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UNSTEADY FLOW OF A DUSTY CONDUCTING FLUID THROUGH A PIPE

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Introduction

The flow of a dusty and electrically conducting fluid through a pipe in the presence of a transverse magnetic field is encountered in a variety of applications such as magnetohydrodynamic generators, pumps, accelerators, and flowmeters. In these devices, the solid particles in form of ash or soot are suspended in the conducting fluid as a result of the corrosion and wear activities and/or the combustion processes in MHD generators and plasma MHD accelerators. The consequent effect of the presence of solid particles on the performance of such devices has led to studies of particulate suspensions in conducting fluids in the presence of externally applied magnetic fields. When the particle concentration becomes high and random particle-particle interaction takes place, the performance or efficiency of these devices deteriorates drastically. Mutual particle interaction leads to higher particle-phase viscous stresses and can be accounted for by endowing the particle phase by the so called particle-phase viscosity.

There have been many articles dealing with theoretical modeling and experimental measurements of the particle-phase viscosity in a dusty fluid (see, for instance, Soo [1], Gidaspow, Tsuo, and Luo [2], Grace [3], and Sinclair and Jackson [4]). The particulate viscosity is needed to account for the energy dissipation between the solid particles due to their interactions. It is of interest in this paper to study the influence of the magnetic field on the flow properties in situations where the particle phase is considered dense enough to include the particulate viscous stresses.

The flow of a conducting fluid in a circular pipe has been investigated by many authors (see, for instance, Shercliff [5] and Gold [6]). Gadiraju, Peddieson, and Munukutla [7] investigated steady two-phase vertical flow in a pipe. Dube and Sharma [8] and Ritter and Peddieson [9] reported solutions for unsteady dusty-gas flow in a circular pipe in the absence of a magnetic field and particle-phase viscous stresses. In this paper, exact solutions which generalize the results reported by Dube and Sharma [8] and Ritter and Peddieson [9] by the inclusion of the magnetic and particle-phase viscous effects are obtained. The carrier fluid is assumed incompressible and electrically conducting. The particle phase is assumed to be incompressible, pressureless and electrically non-conducting. The flow in the pipe starts from rest through the application of a constant axial pressure gradient. The magnetic Reynolds number is assumed to be small and the induced magnetic field is neglected.

Governing Equations

Consider unsteady, laminar, axisymmetric horizontal flow of a dusty conducting fluid through an infinitely long pipe driven by a constant pressure gradient. A uniform magnetic field is applied normal to the flow direction. Let the Hall effect of

Magnetohydrodynamics be negligible and assume that both phases behave as linearly viscous fluids (see, Johnson, Massoudi, and Rajagopal [10]). In addition, assume that the volume fraction of suspended particles be finite and constant. Taking into account these and previously mentioned assumptions, the governing equations for this investigation can be written as

$$\rho\partial_t V = -\partial_z P + \mu(\partial_{rr} V + 1/r\partial_r V) + (\rho_p\phi)/(1-\phi)N(V_p - V) - \sigma B_0^2 V \tag{1}$$

$$\rho_p\partial_t V_p = \mu_p(\partial_{rr} V_p + 1/r\partial_r V_p) + \rho_p N(V - V_p) \tag{2}$$

where t is time, r is the distance in the radial direction, V is the fluid-phase velocity, V_p is the particle-phase velocity, ρ is the fluid-phase density, ρ_p is the particle-phase density, $\partial_z P$ is the fluid pressure gradient, μ is the fluid dynamic viscosity, μ_p is the particle-phase viscosity, ϕ is the particle-phase volume fraction, N is a momentum transfer coefficient (the reciprocal of the relaxation time, the time needed for the relative velocity between the phases to reduce e^{-1} of its original value, see Marble [11]), σ is the fluid electrical conductivity, and B_0 is the magnetic induction. In this work, ρ, ρ_p, μ, μ_p, ϕ, and B_0 are all assumed constant. It should be pointed out that the particle-phase pressure is assumed negligible and that the particles are being dragged along with the fluid phase. This is not necessarily realistic since a particle phase dense enough to have significant effects due to viscosity would have a pressure, or a pressure like term, associated with it. In addition, the assumptions of constant particle-phase density and volume fraction are unlikely to be accurate in a physically realistic system because while the individual particles may be incompressible, the particle phase as a whole will have a variable density distribution and will behave as a compressible fluid. Nevertheless, the closed-form solutions to the simplified Equations (1) and (2) (which will be reported later in this paper) will be useful in verifying numerical schemes used to solve more complex (that is more realistic) problems of this type.

The physics of the problem suggests the following initial, symmetry, and boundary conditions:

$$V(r,0)=0,\ V_p(r,0)=0,\ \partial_r V(0,t)=0,\ \partial_r V_p(0,t)=0,\ V(a,t)=0,\ V_p(a,t)=0 \tag{3}$$

where "a" is the pipe radius.

Equations (1) through (3) constitute an initial-value problem which can be made dimensionless by using

$$r=a\eta,\ t=a^2\rho\tau/\mu,\ \rho_p=\kappa\rho(1-\phi)/\phi,\ \mu=\rho\nu,\ \mu_p=\rho_p\nu_p$$
$$V(r,t)=G_0a^2/\mu\ F(\eta,\tau),\ V_p(r,t)=G_0a^2/\mu\ F_p(\eta,\tau) \quad (4)$$

where $G_0=-\partial_z P$ is a constant.
The resulting dimensionless equations and conditions can be written as

$$\partial_\tau F = \partial_{\eta\eta}F + 1/\eta\partial_\eta F + \kappa\alpha(F_p - F) - M^2F + 1 \quad (5)$$

$$\partial_\tau F_p = \beta(\partial_{\eta\eta}F_p + 1/\eta\partial_\eta F_p) + \alpha(F - F_p) \quad (6)$$

(where $\alpha=Na^2\rho/\mu$, $\beta=\nu_p/\nu$, and $M=(\sigma/\mu)^{1/2}B_0a$ are the inverse Stoke's number, viscosity ratio, and the Hartmann number.)

$$F(\eta,0)=0,\ F_p(\eta,0)=0,\ \partial_\eta F(0,\tau)=0,\ \partial_\eta F_p(0,\tau)=0,\ F(1,\tau)=0,\ F_p(1,\tau)=0 \quad (7)$$

The volumetric flow rates and skin-friction coefficients for both the fluid and particle phases are defined, respectively, as

$$Q=2\int_0^1 F(\eta,\tau)d\eta,\ Q_p=2\int_0^1 F_p(\eta,\tau)d\eta,\ C=-\partial_\eta F(1,\tau),\ C_p=-\beta\kappa\partial_\eta F_p(1,\tau) \quad (8)$$

Results and Discussion

The solutions for F and F_p which satisfy the initial-value problem represented by Equations (5) through (7) can be assumed to take on the forms

$$F(\eta,\tau) = \sum_{n=1}^{\infty} H_n(\tau)\ J_0(\lambda_n\eta),\ F_p(\eta,\tau) = \sum_{n=1}^{\infty} H_{pn}(\tau)\ J_0(\lambda_n\eta) \quad (9)$$

where λ_n are the roots of the equation $J_0(\lambda_n\eta)=0$ (J_0 being the zeroth-order Bessel function of the first kind). Substituting Equations (9) and their derivatives into Equations (5) and (6) (with the non-homogeneous part of these equations represented by a series in $J_0(\lambda_n\eta)$), multiplying by $\eta J_0(\lambda_m\eta)$ (to take advantage of the orthogonality property of Bessel functions), and then integrating with respect to η from 0 to 1 yield

$$\dot{H}_n + (\lambda_n^2 + \kappa\alpha + M^2)\ H_n - \kappa\alpha H_{Pn} = b_n \quad (10)$$

$$\dot{H}_{Pn} + (\beta\lambda_n^2 + \alpha)H_{Pn} - \alpha H_n = 0 \tag{11}$$

where a dot denotes ordinary differentiation with respect to τ and $b_n = 2/(\lambda_n J_1(\lambda_n))$ (J_1 being the first-order Bessel function of the first kind). These equations are combined to give

$$\ddot{H}_n + (\lambda_n^2(1+\beta) + \alpha(1+\kappa) + M^2)\dot{H}_n + (\alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2))H_n = (\alpha + \beta\lambda_n^2)b_n \tag{12}$$

The solution of the above linear, ordinary, non-homogeneous, differential equation subject to $H_n(0)=0$ and $H_{Pn}(0)=0$ can be obtained by the usual method of solving such equations to yield

$$H_n = c_1\exp(s_1\tau) + c_2\exp(s_2\tau) + b_n(\alpha + \beta\lambda_n^2)/(\alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2)) \tag{13}$$

where s_1 and s_2 are the roots of the quadratic auxiliary equation

$$s^2 + (\lambda_n^2(1+\beta) + \alpha(1+\kappa) + M^2)s + \alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2) = 0 \tag{14}$$

$$\begin{aligned} c_1 &= -b_n(1 + s_2(\alpha + \beta\lambda_n^2)/(\alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2)))/(s_2 - s_1) \\ c_2 &= b_n(1 + s_1(\alpha + \beta\lambda_n^2)/(\alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2)))/(s_2 - s_1) \end{aligned} \tag{15}$$

The corresponding solution for H_{Pn} can be shown to be

$$\begin{aligned} H_{pn} &= 1/(\kappa\alpha)(c_1(s_1 + \lambda_n^2 + \kappa\alpha + M^2)\exp(s_1\tau) + c_2(s_2 + \lambda_n^2 + \kappa\alpha + M^2)\exp(s_2\tau)) \\ &\quad + b_n((\lambda_n^2 + \kappa\alpha + M^2)(\alpha + \beta\lambda_n^2)/(\alpha(\lambda_n^2 + M^2) + \beta\lambda_n^2(\lambda_n^2 + \kappa\alpha + M^2)) - 1) \end{aligned} \tag{16}$$

With the solutions for H_n and H_{Pn} known, then F and F_p can be determined from Equations (9). The solutions for Q, Q_p, C, and C_p can be calculated from

$$\begin{aligned} Q &= 4\pi\sum_{n=1}^{\infty} H_n/(b_n\lambda_n^2), \quad Q_p = 4\pi\sum_{n=1}^{\infty} H_{Pn}/(b_n\lambda_n^2) \\ C &= 2\sum_{n=1}^{\infty} H_n/b_n, \quad C_p = 2\beta\kappa\sum_{n=1}^{\infty} H_{pn}/b_n \end{aligned} \tag{17}$$

It should be noted that Equations (17) are obtained by substituting Equations (9) and their derivatives into Equations (8).

The closed-form solutions reported earlier are numerically evaluated and graphically plotted to elucidate the effects of the various physical parameters on the solutions. For brevity, only the transient behavior and the influence of the Hartmann number M on the fluid-phase volumetric flow rate Q, the particle-phase volumetric flow rate Q_p, the fluid-phase skin friction coefficient C, and the particle-phase skin friction coefficient C_p are presented graphically in figures 1 through 4, respectively. Initially, both phases are at rest, and suddenly; they are set to motion through the application of a constant pressure gradient. As a result, the shear stress at the surface of the pipe increases. This explains the obvious increases in Q, Q_p, C, and C_p shown in figures 1 through 4, respectively. These parameters continue to increase until the flow stabilizes and steady-state conditions are attained. The application of a transverse magnetic field has the tendency to retard the flow of both phases causing their average velocities and wall shear stresses in the pipe to decrease. Also, as the magnetic field strength is increased, this retardation effect magnifies. This is clearly depicted in the decreases in Q, Q_p, C, and C_p as the Hartmann number M increases displayed in figures 1 through 4. It can easily be observed from these figures that, for the parametric values employed to produce them, the steady-state conditions are reached in a relatively short period of time. It is also observed from other results not presented herein for brevity that the inclusion of the particle-phase viscous stresses causes Q, Q_p, and C to decrease and C_p to increase (see definition of C_p) at any time. Furthermore, the approach to steady-state conditions is much accelerated than that of the case of inviscid particle phase ($\beta=0$). It should be mentioned that the general exact results obtained herein reduce to those reported by Dube and Sharma [8] and Ritter and Peddieson [9] when $M=0$ and $\beta=0$. Also, The steady-state solutions reported by Gadiraju, Peddieson, and Munukutla [7] for a neutrally buoyant suspension with no particulate wall slip are reproduced by setting $M=0$ in the present results. These comparisons lend confidence in the correctness of the solutions presented in this paper.

Conclusion

The transient flow of a particulate suspension in an electrically conducting fluid in a circular pipe with an applied transverse magnetic field is studied. The governing equations of motion are derived, nondimensionalized, and solved in closed form. Numerical evaluations of the exact solutions are performed and graphical results for the volume flow rates and skin friction coefficients of both phases are presented and discussed to demonstrate the effect of the magnetic field on these physical parameters. It was found that all these parameters decrease as the strength of the applied magnetic field increases. While comparisons with previously published theoretical work on this problem were performed, no comparisons with experimental data were done because, as far as the author is aware, such data are lacking at the present time. It should be noted that the fact that the particle volume fraction was assumed constant allowed the governing equations to be solved analytically. This would not have been possible if the particle-phase volume fraction varied as reported by previous investigators (see, Sinclair and Jackson [4], Johnson, Massoudi, and Rajagopal [10], Wang, Lee, Jones, and Lahey [12], and Drew and Lahey [13]). It is hoped that the results reported herein will serve as a check for investigating suspensions of varying particulate volume fraction and alternate particulate stress models and provide a stimulus for experimental work on this problem.

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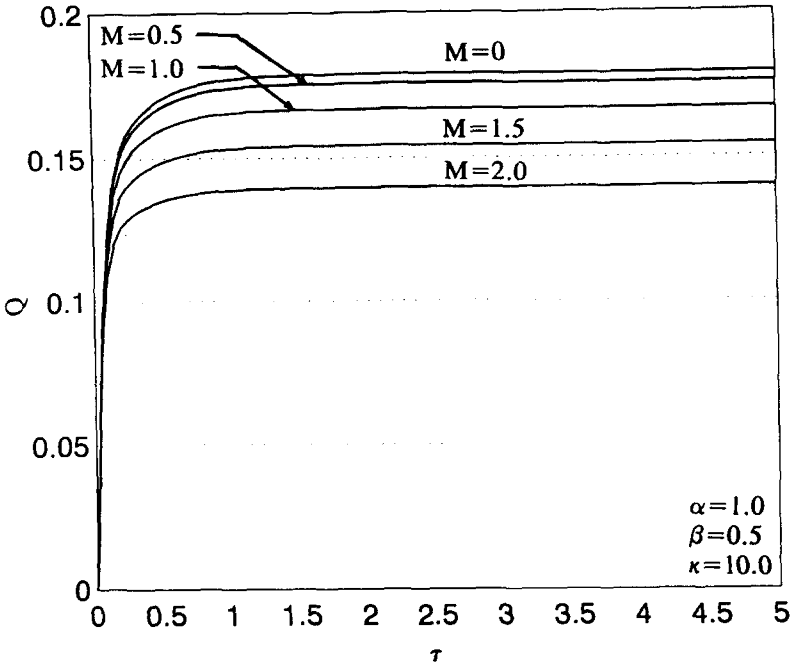

Figure 1. Fluid-Phase Volume Flow Rate Time History

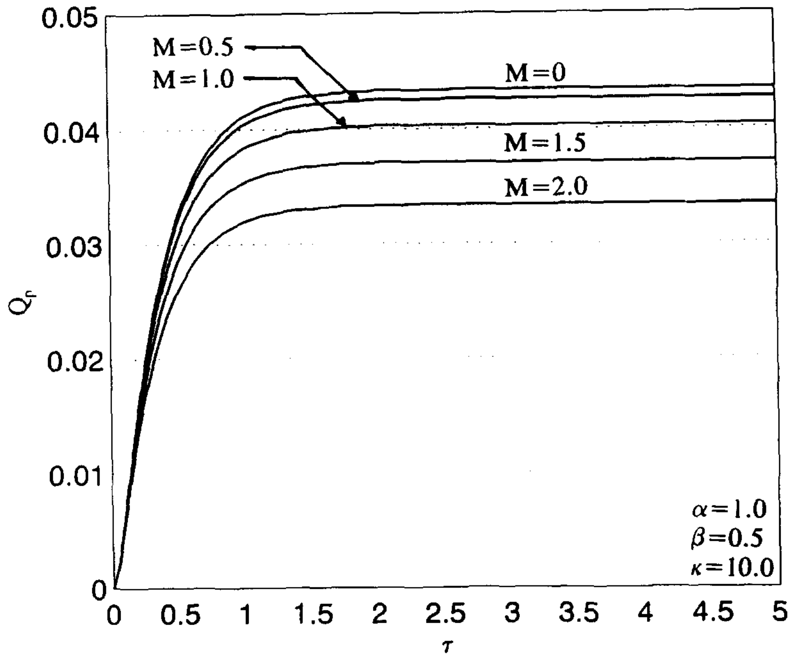

Figure 2. Particle-Phase Volume Flow Rate Time History

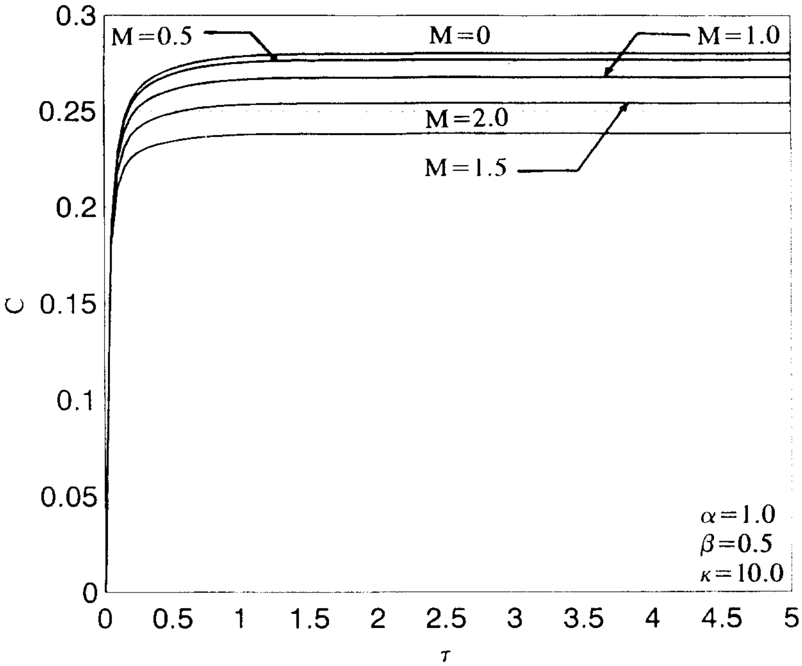

Figure 3. Fluid-Phase Skin Friction Coefficient Time History

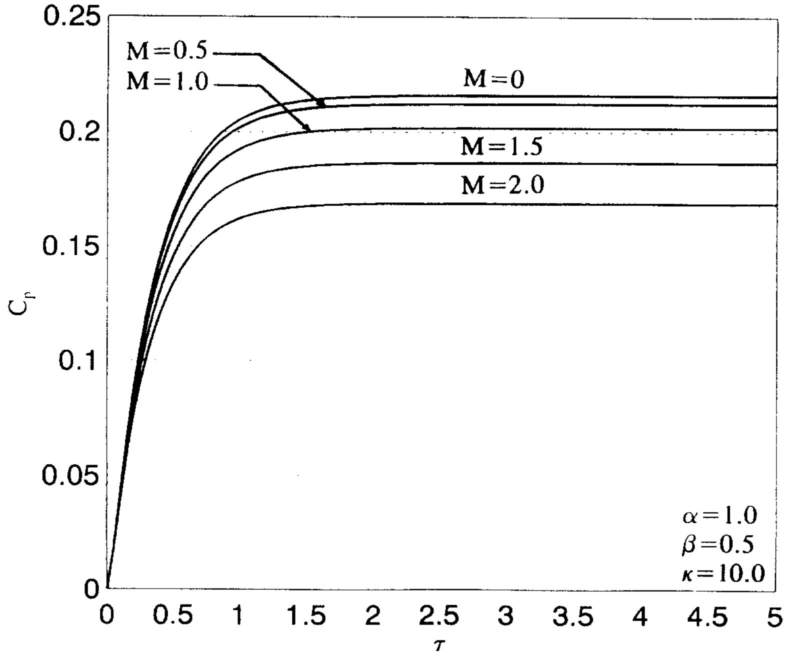

Figure 4. Particle-Phase Skin Friction Coefficient Time History