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Transient natural convection flow of a particulate suspension through a vertical channel

Mansour A. Al-Subaie, Ali J. Chamkha

Abstract Continuum equations governing transient, laminar, fully-developed natural convection flow of a particulate suspension through an infinitely long vertical channel are developed. The equations account for particulate viscous effects which are absent from the original dusty-gas model. The walls of the channel are maintained at constant but different temperatures. No-slip boundary conditions are employed for the particle phase at the channel walls. The general transient problem is solved analytically using trigonometric Fourier series and the Laplace transform method. A parametric study of some physical parameters involved in the problem is performed to illustrate the influence of these parameters on the flow and thermal aspects of the problem.

Nomenclature

c	Fluid-phase specific heat at constant pressure
c_p	Particle-phase specific heat at constant pressure
g	Gravitational acceleration
Gr	Grashof number
h	Channel width
H	Dimensionless buoyancy parameter
k	Fluid-phase thermal conductivity
N	Interphase momentum transfer coefficient
N_T	Interphase heat transfer coefficient
P	Fluid-phase hydrostatic pressure
Pr	Fluid-phase Prandtl number
t	Time
T	Fluid-phase temperature
T_p	Particle-phase temperature
u	Fluid-phase dimensionless velocity
u_p	Particle-phase dimensionless velocity
U	Fluid-phase velocity
U_p	Particle-phase velocity
x, y	Cartesian coordinates

Greek symbols

α	Velocity inverse Stokes number
β	Viscosity ratio
γ	Specific heat ratio
ε	Temperature inverse Stokes number
η	Dimensionless y-coordinate
θ	Dimensionless fluid-phase temperature
κ	Particle loading
μ	Fluid-phase dynamic viscosity
μ_p	Particle-phase dynamic viscosity
ρ	Fluid-phase density
ρ_p	Particle-phase density = product of particle density and particle number density
τ	Dimensionless time

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1 Introduction

Two-phase (fluid-particle) natural convection flow represents one of the most interesting and challenging areas of research in heat transfer. Such flows are found in a wide range of applications including processes in the chemical and food industries, solar collectors where a particulate suspension is used to enhance absorption of radiation, cooling of electronic equipments, cooling of nuclear reactors, and heating of buildings via storage walls (trombe walls). In general, all applications of single-phase flow are valid for two-phase particulate suspension flow because the presence of contaminating dust particles in fluids can occur naturally or deliberately. Actually, most research of natural convection flows within vertical parallel-plate channels is done only for a single phase.

The general area of natural or free convection in vertical parallel-plate channels for a single phase has received a great deal of attention in previous years due to the fact that many practical applications involve natural convection heat transfer. The book by Gebhart et al. (1988) represents a good source of information on many investigations dealing with the subject. Aung et al. (1972) have reported on the development of laminar free convection between vertical flat plates with asymmetric heating. Lee et al. (1982) has considered natural convection in a vertical channel with opposing buoyancy forces. Yang et al. (1974) and Wang (1988) have studied natural convection about vertical plates with oscillatory surface temperature and in vertical channels with periodic heat input, respectively. Joshi (1988) have considered the transient effects in natural convection cooling of vertical parallel plates.

Inspite of its importance, very little work have been reported on the subject of natural convection flow of a particle-fluid suspension over and through different geometries. Recently, Al-Subaie and Chamkha (2001, 2002) have considered steady natural convection flow of a particulate suspension through a parallel-plate channel and reported closed-form solutions. Also, Chamkha and Ramadan (1998) and Ramadan and Chamkha (1999) have reported some analytical and numerical results for natural convection flow of a two-phase particulate suspension over an infinite vertical plate. They found that increases in either of the particle loading or the wall particulate slip coefficient caused reductions in the velocities of both phases. In addition, Okada and Suzuki (1997) have considered buoyancy-induced flow of a two-phase suspension in an enclosure. Ritter and Peddieson (1977) have considered transient two-phase flow in channels and circular pipes. Chamkha (1995) has generalized the work of Ritter and Peddieson (1977) and obtained closed-form solutions for unsteady two-phase fluid-particle flow in a channel under the action of a constant pressure gradient in the presence of particle-phase viscous effects. However, to the best of the authors' knowledge, no work has been reported on transient natural convection flow of a particulate suspension in a vertical channel. Therefore, there is a definite need for investigation of this problem. Thus, the objective of this work is to consider this problem and to solve it analytically under some specified assumptions.

The dusty-gas model (a model meant for small particulate volume fraction and invicid particle phase) discussed by Marble (1970) has been widely used for modelling two-phase fluid-particle flows. It is, however, certainly not the only plausible model of the behavior of small volume fraction fluid-particle suspensions. It is, therefore, of interest to obtain solutions to natural convection flow problems using other models and compare their features to those of the corresponding dusty-gas solutions. The information obtained in this way should facilitate the matching of models with observed physical behavior. In the present work, the problem of transient, laminar, fully-developed natural convection flow of a particulate suspension through a vertical parallel-plate channel which employs a model which retains the small volume fraction assumption but includes the particle-phase viscous effects is solved analytically. The model incorporates ideas that are similar to those to be found in the papers by Soo (1968) and Korjack and Chen (1980).

2 Governing equations

Mathematical modeling of two-phase (fluid-particle) flow and heat transfer in a vertical channel requires the use of a model set of equations based on the balance laws of mass, linear momentum and energy of both phases. The presence of any physical effect in the problem must be reflected in these equations. The basic equations for this problem which treats both the fluid and the particle phases as interacting continua and accounts for the particle-phase viscous effects (not present in the original dusty-gas model discussed by Marble 1970) and the gravity effects can be written in the following vector form as:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{V}) = 0 \tag{1}$$

$$\rho(\partial_t \mathbf{V} + \mathbf{V} \cdot \nabla \mathbf{V}) = -\nabla P + \nabla \cdot (\mu \nabla \mathbf{V}) - \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho \mathbf{g} \tag{2}$$

$$\rho c(\partial_t T + \mathbf{V} \cdot \nabla T) = \nabla \cdot (k \nabla T) + \rho_p c_p N_T (T_p - T) \tag{3}$$

$$\partial_t \rho_p + \nabla \cdot (\rho_p \mathbf{V}_p) = 0 \tag{4}$$

$$\rho_p(\partial_t \mathbf{V}_p + \mathbf{V}_p \cdot \nabla \mathbf{V}_p) = \nabla \cdot (\mu_p \nabla \mathbf{V}_p) + \rho_p N(\mathbf{V} - \mathbf{V}_p) + \rho_p \mathbf{g} \tag{5}$$

$$\rho_p c_p(\partial_t T_p + \mathbf{V} \cdot \nabla T_p) = -\rho_p c_p N_T (T_p - T) \tag{6}$$

where all parameters are defined in the Nomenclature section.

This study considers unsteady or transient, laminar, incompressible, natural convection fully developed two-phase (fluid-particle) flow in a vertical parallel-plate channel. The fluid phase is assumed to be Newtonian, viscous and has constant properties except the density in the buoyancy term. The particle phase is assumed to consist of solid spherical particles having the same size and that its density distribution is uniform. In addition, the particle-phase is assumed viscous and has no analog pressure. The walls of the channel are assumed to be infinitely long. This implies that the dependence of the variables on the x-direction will be negligible compared with that of the y-direction (see Figure 1). Therefore, all dependent variables will only be functions of t and y. Under this and the previous assumptions, equations (1) through (6) become as follows:

$$\rho \partial_t U = -\partial_x P + \mu \partial_{yy} U - \rho_p N(U - U_p) - \rho g \tag{7}$$

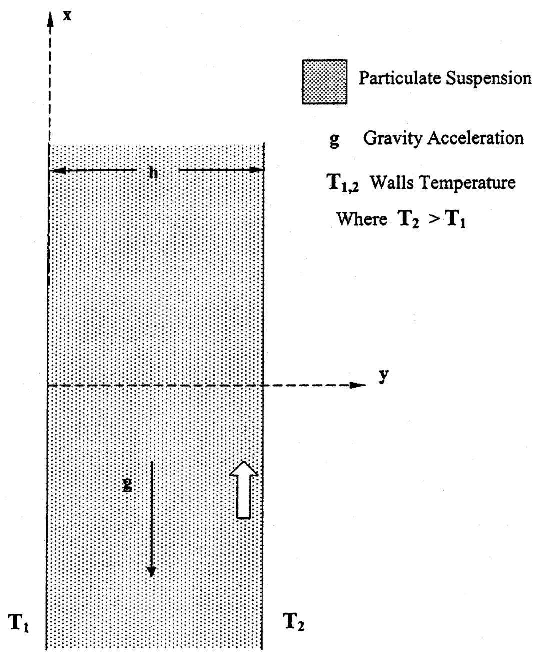

Fig. 1. Problem definition

$$\rho c\partial_t T = k\partial_{yy}T + \rho_p c_p N_T(T_p - T) \quad (8)$$

$$\rho_p\partial_t U_p = \mu_p\partial_{yy}U_p + \rho_p N(U - U_p) - \rho_p g \quad (9)$$

$$\rho_p c_p\partial_t T_p = -\rho_p c_p N_T(T_p - T) \quad (10)$$

It should be noted that the continuity equations of both phases are identically satisfied.

Endowing the particle phase by a viscosity can be thought of as a natural consequence of the averaging processes involved in representing a discrete system of particles as a continuum (see, for example, Drew 1983 and Drew and Segal 1971). Also, the particle-phase viscous effects can be used to model particle-particle interaction and particle-wall interaction in relatively dense suspensions. These effects have been investigated previously by many authors such as Tsuo and Gidaspow (1990) and Gadiraju et al. (1992).

The pressure gradient can be eliminated from the linear momentum equation of the fluid phase by evaluating the governing equations at a reference point at the channel entrance. Let "o" be that reference point such that U = 0, $T = T_o$, $\rho = \rho_o$, $\mu = \mu_o$, $U_p = U_{po}$, $T_p = T_{po}$, $\rho_p = \rho_{po}$ and $\mu_p = \mu_{po}$. Evaluating the governing equations at this reference point gives

$$-\partial_x P = -\rho_{po}NU_{po} + \rho_o g, \quad U_{po} = -g/N, \quad T_{po} = T_o \quad (11)$$

Substituting equations (11) into equation (7) and employing the Boussinesq approximation such that

$$\rho = \rho_o[1 - \beta^*(T - T_o)] \quad (12)$$

results in the following equation:

$$\partial_t U - \rho_{po}/\rho_o g - \mu_o/\rho_o\partial_{yy}U + \rho_{po}/\rho_o N(U - U_p) - \beta^* g(T - T_o) = 0 \quad (13)$$

where β^* is the volumetric expansion coefficient. The linear momentum equation of the fluid phase, equation (7), will now be replaced now by equation (13) in the governing equations.

Initially, the fluid phase is assumed to be at rest while the particle phase is assumed to be falling freely under the gravity effect. Also, the particulate suspension is assumed to be at the reference temperature T_o. This can be expressed by

$$U(y,0) = 0, \quad T(y,0) = T_o, \quad U_p(y,0) = -g/N, \quad T_p(y,0) = T_o \quad (14)$$

The physical boundary conditions for this problem are:

$$U(0,t) = U(h,t) = 0, \quad T(0,t) = T_1, \quad T(h,t) = T_2, \quad U_p(0,t) = -g/N, \quad (15)$$

where h is the channel width, T_1 is the channel wall temperature at y = 0, T_2 is the channel wall temperature at y = h. Equations (15a) and (15b) indicate no slip conditions for the fluid phase at the walls of the channel. Equations (15c) and (15d) suggest that the fluid temperatures at the walls of the channel are some constant values T_1 and T_2 such that $T_2 > T_1$. Equations (15e) and (15f) express slip boundary conditions for the particle phase at the walls of the channel. The value g/N represents the free-fall or terminal velocity of the particle phase. It should be mentioned herein that the exact form of wall boundary conditions for the particulate phase are poorly understood at this time. The condition employed herein represents one of the possible options.

The formulation of the value problem of an infinite vertical parallel-plate channel is now completed. In order to solve this problem, it is convenient to non-dimensionalize the governing equations and conditions. This can be accomplished by using the following parameters:

$$y = h\eta, \quad t = (h^2\rho/\mu)\tau, \quad U = (\mu/\rho h)u, \quad U_p = (\mu/\rho h)u_p, \quad T = (T_2 - T_o)\theta + T_o, \quad T_o = (T_1 + T_2)/2, \quad T_p = (T_2 - T_o)\theta_p + T_o \quad (16)$$

where η is the dimensionless coordinate, u and u_p are the dimensionless fluid- and particle-phase velocities, respectively, and θ and θ_p are the dimensionless fluid- and particle-phase temperatures, respectively. After performing the mathematical operations, the resulting dimensionless governing equations can be written as:

$$\partial_\tau u - \partial_{\eta\eta}u + \alpha\kappa(u - u_p) - Gr\theta - \kappa H = 0 \quad (17)$$

$$\partial_\tau\theta - (1/Pr)\partial_{\eta\eta}\theta - \kappa\gamma\varepsilon(\theta_p - \theta) = 0 \quad (18)$$

$$\partial_\tau u_p - \beta\partial_{\eta\eta}u_p - \alpha(u - u_p) + H = 0 \quad (19)$$

$$\partial_\tau\theta_p + \varepsilon(\theta_p - \theta) = 0 \quad (20)$$

where $\partial_{\eta\eta}$ denotes a second derivative with respect to η, $\alpha = h^2N\rho/\mu$, $\kappa = \rho_p/\rho$, $Gr = g\beta^* h^3\rho^2(T_2 - T_o)/\mu^2$, $H = gh^3\rho^2/\mu^2$, $\beta = \kappa\mu_p/\mu$, $Pr = \mu c/k$, $\gamma = c_p/c$, and $\varepsilon = \rho N_T h^2/\mu$ are the momentum inverse Stokes number, the particle loading, the Grashof number, buoyancy parameter, the viscosity ratio, the Prandtl number, the specific heat ratio, and the temperature inverse Stokes number, respectively.

The dimensionless initial and boundary conditions are:

$$u(\eta,0) = 0, \quad u_p(\eta,0) = -H/\alpha, \quad \theta(\eta,0) = 0, \quad \theta_p(\eta,0) = 0 \quad (21)$$

$$u(0,\tau) = u(1,\tau) = 0, \quad \theta(0,\tau) = -1, \quad \theta(1,\tau) = 1, \quad u_p(0,\tau) = -H/\alpha, \quad u_p(1,\tau) = -H/\alpha \quad (22)$$

It should be mentioned that when $\beta = 0$ (inviscid particle phase), equations (22e,f) are ignored.

3
Results and discussion

The governing equations for the transient, natural convection fully-developed flow of a particulate suspension through vertical parallel-plate channel are be represented by equations (17) through (20). It is convenient for simplicity to divide these equations into two sets of equations by assigning the energy equations of both phases to be the first set, while the momentum equations of both phases to be the second set. This is because the energy equations are uncoupled from the momentum equations. The set of energy equations (18) and (20) can be solved exactly subject to equations (21c,d) and (22c,d) by using assumed solutions of a trigonometric Fourier series form that satisfy both the initial and boundary conditions. For convenience, these solutions can be written as

$$\theta(\eta,\tau) = -1 + 2\eta + \sum_{n=1}^{\infty} F_n(\tau)\sin(n\pi\eta) \tag{23}$$

$$\theta_p(\eta,\tau) = -1 + 2\eta + \sum_{n=1}^{\infty} F_{pn}(\tau)\sin(n\pi\eta) \tag{24}$$

where the Fourier coefficients $F_n(\tau)$ and $F_{pn}(\tau)$ are to be determined.

Substituting equations (23) and (24) into equations (18) and (20) yields

$$\dot{F}_n + A_1F_n - \kappa\gamma\varepsilon F_{pn} = 0 \tag{25}$$

$$\dot{F}_{pn} + \varepsilon F_{pn} - \varepsilon F_n = 0 \tag{26}$$

where a dot represents ordinary differentiation with respect to τ and

$$A_1 = (n\pi)^2/Pr + \kappa\gamma\varepsilon \tag{27}$$

The initial conditions for equations (25) and (26) can be obtained by substituting $\tau = 0$ into equations (23) and (24) then multiplying each side by sin $(m\pi\eta)$ and integrating from 0 to 1. Doing this gives

$$F_n(0) = F_{pn}(0) = 2(1+\cos n\pi)/n\pi \tag{28a,b}$$

The Laplace transform is chosen to solve equations (25) and (26) subject to (28a,b). Doing so yields

$$F_n(\tau) = A_3e^{r_1\tau} + A_4e^{r_2\tau} \tag{29}$$

$$F_{pn}(\tau) = A_6e^{r_1\tau} + A_7e^{r_2\tau} \tag{30}$$

where

$$r_1 = \left[-A_1 - \varepsilon + ((A_1-\varepsilon)^2 + 4\kappa\gamma\varepsilon^2)^{1/2}\right]/2 \tag{31}$$

$$r_2 = \left[-A_1 - \varepsilon - ((A_1-\varepsilon)^2 + 4\kappa\gamma\varepsilon^2)^{1/2}\right]/2 \tag{32}$$

$$A_3 = [2(r_1+\varepsilon)(1+\cos n\pi)/n\pi + A_9]/(r_1 - r_2) \tag{33}$$

$$A_4 = [2(r_2+\varepsilon)(1+\cos n\pi)/n\pi + A_9]/(r_2 - r_1) \tag{34}$$

$$A_6 = A_3(r_1 + A_1)/\kappa\gamma\varepsilon \tag{35}$$

$$A_7 = A_4(r_2 + A_1)/\kappa\gamma\varepsilon \tag{36}$$

$$A_7 = A_4(r_2 + A_1)/\kappa\gamma\varepsilon \tag{37}$$

This completes the exact solutions of the set of energy equations.

Now, the set of momentum equations (17) and (19) can be solved exactly by using assumed solutions of a trigonometric Fourier series form that satisfy both the initial and boundary conditionsas well.

The assumed solutions of a trigonometric Fourier series form can be written as

$$u(\eta,\tau) = \sum_{n=1}^{\infty} U_n(\tau)\sin(n\pi\eta) \tag{38}$$

$$u_p(\eta,\tau) = -H/\alpha + \sum_{n=1}^{\infty} U_{pn}(\tau)\sin(n\pi\eta) \tag{39}$$

where the Fourier coefficients $U_n(\tau)$ and $U_{pn}(\tau)$ are to be determined. Direct substitution of equations (38) and (39) into equations (17) and (19) gives

$$\dot{U}_n + A_{10}U_n - \kappa\alpha U_{pn} = A_{12} + Gr\,F_n \tag{40}$$

$$\dot{U}_{pn} + A_{11}U_{pn} - \alpha U_n = 0 \tag{41}$$

where

$$A_{10} = (n\pi)^2 + \kappa\alpha \tag{42}$$

$$A_{11} = (n\pi)^2\beta + \alpha \tag{43}$$

$$A_{12} = -2\,Gr(1+\cos n\pi)/n\pi \tag{44}$$

The initial conditions for equations (40) and (41) can be obtained by substituting $\tau = 0$ into equations (40) and (41) then multiplying each side by $\sin(m\pi\eta)$ and integrating from 0 to 1. Doing this gives

$$U_n(0) = U_{pn}(0) = 0 \tag{45}$$

The Laplace transform is chosen to solve equations (40) and (41) subject to (45). Doing so yields

$$U_n(\tau) = A_{13}e^{r_1\tau} + A_{14}e^{r_2\tau} + A_{15}e^{r_3\tau} + A_{16}e^{r_4\tau} + A_{17} \tag{46}$$

$$U_{pn}(\tau) = A_{18}e^{r_1\tau} + A_{19}e^{r_2\tau} + A_{20}e^{r_3\tau} + A_{21}e^{r_4\tau} + A_{22} \tag{47}$$

where

$$r_3 = \left[-A_{11} - A_{10} + ((A_{11}-A_{10})^2 + 4\kappa\alpha^2)^{1/2}\right]/2 \tag{48}$$

$$r_4 = \left[-A_{11} - A_{10} - ((A_{11}-A_{10})^2 + 4\kappa\alpha^2)^{1/2}\right]/2 \tag{49}$$

$$A_{13} = A_{18}(r_1 + A_{11})/\alpha \tag{50}$$

$$A_{14} = A_{19}(r_2 + A_{11})/\alpha \tag{51}$$

$$A_{15} = A_{20}(r_3 + A_{11})/\alpha \tag{52}$$

$$A_{16} = A_{21}(r_4 + A_{11})/\alpha \tag{53}$$

$$A_{17} = A_{22}A_{11}/\alpha \tag{54}$$

$$A_{18} = \alpha A_3 Gr/[(r_1 - r_3)(r_1 - r_4)] \tag{55}$$

$$A_{19} = \alpha A_4 Gr/[(r_2 - r_3)(r_2 - r_4)] \tag{56}$$

$$A_{20} = [\alpha A_{12}/r_3 + \alpha Gr A_3/(r_3 - r_1) + \alpha Gr A_4/(r_3 - r_2)]/(r_3 - r_4) \tag{57}$$

$$A_{21} = [\alpha A_{12}/r_4 + \alpha Gr A_3/(r_4 - r_1) + \alpha Gr A_4/(r_4 - r_2)]/(r_4 - r_3) \tag{58}$$

$$A_{22} = \alpha A_{12}/r_3 r_4 \tag{59}$$

This completes the exact solution of the momentum equations of both phases and concludes the results of the transient problem.

Some results for the fluid-phase temperature profile θ and the particle-phase temperature profile θ_p, the fluid-phase velocity u and the particle-phase velocity u_p based on the closed-form solutions reported above are presented in Figures 2 through 11. These results are presented to illustrate the influence of some of the physical parameters on the solutions. The values of the physical parameters employed to obtain the graphical results may or may not represent actual conditions of such applications as heat exchanger technology, cooling processes in electronic devices and solar collectors. Never-the-less, they do give a qualitative behavior of the flow and heat transfer situation in such processes.

Figures 2 and 3 present the evolution of the fluid-phase temperature profile θ and the particle-phase temperature profile θ_p, respectively. The proper transition from transient conditions at small values of the dimensionless time τ to steady conditions at large values of τ is apparent. In the transient range, the temperature profiles of both phases are nonlinear and the degree of nonlinearity decreases as the steady-state condition is approached. The steady-state profiles at $\tau = \infty$ are consistent with the exact solutions reported earlier Al-Subaie and Chamkha (2001).

Figures 4 and 7 present the development of the velocity profiles for the fluid and particle phases in the channel with the dimensionless time τ for inviscid (β =0) and viscous ($\beta \neq 0$) particle phases, respectively. It is seen from Figures 4 and 5 that, the reversed flow condition occurs in a symmetrical fashion as time progresses. A similar behavior is exhibited in the particle-phase

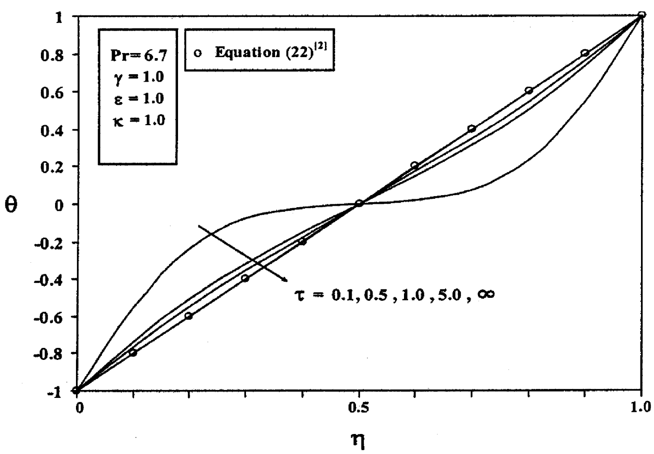

Fig. 2. Transient development of fluid-phase temperature profiles

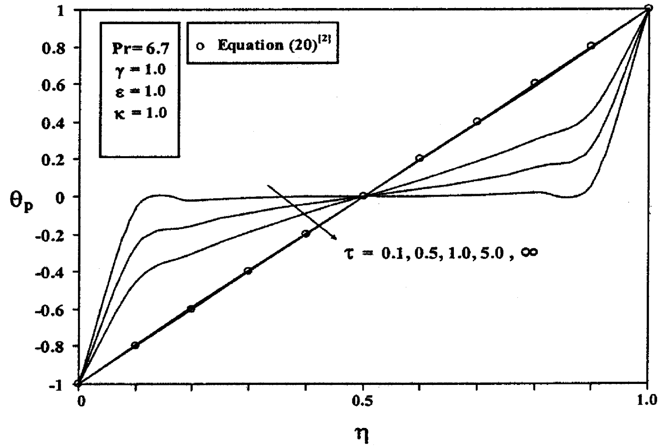

Fig. 3. Transient development of particle-phase temperature profiles

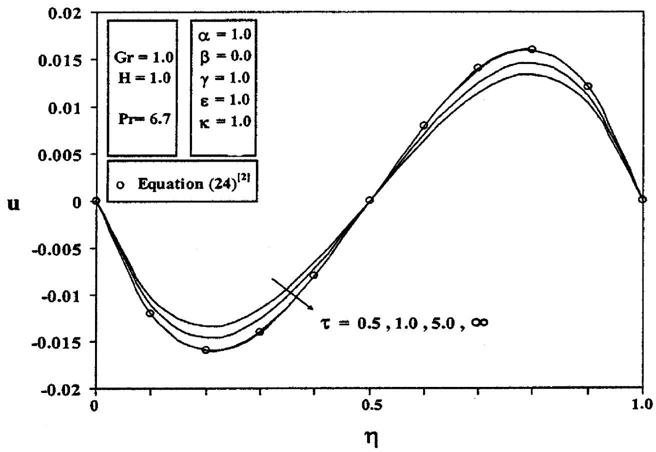

Fig. 4. Transient development of fluid-phase velocity profiles at $\beta = 0.0$

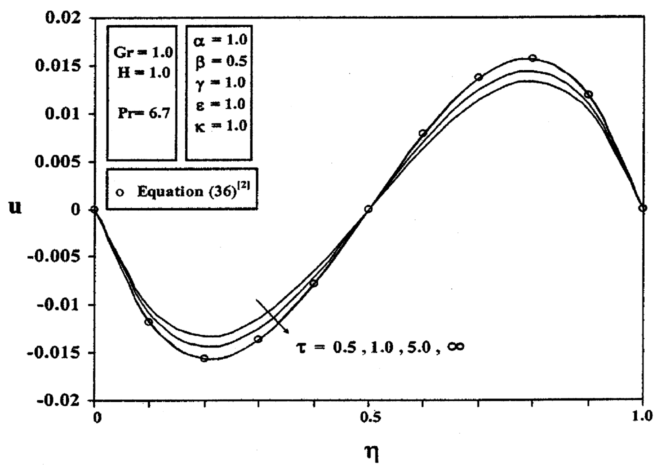

Fig. 5. Transient development of fluid-phase velocity profiles at $\beta = 0.5$

velocity profiles as shown in Figures 6 and 7 except that the profiles become more flattened for $\beta = 0$ as time progresses. For $\beta \neq 0$ the particle-phase velocity

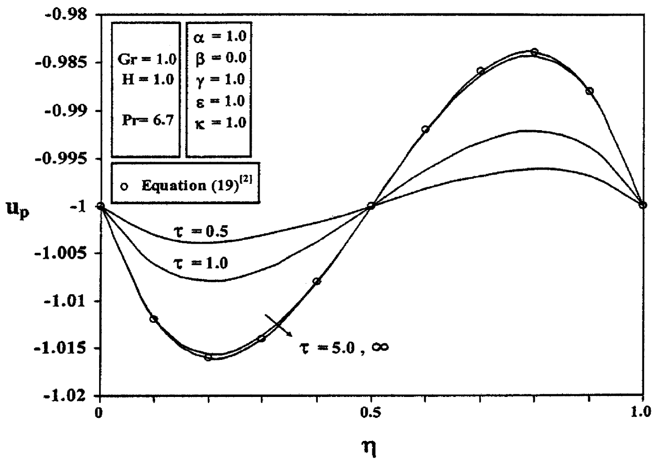

Fig. 6. Transient development of particle-phase velocity profiles at $\beta = 0.0$

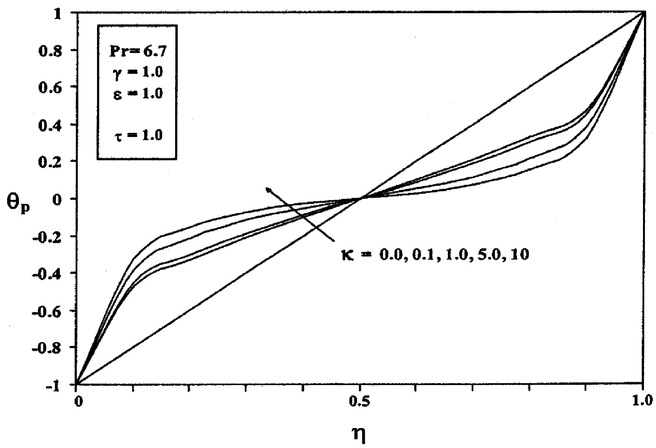

Fig. 9. Effects of κ on particle-phase temperature profiles at $\tau = 1.0$

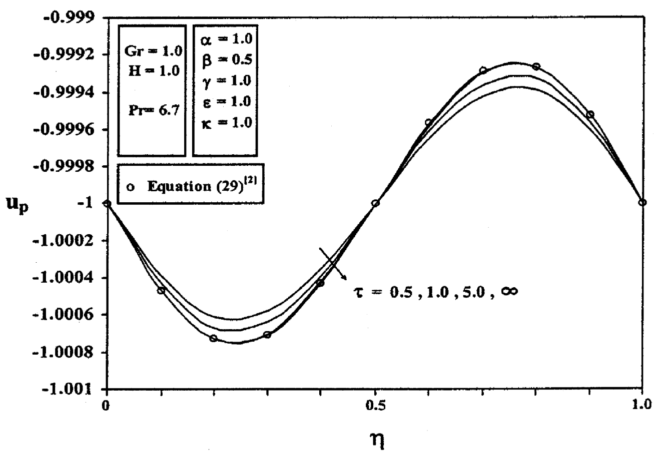

Fig. 7. Transient development of particle-phase velocity profiles at $\beta = 0.5$

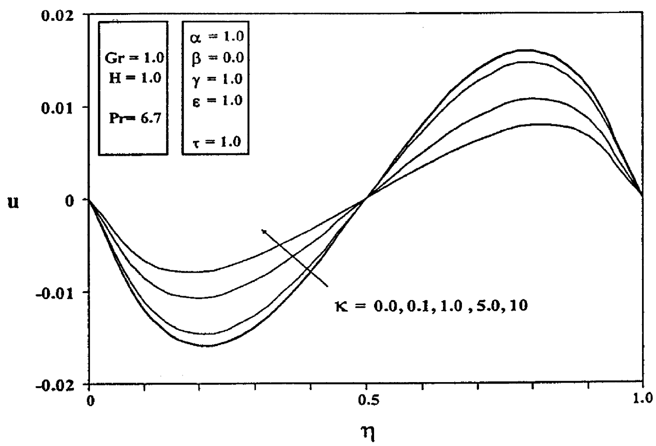

Fig. 10. Effects of κ on fluid-phase velocity profiles at $\tau = 1.0$

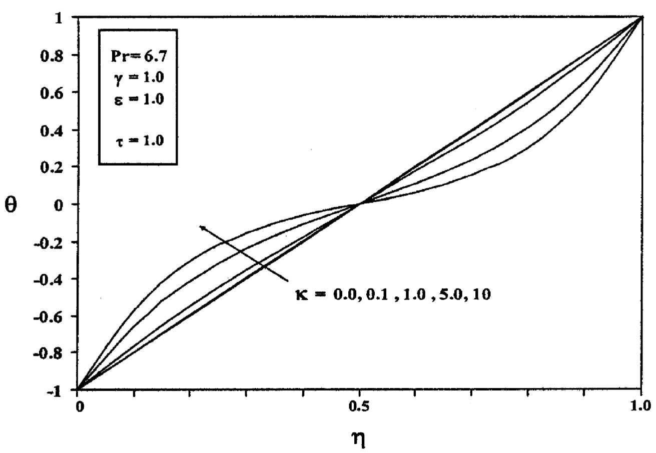

Fig. 8. Effects of κ on fluid-phase temperature profiles at $\tau = 1.0$

profiles reach the steady-state conditions faster than that for $\beta = 0$.

To analyze the effect of the particle loading κ on the solutions developed above, various runs were made to generate data by varying κ at the time $\tau = 1$. The effect of κ on the temperature and velocity profiles of both the fluid and particle phases are shown in Figures 8 through 11. Figures 8 and 9 present the effects of κ at $\tau = 1$ on the fluid-phase temperature θ and the particle-phase temperature θ_p, respectively. These figures show that as the product $\kappa\gamma$ which represents the heat capacity of the particle increases, it takes longer until the temperature distributions of both phases reach steady states. This also causes the fluid-phase temperature profiles to decrease near the hot wall and to increase near the cold wall in a symmetrical fashion. Figures 10 and 11 show the effects of κ on the fluid-and particle-phase velocities (u and u_p) at $\tau = 1$, respectively. As the particle loading increases, more energy exchange between the phases occurs causing an increase in the magnitude of the viscous or frictional effects for both phases in comparison with the buoyancy effects. This has the direct effect of decreasing the velocities of both phases in the channel in the transient range. In fact, increases in the values of κ at $\tau = 1$ have the tendency to flatten the fluid- and particle-phase velocity profiles. These behaviors are depicted in Figures 10 and 11.

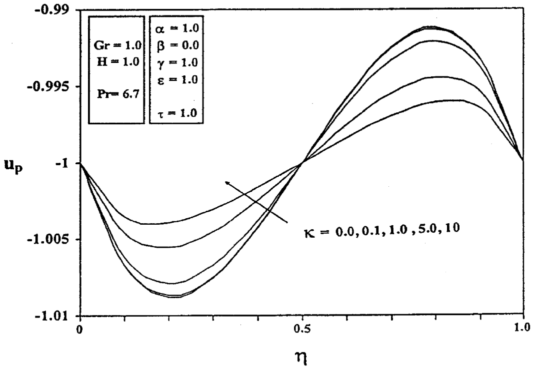

Fig. 11. Effects of κ on particle-phase velocity profiles at $\tau = 1.0$

4 Conclusion

The mathematical modeling of natural convection flow of a particulate suspension was formulated in its general form based on the balance laws of mass, linear momentum, and energy for both the fluid and particle phases. The general formulation took into account the particle-phase viscosity effects. The governing equations were non-dimensionlized and solved analytically. The proper transition from transient conditions at small values of the dimensionless time to steady conditions at large values of τ was predicted. The steady-state profiles were consistent with the exact solutions reported by the same authors in a previous paper. In the transient range, the temperature profiles of both phases were nonlinear and the degree of nonlinearity decreased as the steady state condition was approached. Also, as the particle loading increased, more energy exchange between the phases took place causing the temperature profiles to decrease near the hot wall and to increase near the cold wall in a symmetrical fashion. In addition, increases in the particle loading were predicted to decrease the magnitudes of both the fluid- and particle-phase velocities.

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