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Mixed Convection in a Vertical Porous Channel

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Abstract. A numerical study of mixed convection in a vertical channel filled with a porous medium including the effect of inertial forces is studied by taking into account the effect of viscous and Darcy dissipations. The flow is modeled using the Brinkman–Forchheimer-extended Darcy equations. The two boundaries are considered as isothermal–isothermal, isoflux–isothermal and isothermal–isoflux for the left and right walls of the channel and kept either at equal or at different temperatures. The governing equations are solved numerically by finite difference method with Southwell–Over–Relaxation technique for extended Darcy model and analytically using perturbation series method for Darcian model. The velocity and temperature fields are obtained for various porous parameter, inertia effect, product of Brinkman number and Grashof number and the ratio of Grashof number and Reynolds number for equal and different wall temperatures. Nusselt number at the walls is also determined for three types of thermal boundary conditions. The viscous dissipation enhances the flow reversal in the case of downward flow while it counters the flow in the case of upward flow. The Darcy and inertial drag terms suppress the flow. It is found that analytical and numerical solutions agree very well for the Darcian model.

Key words: mixed convection, vertical channel, porous medium, non-Darcy model, analytical and numerical solutions.

Nomenclature

A	constant defined in Equation (4).
Br	Brinkman number defined in Equation (10).
C_p	specific heat at constant pressure.
C_F	porous media inertia coefficient.
D	$=2L$, hydraulic diameter.
g	acceleration due to gravity.
GR	mixed convection parameter (Gr/Re) defined in Equation (10).
Gr	Grashof number defined in Equation (10).
I	dimensionless porous medium inertia coefficient defined in Equation (10).
K	permeability of the porous media.
k	thermal conductivity.

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k_{eff}	effective thermal conductivity.
L	channel width.
p	pressure.
P	$=p+\rho_0 gX$, difference between the pressure and the hydrostatic pressure.
Pr	Prandtl number defined in Equation (10).
Re	Reynolds number defined in Equation (10).
R_T	temperature difference ratio defined in Equation (10).
T	temperature.
T_1, T_2	prescribed boundary temperatures.
T_0	reference temperature.
u	dimensionless velocity component in the X-direction.
U	velocity component in the X-direction.
U_0	reference velocity defined in Equation (11).
V	velocity component in the Y-direction.
X, Y	space coordinates.
y	dimensionless transverse coordinate.
Greek symbols	
α	$=k/\rho_0 Cp$, thermal diffusivity.
β	thermal expansion coefficient.
ΔT	reference temperature difference defined by Equation (12) or (13).
ε	dimensionless parameter defined by Equation (10).
θ	dimensionless temperature defined in Equation (10).
μ	fluid dynamic viscosity.
μ_{eff}	effective dynamic viscosity.
ν	$=\mu/\rho_0$, kinematic viscosity.
σ	$=h/\sqrt{K}$, Darcy number defined in Equation (10).

1. Introduction

The problem of convective flow in fluid-saturated porous media has been the subject of several recent papers. Interest in understanding the convective transport processes in porous materials is increasing owing to the development of geothermal energy technology, high performance insulation for building and cold storage, renewed interest in the energy efficient drying processes and many other areas. It is also of interest in the nuclear industry, particularly in the evaluation of heat removal from a hypothetical accident in a nuclear reactor and to provide effective insulation. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop (1998), Nield and Bejan (1999), Vafai (2002), Pop and Ingham (2001), Bejan and Kraus (2003).

The present paper studies numerically the steady mixed convection flow in a vertical channel filled with a porous medium including the effect of inertial forces and taking into account the effect of viscous and Darcy dissipations. The main purpose is to reduce heat transfer by means of open-pore insulators, such as glass fibers or rock fibrous material. In this type of porous medium, the macroscopic velocity is not always small and hence the inertial force may not be negligible. Further, the distortion of velocity gives rise to shear stresses which in turn give rise to a viscous force. Therefore, the study of convection in such a special type of coarse porous medium needs the generalized Darcy law which incorporates both inertial and viscous forces in addition to the usual Darcy resistance. It offers a quantitative theory for the details of the transition from the conduction regime to convection and a convenient means for demonstrating the non-linear effects such as the preferred cell pattern, heat transport and so on.

In the literature about fluid flow in domains totally filled with porous material, it has been realized that the local macroscopic inertial term is usually small compared to the microscopic Darcy drag term, and hence can be neglected (Nield, 1991). However, the local inertial term may retain its importance in applications involving very thin porous substrates or at large Darcy numbers. Most early works on porous media have used the Darcy law which represents an empirical relation between the Darcian velocity and the pressure drop across the porous medium. Vafai and Tien (1981) have discussed the importance of boundary and inertia effects of porous media which are not accounted for by the Darcy law. Rudraiah (1984) studied non-linear convection in a porous medium with convective acceleration and viscous force. Also, Vafai and Kim (Date) have studied and reported an exact solution for forced convection in a channel filled with a porous medium. Srinivasan and Vafai (Date) have reported a theoretical study for linear encroachment in two immiscible fluid systems in a porous medium taking into account the non-Darcian boundary and inertia effects. Chen and Vafai (1997) have analyzed free surface momentum and energy transport in porous media in the absence and presence of surface tension effects. Abu-Hijleh and Al-Nimr (2001) have investigated the importance of the local inertial term in forced-convection fluid flow problems in channels partially filled with porous material. Equations governing transient flow and heat transfer aspects of a particulate suspension with a constant finite volume fraction in a porous channel were solved by Chamkha (1996). Also, Chamkha (1997) studied the non-Darcy fully developed mixed convection in a porous medium channel with heat generation/absorption and hydromagnetic effect. Further, Chamkha (2000) studied inertial effects for the flow of two-immiscible fluids in porous and nonporous channel.

Keeping in view the importance of inertial effects for flow through porous medium, it is the objective of this paper to consider the problem of laminar mixed convection flow in a vertical porous channel.

2. Mathematical Model

Consider a steady, laminar, fully developed mixed convection flow in an open-ended vertical parallel plate channel filled with a porous material. The fluid is assumed to be Newtonian and the porous medium is isotropic and homogeneous. It is assumed that the thermal conductivity, the dynamic viscosity and the thermal expansion coefficient are considered constant. The Oberbeck–Boussinesq approximation is supposed to hold. It is worth mentioning that Rajagopal *et al.* (1996) provided a systematic basis for the Oberbeck–Boussinesq approximation. The X-axis is choosen parallel to the gravitational field, but with opposite direction and y-axis is transverse to the plates. The origin is such that the channel walls are at positions $Y = -L/2$ and $Y = L/2$, respectively as shown in Figure 1.

By the condition of fully developed flow, the mass balance equation will be $\frac{\partial U}{\partial X} = 0$ so that U depends only on Y, where U is the velocity along the X-axis. The stream wise and the transverse momentum balance equations including inertial effects for the porous matrix are (see Arpaci and Larsen, 1984):

$$g\beta(T - T_0) - \frac{1}{\rho_0}\frac{\partial P}{\partial X} + \frac{\mu_{\text{eff}}}{\rho_0}\frac{\text{d}^2 U}{\text{d}Y^2} - \frac{\upsilon}{K}U + \frac{\rho C_F}{\rho_0}U^2 = 0, \tag{1}$$

$$\frac{\partial P}{\partial Y} = 0, \tag{2}$$

where $g\beta(T - T_0)$ is the buoyancy term, $\mu_{\text{eff}}/\rho_0\ \text{d}^2U/\text{d}Y^2$ is the viscous term, $(\nu/K)U$ is the Darcy term, $(\rho C_F/\rho_0)U^2$ accounts for the inertia effect and $P = p + \rho_0 g X$ is the difference between the pressure and the

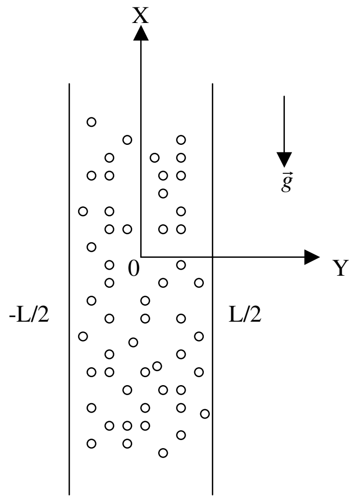

Figure 1. Physical configuration.

hydrostatic pressure. In view of Equation (2), we can write Equation (1) as

$$g\beta(T-T_0)-\frac{1}{\rho_0}\frac{dP}{dX}+\frac{\mu_{\mathrm{eff}}}{\rho_0}\frac{\mathrm{d}^2U}{\mathrm{d}Y^2}-\frac{\upsilon}{K}U+\frac{\rho C_F}{\rho_0}U^2=0. \tag{3}$$

We assume that the temperature of the boundary at $Y=-L/2$ is T_1, while the temperature at $Y=L/2$ is T_2 with $T_2 \geqslant T_1$. These boundary conditions are compatible with Equation (3) if and only if $\mathrm{d}P/\mathrm{d}X$ is independent of X. Let there exists a constant A such that

$$\frac{\mathrm{d}P}{\mathrm{d}X}=A. \tag{4}$$

Evaluating the derivative of Equation (3) with respect to X and using Equation (4), one obtains

$$\frac{\mathrm{d}T}{\mathrm{d}X}=0 \tag{5}$$

which implies that the temperature also depends only on the variable Y.

The energy balance equation, which includes the effects of Darcy and viscous dissipations is

$$\alpha\frac{\mathrm{d}^2T}{\mathrm{d}Y^2}+\frac{\nu}{C_p}\left(\frac{\mathrm{d}U}{\mathrm{d}Y}\right)^2+\frac{\nu}{C_pk}U^2=0. \tag{6}$$

The conservation equations for the porous layer are based on a non-Darcian model, incorporating the Brinkman and Forchheimer extensions. Beckermann *et al.* (1988) have shown in their experiments on natural convection in vertical enclosures with fluid and porous layers that, Brinkman's extension is small compared to the Darcy term. However, Brinkman's extension has been included in all the computations to ensure continuity of the velocities and stresses at the fluid/porous medium interface. Forchheimer's extension serves to model the inertia and, hence, Prandtl number effects on the flow in the porous medium. Although the effect of Forchheimer's extension is insignificant mainly at low Prandtl numbers, it has been utilized in all the computations of the present study. It should be noted that both extensions must be included simultaneously for a high-permeability porous medium (i.e., a high Darcy number). As a first approximation and owing to lack of conclusive information, μ_{eff} is taken equal to the fluid viscosity μ in the present study. Equation (3) using Equation (6) become

$$\frac{\mathrm{d}^4U}{\mathrm{d}Y^4}=\frac{\beta g}{\alpha C_p}\left(\frac{\mathrm{d}U}{\mathrm{d}Y}\right)^2+\frac{1}{K}\frac{\mathrm{d}^2U}{\mathrm{d}Y^2}+\frac{\beta g}{\alpha C_pK}U^2+\frac{C_F}{\nu}\frac{\mathrm{d}^2U^2}{\mathrm{d}Y^2}. \tag{7}$$

The no-slip boundary conditions on U are

$$U\left(-\frac{L}{2}\right) = U\left(\frac{L}{2}\right) = 0. \tag{8}$$

Using Equations (3) and (4), and using the boundary conditions on T one obtains

$$\left(\frac{d^2U}{dY^2}\right)_{y=-L/2} = \frac{A}{\mu} - \frac{\beta g\,(T_1 - T_0)}{\nu}$$
$$\left(\frac{d^2U}{dY^2}\right)_{y=L/2} = \frac{A}{\mu} - \frac{\beta g\,(T_2 - T_0)}{\nu}. \tag{9}$$

Equations (7)–(9) can be written in a dimensionless form by employing the dimensionless quantities

$$u = \frac{U}{U_0};\ \theta = \frac{T - T_0}{\Delta T};\ y = \frac{Y}{D};\ Gr = \frac{g\beta \Delta T D^3}{\nu^2}$$

$$Re = \frac{U_0 D}{\nu};\ Pr = \frac{\nu}{\alpha};\ Br = \frac{\mu U_0^2}{K \Delta T} \tag{10}$$

$$\sigma = \frac{h}{\sqrt{K}};\ I = \frac{C_F U_0 D^2}{\nu};\ GR = \frac{Gr}{Re};\ R_T = \frac{T_2 - T_1}{\Delta T},$$

where $D = 2L$ is the hydraulic diameter. The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0 = -\frac{AD^2}{48\mu}; \qquad T_0 = \frac{T_1 + T_2}{2}. \tag{11}$$

The physical meaning of other quantities defined in (10) are explained in the Nomenclature. Moreover, the reference temperature difference ΔT is given by

$$\Delta T = T_2 - T_1 \quad if\ \ T_1 < T_2 \tag{12}$$
$$\Delta T = \frac{\nu^2}{C_p D^2} \quad if\ \ T_1 = T_2. \tag{13}$$

As a consequence, the dimensionless parameter R_T can only take the values 0 or 1. That is, R_T is 1 for asymmetric heating i.e., $T_1 < T_2$, while R_T is 0 for symmetric heating i.e., $T_1 = T_2$, respectively.

Equation (4) implies that A can be either positive or negative. If $A > 0$, then U_0, Re and Gr are negative, i.e., the flow is downward. On the contrary, if $A < 0$, the flow is upward, so that U_0, Re and GR are positive. Using (10), Equations (6)–(9) become

$$\frac{d^2\theta}{dy^2} = -\sigma^2 Br\, u^2 - Br\left(\frac{du}{dy}\right)^2 \tag{14}$$

$$\frac{d^4 u}{dy^4} = \sigma^2 \frac{d^2 u}{dy^2} + I\frac{d^2 u^2}{dy^2} + \sigma^2 GR\, Br\, u^2 + GR Br\left(\frac{du}{dy}\right)^2 \tag{15}$$

subject to the boundary conditions

$$u\left(-\frac{1}{4}\right) = u\left(\frac{1}{4}\right) = 0 \tag{16}$$

$$\left(\frac{d^2 u}{dy^2}\right)_{y=-\frac{1}{4}} = -48 + \frac{R_T GR}{2}; \quad \left(\frac{d^2 u}{dy^2}\right)_{y=\frac{1}{4}} = -48 - \frac{R_T GR}{2}. \tag{17}$$

Further, Equation (3) using variables (10) and (11) can be written as

$$\theta = -\frac{1}{GR}\left(48 - \sigma^2 u - I u^2 + \frac{d^2 u}{dy^2}\right). \tag{18}$$

We notice that Equation (15) is highly nonlinear through inertia, buoyancy and viscous dissipations. However, they can be solved analytically if the inertial effects of the porous medium are neglected (i.e., $I = 0$). Then Equation (15) becomes

$$\frac{d^4 u}{dy^4} = \sigma^2 \frac{d^2 u}{dy^2} + \sigma^2 GRBr\, u^2 + GRBr\left(\frac{du}{dy}\right)^2 \tag{19}$$

The solution of this equation in the absence of the viscous dissipation term, (i.e., $(du/dy)^2 = 0$ so that $Br = 0$, as seen by Equation (14), with this Equation (19) become linear) and using the boundary conditions (16) and (17) is

$$u = \frac{48}{\sigma^2}\left(1 - \frac{\cosh(\sigma y)}{\cosh(\sigma/4)}\right) + \frac{2GRR_T}{\sigma^2}\left(y - \frac{\sinh(\sigma y)}{4\sinh(\sigma/4)}\right) \tag{20}$$

Using Equation (19) in (18) we obtain

$$\theta = 2R_T y. \tag{21}$$

When buoyancy forces are dominating i.e., when $GR \to \pm\infty$, in the case of asymmetric heating, Equation (20) gives

$$\frac{u}{GR} = \frac{2}{\sigma^2}\left(y - \frac{\sinh(\sigma y)}{4\sinh(\sigma/4)}\right) \tag{22}$$

Another simple solutions of Equations (19) and (14) can be obtained when buoyancy forces are negligible and viscous dissipation is dominating, i.e., $GR = 0$, so that a purely forced convection occurs. For this condition solutions of Equations (19) and (18) are

$$u = \frac{48}{\sigma^2}\left(1 - \frac{\cosh(\sigma y)}{\cosh(\sigma/4)}\right) \tag{23}$$

$$\begin{aligned}\theta = {} & A\left(y^2 - \frac{1}{16}\right) + B\left(\cosh(2\sigma y) - \cosh(\sigma/2)\right) \\ & + C\left(\cosh(\sigma y) - \cosh(\sigma/4)\right) + 2R_T y\end{aligned} \tag{24}$$

where A, B and C are constants. For $GR = 0$ the boundary conditions on θ are $\theta(-1/4) = \theta(1/4) = R_T/2$.

3. Perturbation Method

In this section, Equation (19) along with boundary conditions (16) and (17) are solved analytically by a perturbation series method. Then temperature field is determined by employing Equation (18) in the absence of inertial effects. The dimensionless perturbation parameter is defined by

$$\varepsilon = Br\,GR = Re\,Pr\frac{\beta g D}{C_p}. \tag{25}$$

Thus, the solution of Equation (19) for a fixed value of $GR \neq 0$ can be expressed for small values of $\varepsilon (<< 1)$ in the form of series

$$u(y) = u_0(y) + \varepsilon u_1(y) + \varepsilon^2 u_2(y) + \cdots = \sum_{n=0}^{\infty} \varepsilon^n u_n(y). \tag{26}$$

Case 1: Isothermal–Isothermal Walls

Substituting series (26) into Equations (16), (17) and (19), and equating coefficients of like powers of ε to zero, one obtains the boundary value problem for $n = 0$ and 1 as

$$\frac{d^4 u_0}{dy^4} - \sigma^2 \frac{d^2 u_0}{dy^2} = 0, \tag{27}$$

$$\frac{\mathrm{d}^4 u_1}{\mathrm{d}y^4} - M^2 \frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2} = M^2 u_0^2 + \left(\frac{\mathrm{d}u_0}{\mathrm{d}y}\right)^2, \tag{28}$$

$$u_0\left(-\frac{1}{4}\right) = u_0\left(\frac{1}{4}\right) = 0, \tag{29}$$

$$\left(\frac{\mathrm{d}^2 u_0}{\mathrm{d}y^2}\right)_{y=-\frac{1}{4}} = \frac{GRR_T}{2} - 48; \qquad \left(\frac{\mathrm{d}^2 u_0}{\mathrm{d}y^2}\right)_{y=\frac{1}{4}} = -\frac{GRR_T}{2} - 48, \tag{30}$$

$$u_1\left(-\frac{1}{4}\right) = u_1\left(\frac{1}{4}\right) = 0, \tag{31}$$

$$\left(\frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2}\right)_{y=-\frac{1}{4}} = \left(\frac{\mathrm{d}^2 u_1}{\mathrm{d}y^2}\right)_{y=\frac{1}{4}} = 0 \tag{32}$$

Equation (27), which represents the flow in the absence of viscous and Darcy dissipations can be solved in closed form. Thus its solution, which satisfies the boundary conditions (29) and (30) is

$$u_0 = c_1 + c_2 y + c_3 \cosh(\sigma y) + c_4 \sinh(\sigma y), \tag{33}$$

where $c_i (i = 1, \ldots, 4)$ are constants. Equation (33) is also the solution of Equation (15) in the case of $Br = 0$ and $I = 0$. On the other hand, Equation (28) can be also solved exactly as u_0 is known now and is given by

$$\begin{aligned} u_1 = {} & c_5 + c_6 y + c_7 \cosh(\sigma y) + c_8 \sinh(\sigma y) + \mathrm{d}_1 \cosh(2\sigma y) + \mathrm{d}_2 \sinh(2\sigma y) \\ & + \mathrm{d}_3 y^2 \cosh(\sigma y) + \mathrm{d}_4 y^2 \sinh(\sigma y) + \mathrm{d}_5 y \cosh(\sigma y) + \mathrm{d}_6 y \sinh(\sigma y) \\ & + \mathrm{d}_7 y^4 + \mathrm{d}_8 y^3 + \mathrm{d}_9 y^2, \end{aligned} \tag{34}$$

where $c_i (i = 5, \ldots, 8)$ and $d_i (i = 1, \ldots, 9)$ are constants. We truncate the series in Equation (26) after $n = 1$ and the solution is obtained up to $O(\varepsilon)$ as

$$u = u_0 + \varepsilon u_1. \tag{35}$$

The dimensionless temperature field is evaluated by Equation (18) in the absence of inertial effects, which is given by

$$\theta = -\frac{1}{GR}\left(\begin{array}{l} 48 - \sigma^2 C_1 - \sigma^2 C_2 y + \\ \varepsilon \left(\begin{array}{l} f_1 \cosh(2\sigma y) + f_2 \sinh(2\sigma y) + f_3 y \cosh(\sigma y) + f_4 y \sinh(\sigma y) + \\ f_5 \cosh(\sigma y) + f_6 \sinh(\sigma y) + f_7 y^4 + f_8 y^3 + f_9 y^2 + f_{10} y + f_{11} \end{array} \right) \end{array} \right), \tag{36}$$

where C_1, C_2 and $f_i (i = 1, \ldots, 11)$ are constants.

Case 2: Isoflux–Isothermal Walls $(q_1 - T_2)$

The isoflux and isothermal boundary conditions for the channel walls are given by (see Chamkha, [22])

$$q_1 = -k\left(\frac{\mathrm{d}T}{\mathrm{d}Y}\right)_{Y=\frac{L}{2}}; \qquad T\left(\frac{L}{2}\right) = T_2. \tag{37}$$

In the non-dimensional form Equation (37) using Equation (10) with $\Delta T = q_1 D/k$ becomes

$$\left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)_{y=-1/4} = -1; \qquad \theta\left(\frac{1}{4}\right) = R_{qt}, \tag{38}$$

where $R_{qt} = (T_2 - T_0)/\Delta T$ is the thermal ratio parameter.

To solve Equation (15), apart from the no-slip conditions, two more boundary conditions, are required. These are obtained from Equation (1) using Equation (37) as follows. Differentiating Equation (1) with respect to Y and using Equation (4) we obtain

$$g\beta\frac{\mathrm{d}T}{\mathrm{d}Y} + \frac{\mu_{\mathrm{eff}}}{\rho_0}\frac{\mathrm{d}^3U}{\mathrm{d}Y^3} - \frac{\mu}{\rho_0 K}\frac{\mathrm{d}U}{\mathrm{d}Y} + \frac{\rho C_F}{\rho_0}\frac{\mathrm{d}U^2}{\mathrm{d}Y} = 0. \tag{39}$$

Equation (39) can be written in non-dimensional form using Equation (10) with $\mu_{\mathrm{eff}} = \mu$ as

$$\frac{\mathrm{d}^3u}{\mathrm{d}y^3} + \frac{Gr}{Re}\frac{\mathrm{d}\theta}{\mathrm{d}y} - \sigma^2\frac{\mathrm{d}u}{\mathrm{d}y} + I\frac{\mathrm{d}u^2}{\mathrm{d}y} = 0. \tag{40}$$

Evaluating Equation (40) at the left wall i.e., at $y = -1/4$ it yields

$$\left(\frac{\mathrm{d}^3u}{\mathrm{d}y^3} - \sigma^2\frac{\mathrm{d}u}{\mathrm{d}y} + I\frac{\mathrm{d}u^2}{\mathrm{d}y}\right)_{y=-1/4} = GR. \tag{41}$$

The boundary condition at the right wall is the same as that of isothermal–isothermal case replacing R_t by R_{qt} i.e.,

$$\left(\frac{\mathrm{d}^2u}{\mathrm{d}y^2}\right)_{y=1/4} = -48 - \frac{R_{qt}}{2}GR. \tag{42}$$

In the absence of inertia forces Equation (41) become

$$\left(\frac{\mathrm{d}^3u}{\mathrm{d}y^3} - \sigma^2\frac{\mathrm{d}u}{\mathrm{d}y}\right)_{y=-1/4} = GR. \tag{43}$$

Substituting Equation (26) in Equations (42) and (43) and equating coefficients of like powers of ε to zero, we obtain the boundary conditions for zeroth and first order of ε as follows

$$\left(\frac{d^2u_0}{dy^2}\right)_{y=1/4} = -48 - \frac{R_{qt}}{2}GR, \tag{44}$$

$$\left(\frac{d^3u_0}{dy^3} - \sigma^2\frac{du_0}{dy}\right)_{y=-1/4} = GR, \tag{45}$$

$$\left(\frac{d^2u_1}{dy^2}\right)_{y=1/4} = 0, \tag{46}$$

$$\left(\frac{d^3u_1}{dy^3} - \sigma^2\frac{du_1}{dy}\right)_{y=-1/4} = GR. \tag{47}$$

The solution of Equations (27) and (28) with the boundary conditions given by Equations (29), (31) and (44)–(47) can be easily obtained and the solutions are not presented.

On the other hand, in the absence of inertial force, the temperature field can be obtained by Equation (18) using the solutions of Equations (27) and (28).

Case 3: Isothermal-Isoflux Walls ($T_1 - q_2$)

The thermal boundary conditions for this case are

$$T\left(-\frac{L}{2}\right) = T_1; \qquad q_2 = -K\left(\frac{dT}{dy}\right)_{L/2}, \tag{48}$$

where q_2 is a constant. Equation (48) in the non-dimensional form using Equation (10) with $\Delta T = q_2 D/k$ becomes

$$\theta\left(-\frac{1}{4}\right) = R_{tq}; \qquad \left(\frac{d\theta}{dy}\right)_{y=1/4} = -1, \tag{49}$$

where $R_{tq} = (T_1 - T_0)/\Delta T$ is the thermal ratio parameter for the isothermal–isoflux case.

Following the procedure used for isoflux–isothermal walls, the dimensionless boundary conditions obtained from using Equation (1) and applying Equation (10) can be written as

$$\left(\frac{d^2u}{dy^2}\right)_{y=-1/4} = -48 + \frac{R_{tq}}{2}GR, \tag{50}$$

$$\left(\frac{d^3u}{dy^3} - \sigma^2\frac{du}{dy}\right)_{y=1/4} = GR. \tag{51}$$

Using Equation (26) in Equations (50) and (51) and equating like powers of ε to zero, one obtains the following boundary conditions for zeroth and first order as

$$\left(\frac{d^2u_0}{dy^2}\right)_{y=-1/4} = -48 + \frac{R_{qt}}{2}GR, \tag{52}$$

$$\left(\frac{d^3u_0}{dy^3} - \sigma^2\frac{du_0}{dy}\right)_{y=1/4} = GR, \tag{53}$$

$$\left(\frac{d^2u_1}{dy^2}\right)_{y=1/4} = 0, \tag{54}$$

$$\left(\frac{d^3u_1}{dy^3} - \sigma^2\frac{du_1}{dy}\right)_{y=1/4} = GR. \tag{55}$$

One can obtain easily the solution of Equations (27) and (28) using boundary conditions (29), (31) and (52)–(55). The expression for temperature field in the absence of inertial forces using Equation (18) can be obtained using solutions of Equations (27) and (28).

4. Heat Transfer Aspects

We shall now calculate the heat transfer parameters on the wall expresses in terms of the Nusselt numbers.

1. Isothermal–Isothermal $(T_1 - T_2)$ *Walls*

$$Nu_1 = \frac{D}{\Delta T}\left(\frac{dT}{dY}\right)_{Y=-L/2} = \left(\frac{d\theta}{dy}\right)_{y=-1/4}, \tag{56}$$

$$Nu_2 = \frac{D}{\Delta T}\left(\frac{dT}{dY}\right)_{Y=L/2} = \left(\frac{d\theta}{dy}\right)_{y=1/4}, \tag{57}$$

where Nu_1 and Nu_2 are the Nusselt numbers at the left and right walls, respectively.

2. Isoflux–Isothermal $(q_1 - T)_2$ *Walls*

$$Nu_1 = \frac{h_1 D}{K} = \frac{D q_1}{K\,(T_1 - T_0)} = \frac{1}{\theta\left(-\frac{1}{4}\right)}, \tag{58}$$

$$Nu_2 = \frac{h_2 D}{K} = \frac{D}{\Delta T}\left(\frac{\mathrm{d}T}{\mathrm{d}y}\right) = \left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)_{y=1/4}. \tag{59}$$

3. Isothermal–Isoflux Walls $(T_1 - q_2)$

$$Nu_1 = \frac{h_1 D}{K} = \frac{D}{Dt}\frac{\mathrm{d}T}{\mathrm{d}y} = \left(\frac{\mathrm{d}\theta}{\mathrm{d}y}\right)_{y=-1/4}, \tag{60}$$

$$Nu_2 = \frac{h_2 D}{K} = \frac{D q_2}{K\,(T_2 - T_0)} = \frac{1}{\theta\left(\frac{1}{4}\right)}. \tag{61}$$

The expressions for Nusselt numbers for different wall temperatures are obtained and results are shown in Figure 7.

5. Numerical Method

The general Equation (15) including the inertial effects do not poses an analytical solution. Therefore, a numerical procedure is employed using finite-difference method. Replacing the derivatives with corresponding central difference approximations, lead to m linear algebraic equations where m is the number of divisions from $y = -1/4$ to 1/4. The solutions of reduced algebraic equations are solved by the successive-over-relaxation method. The relaxation parameter is fixed by comparing the numerical values with the analytical results for the case $GR = 0$. The convergence criteria is based on the step size and the previous iterations for the iterative difference to the order 10^{-6}. The comparison of numerical and analytical results is shown in Figure 2 and they agree very well for $\varepsilon = 0$ and vary as ε increases. Figures 3 and 4 display the effects of inertia on the velocity and temperature profiles for asymmetric wall temperatures.

6. Results and Discussion

An analytical solution for mixed convective flow and heat transfer in a vertical porous channel is obtained using regular perturbation method in the absence of inertial forces. A numerical procedure is performed to obtain the flow solutions in the presence of inertial forces. The flow is modeled using Brinkman–Forchhiemer extended Darcy equations. The viscous and Darcy

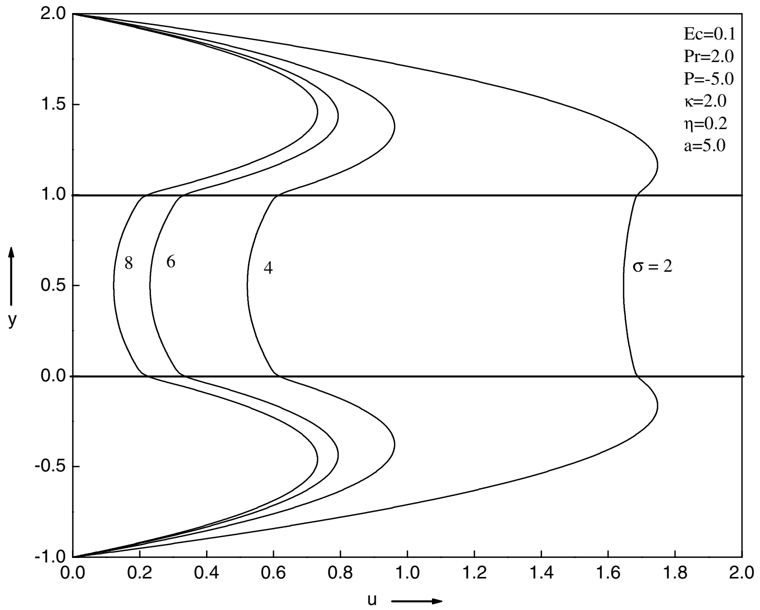

Figure 2. Dimensionless velocity profiles for different values of porous parameter σ.

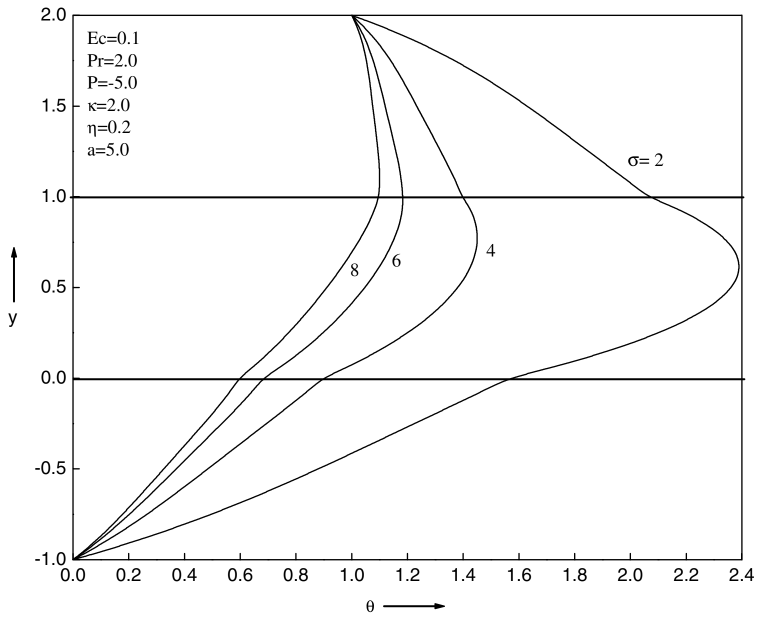

Figure 3. Dimensionless temperature profiles for different values of porous parameter σ.

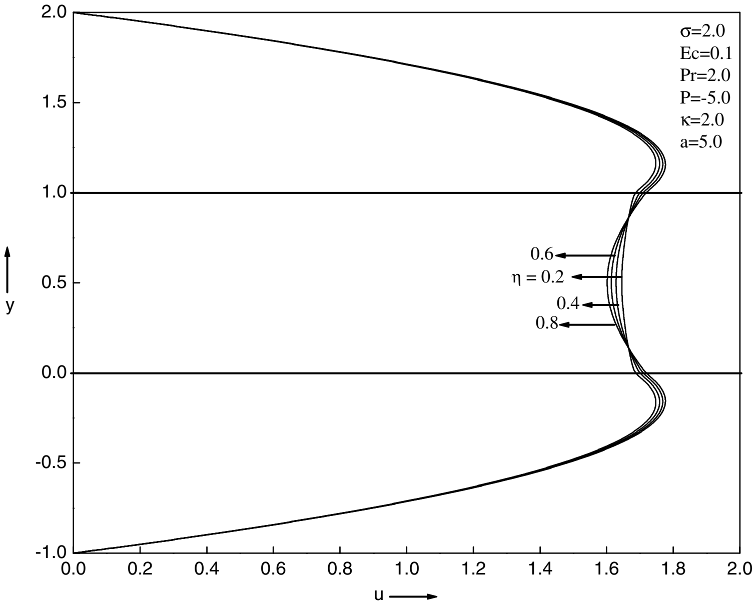

Figure 4. Dimensionless velocity profiles for different values of ratio of viscosity parameter η.

dissipation terms are included in the energy equation. The analytical solutions has been determined using $BrGr/\mathrm{Re}$ as the perturbation parameter. The heat transfer coefficient have been tabulated for three different thermal boundary conditions.

The velocity and temperature fields in the case of asymmetric heating ($R_T = 1$) are obtained and are depicted in Figures 2–4. For asymmetric heating the temperatures at the boundaries are different and hence velocity and temperature fields depend on perturbation parameter ε and *GR*. When the flow is upward, ε and *GR* are positive. On the other hand, when the flow is downward, ε and *GR* are negative. The sign of ε and *GR* are equal but their absolute values are different. It is also seen from Figure 2 that analytical and numerical solutions agree very well in the absence of inertial effects for $\varepsilon = 0$ and vary largely for $\varepsilon = 8$.

The effect of the mixed convection parameter $GR(= Gr/Re)$ on the velocity field with asymmetric isothermal–isothermal wall conditions is shown in Figure 2 in the absence of inertia effects. Figure 2 displays the velocity profiles for $GR = \pm 500$ for fixed σ. The curves with maximum values are on the left for $GR = -500$ and on the right for $GR = 500$. Flow reversal occurs for large values of GR near the cold wall at $y = -1/4$ for positive GR and near the hot wall at $y = 1/4$ for negative *GR*. Also it is interesting to note that flow reversal is less for large ε. The influence of the presence of the porous medium, Darcian and inertial effects on velocity and temperature distributions in the channel is illustrated for asymmetric wall temperatures in Figures 3 and 4. These results are obtained numerically. The presence of porous medium produces flow resistance. In addition, the inertia effects adds on this resistance mechanism which further reduces the flow in the channel. Lai and Kulacki (1987) also proved the similar result that the inertial forces has no effect on the wall heat flux, but it does have the effect of flattening the dimensionless velocity profile. The flow reversal in the absence of inertial forces (Figure 2) is same as in the presence of inertial forces (Figure 3).

Figure 4 shows the plots of θ versus y for different values of ε, σ and I for the asymmetric heating of the walls. This graph shows that temperature field increases with increase in the value of ε and it is suppressed in the presence of inertia for both positive and negative values of *GR*.

Figure 5 illustrates the influence of porous parameter σ on temperature with isoflux–isothermal wall conditions. It is observed that the temperature at the wall with constant heat flux decreases as σ increases for positive and negative values of *GR*. However, the wall temperature is more influenced for positive *GR* than for the negative *GR*. Figure 6 displays the variations on temperature profiles with isothermal–isoflux wall conditions for values of mixed convection parameter. The flow nature is similar to that for

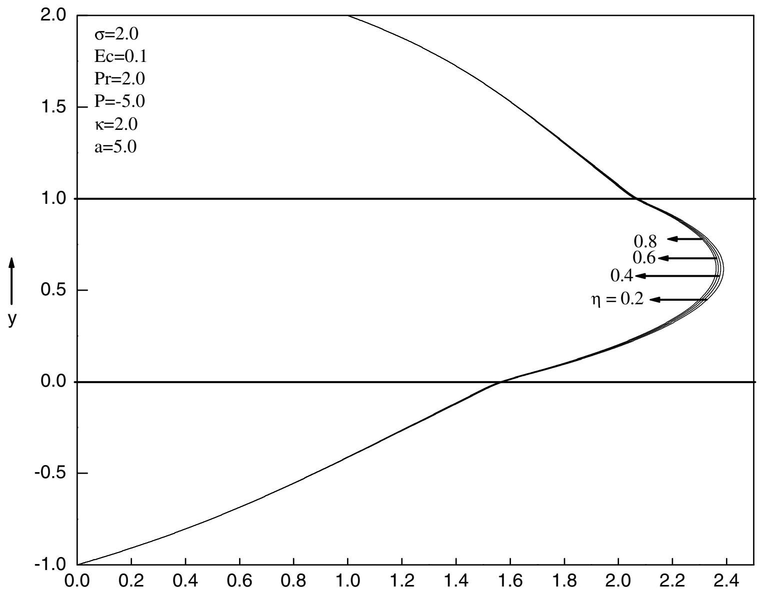

Figure 5. Dimensionless temperature profiles for different values of ratio of viscosity parameter η.

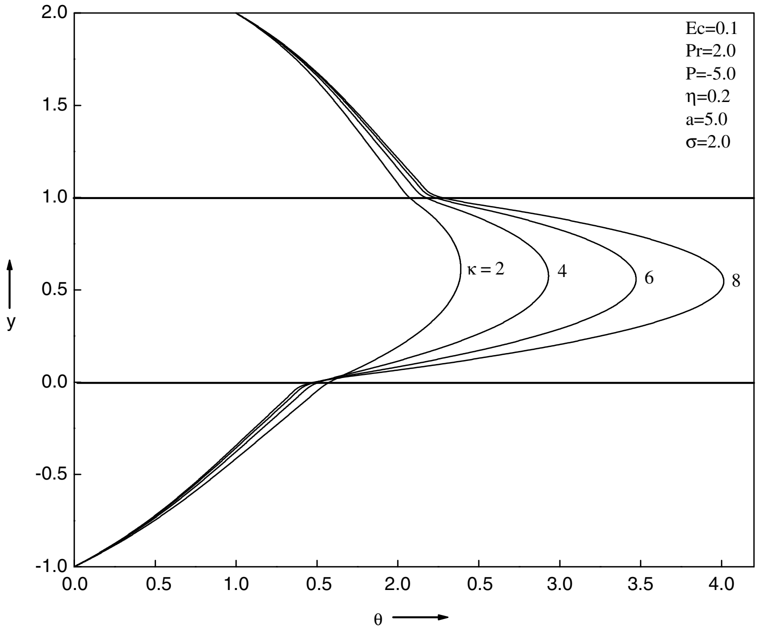

Figure 6. Dimensionless temperature profiles for different values of ratio of thermal conductivity parameter κ.

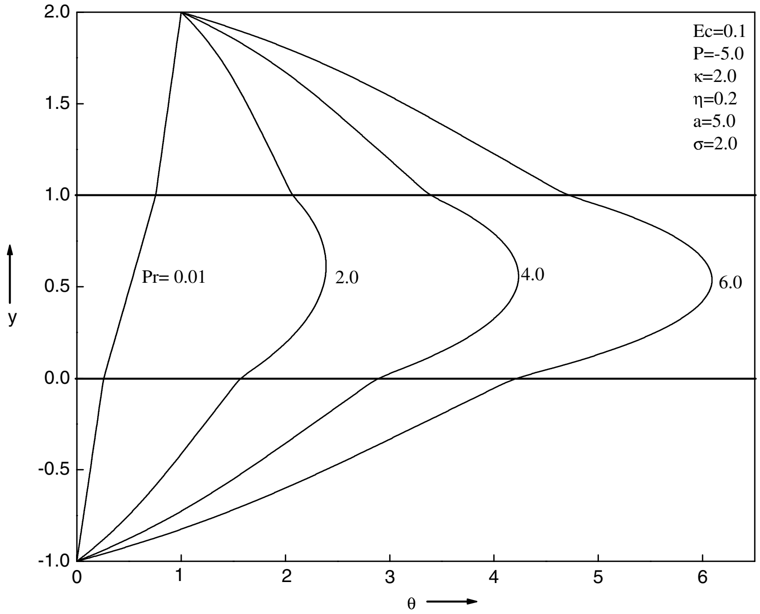

Figure 7. Dimensionless temperature profiles for different values of Prandtl number Pr.

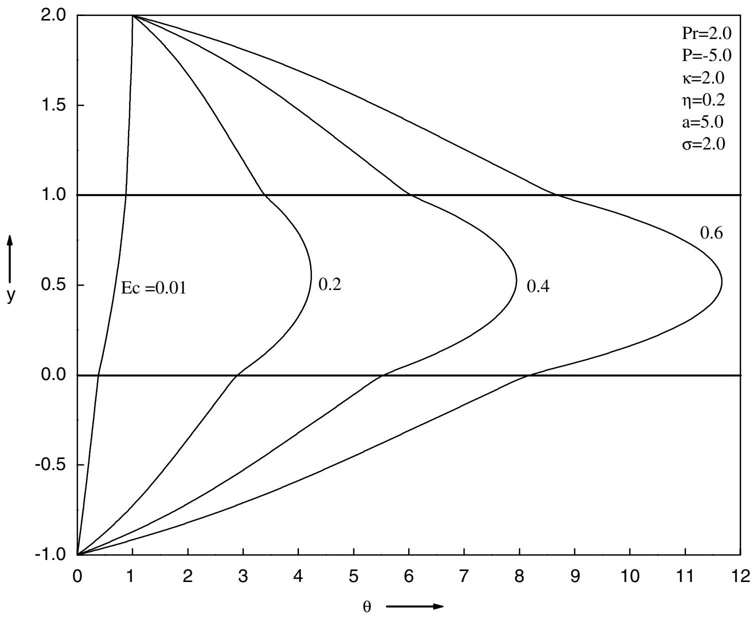

Figure 8. Dimensionless temperature profiles for different values of Eckert number Ec.

isoflux–isothermal wall conditions. Figure 7 displays the rate of heat transfer for $R = 500$ with σ for isothermal-isothermal, isothermal–isoflux and isoflux–isothermal wall temperatures. Rate of heat transfer is less at $y = \frac{1}{4}$ compared to at $y = -\frac{1}{4}$ for different wall temperatures and the difference is very large for isothermal–isoflux and isoflux–isothermal wall temperatures (Figures 8–10).

7. Conclusions

The problem of steady, laminar, mixed convective flow and heat transfer in a vertical channel embedded in a porous media with symmetric and asymmetric wall temperatures is studied by both analytical and numerical methods. Three different combinations of thermal wall conditions such as isothermal–isothermal, isoflux–isothermal and isothermal–isoflux are considered. The nonlinear dimensionless equations are solved analytically using perturbation series using $\varepsilon(= BrGr/Re)$ as the perturbation parameter. The mixed convection problem which includes the inertial effects was solved numerically using finite-difference technique. The numerical solutions are successfully validated by the analytical solutions in the absence of inertial forces. Dimensionless Nusselt numbers at the left and right walls for the above-mentioned three different wall conditions are solved analytically and graphically depicted for different values of porous

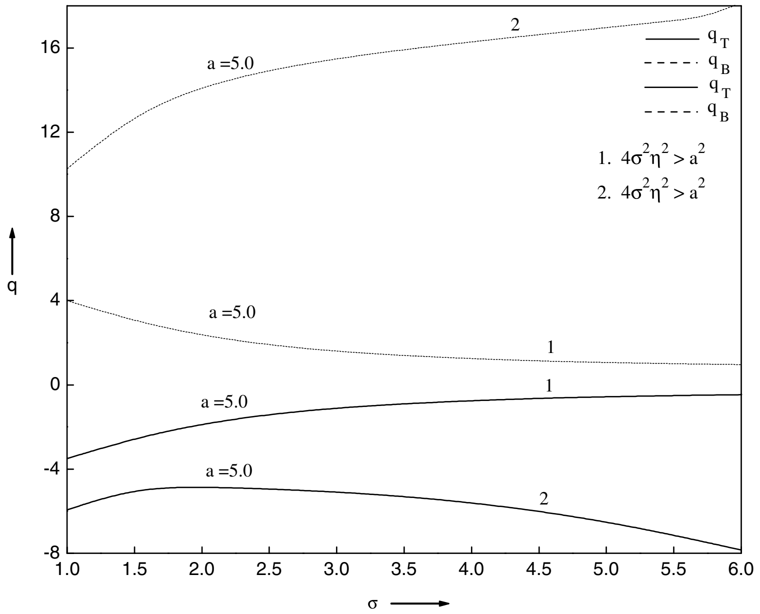

Figure 9. Dimensionless rate of heat transfer for different values of the parameter σ.

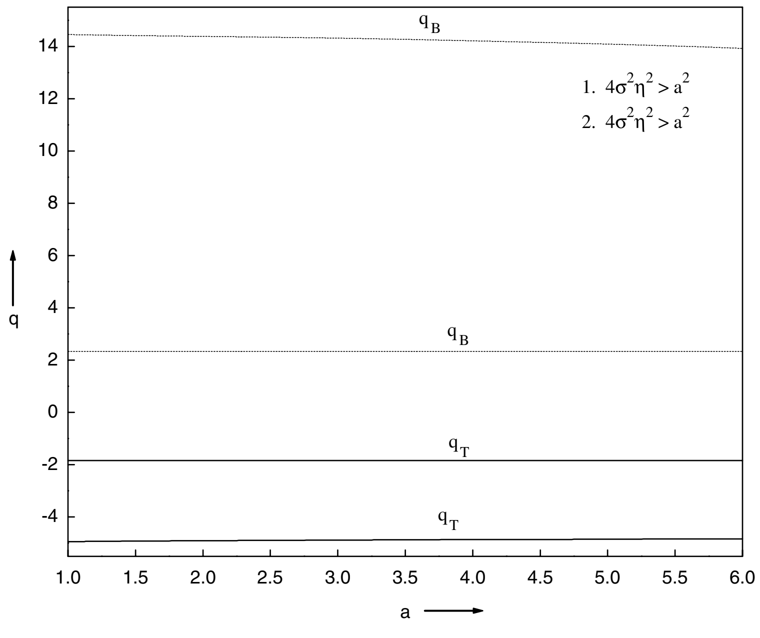

Figure 10. Dimensionless rate of heat transfer for different values of couple stress parameter "*a*".

parameter. The velocity and temperature are also evaluated as shown graphically for asymmetric boundary temperatures. Increase in the values of porous parameter and Forchhiemer drag term produces reduced flow in the channel for unequal wall temperatures. Flow reversal near the walls is obtained for asymmetric wall temperatures and is found to increase in the presence of porous and inertial effects. The viscous dissipation enhances the effect of flow reversal in the case of downward flow while it encounter this effect in the case of upward flow.

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