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Steady Natural Convection Flow of a Particulate Suspension Through a Parallel-Plate Channel

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Abstract A continuum model for two-phase (fluid/particle) flow induced by natural convection is developed and applied to the problem of steady natural convention flow of a particulate suspension through an infinitely long channel. The walls of the channel are maintained at constant but different temperatures. The two-phase model accounts for particle-phase viscous effects. Boundary conditions borrowed from rarefied gas dynamics are employed for the particle-phase wall conditions. Various closed-form solutions for different special cases are obtained. A parametric study of the physical parameters involved in the problem are performed to illustrate the influence of these parameters on the flow and heat transfer aspects of the problem.

List of Symbols

a	Radius of spherical particles
c	Fluid-phase specific heat at constant pressure
c_p	Particle-phase specific heat at constant pressure
g	Gravitational acceleration
Gr	Grashof number
h	Channel width
H	Dimensionless buoyancy parameter
k	Fluid-phase thermal conductivity
m	Particulate mass
N	Interphase momentum transfer coefficient
N_T	Interphase heat transfer coefficient
P	Fluid-phase hydrostatic pressure
R_s	Slip Reynolds number
Pr	Fluid-phase Prandtl number
S	Dimensionless particle-phase wall slip coefficient
t	time
T	Fluid-phase temperature
T_p	Particle-phase temperature
u	Fluid-phase dimensionless velocity
u_p	Particle-phase dimensionless velocity
U	Fluid-phase velocity
U_p	Particle-phase velocity
V	Fluid-phase velocity vector
$\mathbf{V}_p$	Particle-phase velocity vector
V_s	Slip velocity
x, y	Cartesian coordinates

Greek Symbols

α	Velocity inverse Stokes number
β	Viscosity ratio
β^*	Thermal expansion coefficient
γ	Specific heat ratio
ε	Temperature inverse Stokes number
η	Dimensionless y-coordinate
θ	Dimensionless fluid-phase temperature
κ	Particle loading
μ	Fluid-phase dynamic viscosity
μ_p	Particle-phase dynamic viscosity
ρ	Fluid-phase density
ρ_p	Particle-phase density
ω	Particle-phase wall slip coefficient

1 Introduction

Two-phase (fluid-particle) natural convection flow represents one of the most interesting and challenging areas of research in heat transfer. Such flows are found in a wide range of applications including processes in the chemical and food industries, solar collectors where a particulate suspension is used to enhance absorption of radiation, cooling of electronic equipments, cooling of nuclear reactors, and heating of buildings via storage walls (trombe walls). In general, all applications of single-phase flow are valid for two-phase particulate suspension flow because the nature of real life dictates the presence of contaminating solid particles in fluids. Inspite of this fact, all research on natural convection flows within vertical parallel-plate channels are done only for a single phase. For example, Elenbass (1942) analyzed heat dissipation effects of parallel plates by free convection. Aung et al. (1972) investigated the development of laminar free convection between vertical flat plates with asymmetric heating. Akbari and Borgers (1979) studied laminar natural convection heat transfer between the channel surfaces of a trombe wall. Many other works can be found in the book by Gebhart et al. (1988).

On the other hand, very little work have been reported on natural convection flow of a particle-fluid suspension over and through different geometries. Recently, Chamkha and Ramadan (1998) and Ramadan and Chamkha (1999) have developed a mathematical two-phase model

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(fluid/particle) which accounts for the presence of thermal buoyancy effects and reported some analytical and numerical results for natural convection flow of a two-phase particulate suspension over an infinite vertical plate. They found that increases in either of the particle loading or the wall particulate slip coefficient caused reductions in the velocities of both phases. Also, Okada and Suzuki (1997) have considered buoyancy-induced flow of a two-phase suspension in an enclosure. However, to the best of the authors' knowledge, there is no previous work reported on natural convection flow of a particulate suspension through a vertical channel. Thus, there is a definite need for investigation of such a problem. Hence, the objective of this research is to perform an analytical investigation on steady natural convection laminar flow of a particulate suspension in an infinite vertical parallel-plate channel.

2 Governing Equations

In order to investigate the characteristics of two-phase natural convection flow in channels, one must start from the basic equations. These are the fluid-phase continuity equation, fluid-phase balance of linear momentum equation, fluid-phase balance of energy equation, particle-phase continuity equation, particle-phase balance of linear momentum equation and the particle-phase balance of energy equation. These balance laws can be written in the following vector form (see Marble, 1970 and Drew, 1983) as:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0 \quad (1)$$

$$\rho(\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V}) = -\nabla P + \nabla \cdot (\mu \nabla \mathbf{V}) - \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho \mathbf{g} \quad (2)$$

$$\rho c(\partial_t T + \mathbf{V} \cdot \nabla T) = \nabla \cdot (k \nabla T) + \rho_p c_p N_T (T_p - T) \quad (3)$$

$$\partial_t \rho_p + \nabla \cdot (\rho_p \mathbf{V}_p) = 0 \quad (4)$$

$$\rho_p(\partial_t \mathbf{V}_p + \mathbf{V}_p \cdot \nabla \mathbf{V}_p) = \nabla \cdot (\mu_p \nabla \mathbf{V}_p) + \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho_p \mathbf{g} \quad (5)$$

$$\rho_p c_p(\partial_t T_p + \mathbf{V}_p \cdot \nabla T_p) = -\rho_p c_p N_T (T_p - T) \quad (6)$$

where all symbols are defined in the List of Symbols section. It should be mentioned here that the slip Reynolds number $R_s = 2\rho a V_s/\mu$ (where $V_s = V - V_p$) is assumed to be small so that the interphase force is approximated by Stokes drag force on a sphere. In Equations (2) and (5), the interphase momentum transfer coefficient $N = 6\pi a \mu/m$.

This study considers steady, homogeneous with discrete particles, one dimensional, incompressible, laminar, natural convection fully developed two-phase (fluid-particle) flow in an impermeable parallel-plate channel. The walls of the channel are assumed to be infinitely long. This implies that the dependence of the variables on the x-direction will be negligible compared with that of the y-direction (see Fig. 1). Therefore, all dependent variables

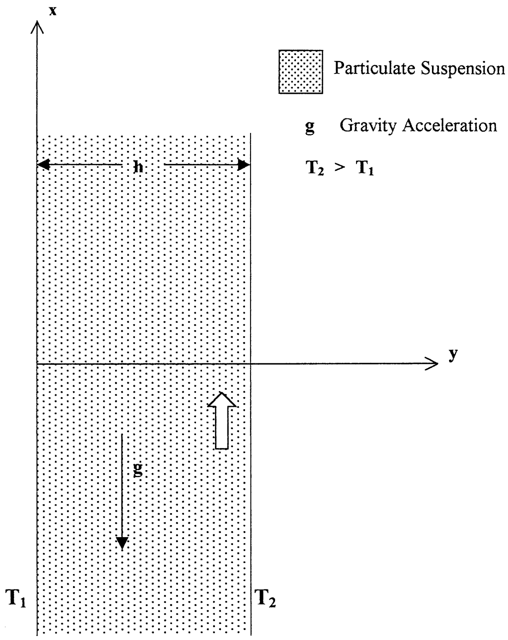

Fig. 1. Problem Definition

in equations (1) through (6) will only be functions of y as follows:

$$-\partial_x P + \mu \partial_{yy} U - \rho_p N(U - U_p) - \rho g = 0 \quad (7)$$

$$k \partial_{yy} T + \rho_p c_p N_T (T_p - T) = 0 \quad (8)$$

$$\mu_p \partial_{yy} U_p + \rho_p N(U - U_p) - \rho_p g = 0 \quad (9)$$

$$\rho_p c_p N_T (T_p - T) = 0 \quad (10)$$

It should be noted that the continuity equations of both phases are identically satisfied.

The pressure gradient can be eliminated from the linear momentum equation of the fluid phase by evaluating the governing equations at a reference point within the channel. Let "o" be a reference point within the channel such that $U = 0$, $T = T_o$, $\rho = \rho_o$, $\mu = \mu_o$, $U_p = U_{po}$, $T_p = T_{po}$, $\rho_p = \rho_{po}$ and $\mu_p = \mu_{po}$. Evaluating the governing equations at this reference point and employing the Boussinesq approximation gives:

$$\rho_{po}/\rho_o g + \mu_o/\rho_o \partial_{yy} U - \rho_{po}/\rho_o N(U - U_p) + \beta^* g(T - T_o) = 0 \quad (11)$$

where β^* is the volumetric expansion coefficient. The linear momentum equation of the fluid phase, equation (7), will now be replaced by equation (11) in the governing equations.

Each of equations (7), (8) and (9) requires two boundary conditions to solve them completely. The physical boundary conditions for this problem are:

$$U(0) = U(h) = 0, \quad T(0) = T_1, \quad T(h) = T_2 \quad (12a-d)$$

$$U_p(0) = \omega\partial_y U_p(0) - g/N,$$
$$U_p(h) = -\omega\partial_y U_p(h) - g/N \quad (12e-h)$$
$$T_p(0) = T_1, \quad T_p(h) = T_2$$

where h is the channel width, T_1 is the channel wall temperature at y = 0, T_2 is the channel wall temperature at y = h and ω is the particle-phase slip coefficient. Equations (12a) and (12b) indicate no slip conditions for the fluid phase at the walls of the channel. Equations (12c) and (12d) suggest that the fluid temperatures at the walls of the channel are some constant values T_1 and T_2 such that $T_2 > T_1$. Equations (12e) and (12f) express proposed wall boundary conditions for the particle phase at the walls of the channel. Equations (12g) and (12h) indicate that the particle phase is in thermal equilibrium with the fluid phase at the walls. It should be mentioned herein that the wall boundary conditions for the particulate phase are poorly understood at present. However, there is an experimental evidence that particles tend to slip at a boundary. Therefore, two idealized conditions will be considered. These are the no-slip condition (ω = 0) and the perfect slip condition ($\omega \rightarrow \infty$: $\partial_y U_p(0,t) = \partial_y U_p(h,t) = 0$). It is expected that the actual behavior would be somewhere between these two extremes.

The formulation of the value problem of an infinite vertical parallel-plate channel is now completed. In order to solve this problem, it is convenient to non-dimensionalize the governing equations and conditions. This can be accomplished by using the following parameters:

$$y = h\eta, \quad U = (\mu/\rho h)u, \quad U_p = (\mu/\rho h)u_p,$$
$$T = (T_2 - T_o)\theta + T_o, \quad T_o = (T_1 + T_2)/2,$$
$$T_p = (T_2 - T_o)\theta_p + T_o \quad (13)$$

where η is the dimensionless coordinate, u and u_p are the dimensionless fluid- and particle-phase velocities, respectively, and θ and θ_p are the dimensionless fluid- and particle-phase temperatures, respectively. After performing the mathematical operations, the resulting dimensionless governing equations can be written as:

$$D^2u - \alpha\kappa(u - u_p) + Gr\theta + \kappa H = 0 \quad (14)$$

$$(1/Pr)D^2\theta + \kappa\gamma\varepsilon(\theta_p - \theta) = 0 \quad (15)$$

$$\beta D^2u_p + \alpha(u - u_p) - H = 0 \quad (16)$$

$$\varepsilon(\theta_p - \theta) = 0 \quad (17)$$

where D^2 denotes a second derivative operator with respect to η, $\alpha = h^2N\rho/\mu$, $\kappa = \rho_p/\rho$, $Gr = g\beta^* h^3\rho^2(T_2 - T_o)/\mu^2$, $H = gh^3\rho^2/\mu^2$, $\beta = \mu_p/(\kappa\mu)$, $Pr = \mu c/k$, $\gamma = c_p/c$ and $\varepsilon = \rho N_T h^2/\mu$ are the momentum inverse Stokes number, the particle loading, the Grashof number, buoyancy parameter, the viscosity ratio, the Prandtl number, the specific heat ratio, and the temperature inverse Stokes number, respectively.

The dimensionless boundary conditions are:

$$u(0) = u(1) = 0, \quad \theta(0) = -1, \quad \theta(1) = 1 \quad (18a-d)$$

$$u_p(0) = SDu_p(0) - H/\alpha, \quad u_p(1) = -SDu_p(1) - H/\alpha$$
$$\theta_p(0) = -1, \quad \theta_p(1) = 1 \quad (18e-h)$$

where S = ω/h is the dimensionless particle-phase slip parameter. It should be mentioned that when β= 0 (inviscid particle phase), equations (18e, f) are ignored.

3 Analytical Results and Discussion

Inviscid Particle Phase

For an inviscid particle phase (β = 0), equation (16) implies that :

$$u_p(\eta) = u(\eta) - H/\alpha \quad (19)$$

which indicates that the particle-phase velocity is the same as the fluid-phase velocity except that it is shifted by the factor H/α below the fluid-phase velocity.

Equation (17) implies that:

$$\theta_p(\eta) = \theta(\eta) \quad (20)$$

By substituting equation (20) into equation (15) one obtains :

$$D^2\theta = 0 \quad (21)$$

The solution of this simple second-order differential equation, which satisfies the boundary conditions (18c, d), is:

$$\theta(\eta) = 2\eta - 1 \quad (22)$$

This indicates that the temperature of both phases has a linear shape of pure conduction. Again, substituting equations (19) and (22) into equation (14) gives:

$$D^2u = -2Gr\eta + Gr \quad (23)$$

The solution of this second-order differential equation, which satisfies the boundary conditions (18a,b), is:

$$u(\eta) = -Gr(\eta - 3\eta^2 + 2\eta^3)/6 \quad (24)$$

This shows that the fluid-phase velocity profile has a cubic relation with the normal distance. The corresponding solution for $u_p(\eta)$ is obtained by substituting equation (24) into equation (19).

Comparisons with previously published work for the case of a clear fluid (single phase) can be made. Equations (22) and (24) are identical to those reported by Aung (1972) without the Gr factor. On the other hand, if the boundary condition of the fluid-phase dimensionless temperature was put equal to –1 at η= –1, instead of η= 0 in the present problem, then the predicted results are essentially identical to those reported by White (1991).

4
Viscous Particle Phase:

In the presence of a particle-phase viscosity ($\beta \neq 0$), equations (14) and (15) can be rearranged and rewritten in matrix form as

$$\begin{bmatrix} D^2-\kappa\alpha & \kappa\alpha \\ \alpha/\beta & D^2-\alpha/\beta \end{bmatrix}\begin{bmatrix} u \\ u_p \end{bmatrix} = \begin{bmatrix} -2Gr\eta+Gr-\kappa H \\ H/\beta \end{bmatrix} \quad (25)$$

This matrix implies that:

$$[D^4-(\alpha/\beta+\kappa\alpha)D^2]u=(2Gr\alpha/\beta)\eta-Gr\alpha/\beta \quad (26)$$

$$[D^4-(\alpha/\beta+\kappa\alpha)D^2]u_p=(2Gr\alpha/\beta)\eta-Gr\alpha/\beta \quad (27)$$

The general solutions of the above equations are:

$$u(\eta)=c_1+c_2\eta+c_3e^{\zeta\eta}+c_4e^{-\zeta\eta}+Gr(\eta^2/2-\eta^3/3)/(1+\beta\kappa) \quad (28)$$

$$u_p(\eta)=d_1+d_2\eta+d_3e^{\zeta\eta}+d_4e^{-\zeta\eta}+Gr(\eta^2/2-\eta^3/3)/(1+\beta\kappa) \quad (29)$$

where ζ is given by

$$\zeta=(\alpha/\beta+\kappa\alpha)^{1/2} \quad (30)$$

The relationships between the c's and d's are:

$$c_1=d_1+H/\alpha-\beta Gr/(\alpha+\beta\kappa\alpha) \quad (31)$$

$$c_2=d_2+2\beta Gr/(\alpha+\beta\kappa\alpha) \quad (32)$$

$$c_3=(1-\zeta^2\beta/\alpha)d_3 \quad (33)$$

$$c_4=(1-\zeta^2\beta/\alpha)d_4 \quad (34)$$

Now, by substituting these relations into equation (26) with the assumption that

$$\psi=1-\zeta^2\beta/\alpha \quad (35)$$

gives the following general solution

$$u(\eta)=H/\alpha-\beta Gr/(\alpha+\beta\kappa\alpha)+d_1+(d_2+2\beta Gr/(\alpha+\beta\kappa\alpha))\eta + \psi d_3e^{\zeta\eta}+\psi d_4e^{-\zeta\eta}+Gr(\eta^2/2-\eta^3/3)/(1+\beta\kappa) \quad (36)$$

In order to determine the constant d's, the boundary conditions (18a,b) and (18e,f) must be applied with equations (28) and (35) to give the following equations :

$$d_1+\psi d_3+\psi d_4=\beta Gr/(\alpha+\beta\kappa\alpha)-H/\alpha \quad (37)$$

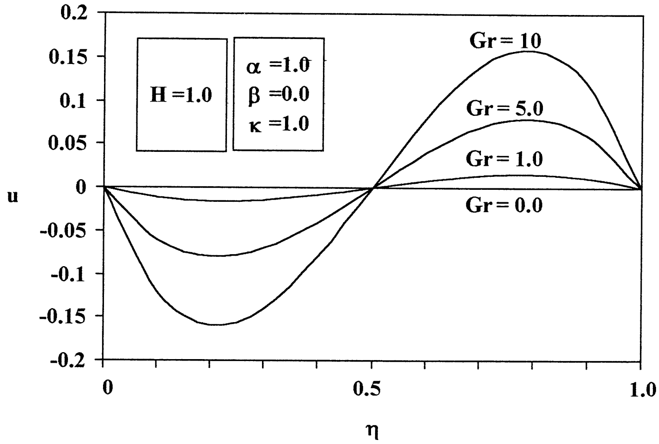

Fig. 2. Effects of Gr on FluidPhase Velocity Profiles

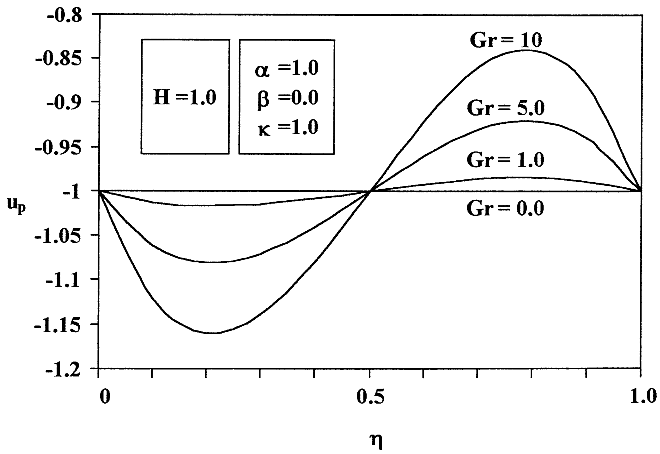

Fig. 3. Effects of Gr on Particle-Phase Velocity Profiles

$$d_1+d_2+\psi e^{\zeta}d_3+\psi e^{-\zeta}d_4=-Gr/(6+6\beta\kappa) - \beta Gr/(\alpha+\beta\kappa\alpha)-H/\alpha \quad (38)$$

$$d_1-Sd_2+(1-S\zeta)d_3+(1+S\zeta)d_4=-H/\alpha \quad (39)$$

$$d_1+(1+S)d_2+(1+S\zeta)e^{\zeta}d_3+(1-S\zeta)e^{-\zeta}d_4 = -Gr/(6+6\beta\kappa)-H/\alpha \quad (40)$$

The above equations (36 to 39) determine d_1, d_2, d_3 and d_4. This concludes the solution and shows the effects of the slip coefficient S on the velocity profiles of both the fluid and particle phases within the range $0 \leq S < \infty$. Moreover, if $S \to \infty$, then equations (38) and (39) will be replaced by:

$$d_2+\zeta d_3-\zeta d_4=0 \quad (41)$$

$$d_2+\zeta e^{\zeta}d_3-\zeta e^{-\zeta}d_4=0 \quad (42)$$

Some results for the velocity profiles of both phases (u and u_p) based on the closed-form solutions for the flow

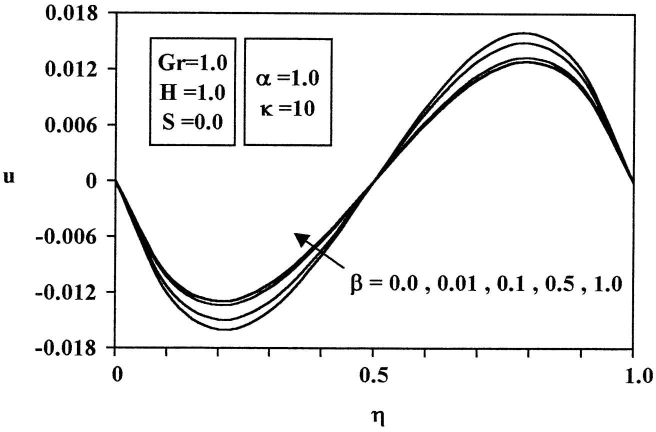

Fig. 4. Effects of β on Fluid-Phase Velocity Profiles

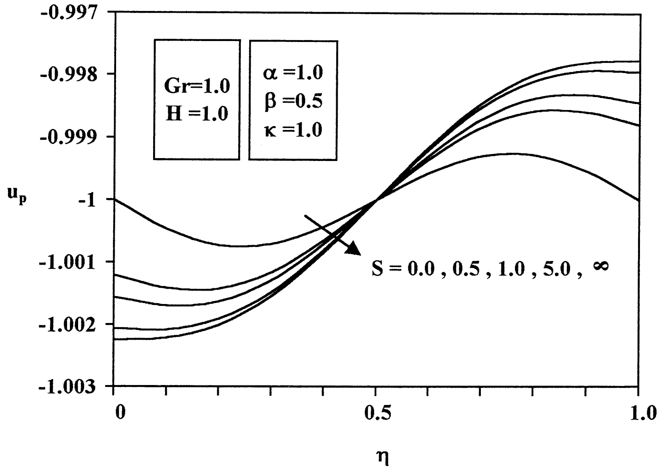

Fig. 6. Effects of S on Particle-Phase Velocity Profiles

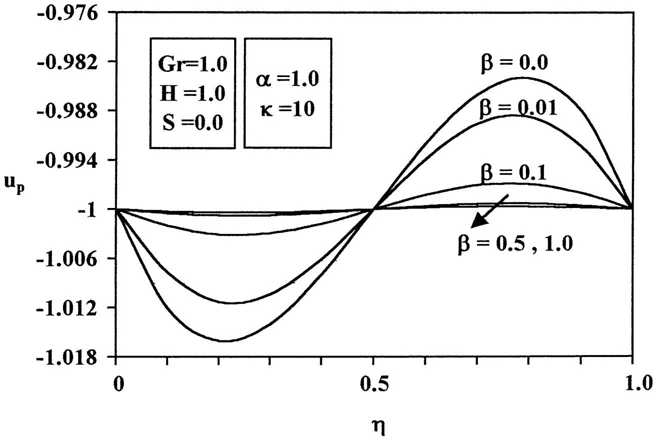

Fig. 5. Effects of β on Particle-Phase Velocity Profiles

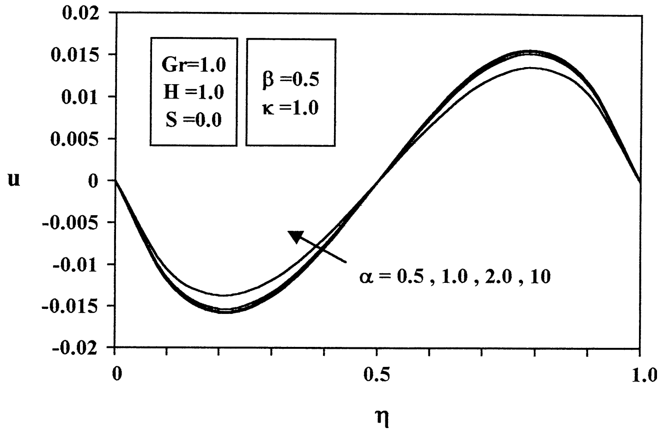

Fig. 7. Effects of α on Fluid-Phase Velocity Profiles

through a vertical channel are presented in Figures 2 through 10. These results are presented to illustrate the influence of the Grashof number Gr, the viscosity ratio β, the particle wall slip coefficient S, the inverse Stokes number α and the particle loading κ, respectively.

Figures 2 and 3 display the effects of increasing the Grashof number Gr on the velocity fields of both the fluid and particle phases, respectively. Increases in the values of Gr have the tendency to increase the thermal buoyancy effect represented by the $Gr\theta$ term of equation (14). This gives rise to an increase in the induced flow of both phases along the hot wall as shown in Figures 2 and 3. A reversed flow situation near the cold wall occurs and is increased as the flow is enhanced near the hot wall in a symmetrical fashion.

Numerical evaluations of the flow solutions given by equations (28) and (35) for various values of the viscosity ratio β are illustrated graphically in Figures 4 and 5. Figures 4 and 5 depict the effect of the ratio of the particle-to-fluid-phase viscosity β on the velocity profiles of both phases. Increases in the viscosity ratio β have the tendency to increase the magnitude of frictional effects for both phases in comparison with the buoyancy effects. This has the effect of decreasing the velocity of both phases as clearly depicted in Figures 4 and 5. In addition, Figure 5 shows that increases in the values of β have the tendency to flatten the particle-phase velocity profiles. Moreover, because the energy equations of both phases are uncoupled from the momentum equations, the temperature profiles for both phases are unaffected by the changes in the values of β.

Figure 6 illustrates the influence of the particle-phase wall slip coefficient S on the particle-phase velocity. In general, as the particle-phase wall slip increases, it becomes easier for the carrier fluid to move it causing the particle-phase velocity to increase. In addition, the slip coefficient S seems to have no significant effect on the fluid-phase velocity and no effect on both fluid- and particle-phases temperature. This was observed from results not presented herein for brevity.

In order to elucidate the influence of the inverse Stokes number α, graphical representation of u and u_p are obtained and presented in Figures 7 and 8. As α increases, the interphase momentum transfer due to the drag mechanism between the phases increases causing the fluid-phase velocity to decrease and the particle-phase velocity to increase as is evident from Figures 7 and 8. According to equations (28) through (35), the limit $\alpha \rightarrow \infty$ will cause $u_p(\eta)$ to approach $u(\eta)$ and equilibrium conditions between the phases occur in which both phases

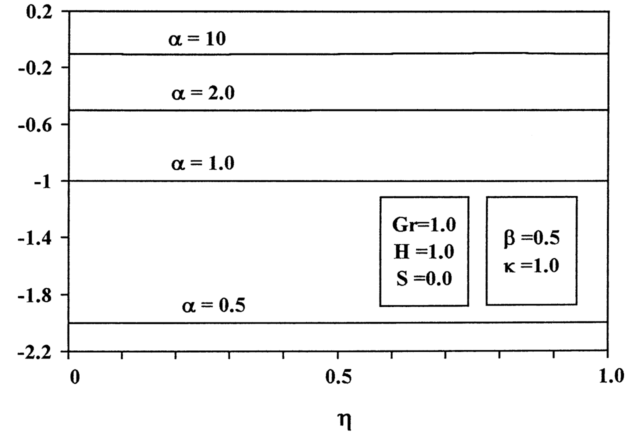

Fig. 8. Effects of α on Particle-Phase Velocity Profiles

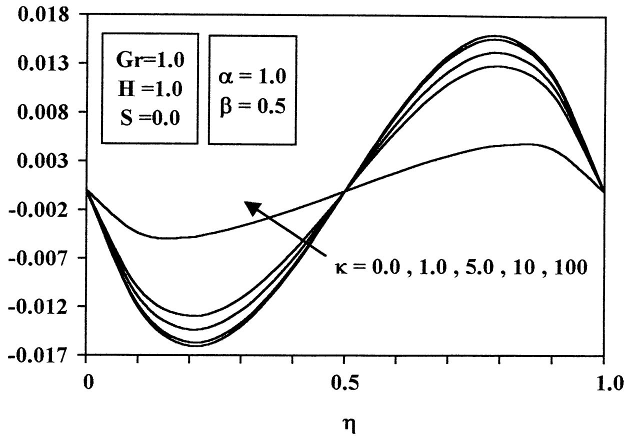

Fig. 9. Effects of κ on Fluid-Phase Velocity Profiles

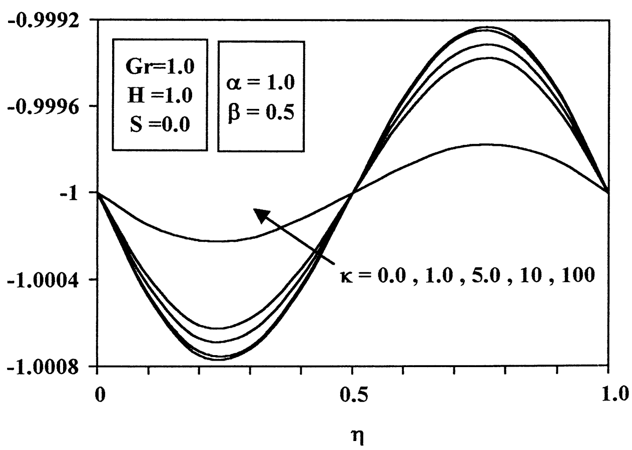

Fig. 10. Effects of κ on Particle-Phase Velocity Profiles

move together with the same velocity as long as the boundary conditions of both phases at the walls are the same.

Figures 9 and 10 show representative velocity profiles for the fluid and particle phases (u and u_p) for various values of the particle loading κ, respectively. Physically speaking, as the particle concentration increases, the drag force between the phases increases causing a slower motion of the fluid. This produces a reduction in the particle-phase velocity since the particle phase is being dragged along by the carrier fluid. In fact, increases in the values of κ have the tendency to flatten the fluid- and particle-phase velocity profiles. These facts are clearly illustrated in Figures 9 and 10.

5
Conclusions

The mathematical modeling of natural convection flow of a particulate suspension was formulated by stating the conservation laws of mass, linear momentum, and energy for both the fluid and particle phases. The governing equations were non-dimensionlized and solved analytically for the problem of steady, laminar buoyancy-induced fully developed two-phase flow through a vertical parallel-plate channel with isothermal walls. Various closed-form solutions were obtained. Representative results were plotted to illustrate the influence of the physical parameters on the solutions. A reverse (back) fluid and particle flow situation near the cold wall was predicted and the velocity profiles had a symmetrical distribution. An increase in the values of the Grashof number increased the thermal buoyancy effect which, consequently, increased the flow of both phases along the hot wall. The reversed flow near the cold wall was increased as the flow was enhanced near the hot wall in a symmetrical fashion. The effect of increasing the values of the viscosity ratio was found to increase the magnitude of the frictional effects for both phases in comparison with the buoyancy effects. The influence of increasing the values of the particle-phase slip coefficient was predicted to increase the magnitude of the particle-phase velocity. Increases in the velocity inverse Stokes number had the effect of increasing the interphase momentum transfer due to the drag mechanism between the phases. This caused the magnitude of thefluid-phase velocity to decrease and the magnitude of the particle-phase velocity to increase. Increases in the particle concentration (particle loading) increased the drag force between the phases causing a slower motion of both the fluid and particle phases. It is hoped that the results reported in this research will serve as a check for further theoretical modeling and a stimulus for experimental work on this problem.

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