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EFFECT OF THERMOPHORESIS PARTICLE DEPOSITION IN FREE CONVECTION BOUNDARY LAYER FROM A VERTICAL FLAT PLATE EMBEDDED IN A POROUS MEDIUM

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ABSTRACT

This work deals with heat and mass transfer by steady laminar boundary layer flow of a Newtonian, viscous fluid over a vertical flat plate embedded in a fluid-saturated porous medium in the presence of the thermophoresis particle deposition effect. The governing partial differential equations are transformed into ordinary differential equations by using special transformations. The resulting similarity equations are solved numerically by an implicit finite-difference method. Comparisons with previously published work are performed and the results are found to be in excellent agreement. Many results are obtained and a representative set is displayed graphically to illustrate the influence of the various physical parameters on the wall thermophoretic deposition velocity and concentration profiles. © 2004 Elsevier Science Ltd

Introduction

Thermophoresis is a phenomenon, which causes small particles to be driven away from a hot surface and toward a cold one. Small particles, such as dust, when suspended in a gas with a temperature gradient, experience a force in the direction opposite to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particle trajectories from combustion devices, and in studying the particulate material deposition on turbine blades. It has been also show that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber perform and is also important in view of its relevance to postulated accidents by radioactive particle deposition in nuclear reactors. In many industries the composition of processing gases may contain any of an unlimited range of particle, liquid, or gaseous contaminants and may be influenced by uncontrolled factors of temperature and humidity. When such an impure gas is bounded by a solid surface, a boundary layer will develop, and energy and momentum transfer gives rise to temperature and velocity gradients. Mass transfer caused by gravitation, molecular diffusion, eddy diffusion, and inertial impact results in deposition of the suspended components onto the surface. In the application of pigments, or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in exploiting deposition mechanisms to improve efficiency. Goren [1] was one of the first to study the role of thermophoresis in laminar flow of a viscous and incompressible fluid. He used the classical problem of flow over a flat plate to calculate deposition rates and showed that substantial changes in surface depositions can be obtained by increasing the difference between the surface and free stream temperatures. This was later followed by similarity solutions of two dimensional laminar boundary layers and stagnation point flows by Gokoglu and Rosner [2], and Park and Rosner [3]. Also, Chiou [4] obtained similarity solutions for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. Garg and Jayaraj [5] discussed the thermophoretic deposition of small particles in forced convection laminar flow over inclined plates. Epstein *et al.* [6] have studied the thermophoretic transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid, while Chiou [7] has considered the particle deposition from natural convection boundary layer flow onto an isothermal vertical cylinder.

Despite the practical importance of thermophoresis there is, to our best knowledge, almost no work devoted to this topic in porous media. Consideration is, therefore, given here to the similarity solutions of the boundary layer free convection thermophoretic deposition of aerosol particles on a vertical isothermal flat plate embedded in a fluid-saturated porous medium. The Darcy and energy equations yield the velocity and temperature distributions in the boundary layer, which are then used in the coupled concentration equation to calculate the rates of particle deposition. Convective flows in porous media have been extensively investigated during the last several decades, due to many practical applications, which can be modeled or approximated as transport phenomena in porous media. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop [8, 9], Nield and Bejan [10], and Pop and Ingham [11].

Governing Equations

Consider the steady free convection boundary layer over a vertical flat plate of constant temperature T_w and concentration C_w, which is embedded in a fluid-saturated porous medium of ambient temperature T_∞ and concentration C_∞, where $T_w > T_\infty$ and $C_w > C_\infty$, respectively. Allowing for both Brownian motion of particles and thermophoretic transport the governing boundary layer equations are, see Nield and Bejan [10] and Chiou [7],

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u = \frac{gK}{\nu}\left[\beta_T (T - T_\infty) + \beta_C (C - C_\infty)\right] \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + \frac{\partial (C v_t)}{\partial y} = D\frac{\partial^2 C}{\partial y^2} \tag{4}$$

where x and y are the Cartesian coordinates along the plate and normal to it, respectively, u and v are the velocity components along x and y axis, respectively, T is the fluid temperature, C is the fluid concentration, g is the gravitational acceleration, K is the permeability of the porous medium, ν is the kinematic viscosity, D is the Brownian diffusion coefficient, α_m is the effective thermal diffusivity, and β_T and β_C are the thermal expansion coefficients of temperature and concentration, respectively. The effect of thermophoresis is usually prescribed by means of the average velocity, which a particle will acquire when exposed to a temperature gradient. In boundary layer flow, the temperature gradient in the y - direction is very much larger than in the x - direction, and therefore only the thermophoretic velocity in y - direction is considered. In consequence the thermophoretic velocity v_t can be expressed in the form

$$v_t = -k\frac{\nu}{T}\frac{\partial T}{\partial y} \tag{5}$$

where k is the thermophoretic coefficient.

The boundary conditions of Eqs. (1) – (5) are

$$\begin{aligned} &v = 0, \quad T = T_w, \quad C = C_w \qquad on \qquad y = 0 \\ &u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty \end{aligned} \tag{6}$$

We now introduce the following non-dimensional variables

$$X = x/l,\ Y = Ra^{1/2}(y/l),\ U = u/U_c,\ V = Ra^{1/2}(v/U_c)$$
$$V_t = Ra^{1/2}(v_t/U_c),\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty} \tag{7}$$

where $U_c = gK\beta_T(T_w - T_\infty)/\nu$ is the characteristic velocity, $Ra = gK\beta_T(T_w - T_\infty)\, l/\alpha_m \nu$ is the Rayleigh number and l is a characteristic length of the plate. Thus, Eqs. (1) – (5) take the following form

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{8}$$

$$U = \theta + N\phi \tag{9}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{\partial^2 \theta}{\partial Y^2} \tag{10}$$

$$U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} + \frac{\partial(\phi V_t)}{\partial Y} = \frac{1}{Le}\frac{\partial^2 \phi}{\partial Y^2} \tag{11}$$

$$V_t = -k\frac{Pr}{N_t + \theta}\frac{\partial \theta}{\partial Y} \tag{12}$$

where Pr and Le are the Prandtl and Lewis numbers for a porous medium, N_t is the therophoresis parameter and N is the buoyancy parameter, which are defined as

$$Pr = \nu/\alpha_m,\ Le = \alpha_m/D,\ N_t = (T_w - T_\infty)/T_\infty,\ N = \beta_C(C_w - C_\infty)/\beta_T(T_w - T_\infty) \tag{13}$$

The boundary conditions (6) now become

$$V = 0,\quad \theta = 1,\quad \phi = 1 \quad on \quad Y = 0$$
$$U \to 0,\quad \theta \to 0,\quad \phi \to 0 \quad as \quad Y \to \infty \tag{14}$$

We now look for a similarity solution of Eqs. (8) – (11) of the form

$$\xi = X,\quad \eta = \frac{y}{\xi^{1/2}},\quad \psi = \xi^{1/2} f(\eta)$$
$$\theta = \theta(\eta),\quad \phi = \phi(\eta) \tag{15}$$

Thus, we get

$$f' = \theta + N\phi \tag{16}$$

$$\theta'' + \frac{1}{2} f\theta' = 0 \tag{17}$$

$$\frac{1}{Le}\phi'' + \frac{1}{2} f\theta' + \frac{k\,Pr}{N_t + \theta}\left[\theta'\phi' + \phi\,\theta'' - \frac{\phi}{N_t + \theta}\theta'^2\right] = 0 \tag{18}$$

subject to the boundary conditions

$$\begin{aligned} &f(0) = 0, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ &f' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad as \quad \eta \to \infty \end{aligned} \tag{19}$$

where a prime denotes ordinary differentiation with respect to η.

Of interest in this problem are the non-dimensional concentration profiles, $\phi(\eta)$ and the wall thermophoretic deposition velocity V_{tw}, which is given by

$$V_{tw} = -\frac{k\,Pr}{1 + N_t}\theta'(0) \tag{20}$$

We notice that for $k = 0$ (absence of thermophoresis), Eqs. (16) – (18) reduce to those of Cheng and Minkowycz [12] when $N = 0$ and to those of Bejan and Khair [13] when $N \neq 0$, respectively.

Results and Discussion

The similarity equations (16) – (18) are non-linear, coupled, ordinary differential equations, which possess no closed-form solution. Therefore, they must be solved numerically subject to the boundary conditions given by Eq. (19). The implicit, iterative finite-difference method discussed by Blottner [14] has proven to be adequate for the solution of this type of equations. For this reason, this method is employed in the present work. Equations (17) and (18) are discretized using three-point central difference quotients. This converts the differential equations into linear sets of algebraic equations, which can be readily solved by the well-known Thomas algorithm (see Blottner [14]). On the other hand, Eq. (16) is discretized and solved subject to the appropriate boundary condition by the trapezoidal rule. The computational domain in the η direction was made up of 196 non-uniform grid points. It is expected that most changes in the dependent variables occur in the region close to the plate where viscous effects dominate. However, small changes in the dependent variables are expected far away from the plate surface. For these reasons, variable step sizes in the η direction are employed. The initial step size $\Delta\eta_1$

and the growth factor K* employed such that $\Delta\eta_{i+1} = K^{*}\Delta\eta_{i}$ (where the subscript i indicates the grid location) were 10^{-3} and 1.03, respectively. These values were found (by performing many numerical experimentations) to give accurate and grid-independent solutions. The convergence criterion employed in the present work was based on the difference between the values of the dependent variables at the current and the previous iterations. When this difference reached 10^{-5}, the solution was assumed converged and the iteration process was terminated. The results are given for several values of the parameters k, Pr, Le, N and N_t. However, to check the present numerical results, we calculate the values of the reduced heat transfer, $-\theta'(0)$, and mass transfer, $-\phi'(0)$, from the plate for $k = 0$, $N = 0$, 1 and $Le = 1$. Thus, for $N= 0$ we obtained $-\theta'(0) = 0.44325$, while the value found by Cheng and Minkowycz [12] is $-\theta'(0) = 0.444$. Also, for $k = 0$ and $N = Le = 1$, we get $-\theta'(0) = -\phi'(0) = 0.62783$, while Bejan and Khair [13] obtained $-\theta'(0) = -\phi'(0) = 0.628$. It is seen that these results are in excellent agreement and we are, therefore, confident that the present numerical results are very accurate.

Typical concentration profiles $\phi\ (\eta)$ and the wall thermophoretic deposition velocity V_{tw} are shown in Figs. 1 to 6 for $Pr = 0.72$ and some values of the governing parameters k, Le, N and N_t. These figures show how the concentration boundary layer and the wall thermophoretic deposition velocity react to changes in the governing parameters. Thus, the concentration profiles indicate the characteristic shape of a non-dimensional concentration with a rapid development close to the plate. This will give rise to a large wall concentration gradient, causing a high deposition on the surface, which increases as the thermophoretic parameter k increases. This is depicted in Figs. 1, 4 and 5, where it can be noticed that the wall thermophoretic deposition velocity becomes sensitive to the variation of the parameters k, Le, N and N_t. This is of particular benefit in processes, which require extreme cleanliness of the surfaces.

Concluding Remarks

Numerical solutions for heat and mass transfer by steady, laminar boundary-flow of a Newtonian fluid over a vertical flat plate embedded in a porous medium in the presence of thermophoresis particle deposition effect were reported. Based on the obtained graphical results, the following conclusions were deduced

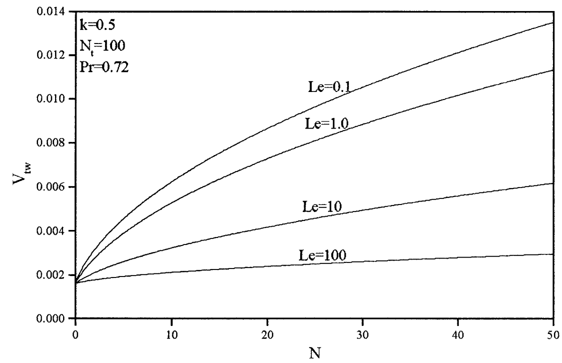

FIG. 1
Effects of Le and N on thermophoretic deposition velocity

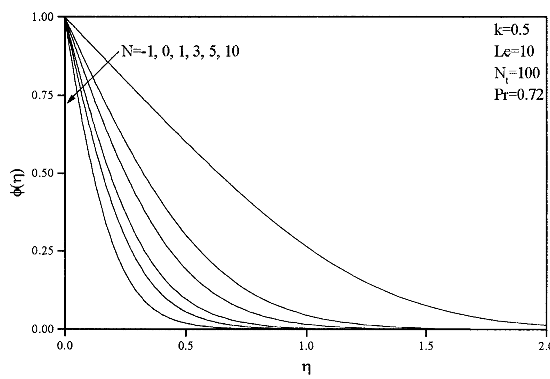

FIG. 2
Effects of N on concentration profiles

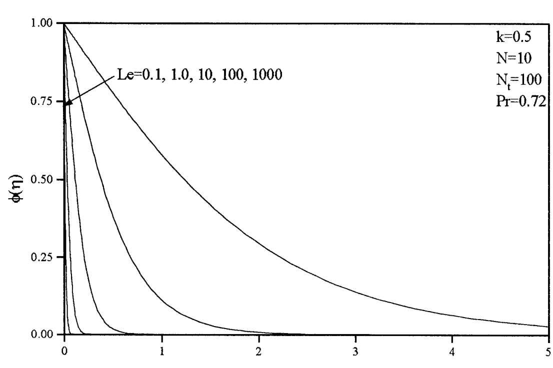

FIG. 3
Effects of Le on concentration profiles

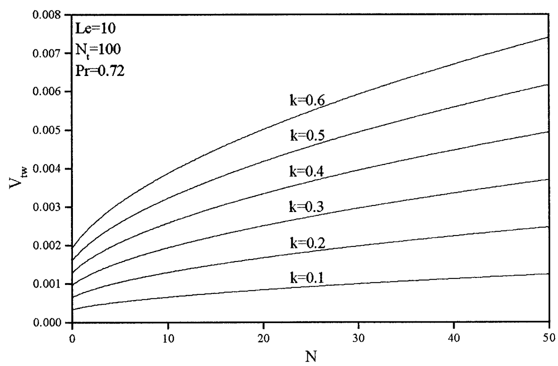

FIG. 4
Effects of k and N on thermophoretic deposition velocity

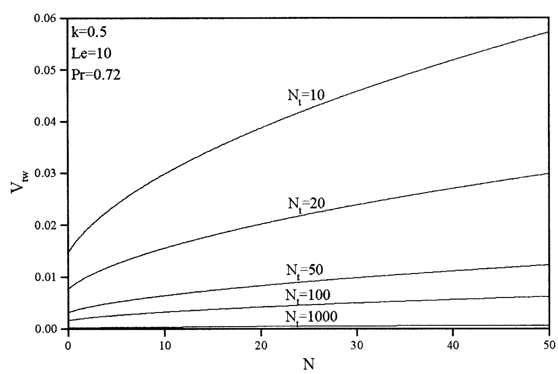

FIG. 5
Effects of N_t and N on thermophoretic deposition velocity

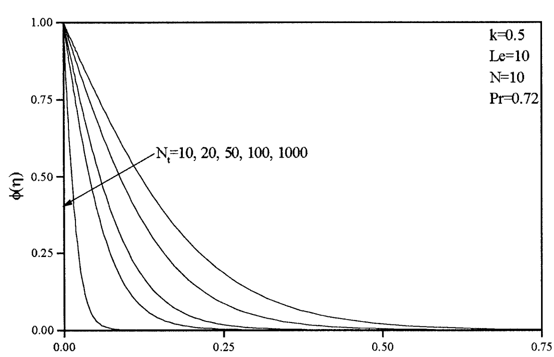

FIG. 6
Effects of N_t on concentration profiles

1. The thermophoretic deposition velocity decreased as either of the Lewis number Le or the thermophoresis parameter N_t was increased and increased as either of the buoyancy ratio N or the thermophoretic coefficient k was increased.
2. The particle concentration, as well as the concentration boundary layer decreased due to increases in either of the Lewis number Le, the thermophoresis parameter N_t, or the buoyancy ratio N. However, for small values of the Prandtl number (Pr = 0.72), slight changes in the concentration profiles are predicted as the thermophoretic coefficient k was altered.

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