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FULLY DEVELOPED FREE CONVECTION OF A MICROPOLAR FLUID IN A VERTICAL CHANNEL

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ABSTRACT

Numerical and analytical solutions of the developing laminar free convection of a micropolar fluid in a vertical parallel plate channel with asymmetric heating are presented. The solutions are obtained for different ratios of the wall temperature differences (above the temperature of the fluid at the channel entrance). The numerical solution is shown to be in excellent agreement with the closed form analytical solution.

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Introduction

The theory of micropolar and thermomicropolar fluids, developed by Eringen [1,2], has received considerable interest in recent years. This theory includes effects of local rotation inertia and couple stresses, and provides a mathematical model of non-Newtonian behaviour observed in certain fluids such as colloidal fluids, polymeric fluids, animal blood and real fluids with suspensions. This subject has been reviewed by Ariman et al. [3,4] and recently by Łukaszewicz [5] and Eringen [6].

Studies of external convective flows of micropolar fluids have focused mainly on free, forced and mixed convection problems. However, there are only few studies investigating the effect of microstructure on the free convection heat transfer in enclosures, see Chiu et al. [7]. Natural convection of an enclosed fluid is a long-standing classical subject. Applications are found in a variety of engineering problems, such as air conditioning of a room, solar energy collecting devices, material processing, and passive cooling of nuclear reactors, to name a few.

The purpose of the present paper is to report both analytical and numerical results of the problem of fully developed free convection flow of a micropolar fluid between a parallel-plate vertical channel with asymmetric wall temperature distribution. It is worth mentioning that for a Newtonian fluid Aung [8] and Aung et al. [9] have studied this problem. They assumed that the two walls of the channel are maintained at uniform but not necessary equal temperatures and this assumption will also be considered here.

Basic Equations

Consider the laminar free convection flow of a micropolar fluid between two vertical plates, the space between the plates being h , as shown in Fig. 1. The flow is assumed steady and fully developed, i.e. the transverse velocity is zero. It is also assumed that the walls are heated uniformly but their temperatures may be different resulting in asymmetric heating situation. Under these assumptions the equations that describe the physical situation are

$$(\mu + \kappa) \frac{d^2 u}{dy^2} + \kappa \frac{dn}{dy} + \rho g \beta (T - T_0) = 0 \quad (1)$$

$$\gamma \frac{d^2 n}{dy^2} - \kappa \left(2n + \frac{du}{dy} \right) = 0 \quad (2)$$

$$\frac{d^2 T}{dy^2} = 0 \quad (3)$$

subject to the boundary conditions:

$$\begin{aligned} u(0) = 0, \quad T(0) = T_1, \quad n(0) = 0 \\ u(h) = 0, \quad T(h) = T_2, \quad n(h) = 0 \end{aligned} \quad (4)$$

where x and y are the axial and transverse co-ordinate, respectively ($x = 0$ is the duct entrance and $y = 0$ is the left wall), u is the axial velocity, T is the fluid temperature, n is the microrotation component of the micropolar fluid normal to (x, y) - plane, T_1 is the temperature of the cooled wall (i. e. at $y = 0$), T_2 is the temperature of the hot wall (i.e. $y = h$) and ρ , g , β , μ , κ and γ are the density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity and spin gradient viscosity respectively. We notice that the condition that microrotation n vanishes on the walls is called strong interaction, see Guram and Smith [10]. Further, we shall assume that γ has the following form as proposed by Ahmadi [11]

$$\gamma = \left(\mu + \frac{\kappa}{2} \right) j \quad (5)$$

where j is the microinertia density.

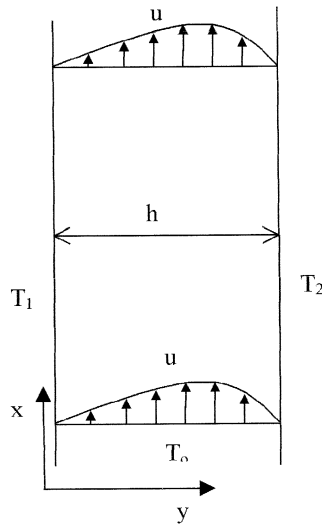


FIG. 1

Schematic diagram of the physical model.

Equations (1) – (3) can be non-dimensionalized using the variables

$$Y = \frac{y}{h}, \quad U = \frac{u}{U_c}, \quad \theta = \frac{T - T_0}{T_2 - T_0}, \quad N = \frac{h}{U_c} n \tag{6}$$

where $U_c = g\beta(T_2 - T_0)h^2/\nu$ is the characteristic velocity. Thus, we get

$$(1 + K) \frac{d^2 U}{dY^2} + K \frac{dN}{dY} + \theta = 0 \tag{7}$$

$$\left(1 + \frac{K}{2}\right) \frac{d^2 N}{dY^2} - K \left(2N + \frac{dU}{dY}\right) = 0 \tag{8}$$

$$\frac{d^2 \theta}{dY^2} = 0 \tag{9}$$

where we set $j = h^2$ and K is the material parameter, which is defined as

$$K = \frac{k}{\mu} \tag{10}$$

The boundary conditions (4) become

$$\begin{aligned} U(0) = 0, \quad \theta(0) = R, \quad N(0) = 0 \\ U(1) = 0, \quad \theta(1) = 1, \quad N(1) = 0 \end{aligned} \tag{11}$$

where

$$R = \frac{T_1 - T_0}{T_2 - T_0} \tag{12}$$

Equation (9) subject to the boundary conditions (11) has the analytical solution

$$\theta = (1 - R)Y + R \tag{13}$$

and for $K = 0$ (Newtonian fluid) we have $N = 0$ and U has the analytical solution given by Aung [8].

Analytical solution for $K \neq 0$

Solving analytically Eqs. (7) and (8) for $K \neq 0$ (micropolar fluid), we get after a long algebra

$$\begin{aligned} \sqrt{2K(1+K)}U = & \frac{K(1+R)}{2(2+K)} \frac{ch(\sqrt{2K/(1+K)}Y)}{sh\sqrt{2K/(1+K)}} \\ & - \frac{\sqrt{2K(1+K)}}{2+K} \left[\frac{1}{3}(1-R)Y^3 + RY^2 \right] + \sqrt{\frac{1+K}{2K}} [(1-R)Y + R] \\ & + \left[-K \exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) \frac{ch(\sqrt{2K/(1+K)}Y)}{ch\left(\frac{1}{2}\sqrt{2K/(1+K)}\right)} + K \exp\left(\sqrt{\frac{2K}{1+K}}Y\right) - 2\sqrt{2K(1+K)}Y \right] A \\ & + 2\sqrt{2K(1+K)}B \end{aligned} \tag{14}$$

$$\begin{aligned} N = & -\frac{1+R}{2(2+K)} \frac{sh(\sqrt{2K/(1+K)}Y)}{sh\sqrt{2K/(1+K)}} + \frac{1-R}{2(2+K)} Y^2 \\ & + \frac{R}{(2+K)} Y + \left[\exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) \frac{sh(\sqrt{2K/(1+K)}Y)}{ch\left(\frac{1}{2}\sqrt{2K/(1+K)}\right)} \right. \\ & \left. - 2 \exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}Y\right) sh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}Y\right) \right] A \end{aligned} \tag{15}$$

where A and B are constants which have to be determined from the following algebraic system of equations

$$\begin{aligned} K th\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) A - \sqrt{2K(1+K)}B = & \frac{K}{2(2+K)} \frac{1+R}{sh\sqrt{2K/(1+K)}} + R\sqrt{\frac{1+K}{2K}} \\ \left[K th\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) - 2\sqrt{2K(1+K)} \right] A + & \sqrt{2K(1+K)}B \\ = & -\frac{K(1+R)}{2(2+K)} cth\left(\sqrt{\frac{2K}{1+K}}\right) + \frac{1+R}{3} \frac{\sqrt{2K(1+K)}}{2+K} - \sqrt{\frac{1+K}{2K}} \end{aligned} \tag{16}$$

Results and Discussion

Equations (7) and (8) subject to the boundary conditions (11) have been solved numerically using the implicit finite-difference method discussed by Blottner [12] for some values of the temperature and material parameters R and K . These equations are discretized using three-point central-difference quotients and, as a consequence, a set of algebraic equations results. These algebraic equations are then solved by the well-known tri-diagonal Thomas algorithm (see Blottner [12]). The computational domain was divided into 201 points and constant step sizes of 0.01 were utilised. These step sizes were found to give accurate grid-independent results as verified by the comparisons shown in the figures below. The numerical results for the axial velocity U and microrotation N are presented in Figs. 2 to 7. The analytical solution given by Eqs. (14) and (15) are also included in these figures and we notice that both the numerical and analytical solutions are in excellent agreement. It is also seen from Fig. 2 for U that for $K = 0$ (Newtonian fluid) the present results are in excellent agreement with those reported by Wang [8]. Figures 3 to 6 show, as expected, that for a fixed value of R the effect of the material parameter K is to reduce the velocity profiles. We also notice that for a fixed value of K the effect of R is to increase the velocity and microrotation profiles.

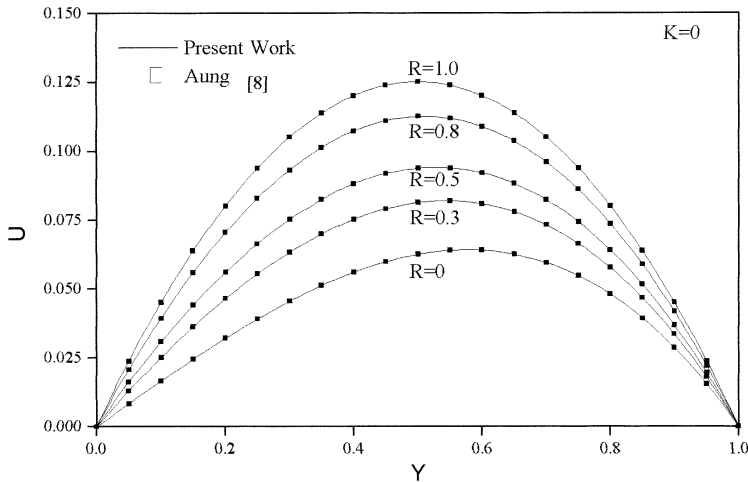


FIG. 2
Comparison of the present work with Aung [8].

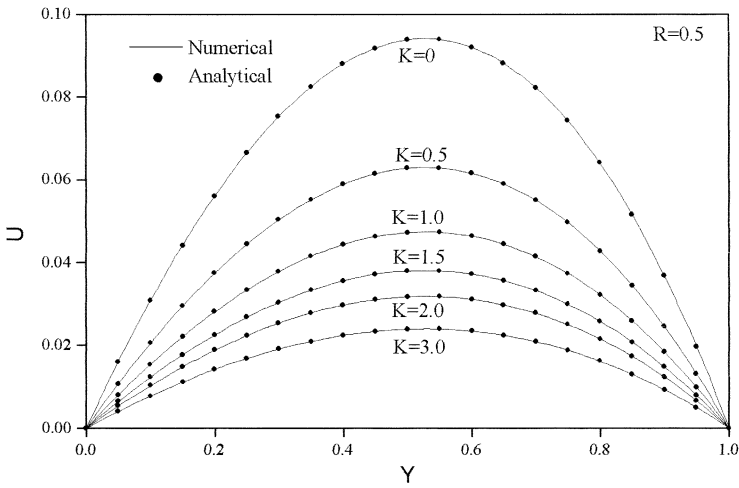


FIG. 3
Effects of K on the velocity profiles.

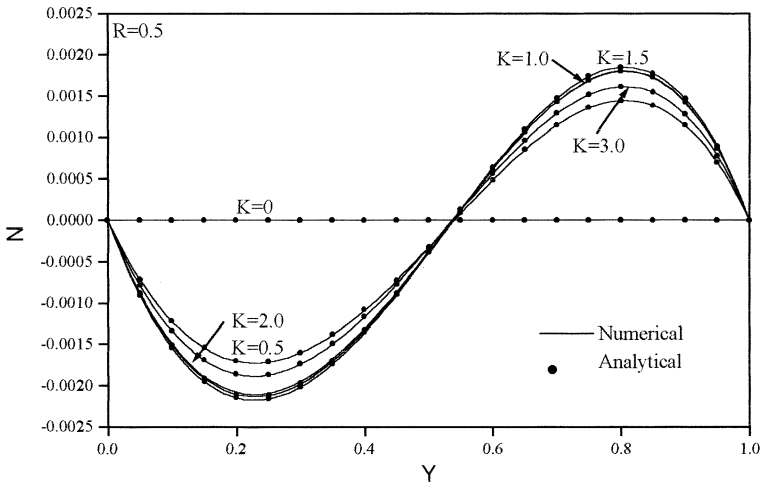


FIG. 4
Effects of K on the microrotation profiles.

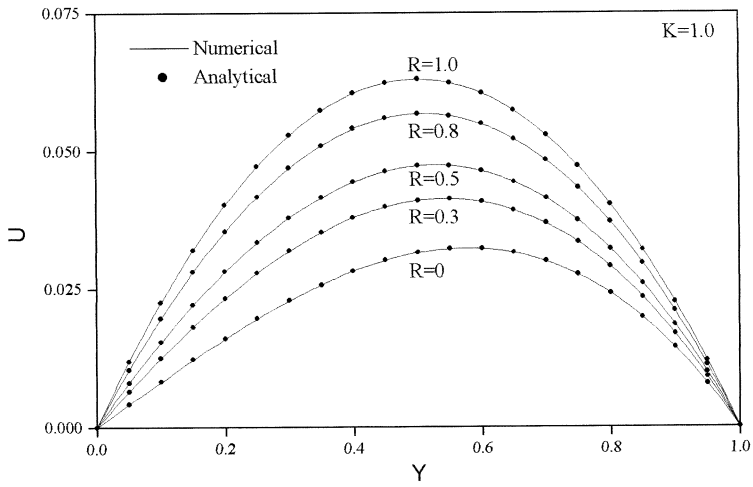


FIG. 5
Effects of R on the velocity profiles.

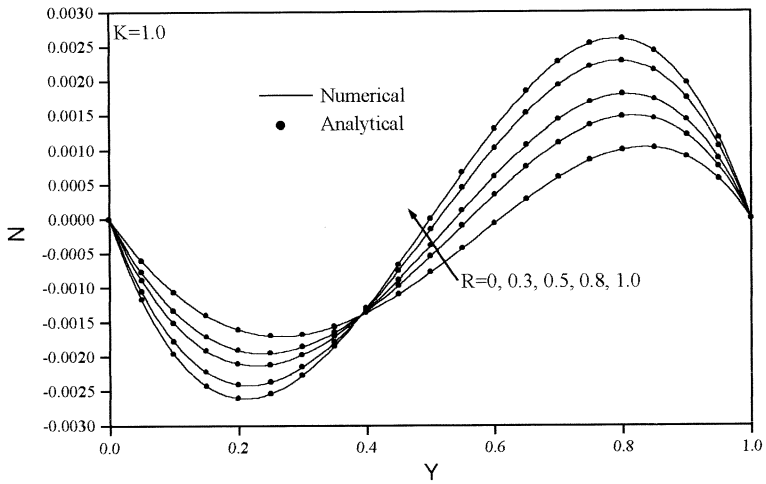


FIG. 6
Effects of R on the microrotation profiles.

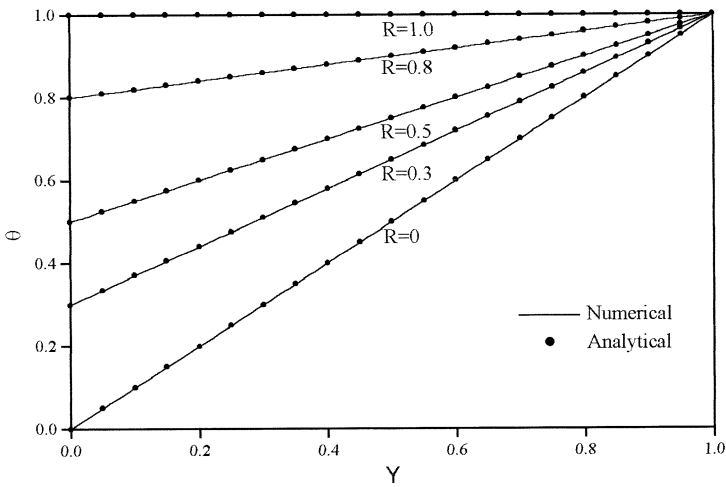


FIG. 7
Effects of R on the temperature profiles.

Nomenclature

- A, B constants
- g acceleration due to gravity
- h spacing between channel walls
- j microinertia density
- K non-dimensional material parameter
- n microrotation
- R wall temperature difference ratio
- T fluid temperature
- T_1 temperature of the cooled wall ($y = 0$)
- T_2 temperature of the hot wall ($y = h$)
- u axial velocity
- x, y Cartesian co-ordinates

β	coefficient of thermal expansion
γ	spin-gradient viscosity
κ	vortex viscosity
μ	dynamic viscosity
ρ	density
θ	non-dimensional temperature

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