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Fully Developed Mixed Convection of a Micropolar Fluid in a Vertical Channel†

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A theoretical study of the fully developed mixed convection flow of a micropolar fluid in a parallel plate vertical channel with an asymmetric wall temperature distribution has been presented. Solutions of the governing equations are obtained both analytically and numerically and it is shown that they are in excellent agreement. A reverse flow is observed in some cases and is based on the analytical solution. Criteria for the occurrence of this flow are presented.

* * *

Nomenclature

A, B	constants;
g	acceleration due to gravity;
h	spacing between channel walls;
j	microinertia density;
K	non-dimensional material parameter;
n	microrotation;
R	wall temperature difference ratio;
T	fluid temperature;
T_0	temperature at the channel entrance;
T_1	temperature of the cold wall ($y = 0$);
T_2	temperature of the hot wall ($y = h$);
u	axial velocity;
U_0	velocity at the channel entrance;
x, y	Cartesian coordinates;
β	coefficient of thermal expansion;

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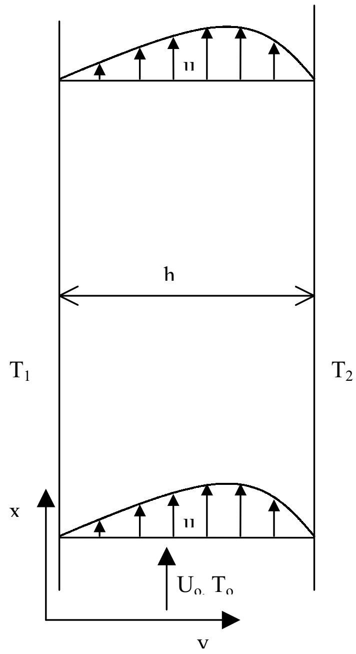

Fig. 1. Schematic diagram of the physical model.

γ	spin-gradient viscosity;
κ	vortex viscosity;
μ	dynamic viscosity;
ρ	density;
θ	non-dimensional temperature.

Introduction

The theory of micropolar fluids and its extension to thermo-micropolar fluids was formulated by Eringen [1, 2] and has drawn considerable interest in recent years due to its practical application in many fields. It can be used to study the behavior of exotic lubricants, colloidal suspensions or polymeric additives, blood flow, liquid crystals and dirty oils, to name just a few practical applications of these fluids. A detailed review of the published papers on these fluids can be found in the review article by Ariman et al. [3] and in the recent books by Łukaszewicz [4] and Eringen [5].

However, to the best of our knowledge, there exist only a little number of works published covering forced, free and mixed convection flows of a micropolar fluid in a vertical channel. Therefore, the purpose of this paper is to study the fully developed flow of a micropolar fluid in a mixed convection between a parallel-plate vertical channel with asymmetric wall temperature distribution. It is worth pointing out that for a Newtonian fluid this problem was studied in a series of papers by Aung and Worku [6–8]. They assumed that the two walls of the channel are maintained at uniform but not necessarily equal temperatures and this assumption will also be considered here. It is shown that for a fixed value of the material parameter K the buoyancy effects are at largest adjacent to heated wall and, therefore, velocities and microrotation profiles increase in near-wall regions with a concomitant decrease elsewhere due to the fixed flow rate.

1. Basic equations

Let us consider the laminar mixed convection flow of a micropolar fluid between two vertical plates, the space between the plates being h, as shown in Fig. 1. The flow is assumed to be steady and fully developed, i. e., the transverse velocity is zero. It is also assumed that the fluid has a uniform velocity U_0 and temperature T_0 at the channel entrance. The walls are heated uniformly but their temperatures may be different resulting in an asymmetric heating situation. Under these assumptions, the equations that describe the physical situation are

$$-\frac{dp}{dx}+(\mu+\kappa)\frac{d^2u}{dy^2}+\kappa\frac{dn}{dy}+\rho\mathrm{g}\beta(T-T_0)=0, \tag{1}$$

$$\gamma\frac{d^2n}{dy^2}-\kappa\left(2n+\frac{du}{dy}\right)=0, \tag{2}$$

$$\frac{d^2T}{dy^2}=0, \tag{3}$$

subjected to the boundary conditions:

$$\begin{aligned} &u(0)=0, \qquad T(0)=T_1, \qquad n(0)=0,\\ &u(h)=0, \qquad T(h)=T_2, \qquad n(h)=0, \end{aligned} \tag{4}$$

where x and y are axial and transverse co-ordinates, respectively ($x=0$ is the duct entrance and $y=0$ is the left wall); u is an axial velocity; T is a fluid temperature; n is a microrotation component of the micropolar fluid normal to (x,y)-plane; p is a pressure; T_1 is a temperature of the cold wall (i. e., at $y=0$); T_2 is a temperature of the hot wall (i. e., $y=h$) and ρ, g, β, μ, κ, and γ are density, gravitational acceleration, coefficient of thermal expansion, dynamic viscosity, vortex viscosity, and spin gradient viscosity, respectively. We may notice that the condition that microrotation n vanishes on the walls, which is called strong interaction, see Guram and Smith [9]. Further, we shall assume that γ has the following form as proposed by Ahmadi [10]:

$$\gamma=\left(\mu+\frac{\kappa}{2}\right)j, \tag{5}$$

where j is a microinertia density.

Eqs (1) – (3) can be non-dimensionalized using the variables

$$Y=\frac{y}{h}, \qquad U=\frac{u}{U_c}, \quad \theta=\frac{T-T_0}{T_2-T_0}, \qquad N=\frac{h}{U_c}\,n, \tag{6}$$

where U_c is a characteristic velocity. After employing Eq. (6) and taking $j=h^2$, the dimensionless form of the governing equations become the following ones:

$$(1+K)\frac{d^2U}{dY^2}+K\frac{dN}{dY}+\frac{\mathrm{Gr}}{\mathrm{Re}}\,\theta+\alpha=0, \tag{7}$$

$$\left(1+\frac{K}{2}\right)\frac{d^2N}{dY^2}-K\left(2N+\frac{dU}{dY}\right)=0, \tag{8}$$

$$\frac{d^2\theta}{dY^2}=0, \tag{9}$$

where $\mathrm{Gr} = \mathrm{g}\beta(T_2 - T_0)h^3/\nu^2$ is the Grashof number; $\mathrm{Re} = U_0 h/\nu$ is the Reynolds number; α is the pressure gradient parameter (assumed constant); K is a material parameter. Those are defined as

$$\alpha = -\frac{dP}{dX}\,, \qquad K = \frac{k}{\mu}\,. \tag{10}$$

The dimensionless boundary conditions Eq. (4) become:

$$\begin{aligned} &U(0) = 0, \qquad \theta(0) = R, \qquad N(0) = 0, \\ &U(1) = 0, \qquad \theta(1) = 1, \qquad N(1) = 0, \end{aligned} \tag{11}$$

where

$$R = \frac{T_1 - T_0}{T_2 - T_0}\,. \tag{12}$$

2. Solution

Eq. (9) subjected to Eq. (11) has the analytical solution

$$\theta = (1 - R)Y + R \tag{13}$$

and for $K = 0$ (Newtonian fluid) we have $N = 0$ and U has the analytical solution obtained by Aung and Worku [7].

On the other hand, Eqs (7) and (8) possess for $K \neq 0$ (micropolar fluid) the following analytical solution:

$$\begin{aligned} \sqrt{2K\,(1+K)}\,U = {} & \frac{K}{2\,(2+K)} \frac{\cosh\left(\sqrt{2K/\,(1+K)}Y\right)}{\sinh\left(\sqrt{2K/\,(1+K)}\right)} \left[\frac{\mathrm{Gr}}{\mathrm{Re}}\,(1+R) + 2\alpha\right] \\ & - \frac{\sqrt{2K\,(1+K)}}{2+K} \left[\frac{1}{3}\frac{\mathrm{Gr}}{\mathrm{Re}}\,(1-R)\,Y^3 + \left(\frac{\mathrm{Gr}}{\mathrm{Re}}R + \alpha\right)Y^2\right] \\ & + \sqrt{\frac{1+K}{2K}} \left[\frac{\mathrm{Gr}}{\mathrm{Re}}\,(1-R)\,Y + \frac{\mathrm{Gr}}{\mathrm{Re}}R + \alpha\right] \\ & + \left[-K\exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) \frac{\cosh\left(\sqrt{2K/\,(1+K)}\,Y\right)}{\cosh\left(\frac{1}{2}\sqrt{2K/\,(1+K)}\right)} + K\exp\left(\sqrt{\frac{2K}{1+K}}\,Y\right) \right. \\ & \left. -2\sqrt{2K(1+K)}\,Y\right] A + 2\sqrt{2K(1+K)}\,B, \end{aligned} \tag{14}$$

$$N = -\frac{1}{2\left(2+K\right)} \frac{\sinh\left(\sqrt{2K/\left(1+K\right)}\,Y\right)}{\sinh\left(\sqrt{2K/\left(1+K\right)}\right)} \left[\frac{\mathrm{Gr}}{\mathrm{Re}}\left(1+R\right)+2\alpha\right]$$
$$+\frac{1}{2\left(2+K\right)} \frac{\mathrm{Gr}}{\mathrm{Re}} \left(1-R\right)Y^2 + \frac{1}{\left(2+K\right)}\left(\frac{\mathrm{Gr}}{\mathrm{Re}}R+\alpha\right)Y$$
$$+\left[\exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right)\frac{\sinh\left(\sqrt{2K/\left(1+K\right)}\,Y\right)}{\cosh\left(\frac{1}{2}\sqrt{2K/\left(1+K\right)}\right)} \right.$$
$$\left. -2\exp\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\,Y\right)\sinh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right)Y\right]A, \tag{15}$$

where A and B are constants which have to be determined from the following algebraic system of equations:

$$K\tanh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right)A - \sqrt{2K(1+K)}\,B$$
$$= \frac{K}{2(2+K)}\frac{1}{\sinh\left(\sqrt{2K/(1+K)}\right)}\left[\frac{\mathrm{Gr}}{\mathrm{Re}}\left(1+R\right)+2\alpha\right]+\sqrt{\frac{1+K}{2K}}\left(\frac{\mathrm{Gr}}{\mathrm{Re}}R+\alpha\right),$$

$$\left[K\tanh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right)-2\sqrt{2K(1+K)}\right]A+\sqrt{2K(1+K)}B \tag{16}$$
$$= -\frac{K}{2(2+K)}\coth\left(\sqrt{\frac{2K}{1+K}}\right)\left[\frac{\mathrm{Gr}}{\mathrm{Re}}\left(1+R\right)+2\alpha\right]$$
$$+\frac{\sqrt{2K(1+K)}}{2+K}\left[\frac{1}{3}\frac{\mathrm{Gr}}{\mathrm{Re}}(1+2R)+\alpha\right]-\sqrt{\frac{1+K}{2K}}\left(\frac{\mathrm{Gr}}{\mathrm{Re}}+\alpha\right).$$

However, the pressure gradient parameter α is still undetermined. It can be evaluated from the equation expressing the conservation of mass at any cross-section of the channel, which is given by

$$\int_0^1 U\,dY = 1. \tag{17}$$

Using Eqs (14) and (17), we get

$$\frac{3+2K}{3K(2+K)}\alpha+\frac{1}{6(2+K)}\frac{\mathrm{Gr}}{\mathrm{Re}}+\frac{1}{4K}\frac{\mathrm{Gr}}{\mathrm{Re}}(1+R)-A+B=1. \tag{18}$$

In the parametric evaluation of the above analytical results, Eqs (16) and (18) are solved simultaneously for the unknowns A, B and α.

It should be noted that the forced convection limit in this problem is obtained by setting $\mathrm{Gr}/\mathrm{Re} = 0$ in Eqs (7), (14) – (16) and (18), while the free convection limit is recovered by taking $\alpha = 0$ and $\mathrm{Gr}/\mathrm{Re} = 1$ in Eqs (7), (14) – (16).

3. Results and Discussion

Eqs (7) and (8) subjected to the boundary conditions Eq. (11) have been solved numerically using the implicit finite-difference method discussed by Blottner [11] for some values of temperature, material and mixed convection parameters R, K and Gr/Re. These equations were discretized using three-point central-difference quotients and, as a consequence, a set of algebraic equations resulted. These algebraic equations were then solved by the well-known tri-diagonal Thomas algorithm (see Blottner [11]). The computational domain was divided into 201 points and constant step sizes of 0.01 were utilized. These step sizes were found to give accurate grid-independent results as verified by the comparisons shown in the figures below. The numerical results for the axial velocity U, microrotation n and pressure gradient α profiles are presented in Figs 2 to 10. The analytical solution given by Eqs (14) and (15) are also included in these figures and it is seen that both the numerical and analytical solutions are in excellent agreement. In addition, Fig. 2 shows that for $K = 0$ (Newtonian fluid) the present results are in excellent agreement with those reported by Aung and Worku [7]. On the other hand, Figs 2 to 10 show that at each R and K values, the profiles become increasingly skewed as Gr/Re increases. The skewness is characterized by an increased positive velocity and positive microrotation profiles near the hot wall ($Y = 1$) and decreased values of these profiles near the cold wall ($Y = 0$). It is also seen from these figures that at sufficiently large values of Gr/Re, the velocity profiles adjacent to the cold wall become negative, i. e., there is a flow reversal condition. Both the magnitudes and extents of the reversed flow increase with Gr/Re and K. At fixed values of Gr/Re, flow separation is observed to move upstream (i. e., down the vertical channel) as the parameters R and K decrease. Further, we can see that all the velocity profiles U intersect at $Y = 0.5$; at this location the velocity is positive for each value of the parameter K and has a numerical value close to 1.5. However, the microrotation profiles remain negative in the most part of the channel but become positive near the hot wall.

The variation of the pressure gradient parameter α as a function of Gr/Re is shown in Fig. 10 for $R = 0$, 0.5 and 1 when $K = 1$. It can be seen from this figure that for all R, the parameter α ranges from 0 for a pure forced convection flow ($\mathrm{Gr}/\mathrm{Re} = 0$) to negative values as Gr/Re increases.

The examination of Figs 2, 3, 5 and 8 suggests that for $R < 1$, the occurrence of the reversal flow is given by the condition

$$\left(\frac{dU}{dY}\right)_{Y=0} < 0. \tag{19}$$

Using Eq. (14), we get

$$(1-R)\frac{\mathrm{Gr}}{\mathrm{Re}} < \frac{2K(2+K)}{1+K}\,A \quad \text{for} \quad R < 1, \tag{20}$$

that is

$$\left(\frac{\mathrm{Gr}}{\mathrm{Re}}\right)_{\min} = \frac{2K(2+K)}{(1-R)(1+K)}\,A \quad \text{for} \quad R < 1. \tag{21}$$

This equation gives the minimum value for Gr/Re for which a reversal flow exists or it is the maximum value of Gr/Re for which no reversal flow occurs. Thus, for any value of Gr/Re greater than $(\mathrm{Gr}/\mathrm{Re})_{\min}$ a reversed flow appears. It should be mentioned that for the evaluation of $(\mathrm{Gr}/\mathrm{Re})_{\min}$, Eqs (16), (18) and (21) are solved simultaneously for the unknowns A, B and α. The values of $(\mathrm{Gr}/\mathrm{Re})_{\min}$ for some values of the parameters R and K are given in the Table, showig that the value of $(\mathrm{Gr}/\mathrm{Re})_{\min}$ for which a reversal flow occurs is greater for a micropolar fluid ($K = 0$) than for a Newtonian fluid ($K = 0$).

Table
Values of $(\mathrm{Gr}/\mathrm{Re})_{\min}$ for occurrence of reversed flow at $Y = 0$.

K / R	0	0.5	1.0	1.5	2.0	3.0
0	72.0000	107.5436	142.5256	177.2169	211.7379	280.4892
0.1	80.0000	119.4928	158.3618	196.9072	235.2643	311.6546
0.3	102.8571	153.6337	203.6080	253.1670	302.4827	400.6988
0.5	144.0000	215.0871	285.0512	354.4339	423.4758	560.9783
0.8	360.0000	537.7178	712.6280	886.0847	1058.7000	1402.4000

For the limiting case of free convection, recently studied by Chamkha et al. [12], we have

$$\left(\frac{dU}{dY}\right)_{Y=0} = \frac{2K(2+K)}{(1-R)(1+K)}\, A < 0 \quad \text{for} \quad R < 1, \tag{22}$$

where the constant A is now obtained from the following system of algebraic equations:

$$\begin{gathered}
K \tanh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) A - \sqrt{2K(1+K)}\, B \\
= \frac{K}{2(2+K)} \frac{1+R}{\sinh\sqrt{2K/(1+K)}} + R\sqrt{\frac{1+K}{2K}}, \\
\left[K \tanh\left(\frac{1}{2}\sqrt{\frac{2K}{1+K}}\right) - 2\sqrt{2K(1+K)}\right] A + \sqrt{2K(1+K)}\, B \\
= -\frac{K(1+R)}{2(2+K)} \coth\left(\sqrt{\frac{2K}{1+K}}\right) + \frac{(1+2R)}{3}\frac{\sqrt{2K(1+K)}}{2+K} - \sqrt{\frac{1+K}{2K}}.
\end{gathered} \tag{23}$$

For $K = 0$, we can compare the values of $(\mathrm{Gr}/\mathrm{Re})_{\min}$ with Eq. (14) of Aung and Worku [6]. Thus, for $R = 0.1$, 0.3, 0.5 and 0.8, we have $(\mathrm{Gr}/\mathrm{Re})_{\min} = 80.0$, 102.85714, 144.0 and 360.0, which are the exact values as those given in Table .

Conclusions

The theoretical results obtained in this study show that both the velocity and microroration profiles in the developing regions can become highly distorted in mid convection flow. The asymmetric wall temperatures lead to skewness in the velocity and microrotation profiles. It is also concluded the the value of $(\mathrm{Gr}/\mathrm{Re})_{\min}$ for which flow reversal occurs is greater for a micropolar fluid ($K = 0$) than for a Newtonian fluid ($K = 0$).

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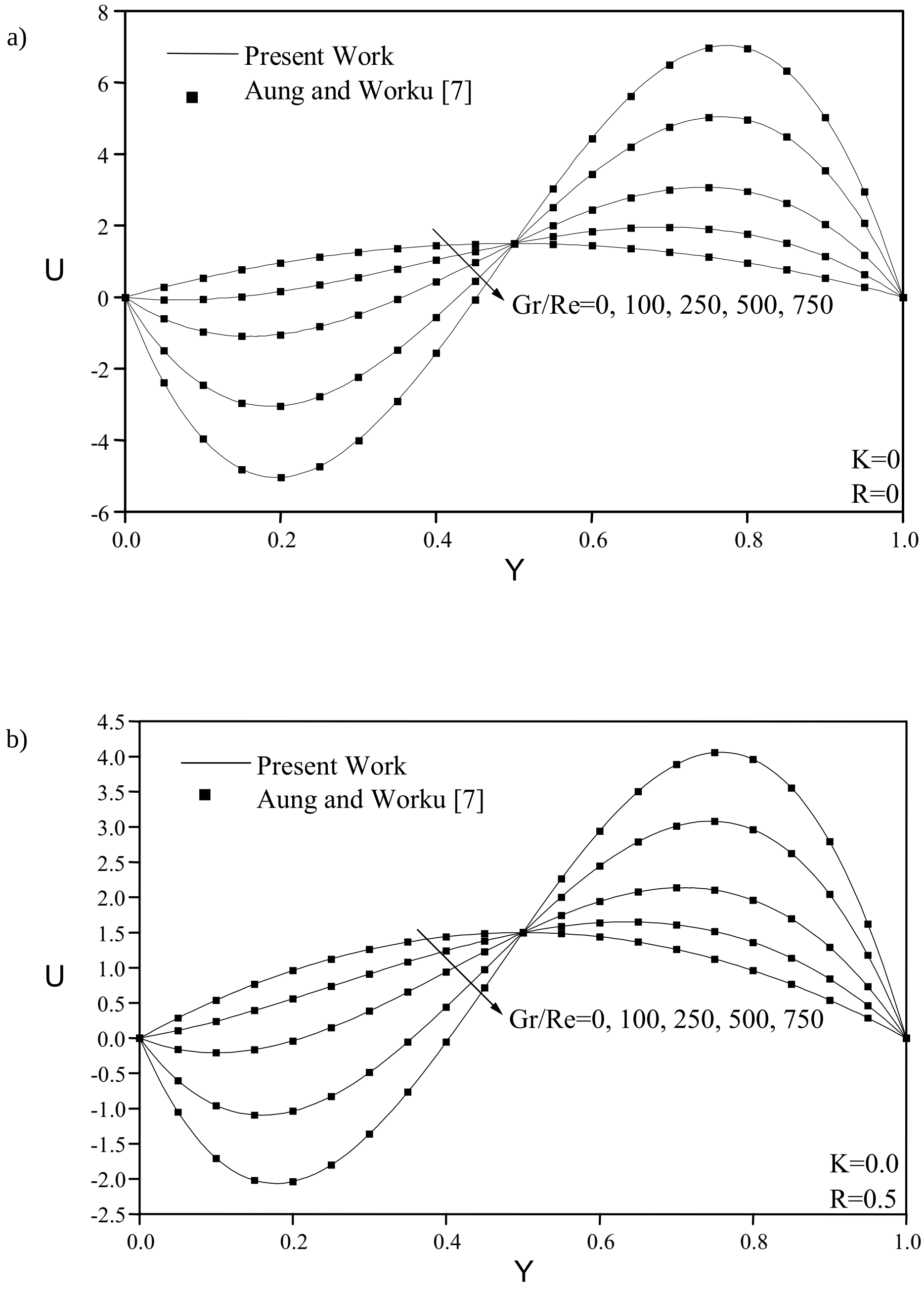

Fig. 2. Comparison of present work with those by Aung and Worku [7]:
a) for $R = 0$, b) for $R = 0.5$.

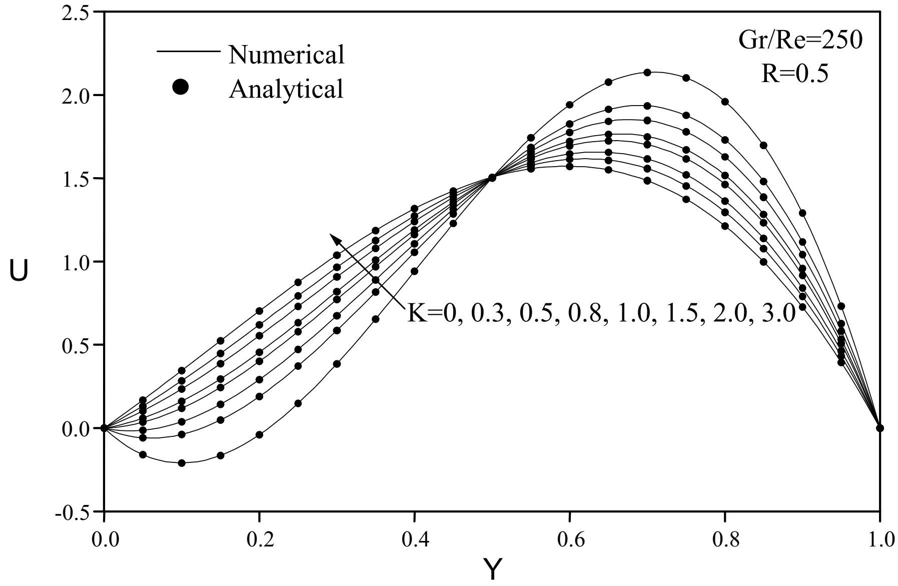

Fig. 3. The effects of K on the velocity profiles.

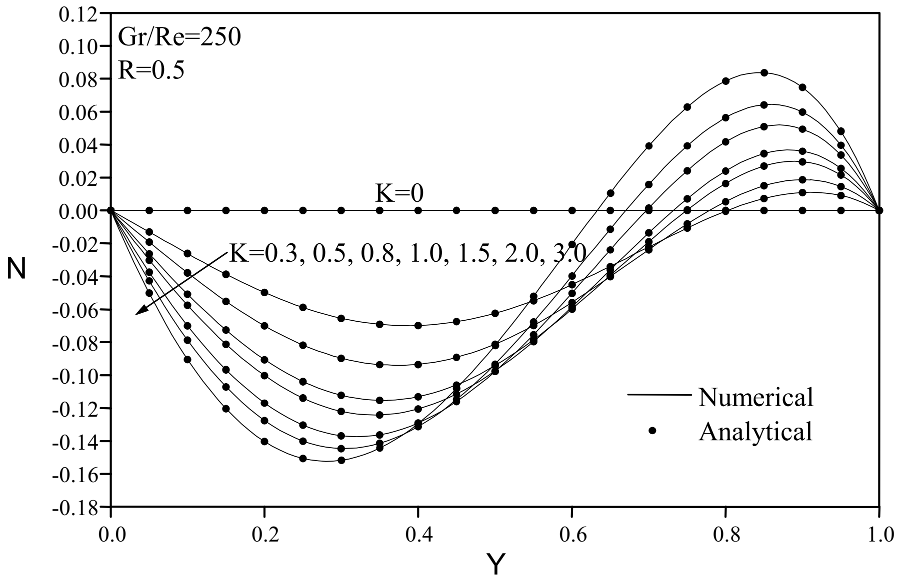

Fig. 4. The effects of K on the microrotation profiles.

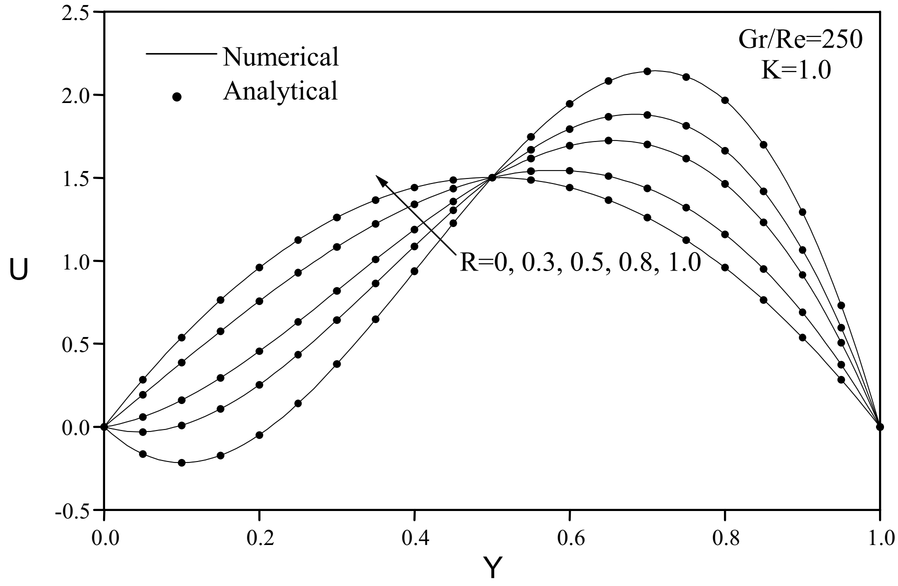

Fig. 5. The effects of R on the velocity profiles.

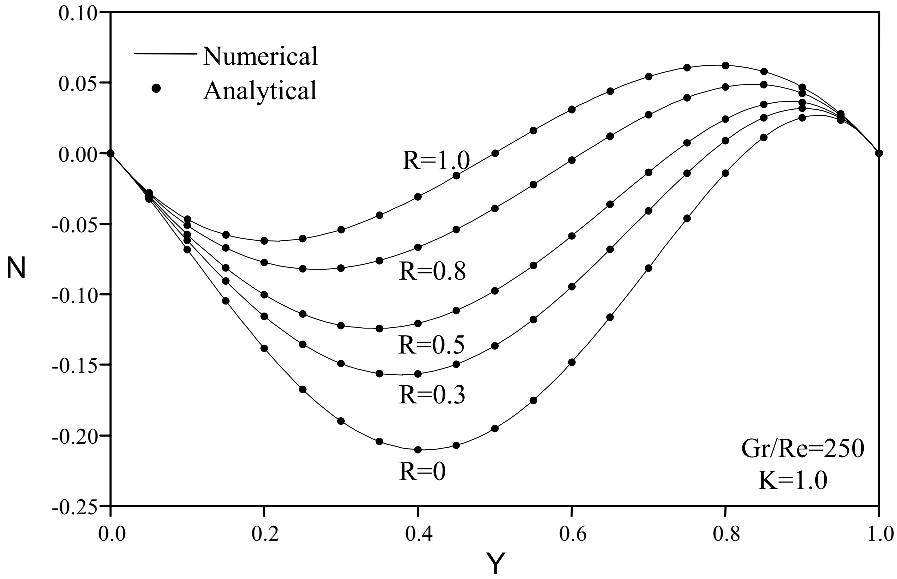

Fig. 6. The effects of R on the microrotation profiles.

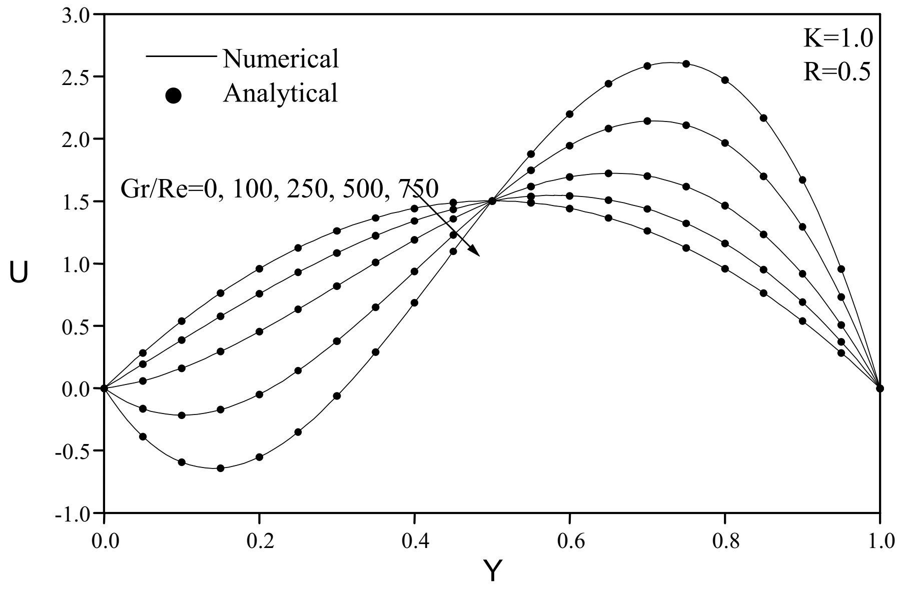

Fig. 7. The effects of Gr/Re on the velocity profiles.

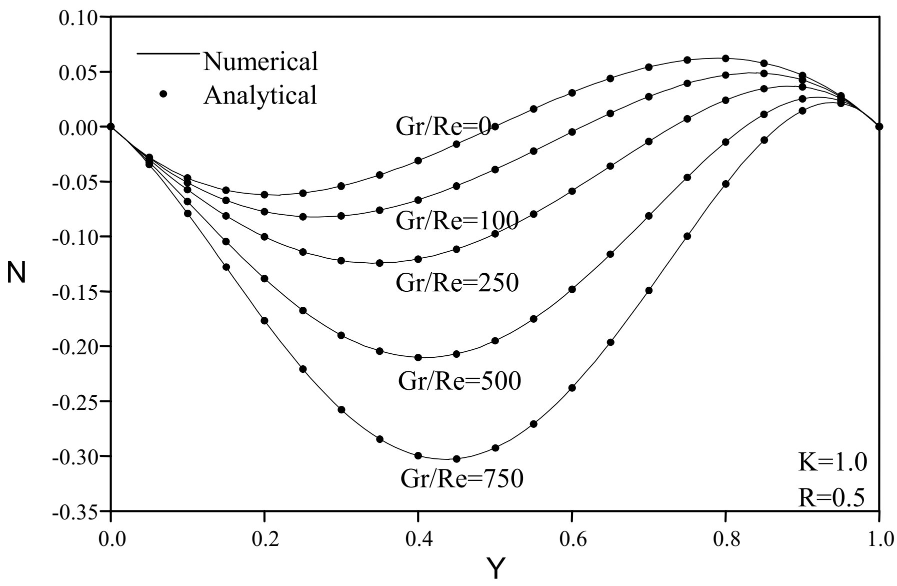

Fig. 8. The effects of Gr/Re on the microrotation profiles.

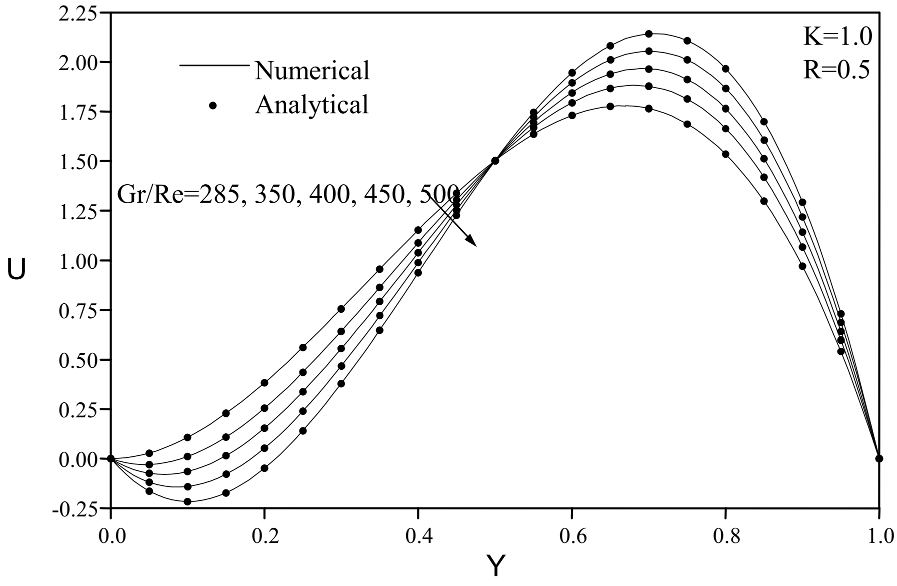

Fig. 9. Observation of the reversal flow conditions for various Gr/Re values.

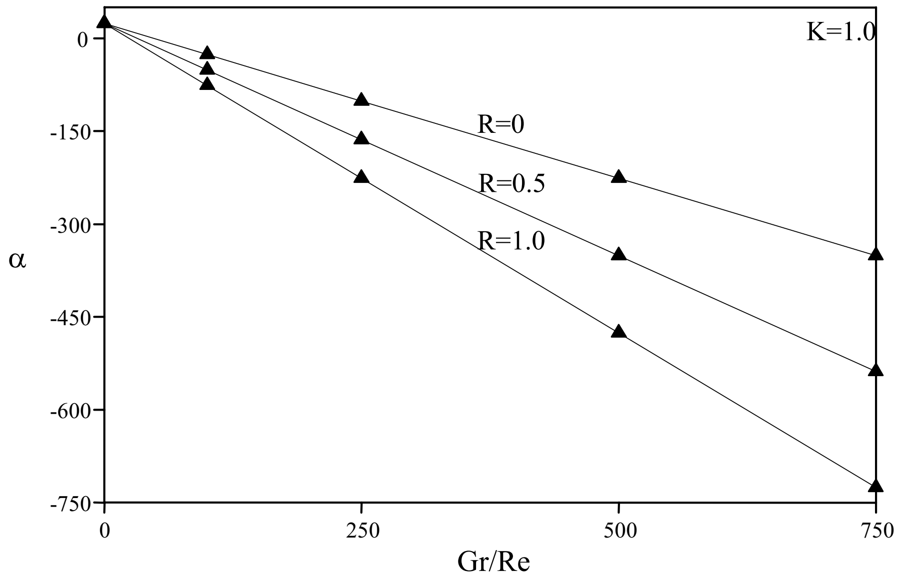

Fig. 10. The effects of R on the pressure gradient relation.

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