Mixed convection inside a fluid-porous composite cavity with centrally rotating cylinder

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Abstract
The mixed convection in a fluid-porous composite medium lying inside a square cavity with a centrally rotating cylinder has been investigated in the present work. The bottom half of the cavity is filled with a porous material and the top half is filled with a clear fluid. The bottom wall of the cavity is at a higher temperature, and the top wall is at a lower temperature. The vertical walls are thermally insulated. The convection inside the cavity sets through the combined mechanisms of the thermal buoyancy force and the shearing action of the centrally rotating cylinder. The relative importance of each driving mechanism over the other is featured through the Richardson number. The Darcy–Brinkman–Forchheimer equation is used for the flow modeling in the porous medium, and a single-domain approach is adopted for the numerical solution in the fluid-porous composite medium. The simulation is carried out with ANSYS Fluent software, and a parametric analysis involving the Rayleigh
number (Ra), Richardson number (Ri), and the Darcy number (Da) is conducted showing their effects on the flow and heat transfer. The phenomena are quite interesting at higher Darcy number and Rayleigh number. The distributions of isotherms, streamlines, and vector plots are plotted, along with the local Nusselt numbers for different parameters, to explore the underlying physics of the phenomenon. The system is found stable at lower Darcy number, and the heat transfer is minimum around Ri = 10. From the numerical study, an empirical correlation for the average Nusselt number is developed as a function of the other dimensionless numbers.

KEYWORDS
ANSYS Fluent, composite medium, Darcy–Brinkman–Forchheimer equation, mixed convection, rotating cylinder, single-domain approach

1 | INTRODUCTION

Mixed convection implies the coupled fluid flow and heat transfer by the combined action of pressure, shear, and buoyancy force. Mixed convection inside a confined porous medium has been a subject of much interest owing to its wide range of industrial applications such as cooling of electronic systems, thermal hydraulics of nuclear reactors, chemical processing equipment, geothermal energy systems, extraction of oil from underground reservoirs, flow in biological materials and so on. The heat transfer and the flow mechanisms in a confined porous medium, where mixed convection occurs, do depend on the nondimensional parameters (i.e., Richardson number, Ri and Reynolds number, Re) and the porous medium suppresses the convective currents.\(^1\),\(^2\) Mixed convection parameters such as Péclet number (Pe) and Rayleigh number (Ra) characterizes the geometrical configurations (i.e., the size of the square vent) provided to impose forced convection inside the domain.\(^3\)

Many previous studies on convection inside a confined porous medium have considered either forced convection or natural convection. Heat transfer enhancement is affected by the porous layer thickness and permeability of the medium, when forced convection occurs inside an annular duct partially filled with a non-Darcian porous medium.\(^4\) The non-Darcian natural convective flow is influenced by porosity, Darcy number (Da), and Rayleigh number (Ra).\(^5\) It is reported that at lower Da and higher Ra, the variation of average Nusselt number, \(\overline{Nu}\), with porosity is significant.\(^6\) The inertial effects enhance the momentum and energy transport in a lid-driven cavity filled with porous medium, and the flow strength increases with Da.\(^7\) The time required to reach the steady state in a confined porous medium is longer for low Ra, and shorter for higher Ra.\(^8\) The location of the heater inside a lid-driven porous enclosure is also important for the convective heat transfer.\(^9\) The Darcy–Brinkman–Forchheimer model, adopted for solution of natural convection in a porous medium problem, produces high accuracy results even at high Ra. The same is experimentally validated by Kladias and Prasad.\(^10\) The Forchheimer model underpredicts the velocity magnitude whereas the Darcy model overpredicts the same at all
Double-diffusive buoyancy opposed natural convection in a porous cavity for different configurations is investigated by Jena and colleagues and they reported the dependency of flow strength on the permeability of porous medium. They have also reported that at low $Da$, the Darcy–Brinkman–Forchheimer model reduces to the Darcy model as the flow significantly loses its strength, and heat transfer approaches to conduction regime.

One of the major debates for flow in porous media concerns with the local thermal equilibrium (LTE) and the local thermal nonequilibrium (LNTE) approaches. In LTE approach, the circulating fluid in the pore and the solid pore matrix are assumed to be in local thermal equilibrium with each other such that heat transfer is inhibited. The LTE approach is the classical approach for solving the porous media problems over past several decades. However, in last few decades, researchers focused on local thermal nonequilibrium condition between the circulating fluid in the pore and the solid porous matrix, which results in local heat transfer between the fluid and the solid. Thus, to address the local thermal nonequilibrium condition, governing energy equations for the circulating fluid and the solid matrix are solved separately. Minkowycz and colleagues are the pioneers in differentiating the role of LNTE approach from LTE approach based on Sparrow number, which suggests the existence of a thermal equilibrium between the fluid and the porous matrix for a wide range of applications. They showed that for large Sparrow numbers (greater than 10) both the LNTE and the LTE approaches produced equal results, as the fluid and the porous matrix resided in a thermal equilibrium state. The thermal nonequilibrium condition exists when the flow occurs in a narrow geometry, that is, the ratio of the characteristic length to the mean pore diameter is small, and the thermal conductivity of the porous matrix is higher than that of the fluid. Many studies such as Vafai and Sozen, Amiri and Vafai, Nield, Lee and Vafai, Bortolozzi and Daiber, Baytas and Pop, Khashan and colleagues, Ouyang and colleagues, Mahmoudi, and Torabi and colleagues focused on LNTE approach. In a thumb rule, if the thermal conductivities of the fluid and the porous matrix are of equal order, then the LTE model is sufficient to produce accurate results.

All the works discussed above are related to a macroscopic approach for flow within a solid porous matrix. However, in many natural and artificial systems, the flow occurs through a composite layer. Such composite layers are encountered in groundwater seepage, dispersion of chemical contaminants through water-saturated soil, bio-facilitated transport of pollutants, disposal of pathogenic organisms, thermal insulation, storage of nuclear waste, fuel cells, heat removal from nuclear fuel debris in nuclear reactors, thermal energy storage systems, solar collectors with a porous absorber, and so on. The composite layer consists of a fluid saturated porous layer followed by an overlaying free fluid layer, called as “plane media,” “free fluid media,” or “clean media.” Beavers and Joseph are the pioneers to describe the flow, heat, and mass transfer in a composite system. They proposed a slip velocity boundary condition at the interface between the clear fluid and the porous medium. Saffman and Taylor reported on the boundary conditions at the surface of the porous medium. Larson and Higdon performed microscopic analysis of flow near the surface of two-dimensional (2D) porous media. Salinger and colleagues, Gartling and colleagues, Nassehi and colleagues used finite element method to solve Navier–Stokes and Darcy law equations, governing the composite media flow. Salinger and colleagues and Gartling and colleagues used the slip velocity condition at the interface, while Nassehi and colleagues avoided the slip velocity interfacial condition.

In the studies of Neale and Nader and Costa and colleagues, continuous stress across the interface was assumed. Using a rigorous volume-averaging method, which accounts for the stress jump at the interface, Ochoa–Tapia and Whitaker developed a new interfacial boundary condition. Sahraoui and Kaviany studied the effect of slip and no-slip boundary conditions at the interface of plain and porous media. Kuznetsov investigated the influence of stress jump at the interface of porous and
clear media on flow. Chandresir and colleagues\textsuperscript{43,44} estimated the prior stress jump coefficient for interfacial boundary condition. Several studies such as Beckermann and colleagues,\textsuperscript{45,46} Song and Viskanta,\textsuperscript{47} Layton and colleagues,\textsuperscript{48} Gobin and colleagues,\textsuperscript{49} Mharzi and colleagues,\textsuperscript{50} and Bennacer and colleagues\textsuperscript{51} also worked on natural convection in an enclosure filled with the composite fluid porous layer. Recently, Chamkha and colleagues\textsuperscript{52} investigated the mixed convection in a partially layered porous cavity utilizing the Galerkin weighted residual method to solve the governing equations.

The above studies indicate that the fluid circulation induced by a rotating cylinder and the heat transfer in the fluid-porous composite medium has not been thoroughly investigated. In the present work, a detailed parametric study investigating the effect of Richardson number, Darcy number, and Rayleigh number on the combined flow and heat transfer mechanisms is delineated. Further, the role of conducting rotating cylinder on fluid flow and heat transfer inside a fluid-porous composite medium is reported. A correlation between average heat transfer as a function of affecting parameters ($Ri$, $Da$, and $Ra$) is also established.

2 | MATHEMATICAL MODELING AND GOVERNING EQUATIONS

In the present work, a 2D computational domain is divided horizontally into two parts as shown in Figure 1. The bottom wall is at a higher temperature $T_H$, and the top wall is at a lower temperature $T_L$. The vertical walls are thermally insulated. The entire domain is assumed to be isotropic with constant physical properties except for the density, which is assumed to follow Boussinesq approximation. The Darcy–Brinkman–Forchheimer model is used to model the fluid saturated porous medium, in which the linear viscous term represents the solid matrix drag, and the quadratic term represents the inertial drag. The forced convection is realized through the rotation of a concentric conducting cylinder. The LTE
approach is considered since the ratio of the characteristic length to the mean pore diameter is high. And the thermal conductivities of the fluid and the porous matrix are of the equal order of magnitudes, which leads to a large Sparrow number as discussed by Minkowycz and colleagues. A single-domain approach described by Goyeau and colleagues, and Gobin and Goyeau is taken for the numerical solution.

The dimensional equations for mass continuity, momentum, and energy transport is represented as

**Mass continuity equation**

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(1)

**X-momentum equation**

\[
\frac{\rho_f}{\varphi^2} \left( \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} \right) = -\frac{\partial p}{\partial x} + \frac{\mu_f}{\varphi} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \delta \left( \frac{\mu_f}{K} + \rho_f \frac{F_	ext{c}}{\sqrt{K}} \sqrt{u_v^2 + v^2} \right) u
\]  

(2)

**Y-momentum equation**

\[
\frac{\rho_f}{\varphi^2} \left( \frac{\partial u v}{\partial x} + \frac{\partial v^2}{\partial y} \right) = -\frac{\partial p}{\partial y} + \frac{\mu_f}{\varphi} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \delta \left( \frac{\mu_f}{K} + \rho_f \frac{F_	ext{c}}{\sqrt{K}} \sqrt{u_v^2 + v^2} \right) v
\]  

\[+\rho_{f_0} g \left( 1 - \beta (T - T_c) \right) \]

(3)

**Energy equation**

\[
\left( \frac{\partial u T}{\partial x} + \frac{\partial v T}{\partial y} \right) = \pi \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  

(4)

where

\[
\varphi = \begin{cases} 
1 & \text{for pure fluid layer} \\
\epsilon & \text{for porous layer}
\end{cases}, \quad \delta = \begin{cases} 
0 & \text{for pure fluid layer} \\
1 & \text{for porous layer}
\end{cases}, \quad \pi = \begin{cases} 
\alpha_f & \text{for pure fluid layer} \\
\alpha_{\text{eff}} & \text{for porous layer}
\end{cases}
\]

Adopting the LTE condition between the fluid and the porous matrix, the effective thermal diffusivity and the thermal conductivity are represented as

\[
\alpha_{\text{eff}} = \frac{k_{\text{eff}}}{(\rho C_p)_f} \quad \text{and} \quad k_{\text{eff}} = (1 - \epsilon) k_s + \epsilon k_f
\]

(6)

The following scales have been used for making the flow field variable dimensionless:

\[
X, Y = \frac{x, y}{L}, \quad U, V = \frac{u, v}{\alpha_f / L}, \quad \Theta = \frac{T - T_c}{T_H - T_c}, \quad P = \frac{(p + \rho_{f_0} g y)L^2}{\rho_f \alpha_f^2}, \quad R = \frac{r}{L}, \quad \Omega = \frac{\omega L^2}{\alpha_f}
\]

The nondimensional forms of the governing equations are as follows:
Mass continuity equation:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{7}
\]

X-momentum equation:

\[
\frac{1}{\varphi^2} \left( \frac{\partial U^2}{\partial X} + \frac{\partial UV}{\partial Y} \right) = -\frac{\partial P}{\partial X} + \frac{\Pr}{\varphi} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) - \delta \left( \frac{\Pr}{Da} + \frac{Fc}{\sqrt{Da}} \sqrt{U^2 + V^2} \right) U \tag{8}
\]

Y-momentum equation:

\[
\frac{1}{\varphi^2} \left( \frac{\partial UV}{\partial X} + \frac{\partial V^2}{\partial Y} \right) = -\frac{\partial P}{\partial Y} + \frac{\Pr}{\varphi} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) - \delta \left( \frac{\Pr}{Da} + \frac{Fc}{\sqrt{Da}} \sqrt{U^2 + V^2} \right) V + Ra \Pr \Theta \tag{9}
\]

Energy equation:

\[
\left( \frac{\partial U \Theta}{\partial X} + \frac{\partial V \Theta}{\partial Y} \right) = \frac{\alpha_{\text{eff}}}{\pi} \left( \frac{\partial^2 \Theta}{\partial X^2} + \frac{\partial^2 \Theta}{\partial Y^2} \right) . \tag{10}
\]

The dimensionless variables are expressed as

\[
\Pr = \frac{v_f}{\alpha_f}, \quad Da = \frac{K}{L^2}, \quad Re = \frac{2 \omega r^2}{v_f}, \quad Ra = \frac{g \beta_f (T_h - T_c) L^3}{v_f \alpha_f}, \quad Gr = \frac{Ra}{\Pr}, \quad Ri = \frac{Gr}{Re^2}.
\]

No-slip and no penetration, velocity boundary conditions are considered for all solid walls and along the rotating cylinder wall:

\[
U, V = 0, \quad \forall \left\{ \begin{array}{l} X, Y = 0, 1 \\ X, Y \in (X + 0.5)^2 + (Y + 0.5)^2 = R^2 \end{array} \right. \tag{11}
\]

A single-domain approach described by Goyeau and colleagues\textsuperscript{53} and Gobin and Goyeau\textsuperscript{54,55} is used for solution. In the single-domain approach, the velocity continuity and the mass conservation are taken across the porous and clean media. Mathematically, the same is expressed as

\[
\left\{ \begin{array}{l} U_f = U_p, V_f = V_p \\ \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)_f = \left( \frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X} \right)_p \end{array} \right\} \forall Y = 0.5, X \in (0, 0.5 - 0.5R] \cup [0.5 + 0.5R, 1) . \tag{12}
\]

The base wall is at a higher temperature, and the top wall is at a lower temperature. The vertical boundary walls are thermally adiabatic.

\[
\left\{ \begin{array}{l} \Theta = 1 \forall Y = 0 \\ \Theta = 0 \forall Y = 1 \\ \frac{\partial \Theta}{\partial X} = 0 \forall X = 0, 1, Y \in (0, 1) \end{array} \right\} X \in (0, 1) \tag{13a}
\]
Temperature continuity and heat flux continuity boundary conditions are taken along the solid cylinder perimeter:

\[
\frac{\partial \Theta_f}{\partial n} = K_1 \frac{\partial \Theta_c}{\partial n}, \Theta_f = \Theta_c \quad X, Y \in (X + 0.5)^2 + (Y + 0.5)^2 = R^2 \quad (13b)
\]

\[
\frac{\partial \Theta_p}{\partial n} = K_2 \frac{\partial \Theta_c}{\partial n}, \Theta_p = \Theta_c
\]

\[
\begin{cases}
X, Y \in (X + 0.5)^2 + (Y + 0.5)^2 = R^2
\end{cases}
\]

\[\hat{n}\] represents the unit normal direction at the cylinder surface. At the fluid porous interface, temperature continuity and heat flux continuity are maintained.

\[
\begin{align*}
\Theta_f &= \Theta_p, \\
\frac{\partial \Theta_f}{\partial Y} &= \frac{k_{eff}}{k_f} \frac{\partial \Theta_p}{\partial Y}
\end{align*}
\]

\[\forall Y = 0.5, X \in (0, 0.5 - 0.5R] \cup [0.5 + 0.5R, 1). \quad (14)
\]

The dimension of the cavity is 1 m $\times$ 1 m, and the diameter of the rotating cylinder is 0.4 m. Water is taken as the circulating fluid with $\rho = 1000 \text{ kg m}^{-3}$, $C_p = 4.182 \text{ KJ kg}^{-1} \text{ K}^{-1}$, $k = 0.6 \text{ WmK}$, and $\mu = 5.5 \times 10^{-4} \text{ Pa}s$. All properties are taken at a mean temperature ($T_m = 0.5 \times (T_H + T_C)$). Aluminum with $k = 202.4 \text{ WmK}$ is used as a material for rotating cylinder. The thermal conductivity of the porous material is 0.173 W m$^{-1}$ K$^{-1}$, and the porosity of the porous medium is 0.6.

The local Nusselt numbers at the bottom and the top walls are expressed as\(^{45}\)

\[
Nu = \frac{\bar{h}_Y L}{k_f} = \left[ \xi \left( \frac{k_{eff}}{k_f} - 1 \right) + 1 \right] \frac{\partial \Theta}{\partial Y}, \quad X \in (0, 1).
\]

The average Nusselt number is defined as

\[
\overline{Nu} = \int_{X=0}^{1} Nu dX, \quad \forall Y = 0, 1 \quad (16)
\]

where $\xi$ is a binary parameter, and is defined as

\[
\xi = \begin{cases} 
1, & \forall Y = 0, \text{ i.e., porous layer} \\
0, & \forall Y = 1, \text{ i.e., fluid layer}
\end{cases}
\]

3 | NUMERICAL TECHNIQUES AND VALIDATION

ANSYS Workbench 15.0 is used for both geometry formation and meshing. Coupled flow algorithm of the Fluent solver is taken for pressure and velocity coupling. Least square cell-based spatial gradient formulation is used for the gradient computation in the generated unstructured mesh. Second-order upwinding is adopted for the pressure, momentum, and the energy equations. Underrelaxation factor value of 0.75 is used for both the momentum and the pressure equations. A mesh size of 0.013 m is taken with 6463 nodes and 6152 elements, after a rigorous mesh sensitivity study (Table 1). A residual of $10^{-6}$ is taken as convergence criteria for all the variables. The adopted mesh structure is shown in Figure 2, and the results of the grid sensitive analysis are shown in Table 1.
<table>
<thead>
<tr>
<th>Element size (m)</th>
<th>Nodes</th>
<th>Elements</th>
<th>Error convergence criteria</th>
<th>Heat flux Bottom</th>
<th>Heat flux Top</th>
<th>Nu</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.09</td>
<td>230</td>
<td>182</td>
<td>$10^{-6}$</td>
<td>111.164</td>
<td>–111.20</td>
<td>8.81</td>
<td>68</td>
</tr>
<tr>
<td>0.08</td>
<td>266</td>
<td>214</td>
<td>$10^{-5}$</td>
<td>117.5</td>
<td>–117.45</td>
<td>9.47</td>
<td>69</td>
</tr>
<tr>
<td>0.04</td>
<td>859</td>
<td>752</td>
<td>$10^{-6}$</td>
<td>113.39</td>
<td>–113.39</td>
<td>9.449</td>
<td>96</td>
</tr>
<tr>
<td>0.03</td>
<td>1449</td>
<td>1324</td>
<td>$10^{-6}$</td>
<td>110.536</td>
<td>–110.536</td>
<td>9.21</td>
<td>139</td>
</tr>
<tr>
<td>0.02</td>
<td>2818</td>
<td>2617</td>
<td>$10^{-6}$</td>
<td>109.3</td>
<td>–109.3</td>
<td>9.104</td>
<td>224</td>
</tr>
<tr>
<td>0.015</td>
<td>4862</td>
<td>4593</td>
<td>$10^{-6}$</td>
<td>110.41</td>
<td>–110.41</td>
<td>9.2</td>
<td>353</td>
</tr>
<tr>
<td>0.013</td>
<td>6463</td>
<td>6152</td>
<td>$10^{-6}$</td>
<td>107.1</td>
<td>–107.1</td>
<td>8.93</td>
<td>405</td>
</tr>
<tr>
<td>0.012</td>
<td>7305</td>
<td>6972</td>
<td>$10^{-6}$</td>
<td>106.97</td>
<td>–106.97</td>
<td>8.9</td>
<td>446</td>
</tr>
<tr>
<td>0.011</td>
<td>8710</td>
<td>8347</td>
<td>$10^{-6}$</td>
<td>107.375</td>
<td>–107.375</td>
<td>8.94</td>
<td>524</td>
</tr>
</tbody>
</table>

**FIGURE 2**  Mesh structure used for computation

Before analyzing the results, validations are made against the relevant works from the literature for two cases, that is, for mixed convection inside a domain with rotating cylinder and a differentially heated rectangular cavity filled with porous media. For mixed convection inside the cavity with the centrally rotating cylinder, validation is done with the work of Liao and Lin. The variation of the Nu around the cylinder is calculated for different Ri and Ra. The same is presented in Figures 3(a) and (b), which indicate the close agreement of the present results with Liao and Lin.

The default fluent solver considers modified Darcy equation for modeling in porous media. In the present case, Forchheimer term is added to fluent solver library as a source term in the momentum equation, through a user-defined function (UDF). The semiempirical correlation developed by Ergun for packed bed porous media is used for computation of viscous and inertial drag. Irmay has shown
the applicability of the Ergun equation for a wide range of Reynolds number. Ergun correlation is expressed as

$$\frac{|\nabla p|}{\Delta x} = \frac{150 \mu (1 - \epsilon)^2}{D_p^2 \epsilon^3} v_\infty + \frac{1.75 \rho (1 - \epsilon)}{D_p \epsilon^3} v_\infty^2.$$  \hspace{1cm} (18)

From comparison of the above correlation with the source term in momentum equations, permeability and inertial loss coefficients are calculated as follows:

$$K = \frac{D_p^2 \epsilon^3}{150 (1 - \epsilon)^2}.$$  \hspace{1cm} (19)

$$F_C = \frac{1.75}{\sqrt{150 \epsilon^{3/2}}}. $$  \hspace{1cm} (20)

The same is also found in the work of Alazmi and Vafai, Du Plessis, and Mahmoudi. The validation for natural convection in a differentially heated cavity containing porous medium is done with the experimental work of Kladias and Prasad. The computed $\overline{Nu}$ matches closely with the experimental values for different $Ra$ and is shown in Figure 4.

4 | RESULTS AND DISCUSSION

Results of the parametric study involving the variation of Rayleigh number, Darcy number, and Richardson number with a meaningful combination and their effect on flow and heat transfer mechanism are discussed in the subsequent subsections.

4.1 | Effect of Richardson number

The Richardson number signifies the relative importance of natural convection with respect to the forced convection. It is specified with respect to the externally imposed velocity. Here the external velocity implies the rotation of the cylinder. Therefore, the variation in $Ri$ is brought by varying the
FIGURE 4  Comparison of average Nusselt number values for different Rayleigh number with the experimental work of Kladias and Prasad\textsuperscript{10}

![Nusselt number vs Rayleigh number graph]

FIGURE 5  Streamlines, vector plots and isotherms contour for different $R_i$ for $Da = 10^{-2}$ and $Ra = 10^5$

![Streamlines, vector plots and isotherms grid]

angular rotation of the cylinder about its center. The effect of $R_i$ for $Ra = 10^5$ and $Da = 10^{-2}$ is shown in Figure 5. At low Richardson number, the forced convection dominates over the free convection, which is due to the high-speed rotation of the central cylinder. Figure 5(f) shows that more than 75% of the flow in the cavity is dominated by forced convection due to the cylinder rotation. At $R_i = 0.1$, the rotational speed of the cylinder is high, and the surrounding fluids start rotating with it in an anticlockwise manner. The flow strength is intensive near the cylinder wall. At the same time, due
FIGURE 6 Local Nusselt number distribution along the isothermal walls for different Richardson number ($Da = 10^{-2}$, $Ra = 10^5$)

to the imposed vertical thermal gradient, a free convection current emerges from the heated bottom plate and tries to reach the top cool plate to exchange heat. However, the forced convection current due to cylinder rotation opposes it, and thus a squeezed clockwise circulation appears at the left part of the cavity, which occupies 20% to 25% of the cavity area. Buoyancy circulation exists at the top-right corner where the shearing effect of cylinder rotation loses its intensity.

The corresponding isotherms and streamlines are shown in Figures 5(k) and (a), respectively. It is marked from the isotherm distribution (Figure 5k) that the temperature is greater near the vertical walls compared to the core of the cavity, where it is almost constant due to the continuous stirring effect of the cylinder. At $Ri = 1$, the strength of the free and forced convection are of the equal order of magnitude, and they try to counterbalance each other. The bottom of the cavity is driven by the shearing action of the cylinder, which results in an anticlockwise circulation. However, at the top corners, the shearing action loses its strength and unable to penetrate. At these locations, buoyancy current drives the flow (Figure 5b). From the isotherm pattern (Figure 5g), it is seen that the vicinity of the right wall is heated, while the vicinity of left wall is relatively cooled. Further increase in $Ri$ results, decrease in the angular speed of the cylinder, and the natural convection takes a dominant role in fluid circulation. For $Ri = 10$ and $Ri = 100$, both the free and forced convection current mix with each other and forms an anticlockwise rotating mixed convection circulation (Figure 5h and i). For the same case, the isotherms are stratified near the top and bottom isothermal wall (Figure 5m and n).

For $Ri = \infty$, the rotating cylinder becomes static, and its shearing action disappears. The free convection is responsible for entire fluid circulation, which is also noticed in Figure 5(j). The clockwise circulation due to buoyancy effect fills the entire cavity and isotherms are oriented accordingly (Figure 5o). Figure 6 shows the local $Nu$ variation with respect to $Ri$ along the bottom wall and the top wall for $Da = 10^{-2}$ and $Ra = 10^5$. Both the heat flux and the $\overline{Nu}$ decrease as the $Ri$ is increased, however, after $Ri = 10$, $\overline{Nu}$ and heat flux increase again, which signifies the dominance of free convection after $Ri = 10$ (Table 2). Figure 6 also shows the same trend for local $Nu$. The distribution of local $Nu$, switching from shearing action to buoyancy effect is shown in Figure 6.

4.2 Effect of Darcy number

The Darcy number deals with the relative permeability of the medium. The lower $Da$ indicates low permeable medium, and higher $Da$ indicates high permeable medium. The variation of flow structure with $Da$ is shown in Figure 7 for $Ri = 1$ and $Ra = 10^6$. The flow gradually loses its strength in the
TABLE 2 Heat transfer record for different cases

<table>
<thead>
<tr>
<th>Ri</th>
<th>Ra</th>
<th>Da = 10^{-2} Heat flux (W/m²)</th>
<th>Da = 10^{-4} Heat flux (W/m²)</th>
<th>Da = 10^{-6} Heat flux (W/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Bottom wall</td>
<td>Top wall</td>
<td>Bottom wall</td>
</tr>
<tr>
<td>0.1</td>
<td>10^3</td>
<td>54.64</td>
<td>−54.59</td>
<td>4.55</td>
</tr>
<tr>
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saturated porous medium with a decrease in $Da$. The isotherms stratify in the saturated porous medium with a decrease in $Da$, which is the feature of the conductive mode of heat transfer (Figure 7i). The variation of local $N_u$ with $Da$ is shown in Figure 8. The trend shows a significant decrease in heat flux and $N_u$ as $Da$ decreases. For $Ri = 0.1$, $Ri = 100$, and $Ri = \infty$ with $Ra = 10^6$, oscillations are observed in the domain for higher permeability, however, as $Da$ decreases, the flow becomes stable. Formation of secondary loops near the wall, lead the variation in local $N_u$ and the heat transfer coefficient. For $Da = 10^{-6}$ and $Ri = \infty$, the flow is not able to permeate out of the porous structure, and four loops are observed at four quadrature of the domain irrespective of $Ra$ as shown in Figure 9(j). A slight rotation of cylinder induces a thinner velocity boundary layer around the cylinder as shown in Figure 9(h). Due to lower $Da$, the fluid particles experience high resistance in porous medium, resulting in quite nominal activities compared to their strength in the plain medium. The fluid particles are injected into the left side of the domain for a thin layer owing to the anticlockwise rotation of the cylinder. This thin layer divides the porous zone by creating two loops inside the porous zone. For lower $Da$, the isotherms stratify at the porous zone, indicating the conductive mode of heat transfer (Figure 9k–o). The decrease in $Da$ decreases the heat flux and $N_u$.

4.3 | Effect of Rayleigh number

$Ra$ signifies the dominance of buoyancy-driven flow. Keeping $Ri$ constant, increasing $Ra$ increases the natural convection strength. As a general trend, the heat flux and the $N_u$ increase with an increase in $Ra$ for all cases of $Ri$ and $Da$. At $Ra = 10^3$ and $Ra = 10^4$, heat transfer is dominated by conduction.
**FIGURE 7** Streamlines and temperature contours for different $Da$ for $Ri = 1$ and $Ra = 10^6$

**FIGURE 8** Local Nusselt number distribution along the isothermal walls for different Darcy number ($Ri = 1, Ra = 10^6$)
As $Ra$ increases to $10^5$ and $10^6$, convective flow strengthens and elevates. For static cylinder case, the temperature contours form a plume at the top (Figure 9o). This plume structure is disrupted when the cylinder is rotated as shown in Figures 9(k–n). In case of $Da = 10^{-2}$, the convective flow has greater strength owing to which the rise in $Nu$ is significant for higher $Ra$. However, as $Da$ decreases, flow permeability decreases. The strength of natural convection current loses its significance. The percentage rise in $Nu$ also diminishes at higher $Da$ even for higher $Ra$ as seen from Figure 10.
Heat transport occurs in the solid rotor through conduction mechanism. A small temperature variation across the cylinder is noticed because of its small dimension and continuous stirring effect. Based on the readings obtained from the above computational work, a correlation is developed, which computes \( \overline{Nu} \) as a function of \( Ra, Da, \) and \( Ri \), for rotational cylinder case, in the present range of working parameters. The correlation is expressed as

\[
\overline{Nu} = 2.048Ra^{0.149}Da^{0.094}Ri^{-0.071}.
\]  

(21)

The parity plot for the correlation is provided in Figure 11, which shows that the computed results closely agree with the correlated results.

5 | CONCLUSIONS

Mixed convection inside a 2D square composite fluid-porous domain with a rotating conducting cylinder at the center is simulated using ANSYS Fluent software. The Darcy–Brinkman–Forchheimer equation is used for the flow in porous media, and a single-domain approach is used for the solution in the fluid-porous composite medium. Heat transfer analysis is done inside the 2D cavity with different rotational speeds of the cylinder and permeability of the porous medium. Based on the obtained results, the following conclusions are outlined.

1. The average Nusselt number \( \overline{Nu} \) increases on either side of \( Ri = 10 \) signifying the dominance of forced convection with a decrease in the \( Ri \) and the dominance of natural convection with an increase in the \( Ri \).

2. The rotation of the cylinder increases \( Nu \) for a large portion of the bottom wall.

3. The flow shows the symmetric behavior around the vertical axis for higher \( Ri \) and lower \( Da \).
4. The intensity of flow is quite high in the fluid medium compared to the porous medium.
5. Decreasing the \( Da \), brings stability to the system. As \( Da \) decreases, the heat transfer decreases.
6. The average Nusselt number (\( \overline{Nu} \)) increases at a higher rate with increase in \( Ra \) at higher \( Da \).
7. A small temperature variation across the cylinder is noticed because of its small dimension and its continuous stirring effect.
8. The developed correlation \( \overline{Nu} = 2.048Ra^{0.149}Da^{-0.094}Ri^{-0.071} \) predicts the heat transfer in the working range of parameters with the cylinder rotation.

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