that all the profiles worsen throughout the boundary layer regime with rising values of suction parameter (Vo > 0) in the both nanofluids. This is because of the veracity that suction is taken away the warm fluid from the boundary layer regime, which causes the deceleration in the thickness of all the boundary layers. Moreover, the deterioration in the velocity and temperature of the fluid is more in MWCNTs-water than the SWCNTs-water nanofluids. However, the worsening in concentration profiles is higher in SWCNTs-water than the MWCNTs-water nanofluids (Fig. 17).

The influence of Biot number (Bi) on the momentum, thermal and solutal boundary layers is displayed in Figs. 18–20 for the both nanofluids. It is perceived from Figs. 18 and 19 that, the velocity and temperature of the fluid remarkably enhances and is more in SWCNTs-water than MWCNTs-water based nanofluid with higher values of (Bi). This is because of the fact that as (Bi) values improve means there exists higher internal thermal resistance of the cone than the thermal boundary layer; as a result the temperature of the fluid is enhanced in the boundary layer regime. However, the concentration profiles of the both nanofluids diminish as (Bi) values intensifying (Fig. 20).

Fig. 21 is a plot of solutal boundary layer in the both SWCNTs-water and MWCNTs-water nanofluids with various values of chemical reaction parameter (Cr). It is observed that the depreciation in concentration
profiles of SWCNTs-water nanofluid is slightly more than the SWCNTs-water based nanofluid.

The values of skin-friction coefficient (− f(0)), Nusselt number (θ(0)) and Sherwood number (S′(0)) for diverse values of pertinent parameters in the both SWCNTs-water and MWCNTs-water nanofluids are summarized in Table 3. It is reported that the rates of velocity declines in the both nanofluids, whereas, heat and mass transfer rates enhances in the both nanofluids with higher values of (φ). It is clear that the rate heat transfer is highly influenced by the volume fraction of nanoparticles and is enhanced in the boundary layer regime. It is found that, with increase in the values of magnetic field parameter (M), the skin-friction coefficient and Nusselt number impedes in SWCNTs-water nanofluid, whereas, escalates in MWCNTs-water nanofluid. However, the Sherwood number upsurge in the both nanofluids with increasing values of (M). It is also evident from this table that the values of both skin-friction coefficient and Sherwood number rises in the fluid regime as the values of radiation parameter (R) heightens in the both SWCNTs-water and MWCNTs-water nanofluid. However, the dimensionless heat transfer rates deteriorate with higher values of (R). The values of skin-friction coefficient, Nusselt number and Sherwood number are elevates in the fluid regime with improving values of chemical reaction parameter (Cr) in the both nanofluids. It is also evident that (− f(0)) rises as the values of suction/injection parameter (Vp > 0) increases in the SWCNTs-water nanofluid, whereas, (− f(0)) retards in the MWCNTs-water nanofluid. The non-dimensional heat and mass transfer rates elevates in the both nanofluids with increasing values of (Vp). It is found from this table that the skin-friction coefficient, Nusselt number and Sherwood number enhances with higher values Biot number (B1) in both SWCNTs-water and MWCNTs-water based nanofluids.

5. Conclusion

We have analyzed the boundary layer flow, heat and mass transfer analysis of SWCNTs-water and MWCNTs-water nanofluids through porous medium over a vertical cone by taking magnetic field, chemical reaction and suction/injection into the consideration. The important conclusions given by numerical solutions of the problem are as follows:

(i) Heat transfer enhancement is clearly noticed in the both nanofluids as the values of (φ) rises and this enhancement in heat transfer is higher in MWCNTs-water nanofluid than the SWCNTs-water based nanofluid.

(ii) The velocity of the fluid diminishes, whereas, temperature of the fluid boosted in the both nanofluids with rising values of (M).

(iii) The heat and mass transfer rates in the both nanofluids upsurges with higher values of (Vp > 0). Furthermore, the remarkable heat transfer enhancement is noticed in MWCNTs-water nanofluid than the SWCNTs-water based nanofluid.

(iv) Higher the values of Biot number (B1), enhances the temperature of the both fluids.

(v) Both heat and mass transfer rates escalate in the both nanofluids as the values of (Cr) rises.

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