Control volume finite element method for nanofluid MHD natural convective flow inside a sinusoidal annulus under the impact of thermal radiation

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Abstract

Control Volume Finite Element Method (CVFEM) is a new approach in which all advantages of finite volume method and finite element method are combined together. In this research, CVFEM is employed to simulate the impact of radiative heat transfer on magnetohydrodynamic free convection of water based nanofluid. Fe\textsubscript{3}O\textsubscript{4}–H\textsubscript{2}O ferrofluid has been used and viscosity of nanofluid is variable respect to magnetic field. Roles of the radiation parameter (Rd), numbers of undulations (N), Fe\textsubscript{3}O\textsubscript{4}–water volume fraction (\(\phi\)), Hartmann number (Ha) and the Rayleigh number (Ra) are depicted. Nu\textsubscript{ave} is present as a formula according to effect of various parameters. Results prove that the inner surface temperature decreases with the augment of buoyancy forces. Nu\textsubscript{ave} enhances with the augmentation of the thermal radiation parameter while it decreases with the augment of Ha and N.

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1. Introduction

Innovative kind of working fluid which can enhance thermal properties of fluid is Nanofluid. Wacker et al. [1] employed finite element method for linearized incompressible magnetohydrodynamic. Sheikholeslami et al. [2]...

Nayak et al. [11] reported the roles of nanofluid radiative heat transfer. Effect of shape factor on nanofluid properties has been considered by Sheikholeslami and Bhatti [12]. Shah et al. [13] investigated the heat transfer analysis in a second grade fluid over and oscillating vertical plate. Sheikholeslami and Rokni [14] demonstrated Brownian motion effects on nanofluid convective heat transfer in a porous cavity. Casson fluid flow over an infinite oscillating plate was investigated by Saqib et al. [15] in existence of chemical reaction and slip effect. Newly researchers used nanofluid as effective working fluid [16–44].

This article deals with the effect of a radiation source term on magnetohydrodynamic nanofluid flow in a sinusoidal annulus. The Control Volume Finite Element Method (CVFEM) is chosen to numerically simulate this problem. The roles of the radiation parameter, Fe$_3$O$_4$–water volume fraction, and the Hartmann and Rayleigh numbers are examined.
2. Problem statement

Fig. 1 demonstrates the boundary condition, geometry and sample element. The shape of inner cylinder can be generated with:

\[ r = r_{in} + A \cos (N (\zeta - \zeta_0)) \]  

(1)

where \( r_{in} \) is the radius of the base circle. The inner cylinder has a constant heat flux condition.

3. Governing equation and simulation

3.1. Governing formulation

Nanofluid free convection in a sinusoidal annulus is investigated in presence of Lorentz forces. Buoyancy force is considered according to Boussinesq approximation. The PDEs are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(2)

\[
\left( v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial x} \right) = \left[ -\sigma_{nf} B_y^2 u + \sigma_{nf} B_x B_y v + \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial x^2} \right) \mu_{nf} - \frac{\partial P}{\partial x} \right] (\rho_{nf})^{-1}
\]

(3)

\[
\rho_{nf} \left( \frac{\partial v}{\partial x} u + \frac{\partial v}{\partial y} v \right) = \mu_{nf} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{\partial P}{\partial y}
\]

\[ + B_y \sigma_{nf} B_x u - B_x \sigma_{nf} B_y v + (T - T_c) \beta_{nf} g \rho_{nf}, \]

(4)

\[ B_x = B_o \cos \lambda, \quad B_y = B_o \sin \lambda \]

\[
(\rho C_p)_{nf} \left( \frac{\partial T}{\partial y} + u \frac{\partial T}{\partial x} \right) = k_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial q_r}{\partial y},
\]

\[ q_r = -\frac{4 \sigma_e \partial T^4}{3 \beta_R} \frac{\partial T}{\partial y}, \quad T^4 \approx 4 T_c^3 T - 3 T_c^4 \]

(5)

\[(\rho C_p)_{nf}, (\rho \beta)_{nf}, \rho_{nf}, k_{nf} \text{ and } \sigma_{nf} \text{ are defined as:} \]

\[
(\rho C_p)_{nf} = (\rho C_p)_f (1 - \phi) + (\rho C_p)_s \phi
\]

(6)

\[
(\rho \beta)_{nf} = (\rho \beta)_f (1 - \phi) + (\rho \beta)_s \phi
\]

(7)

\[
\rho_{nf} = \rho_f (1 - \phi) + \rho_s \phi
\]

(8)

\[
k_{nf} = k_f \left( \frac{k_s + 2 k_f + 2 \phi(k_s - k_f)}{k_s - \phi(k_s - k_f) + 2 k_f} \right)
\]

(9)

\[
\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3 \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi}{\left( \frac{\sigma_s}{\sigma_f} + 2 \right) - \left( \frac{\sigma_s}{\sigma_f} - 1 \right) \phi}
\]

(10)

\( \mu_{nf} \) is estimated as [45]:

\[
\mu_{nf} = \left( 0.035 B^2 + 3.1 B - 27886.4807 \phi^2 + 4263.02 \phi + 316.0629 \right) e^{-0.01 T}.
\]

(11)

Properties of water and Fe\textsubscript{3}O\textsubscript{4} nanoparticles are presented in Table 1. The pressure source terms can be eliminated by using vorticity and stream function formulation:

\[
\omega + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = 0, \quad \frac{\partial \psi}{\partial x} = -v, \quad \frac{\partial \psi}{\partial y} = u.
\]

(12)
Fig. 1. (a) Geometry and the boundary conditions with (b) the mesh of Geometry considered in this work; (c) A sample triangular element and its corresponding control volume.
Introducing the following dimensionless quantities:

\[ P = \frac{p}{\rho_f (\alpha_f / L)^2}, \quad U = \frac{uL}{\alpha_f}, \quad V = \frac{vL}{\alpha_f}, \quad \Theta = \frac{T - T_c}{\Delta T}, \quad \Delta T = q'' L / k_f, \quad (X, Y) = \frac{(x, y)}{L}. \]  \hspace{1cm} (13)

In Eqs. (2)–(5) yields the following dimensional governing equations:

\[ \frac{\partial V}{\partial Y} + \frac{\partial U}{\partial X} = 0, \]  \hspace{1cm} (14)

\[ U \frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} V = \text{Pr} \left[ \frac{\mu_{nf} / \mu_f}{\rho_{nf} / \rho_f} \right] \left( \frac{\partial^2 U}{\partial Y^2} + \frac{\partial^2 U}{\partial X^2} \right) - Ha^2 \text{Pr} \left[ \frac{\sigma_{nf} / \sigma_f}{\rho_{nf} / \rho_f} \right] \left( B_y^2 U - B_x B_y V \right) - \frac{\partial P}{\partial X}. \]  \hspace{1cm} (15)

\[ V \frac{\partial V}{\partial Y} + U \frac{\partial V}{\partial X} = \text{Pr} \left( \frac{\partial^2 V}{\partial Y^2} + \frac{\partial^2 V}{\partial X^2} \right) \left[ \frac{\mu_{nf} / \mu_f}{\rho_{nf} / \rho_f} \right] \]  \hspace{1cm} (16)

\[ - Ha^2 \text{Pr} \left[ \frac{\sigma_{nf} / \sigma_f}{\rho_{nf} / \rho_f} \right] \left( B_x^2 V - B_y B_y U \right) - \frac{\partial P}{\partial Y} + Ra \text{Pr} \left[ \frac{\beta_{nf}}{\beta_f} \right] \Theta. \]
Fig. 3. Effect of radiation parameter on isotherms (left) and streamlines (right) contours \((Rd = 0 (\cdots), Rd = 0.8 (\cdots))\) when \(N = 3, \phi = 0.04.\)

\[
\frac{\partial \Theta}{\partial X} U + \frac{\partial \Theta}{\partial Y} V = \left[ \frac{(\rho C_P)_{nf} k_{nf}}{(\rho C_P)_{nf} k_f} \right] \left( 1 + \frac{4}{3} \left( \frac{k_{nf}}{k_f} \right)^{-1} Rd \right) \frac{\partial^2 \Theta}{\partial Y^2} + \left[ \frac{(\rho C_P)_{nf} k_{nf}}{(\rho C_P)_{nf} k_f} \right] \left( \frac{\partial^2 \Theta}{\partial X^2} \right)
\]

(17)

where the dimensionless and constant parameters are defined as follows:

\[
Pr = \frac{\nu_f}{\alpha_f}, Ra = \frac{g \beta_f q'' L^4}{(k_f \nu_f \alpha_f)}, Ha = LB_0 \sqrt{\sigma_f / \mu_f}, Rd = 4\sigma_e T_e^3 / (\beta_k k_f),
\]

(18)

and the boundary conditions are:

\[
\frac{\partial \Theta}{\partial n} = 1.0 \quad \text{on inner wall}
\]

\[
\Theta = 0.0 \quad \text{on outer wall}
\]

\[
\Psi = 0.0 \quad \text{on all walls}
\]

(19)

\(Nu_{loc}\) and \(Nu_{ave}\) over the inner wall can be obtained as:

\[
Nu_{loc} = \left( 1 + \frac{4}{3} Rd \left( \frac{k_{nf}}{k_f} \right)^{-1} \right) \frac{1}{\partial} \left( \frac{k_{nf}}{k_f} \right)
\]

(20)

\[
Nu_{ave} = \frac{1}{S} \int_0^S Nu_{loc} ds.
\]

(21)
Fig. 4. Effect of Rayleigh and Hartmann numbers on isotherms (left) and streamlines (right) contours when $N = 3$, $\phi = 0.04$, $Rd = 0.8$.

3.2. Numerical procedure

Combination of FEM and FVM leads to generate new powerful method namely CVFEM. Linear interpolation is employed for the estimation of scalars in the triangular element which is considered as a building block (Fig. 1(b)). Gauss–Seidel method is utilized to solve the algebraic equations. More information exists in the new suitable book [46].
Fig. 5. Effect of Rayleigh and Hartmann numbers on isotherms (left) and streamlines (right) contours when $N = 4$, $\phi = 0.04$, $Rd = 0.8$. 
Fig. 6. Effect of Rayleigh and Hartmann numbers on isotherms (left) and streamlines (right) contours when $N = 5, \phi = 0.04, Rd = 0.8$.

4. Grid independent test and validation

In order to carry out mesh independent outputs, different meshes are tested. As demonstrated in Table 2, a grid size of $81 \times 241$ is selected to complete this research. Fig. 2 and Table 3 demonstrate the correctness of the CVFEM code for a nanofluid magnetohydrodynamic flow [47–49].
Fig. 7. Effects of the number of undulations, Hartmann and Rayleigh numbers on local Nusselt number when $Ra = 0.8$, $\phi = 0.04$. 
Table 1
Thermo physical properties of water and nanoparticles.

<table>
<thead>
<tr>
<th></th>
<th>( \rho ) (kg/m(^3))</th>
<th>( C_p ) (J/kg K)</th>
<th>( k ) (W/m K)</th>
<th>( \beta \times 10^5 ) (K(^{-1}))</th>
<th>( \sigma ) ((\Omega) m(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure water</td>
<td>997.1</td>
<td>4179</td>
<td>0.613</td>
<td>21</td>
<td>0.05</td>
</tr>
<tr>
<td>Fe(_3)O(_4)</td>
<td>5200</td>
<td>670</td>
<td>6</td>
<td>1.3</td>
<td>25000</td>
</tr>
</tbody>
</table>

Table 2
Comparison of the average Nusselt number \( \bar{N}_u \) for different grid resolution at \( Ra = 10^5 \), \( N = 5 \), \( Ha = 60 \), \( Rd = 0.8 \) and \( \phi = 0.04 \).

<table>
<thead>
<tr>
<th>Mesh size in radial direction ( \times ) angular direction</th>
<th>( \bar{N}_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>61 ( \times ) 181</td>
<td>2.5665</td>
</tr>
<tr>
<td>71 ( \times ) 211</td>
<td>2.5188</td>
</tr>
<tr>
<td>81 ( \times ) 241</td>
<td>2.2234</td>
</tr>
<tr>
<td>91 ( \times ) 271</td>
<td>1.0856</td>
</tr>
<tr>
<td>101 ( \times ) 301</td>
<td>1.0110</td>
</tr>
</tbody>
</table>

Table 3
\( \bar{N}_u \) for various \( Gr \) and \( Ha \) at \( Pr = 0.733 \).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>( Gr = 2 \times 10^4 )</th>
<th>( Gr = 2 \times 10^5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Ha )</td>
<td>Present</td>
<td>Rudraiah et al. [49]</td>
<td>Present</td>
<td>Rudraiah et al. [49]</td>
</tr>
<tr>
<td>0</td>
<td>2.5665</td>
<td>2.5188</td>
<td>5.093205</td>
<td>4.9198</td>
</tr>
<tr>
<td>10</td>
<td>2.26626</td>
<td>2.2234</td>
<td>4.9047</td>
<td>4.8053</td>
</tr>
<tr>
<td>50</td>
<td>1.09954</td>
<td>1.0856</td>
<td>2.67911</td>
<td>2.8442</td>
</tr>
<tr>
<td>100</td>
<td>1.02218</td>
<td>1.0110</td>
<td>1.46048</td>
<td>1.4317</td>
</tr>
</tbody>
</table>

Fig. 8. Effects of Hartmann number and number of undulations on local Nusselt number when \( Rd = 0 \).

5. Results and discussion

In this research, the impact of the Lorentz forces on Fe\(_3\)O\(_4\)–H\(_2\)O nanofluid natural convection is simulated considering thermal radiation source term. Both the inner and outer cylinders are sinusoidal with the same shape and the inner one has a constant heat flux condition. Non-uniform viscosity has been taken into consideration. Numerical simulations are examined for various values of radiation parameter \( (Rd = 0 \) to \( 0.8 \)) , volume fraction of Fe\(_3\)O\(_4\)–H\(_2\)O \((\phi = 0 \) to \( 0.04 \)) , Rayleigh number \((Ra = 10^3, 10^4 \) and \( 10^5 \)) , number of undulations \((N = 3, 4 \) and \( 5 \)) and Hartmann number \((Ha = 0 \) to \( 60 \)).
The impact of the radiation parameter on the nanofluid treatment is demonstrated in Fig. 3. The inner wall temperature augments with increase of radiation parameter. Also, augmenting $Rd$ makes the nanofluid motion to enhance. This impact is more evident when the convection mechanism is more prominent.
Figs. 4–6 depict the impact of $Ha$, $N$ and $Ra$ on isotherms and streamlines. Conduction is dominant at low Rayleigh number. $|\Psi_{\text{max}}|$ augments as $Ra$ augments and it decreases as the Lorentz force enhances. As Lorentz forces increase, the center of the vortices move to $x = 0$. As $Ra$ enhances, convection becomes stronger. At $N = 3$, three
main cells exist in the streamlines and their centers move downward in the existence of the Lorentz forces. At $N = 4$, the middle cell disappears in presence of the magnetic forces. As $N$ increases, the distortion of the isotherms decreases. Increasing the Hartmann number causes the thermal plume to diminish. So, $Nu$ decreases with the augment of Lorentz forces. At $N = 5$, increasing the Lorentz force causes the main vortices to convert into three smaller ones.

Figs. 7–9 depict the impact of $N$, $Rd$, $Ra$ and $Ha$ on $Nu_{loc}$, $Nu_{ave}$. The correlation corresponding to the active parameters is:

\[
\begin{align*}
Nu_{ave} &= 2.93 - 0.07N - 1.15\log (Ra) + 0.19Rd + 0.02Ha \\
& - 0.019N \log (Ra) - 0.1N Rd + 4.2 \times 10^{-4}N Ha + 0.36 \log (Ra) Rd \\
& - 0.015\log (Ra) Ha - 0.8 \times 10^{-2}Ra Ha + 0.01N^2 + 0.25(\log (Ra))^2 + 1.23Rd^2 \\
& + 4.3 \times 10^{-4}Ha^2.
\end{align*}
\] (22)

The number of extrema in the $Nu_{loc}$ profile relies on the presence of the thermal plume and the number of undulations. $Nu_{ave}$ augments with the rise of the radiation parameter. As the Rayleigh number increases, the temperature reduces and in turn, $Nu_{ave}$ augments.

6. Conclusions

In this article, impact of magnetic field on thermal radiation and convection heat transfer of ferrofluid is examined. CVFEM is used for simulating the effect of $Fe_2O_3$–water volume fraction, number of undulations, Rayleigh number, Hartmann number and the radiation parameter on flow style and heat transfer rate. Results depict that the inner wall temperature reduces with the augment of the buoyancy forces while it increases with the rise of $Rd$, $Ha$. As the number of undulations augments, the convective heat transfer decreases.

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