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Steady natural convection flow of a particulate suspension through a circular pipe

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Abstract A continuum model for two-phase (fluid/particle) flow induced by natural convection is developed and applied to the problem of steady natural convention flow of a particulate suspension through an infinitely long pipe. The wall of the pipe is maintained at a constant temperature. The particle phase is endowed by an artificial viscosity which may be used to model particle-particle interaction in dension suspensions. Boundary conditions borrowed from rarefied gas dynamics are employed for the particle-phase wall conditions. Closed-form solutions for the velocity and temperature profiles are obtained. For the assumptions employed in the problem, the temperatures of both phases in the pipe are predicted to be uniform. A parametric study of some physical parameters involved in the problem is performed to illustrate the influence of these parameters on the velocity profiles of both the fluid and particle phases.

Nomenclature

c Fluid-phase specific heat at constant pressure
c_p Particle-phase specific heat at constant pressure
g Gravitational acceleration
Gr Grashof number
h Channel width
H Dimensionless buoyancy parameter
k Fluid-phase thermal conductivity
N Interphase momentum transfer coefficient
N_T Interphase heat transfer coefficient
P Fluid-phase hydrostatic pressure
Pr Fluid-phase Prandtl number
r Radial direction
R Radius of the pipe
S Dimensionless particle-phase wall slip coefficient
T Fluid-phase temperature
T_p Particle-phase temperature
T_w Wall temperature
u Fluid-phase dimensionless velocity
u_p Particle-phase dimensionless velocity
U Fluid-phase velocity
U_p Particle-phase velocity
x Vertical direction

Greek Symbols

α Velocity inverse Stokes number
β Viscosity ratio
β^* Volumetric expansion coefficient
γ Specific heat ratio
ε Temperature inverse Stokes number
η Dimensionless radial direction
θ Dimensionless fluid-phase temperature
κ Particle loading
μ Fluid-phase dynamic viscosity
μ_p Particle-phase dynamic viscosity
ρ Fluid-phase density
ρ_p Particle-phase density
ω Particle-phase wall slip coefficient

Subscript

r First derivative with respect to r
x First derivative with respect to x

1 Introduction

A two-phase (fluid-particle) flow and heat transfer can be found in many industrial processes such as chemical, petrochemical, biochemical and environmental processes. Some applications include mixing, pneumatic conveying, fluidized bed, pollution, food industry, cooling of nuclear reactors, heat exchanger technology, cooling processes in electronic devices and solar collectors where a particulate suspension is used to enhance absorption of radiation. The objective of this work is to consider steady natural convection flow of a particulate suspension through a vertical circular pipe.

The general area of natural or free convection in channels and annular ducts for a single phase has received a great deal of attention in recent years due to the fact that many applications involve natural convection. The book by Gebhart et al. (1988) represents a good source of information on many investigations dealing with the subject. Aung et al. (1972) have reported on the development of laminar free convection between vertical flat plates

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with asymmetric heating. Lee et al. (1982) has considered natural convection in a vertical channel with opposing buoyancy forces. Yang et al. (1974) and Wang (1988) have studied natural convection about vertical plates with oscillatory surface temperature and in vertical channels with periodic heat input, respectively. Al-Arabi et al. (1987) and Oosthuizen and Paul (1986) have studied natural convection through vertical annular ducts. Joshi (1987) has provided analytical solutions for steady fully developed free convection flow in vertical annuli. El-Shaarawi and Al-Nimr (1990) have reported analytical solutions for the problem of steady fully developed laminar natural convection in open-ended vertical concentric annuli.

The above references have dealt with natural or free convection flow of a clean fluid. However, in real situations, it is hardly possible to find a totally clean dust-free fluid. In many applications the dust particles may be added deliberately or may be present naturally. Depending on the level of particle contamination, the presence of solid particles in fluids has proven to alter the heat transfer characteristics significantly. This has a direct effect on the efficiencies of devices and systems. Therefore, it is of great interest to study natural convection in a two-phase fluid-particle suspension.

Very little work have been reported on natural convection flow of a particle-fluid suspension over and through different geometries. Recently, Al-Subaie and Chamkha (2001) have considered the problem of steady natural convection flow of a particulate suspension through a parallel-plate channel using a contiuum two-phase fluid-particle model. Chamkha and Ramadan (1998) and Ramadan and Chamkha (1999) have reported some analytical and numerical results for natural convection flow of a two-phase particulate suspension over an infinite vertical plate. They found that increases in either of the particle loading or the wall particulate slip coefficient caused reductions in the velocities of both phases. Also, Okada and Suzuki (1997) have considered buoyancy-induced flow of a two-phase suspension in an enclosure. However, the present authors were unable to locate any theoretical or experimental work in the literature dealing with natural convection laminar flow of a particulate suspension in vertical pipes. Thus, there is a definite need for investigation of such a problem since it is almost impossible to find non-contaminated fluid in real applications. Due to the compexity involved in solving a real two-phase particulate suspension, some assumptions will be made to obtain analytical solutions for steady natural convection laminar flow of particulate suspensions through an infinitely-long vertical pipe.

2 Governing equations

Consider steady, laminar, natural convection fully-developed two-phase (fluid-particle) flow through an infinitely-long vertical circular pipe. The fluid phase is assumed to be incompressible, viscous, and Newtonian while the particle phase is assumed to be made up of small spherical solid nondeformable particles of one size and uniform density. The particle phase is assumed to be somewhat dense so that light particle-particle interaction exists. This can be modeled by endowing the particle phase by an artificial viscosity. Since the circular pipe is assumed to be infinitely long, the dependence of variables on the x-direction (vertical direction) will be negligible compared with that of the r-direction (radial direction) (see Fig. 1). The governing equations for this investigation are based on the balance laws of mass, linear momentum and energy of both phases. Under the assumptions made above, these equations reduce to:

$$-\partial_x P + \mu[\partial_{rr} U + (1/r)\partial_r U] - \rho_p N(U - U_p) + \rho g = 0 \tag{1}$$

$$k[\partial_{rr} T + (1/r)\partial_r T] + \rho_p c_p N_T (T_p - T) = 0 \tag{2}$$

$$\mu_p[\partial_{rr} U_p + (1/r)\partial_r U_p] + \rho_p N(U - U_p) - \rho_p g = 0 \tag{3}$$

$$\rho_p c_p N_T (T_p - T) = 0 \tag{4}$$

where the continuity equations are not written because it is clear that they are identically satisfied. All parameters are defined in the Nomenclature section. It should be noted that equations (1) through (4) represent a generalization to the original dusty-gas model discussed by Marble (1970) through the inclusion of the particle-phase viscous stresses.

The particle-phase viscous stresses can be used to model particle-particle interaction in dense suspensions. They can be thought of as a natural consequence of the averaging processes employed to model a discrete system of particles as a contiuum (see, for instance, Drew and Segal 1971 and Drew 1983).

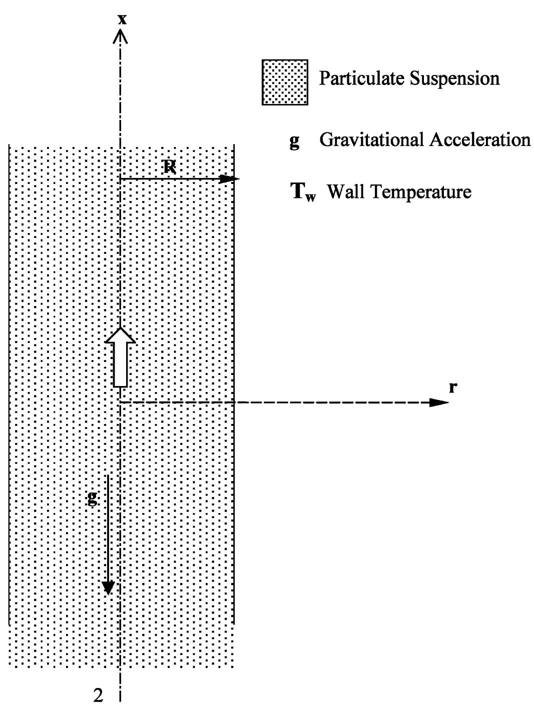

Fig. 1. Problem definition

The pressure gradient term $\partial_x P$ can be eliminated from the linear momentum equation of the fluid phase by evaluating the governing equations at a reference point at the inlet of the pipe. Let subscript "o" be a reference point at the inlet of the pipe (before the flow starts due to buoyancy effects) such that U = 0, $T = T_o$, $\rho = \rho_o$, $\mu = \mu_o$, $U_p = U_{po}$, $T_p = T_{po}$, $\rho_p = \rho_{po}$ and $\mu_p = \mu_{po}$. Evaluating the governing equations at this reference point and employing the Boussinesq approximation gives:

$$\rho_{po}/\rho_o g + \mu_o/\rho_o[\partial_{rr}U + (1/r)\partial_r U] - \rho_{po}/\rho_o N(U - U_p) + \beta^* g(T - T_o) = 0 \quad (5)$$

where β^* is the volumetric expansion coefficient. The linear momentum equation of the fluid phase, equation (1), will now be replaced by equation (5) in the governing equations.

Each of equations (2), (3) and (5) requires two boundary conditions to solve them completely. These can be written as:

$$\partial_r U(0) = U(R) = 0, \quad \partial_r T(0) = 0, \; T(R) = T_w \quad (6a-d)$$

$$\partial_r U_p(0) = 0, \; U_p(R) = -\omega \partial_r U_p(R) - g/N \quad (6e-f)$$

where R is the pipe radius, T_w is the pipe wall temperature at r = R and ω is the particle-phase wall slip coefficient. Equations (6a) and (b) indicate a symmetry condition and a no slip condition for the fluid phase at the pipe wall. Equation (6c) indicates a temperature symmetry condition and equation (6d) indicates that the fluid temperature at the wall of the pipe is some constant value T_w. Equations (6e) and (6f) express a symmetry condition and a proposed wall boundary condition for the particle phase at the pipe surface. The exact form of wall boundary conditions for the particulate phase are not known at present. Therefore, a form which includes two idealized conditions will be considered for the particle phase wall boundary condition of the pipe. These are the no-slip condition ($\omega = 0$) and the perfect slip condition ($\omega \rightarrow \infty$): $\partial_r U_p(R) = 0$). The actual behavior is expected to be somewhere between these two extremes.

The formulation of the value problem of an infinite vertical pipe is now completed. It is now convenient to non-dimensionalize the governing equations and conditions. This can be accomplished by using the following parameters:

$$\begin{aligned} & r = R\eta, \; U = (\mu/\rho R)u, \; U_p(\mu/\rho R)u_p, \\ & T = T_w - T_o)\theta + T_o, \\ & T_p = (T_w - T_o)\theta_p + T_o \end{aligned} \quad (7)$$

where η is the dimensionless transverse coordinate, u and u_p are the dimensionless fluid- and particle-phase velocities, respectively, and θ and θ_p are the dimensionless fluid- and particle-phase temperatures, respectively. After performing the mathematical operations, the resulting dimensionless governing equations and conditions can be written as:

$$D^2 u + (1/\eta)Du - \alpha\kappa(u - u_p) + Gr\,0 + \kappa H = 0 \quad (8)$$

$$(1/Pr)[D^2\theta + (1/\eta)D\theta] + \kappa\gamma\varepsilon(0_p - 0) = 0 \quad (9)$$

$$\beta[D^2 u_p + (1/\eta)Du_p] + \alpha(u - u_p) - H = 0 \quad (10)$$

$$\varepsilon(\theta_p - \theta) = 0 \quad (11)$$

where D and D^2 denote a first and a second-order ordinary derivative operators with respect to η, respectively. Also, $\alpha = R^2 N\rho/\mu$, $\kappa = \rho_p/\rho$, $Gr = g\beta^* R^3 \rho(T_w - T_o)/\mu$, $H = gR^3\rho^2/\mu^2$, $\beta = \mu_p/\mu$, $Pr = \mu c/k$, $\gamma = c_p/c$ and $\varepsilon = \rho N_T R^2/\mu$ are the momentum inverse Stokes number, the particle loading, the Grashof number, buoyancy parameter, the viscosity ratio, the fluid-phase Prandtl number, the specific heat ratio and the temperature inverse Stokes number, respectively.

The dimensionless boundary conditions become:

$$Du(0) = 0, \; u(1) = 0, \; D\theta(0) = 0, \; \theta(1) = 1 \quad (12a-d)$$

$$Du_p(0) = 0, \; u_p(1) = -SD\,u_p(1) - H/\alpha \quad (12e, f)$$

where $S = \omega/R$ is the dimensionless particle-phase slip coefficient. It should be mentioned that, for an inviscid particle phase ($\beta = 0$), equations (12e,f) are ignored.

3 Analytical results and discussion

Inviscid Particle Phase: For an inviscid particle phase ($\beta = 0$), equation (10) implies that:

$$u_p(\eta) = u(\eta) - H/\alpha \quad (13)$$

which indicates that the particle-phase velocity is the same as that of the fluid phase except that it is shifted by the factor H/α below the fluid-phase velocity. Equation (11) implies that:

$$\theta_p(\eta) = \theta(\eta) \quad (14)$$

Substituting equation (14) into equation (9) yields

$$D^2\theta + (1/\eta)D\theta = 0 \quad (15)$$

The solution of equation (15) that satisfies the boundary conditions (12c,d) is

$$\theta(\eta) = 1 \quad (16)$$

This indicates that the temperature of both phases has a constant shape of pure conduction.

The fluid-phase velocity can be found by substituting equations (13) and (16) into equation (8) to yield

$$D^2 u + (1/\eta)Du + Gr = 0 \quad (17)$$

The solution of this second-order differential equation that satisfies the boundary conditions (12a,b) can be shown to be

$$u(\eta) = Gr(1 - \eta^2)/4 \quad (18)$$

This shows that the fluid-phase velocity profile has a quadratic relation with the dimensionless radial distance.

The corresponding solution for $u_p(\eta)$ is obtained by substituting equation (18) into equation (13).

Viscous Particle Phase: In the presence of a particle-phase viscosity ($\beta \neq 0$), equations (8) and (10) can be solved subject to equations (12a,b) and (12e,f) in closed form by using assumed solutions of a Fourier-Bessel series form that satisfy the boundary conditions. For convenience, these solutions can be written as:

$$u(\eta) = \sum_{n=1}^{\infty} U_n J_o(\lambda_n \eta) \tag{19}$$

$$u_p(\eta) = -H/\alpha + \sum_{n=1}^{\infty} U_{pn}[J_o(\lambda_n \eta) + S\lambda_n J_i(\lambda_n)] \tag{20}$$

where the constant λ_n is the n-th root of $J_o\,(\lambda_n) = 0$. The goal is to find the Fourier coefficients U_n and U_{pn}. This can be done by substituting equations (19) and (20) into equations (8) and (10) and applying the orthogonality condition of Bessel functions to give

$$(\alpha/\beta)U_n - [\lambda_n^2 + 2S\,\alpha/\beta + \alpha/\beta]U_{pn} = 0 \tag{21}$$

$$U_n + \kappa\beta U_{pn} = 2Gr/[\lambda_n^3 J_1(\lambda_n)] \tag{22}$$

Solving the above two equations for U_n and U_{pn} yields

$$U_n = 2Gr\{1 - \alpha\kappa/[\lambda_n^2 + 2S\,\alpha/\beta + \alpha/\beta + \kappa\alpha]\}/[\lambda_n^3 J_1(\lambda_n)] \tag{23}$$

$$U_{pn} = 2\alpha\,Gr/\{\beta\lambda_n^2 J_1(\lambda_n)[\lambda_n^2 + 2S\,\alpha/\beta + \alpha/\beta + \kappa\alpha]\} \tag{24}$$

The general solutions for both the fluid- and particle-phase velocities are obtained by simply substituting the Fourier coefficients U_n and U_{pn} in equations (23) and (24) into equations (19) and (20), respectively. It should be noted that these results are valid for the range of values of S such that $0 = S \leq \infty$. In the limit $S \to \infty$ (perfect particulate wall slip), the fluid- and particle-phase velocities will approach the following solutions:

$$u(\eta) = 2Gr\sum_{n=1}^{\infty} \{J_o(\lambda_n \eta)/[\lambda_n^3 J_1(\lambda_n)]\} \tag{25}$$

$$u_p(\eta) = -H/\alpha + \tilde{G}r\sum_{n=1}^{\infty} (1/\lambda_n^2) \tag{26}$$

Some results for the fluid-phase velocity (u) and the particle-phase velocity (u_p) based on the closed-form solutions for the flow through an infinitely long vertical pipe are presented in Figs. 2 through 11. These results are presented to illustrate the influence of the Grashof number Gr, the viscosity ratio β, the particle wall slip coefficient S, the inverse Stokes number α and the particle loading κ, respectively. The values of the physical parameters employed to obtain the graphical results may or may not represent actual conditions of such applications as heat exchanger technology, cooling processes in electronic devices and solar collectors. Never-the-less, they do give a qualitative behavior of the flow and heat transfer situation in such processes.

Figures 2 and 3 display the effects of increasing the Grashof number Gr on the velocity fields of both phases. Increases in the values of Gr increase the thermal buoyancy effects. This gives rise to an increase in the flow of both phases in the pipe as shown in Figs. 2 and 3.

To demonstrate the influence of the viscosity ratio β on the fluid-phase velocity u and the particle-phase velocity u_p, equations (19) and (20) are evaluated numerically and the results are plotted in Figs. 4 and 5. In these figures, all parameters are kept constant while β is allowed to vary. Increases in the viscosity ratio β have the tendency to increase the magnitude of the viscous or frictional effects for both phases in comparison with the buoyancy effects. This

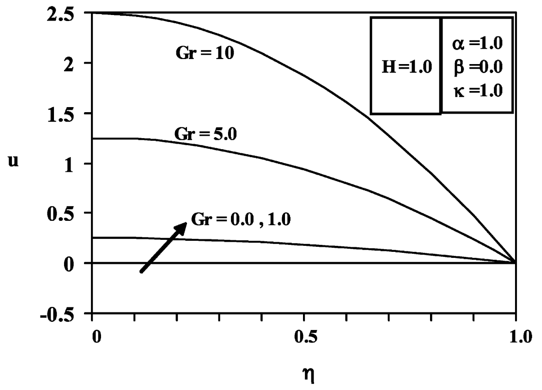

Fig. 2. Effects of Gr on fluid-phase velocity profiles

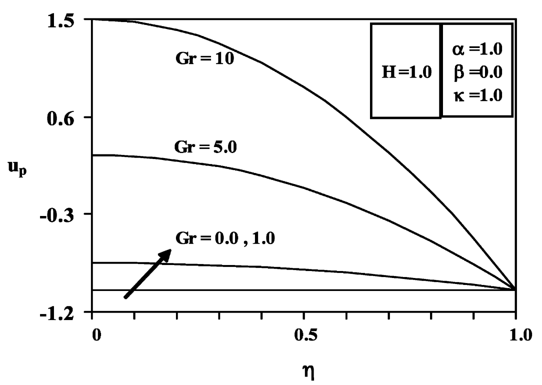

Fig. 3. Effects of Gr on particle-phase velocity profiles

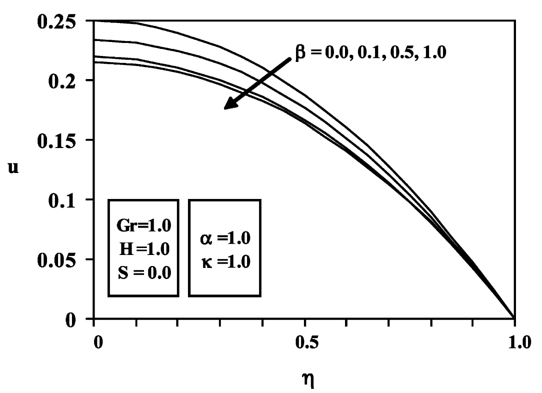

Fig. 4. Effects of β on fluid-phase velocity profiles

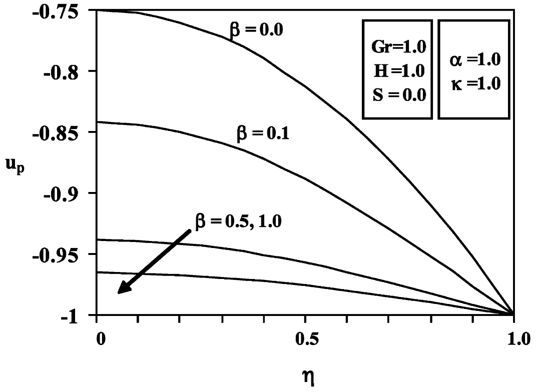

Fig. 5. Effects of β on particle-phase velocity profiles

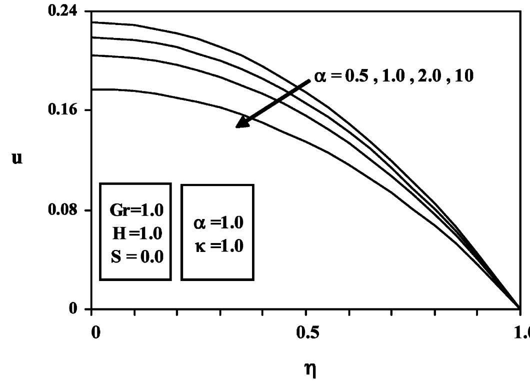

Fig. 8. Effects of α on fluid-phase velocity profiles

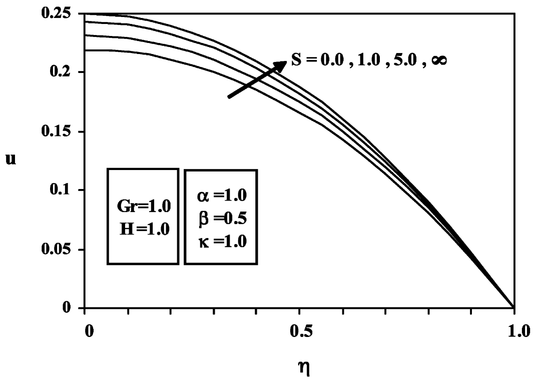

Fig. 6. Effects of S on fluid-phase velocity profiles

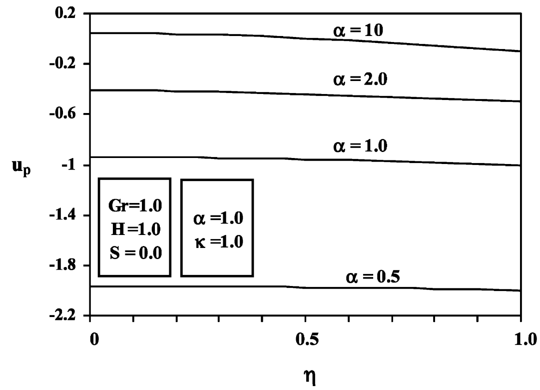

Fig. 9. Effects of α on particle-phase velocity profiles

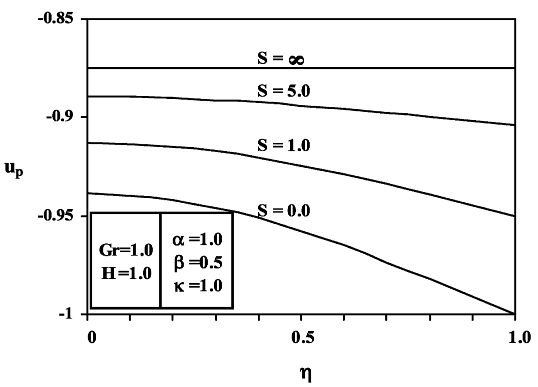

Fig. 7. Effects of S on particle-phase velocity profiles

has the direct effect of decreasing the velocity of both phases as clearly depicted in Figs. 4 and 5. In addition, Fig. 5 shows that increases in the values of β have the tendency to flatten the particle-phase velocity profiles.

Figures 6 and 7 illustrate the influence of the particle-phase slip coefficient S on the fluid- and particle-phase velocities, respectively. Physically speaking, as S increases, the particle-phase viscous or the frictional effects decrease in comparison with the buoyancy effects. This makes it easier for the carrier fluid to move the particles causing the fluid- and particle-phase velocities to increase as shown Figs. 6 and 7. In addition, as S increases, the particle-phase velocity profile becomes more flat as shown in Fig. 7.

In order to elucidate the influence of the velocity inverse Stokes number α, graphical representations of u and u_p are obtained and presented in Figs. 8 and 9. As α increases, the interphase momentum transfer due to the drag mechanism between the phases increases causing the fluid-phase velocity to decrease and the particle-phase velocity to increase as is evident from Figs. 8 and 9. According to equation (20) and equation (24), the limit as $\alpha \rightarrow \infty$ will cause $u_p(\eta)$ to approach $u(\eta)$ and equilibrium conditions between the phases occur in which both phases move together with the same velocity as long as the boundary conditions of both phases at the wall of the pipe are the same.

To analyze the effect of the particle loading κ on the solutions developed above, various runs were made to generate data by varying κ in a suitable range. The effects of κ are shown in Figs. 10 and 11. These figures show representative velocity profiles for the dimensionless fluid and particle phases (u and u_p) for various values of κ, respectively. Increases in the particle concentration or loading have the effect of increasing the drag force between the phases. This causes a slower motion of the fluid phase. This has the effect of reducing the particle-phase velocity since the particle phase is being dragged along by the fluid phase. Furthermore, increases in the values of κ have the tendency to flatten both the fluid- and particle-phase velocity profiles as depicted clearly in Figs. 10 and 11.

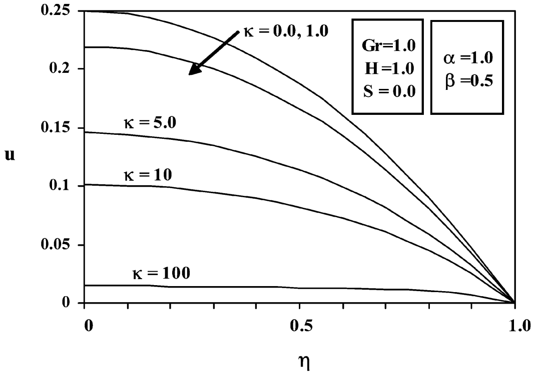

Fig. 10. Effects of κ on fluid-phase velocity profiles

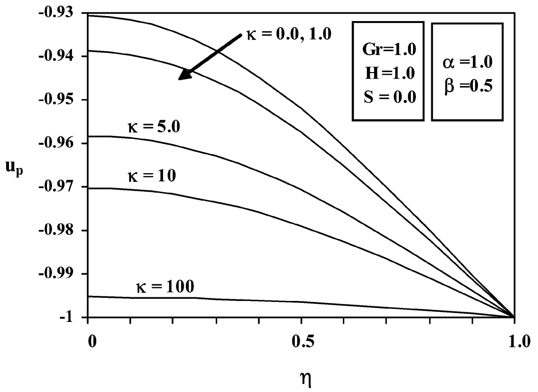

Fig. 11. Effects of κ on particle-phase velocity profiles

4 Conclusions

Continuum modeling of natural convection flow of a particulate suspension was done based on the balance laws of mass, linear momentum, and energy for both the fluid and particle phases. The formulation of the governing equations took into account the effects of particle-phase viscosity. These equations were non-dimensionlized and then solved in closed form for the cases of inviscid and viscous particle phases. Representative results were plotted to illustrate the influence of the physical parameters on the solutions. An increase in the values of the Grashof number increased the thermal buoyancy effects which consequently increased the flow of both phases. The effect of increasing the value of the viscosity ratio was found to increase the magnitude of the frictional or viscous effects for both phases in comparison with the buoyancy effects which produced reductions in the velocities of both phases. The influence of increasing the value of the particle-phase slip coefficient was predicted to increase the magnitudes of the fluid- and particle-phase velocities. Increases in the velocity inverse Stokes number had the effect of increasing the interphase momentum transfer due to the drag mechanism between the phases. This caused the fluid-phase velocity to decrease and the particle-phase velocity to increase. Increases in the particle concentration (particle loading) increased the drag force between the phases causing a slower motion of both the fluid and particle phases.

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