Heat Mass Transfer (2005) 42: 81–90
DOI 10.1007/s00231-004-0565-x

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Unsteady two-fluid flow and heat transfer in a horizontal channel

Received: 17 February 2004 / Accepted: 13 August 2004 / Published online: 12 October 2005

Abstract The problem of unsteady oscillatory flow and heat transfer of two viscous immiscible fluids through a horizontal channel with isothermal permeable walls has been considered. The partial differential equations governing the flow and heat transfer are solved analytically using two-term harmonic and non-harmonic functions in both fluid regions of the channel. Effects of physical parameters such as viscosity ratio, conductivity ratio, Prandtl number and frequency parameter on the velocity and temperature fields are shown graphically. It is observed that the velocity and temperature decrease as the viscosity ratio increases, while they increase with increases in frequency parameter. The effect of increasing the thermal conductivity ratio also suppresses the temperature in both fluid regions. The effect of periodic frequency on the flow is depicted in tabular form. It is predicted that both the velocity and temperature profiles decrease as the periodic frequency increases.

List of symbols

A	Real positive constant
C_p	Specific heat at constant pressure
g	Gravitational acceleration
K	Thermal conductivity
P	Non-dimensional pressure gradient
p	Pressure
Pr	Prandtl number
T	Temperature
T_w	Wall temperature
t	Time
u, v	Velocity components of velocity along and perpendicular to the plates, respectively
$\bar{u}_1$	Average velocity
v_0	Scale of suction
x, y	Cartesian coordinates

Greek letters

α	Ratio of viscosities
β	Ratio of thermal conductivities
ε	Coefficient of periodic parameter
ρ	Fluid density
μ	Viscosity of fluid
ω	Frequency parameter
ωt	Periodic frequency parameter
ν	Kinematic viscosity
θ	Non-dimensional temperature

Subscripts

1, 2	Quantities for Region-I and Region-II, respectively

Superscripts

*	Dimensionless quantity

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1 Introduction

The flow and heat transfer aspects of immiscible fluids is of special importance in the petroleum extraction and transport. For example, the reservoir rock of an oil field always contains several immiscible fluids in its pores. Part of the pore volume is occupied by water and the rest may be occupied either by oil or gas or both. Crude oils often contain dissolved gases which may be released into the reservoir rock when the pressure decreases. These examples show the importance of knowledge of the laws governing immiscible multi-phase flows for proper understanding of the processes involved. In modeling such problems, the presence of a second immiscible fluid phase adds a number of complexities as to the nature of interacting transport phenomena and interface conditions between the phases. In general, multi-phase flows are driven by gravitational and viscous forces. There has been some theoretical and experimental work on stratified laminar flow of two immiscible fluids in a horizontal pipe [1, 10, 11, 16]. Loharsbi and Sahai [8] studied two-phase MHD flow and heat transfer in a parallel plate channel with one of the fluids being electrically conducting. Two-phase MHD flow and heat transfer in an inclined channel was investigated by Malashetty and Umavathi [12]. Chamkha [4] reported analytical solutions for flow of two-immiscible fluids in porous and non-porous parallel-plate channels. Later on, magneto-hydrodynamic two-fluid convective flow and heat transfer in composite porous medium was analyzed by Malashetty et al. [13–15].

The flow and heat transfer problems of viscous fluids through permeable-walled passages is important in many branches of science and engineering. The permeable-walled ducts are used in heat exchangers, solar energy collectors, transpiration cooling of gas-turbine blades, combustion chambers, exhaust nozzles, porous-walled flow reactors [21]. Also, porous or permeable walls have been used in the past to simulate a variety of surface mechanisms. These include natural transpiration, phase sublimation, propellant burning, ablation cooling, and uniformly distributed irrigation. Such mechanisms take place in a number of interesting models of bio-circulatory systems, flow filtration, chemical dispensing, rocket propellant combustion, and other membrane separation processes [9]. Investigations of laminar porous channel flows appear to have been initiated by Berman [2]. After Berman's work, several studies on the topic followed. For example, Zaturska et al. [22] reported on the flow of a viscous fluid driven along a channel by suction at porous walls. Cox [5] considered two-dimensional flow of a viscous fluid in a channel with porous walls. More recently, King and Cox [7] performed an asymptotic analysis of the steady-state and time-dependent Berman problem.

All of the above studies pertain to steady flow. However, a significant number of problems of practical interest are unsteady. The flow unsteadiness may be caused by a change in the free stream velocity or in the surface temperature (surface heat flux) or in both. When there is an impulsive change in the velocity field, the inviscid flow is developed instantaneously, but the flow in the viscous layer near the wall is developed slowly which becomes fully developed steady flow after some time. Soundelgekar [17, 18] studied the effects of free stream oscillations and free convection currents on the flow past an infinite vertical plate with constant suction. Soundelgekar and Bhatt [19] analyzed oscillatory MHD channel flow and heat transfer under a transverse magnetic field. Goto and Uchida [6] considered unsteady flows in a semi-infinite expanding pipe with injection through the wall. Umavathi and Palaniappan [20] studied oscillatory flow of unsteady convective fluid in an infinite vertical porous stratum. Recently, Chamkha [3] studied unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Keeping in view the wide area of practical importance of multi-fluid flows as mentioned above, it is the objective of the present study to investigate unsteady flow and heat transfer of two immiscible fluids in a horizontal channel with permeable walls with time-dependent oscillating normal velocity.

2 Mathematical formulation

Consider a two-dimensional unsteady flow of two immiscible fluids in horizontal parallel permeable plates, extending in the x and $y$directions. The region $0 \leq y \leq h$ (Region-I) is filled with a viscous fluid having density ρ_1, dynamic viscosity μ_1, specific heat at constant pressure C_{P_1}, thermal conductivity K_1 and the region $-h \leq y \leq 0$ (Region-II) is filled with a different viscous fluid having density ρ_2, dynamic viscosity μ_2, specific heat at constant pressure C_{P_2}, and thermal conductivity K_2.

The flow in both regions of the channel is assumed to be fully developed and is driven by a common pressure gradient $(-\partial p/\partial x)$. The two plates are maintained at constant temperatures T_{w_1} at $y = h$ and T_{w_2} at $y = -h$ All fluid properties are assumed constant.

Under these assumptions and taking $\rho_1 = \rho_2 = \rho_0$ and $C_{p_1} = C_{p_2} = C_p$, the governing equations of motion and energy [8] are given by

Region-I

$$\frac{\partial v_1}{\partial y} = 0 \tag{1}$$

$$\rho_0\left(\frac{\partial u_1}{\partial t} + v_1\frac{\partial u_1}{\partial y}\right) = \mu_1\frac{\partial^2 u_1}{\partial y^2} - \frac{\partial p}{\partial x} \tag{2}$$

$$\rho_0 C_p\left(\frac{\partial T_1}{\partial t} + v_1\frac{\partial T_1}{\partial y}\right) = K_1\frac{\partial^2 T_1}{\partial y^2} + \mu_1\left(\frac{\partial u_1}{\partial y}\right)^2 \tag{3}$$

Region-II

$$\frac{\partial v_2}{\partial y} = 0 \tag{4}$$

$$\rho_0\left(\frac{\partial u_2}{\partial t} + v_2\frac{\partial u_2}{\partial y}\right) = \mu_2\frac{\partial^2 u_2}{\partial y^2} - \frac{\partial p}{\partial x} \tag{5}$$

$$\rho_0 C_p\left(\frac{\partial T_2}{\partial t} + v_2\frac{\partial T_2}{\partial y}\right) = K_2\frac{\partial^2 T_2}{\partial y^2} + \mu_2\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{6}$$

where u is the x-component of fluid velocity, v is the y-component of fluid velocity and T is the fluid temperature. The subscripts 1 and 2 correspond to Region-I and Region-II, respectively. The boundary conditions on velocity are the no-slip boundary conditions which require that the x-component of velocity must vanish at the wall. The boundary conditions on temperature are isothermal conditions. We also assume the continuity of velocity, shear stress, temperature and heat flux at the interface between the two fluid layers at $y=0$.

The hydrodynamic boundary and interface conditions for the two fluids can then be written as

$$u_1(h) = 0$$
$$u_2(-h) = 0$$
$$u_1(0) = u_2(0)$$
$$\mu_1 \frac{\partial u_1}{\partial y} = \mu_2 \frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \tag{7}$$

The thermal boundary and interface conditions on temperature for both fluids are given by

$$T_1(h) = T_{w1}$$
$$T_2(-h) = T_{w2}$$
$$T_1(0) = T_2(0)$$
$$K_1 \frac{\partial T_1}{\partial y} = K_2 \frac{\partial T_2}{\partial y} \quad \text{at } y = 0 \tag{8}$$

The continuity equations of both fluids (Eqs. 1 and 4) imply that, v_1 and v_2 are independent of y, they can be utmost functions of time alone. Hence, we can write (assuming $v_1 = v_2 = v$)

$$v = v_0\left(1 + \varepsilon A\, e^{i\omega t}\right) \tag{9}$$

where A is a real positive constant. ω is the frequency parameter and ε is small such that $\varepsilon A \leq 1$ Here it is assumed that the transpiration velocity v varies periodically with time about a non-zero constant mean v_0. When $\varepsilon A = 0$, the case of constant transpiration velocity is recovered. By use of the following non-dimensional quantities:

$$u_i = \bar{u}_1 u_i^* \quad y = hy^* \quad t = \frac{h^2}{v} t^* \quad v = \frac{v}{h} \quad v^* = \frac{v}{|v_0|}$$
$$\omega = \frac{v}{h^2}\,\omega^* \quad P = \frac{h^2}{\mu_1 \bar{u}_1}\left(\frac{\partial p}{\partial x}\right) \theta = \frac{T - T_w}{\bar{u}_1 \mu_1 / K_1} \tag{10}$$

and for simplicity dropping the asterisks, Eqs. 2, 3, 4, 5 and 6 become

Region-I

$$\frac{\partial u_1}{\partial t} + v\frac{\partial u_1}{\partial y} = P + \frac{\partial^2 u_1}{\partial y^2} \tag{11}$$

$$\frac{\partial \theta_1}{\partial t} + v\frac{\partial \theta_1}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 \theta_1}{\partial y^2} + \frac{1}{\Pr}\left(\frac{\partial u_1}{\partial y}\right)^2 \tag{12}$$

Region-II

$$\frac{\partial u_2}{\partial t} + v\frac{\partial u_2}{\partial y} = P + \alpha\frac{\partial^2 u_2}{\partial y^2} \tag{13}$$

$$\frac{\partial \theta_2}{\partial t} + v\frac{\partial \theta_2}{\partial y} = \frac{\beta}{\Pr}\frac{\partial^2 \theta_2}{\partial y^2} + \frac{\alpha}{\Pr}\left(\frac{\partial u_2}{\partial y}\right)^2 \tag{14}$$

where $\Pr = \rho_0 v C_p / K_1$is the Prandtl number, $\alpha = \mu_2/\mu_1$ is the ratio of viscosities and $\beta = K_2/K_1$ is the ratio of thermal conductivities.

The hydrodynamic and thermal boundary and interface conditions for both fluids in non-dimensional form become

$$u_1(1) = 0$$
$$u_2(-1) = 0$$
$$u_1(0) = u_2(0)$$
$$\frac{\partial u_1}{\partial y} = \alpha\frac{\partial u_2}{\partial y} \quad \text{at } y = 0 \tag{15}$$
$$\theta_1(1) = 1$$
$$\theta_2(-1) = 0$$
$$\theta_1(0) = \theta_2(0)$$
$$\frac{\partial \theta_1}{\partial y} = \beta\frac{\partial \theta_2}{\partial y} \quad \text{at } y = 0 \tag{16}$$

3 Closed-form solutions

The governing momentum equations (11 and 13) along with the energy equations (12and 14) are solved subject to the boundary and interface conditions (Eqs. 15 and 16) for the velocity and temperature distributions in both fluid regions. These equations are coupled partial differential equations that cannot be solved in closed form. However, they can be reduced to ordinary differential equations by assuming

$$u_1(y,t) = u_{10}(y) + \varepsilon e^{i\omega t} u_{11}(y), \quad u_2(y,t) = u_{20}(y) + \varepsilon e^{i\omega t} u_{21}(y) \tag{17}$$

$$\theta_1(y,t) = \theta_{10}(y) + \varepsilon e^{i\omega t} \theta_{11}(y), \quad \theta_2(y,t) = \theta_{20}(y) + \varepsilon e^{i\omega t} \theta_{21}(y) \tag{18}$$

This is a valid assumption because of the choice of v as defined in Eq. 9 that the amplitude $\varepsilon A << 1$.

By substituting Eqs. 17 and 18into Eqs. 11, 12, 13, 14, 15 and 16, keeping the harmonic and non-harmonic terms and neglecting the higher-order terms of ε^2, one obtains the following pairs of equations:

Region-I

Non-periodic coefficients

$$\frac{d^2 u_{10}}{dy^2} - \frac{du_{10}}{dy} + P = 0 \tag{19}$$

$$\frac{d^2\theta_{10}}{dy^2} - \mathrm{Pr}\frac{d\theta_{10}}{dy} = -\left(\frac{du_{10}}{dy}\right)^2 \tag{20}$$

Periodic coefficients

$$\frac{d^2u_{11}}{dy^2} - \frac{du_{11}}{dy} - i\,\omega\,u_{11} = A\frac{du_{10}}{dy} \tag{21}$$

$$\frac{d^2\theta_{11}}{dy^2} - \mathrm{Pr}\frac{d\theta_{11}}{dy} - i\omega\,\mathrm{Pr}\,\theta_{11} = \mathrm{Pr}A\frac{d\theta_{10}}{dy} - 2\frac{du_{10}}{dy}\frac{du_{11}}{dy} \tag{22}$$

Region -II

Non-periodic coefficients

$$\alpha\frac{d^2u_{20}}{dy^2} - \frac{du_{20}}{dy} + P = 0 \tag{23}$$

$$\beta\frac{d^2\theta_{20}}{dy^2} - \mathrm{Pr}\frac{d\theta_{20}}{dy} = -\alpha\left(\frac{du_{20}}{dy}\right)^2 \tag{24}$$

Periodic coefficients

$$\alpha\frac{d^2u_{21}}{dy^2} - \frac{du_{21}}{dy} - i\,\omega\,u_{21} = A\frac{du_{20}}{dy} \tag{25}$$

$$\beta\frac{d^2\theta_{21}}{dy^2} - \mathrm{Pr}\frac{d\theta_{21}}{dy} - i\omega\,\mathrm{Pr}\,\theta_{21} = \mathrm{Pr}\,A\frac{d\theta_{20}}{dy} - 2\alpha\frac{du_{20}}{dy}\frac{du_{21}}{dy} \tag{26}$$

The corresponding boundary and interface conditions become as follows:

Non-periodic coefficients

$$u_{10}(1) = 0$$
$$u_{20}(-1) = 0$$
$$u_{10}(0) = u_{20}(0)$$
$$\frac{\partial u_{10}}{\partial y} = \alpha\frac{\partial u_{20}}{\partial y} \quad \text{at } y = 0 \tag{27}$$

Periodic coefficients

$$u_{11}(1) = 0$$
$$u_{21}(-1) = 0$$
$$u_{11}(0) = u_{21}(0)$$
$$\frac{\partial u_{11}}{\partial y} = \alpha\frac{\partial u_{21}}{\partial y} \quad \text{at } y = 0 \tag{28}$$

Non-periodic coefficients

$$\theta_{10}(1) = 1$$
$$\theta_{20}(-1) = 0$$
$$\theta_{10}(0) = \theta_{20}(0)$$
$$\frac{\partial\theta_{10}}{\partial y} = \beta\frac{\partial\theta_{20}}{\partial y} \quad \text{at } y = 0 \tag{29}$$

Periodic coefficients

$$\theta_{11}(1) = 0$$
$$\theta_{21}(-1) = 0$$
$$\theta_{11}(0) = \theta_{21}(0)$$
$$\frac{\partial\theta_{11}}{\partial y} = \beta\frac{\partial\theta_{21}}{\partial y} \quad \text{at } y = 0 \tag{30}$$

Equations 19, 20, 21, 22, 23, 24, 25 and 26 along with boundary and interface conditions 27, 28, 29 and 30 represent a system of ordinary differential equations and conditions that can be solved in closed form.

Solutions of the non-periodic (harmonic) terms lead to steady flow solutions for both fluids. Without going into detail, the steady-state velocity and temperature profiles can be shown to be

$$u_{10} = C_1 + C_2 e^{y} \tag{31}$$
$$u_{20} = C_3 + C_4 e^{m_1 y} \tag{32}$$
$$\theta_{10} = C_5 + C_6 e^{\mathrm{Pr}\,y} + k_1 e^{2y} + k_2 e^{y} + k_3 y \tag{33}$$
$$\theta_{20} = C_7 + C_8 e^{m_2 y} + k_4 e^{2m_1 y} + k_5 e^{m_1 y} + k_6 y \tag{34}$$

Solutions of the periodic (non-harmonic) terms or transient velocity and temperature profile solutions in both regions of the channel (Region-I and Region-II) are given by

$$u_{11} = e^{A_1 y}(XC_9\cos B_1 y + XC_{10}\sin B_1 y) + i\left[e^{A_1 y}(YC_9\cos B_1 y + YC_{10}\sin B_1 y) + \frac{A(C_2 e^{y} + P)}{\omega}\right] \tag{35}$$

$$u_{21} = e^{A_2 y}(XC_{11}\cos B_2 y + XC_{10}\sin B_2 y) + K_7 E_1 e^{m_1 y} + i\left[e^{A_2 y}(YC_{11}\cos B_2 y + YC_{12}\sin B_2 y) + K_7\omega e^{m_1 y} + K_8\right] \tag{36}$$

$$\begin{aligned}\theta_{11} = {} & e^{A_3 y}(XC_{13}\cos B_3 y + XC_{14}\sin B_3 y) + K_{25}e^{2y} + K_{26}e^{y} \\ & + K_{11}e^{m_3 y}(K_{21}\cos B_2 y + K_{22}\sin B_2 y) + K_{14}e^{A_1 y}(K_{23}\cos B_1 y + K_{24}\sin B_1 y) \\ & + i\begin{bmatrix} e^{A_3 y}(YC_{13}\cos B_3 y + YC_{14}\sin B_3 y) + k_{25}e^{2y} + k_{26}e^{y} + k_1 e^{\mathrm{Pr}\,y} + k_2 \\ + K_{11}e^{m_3 y}(k_{21}\cos B_2 y + k_{22}\sin B_2 y) + K_{14}e^{A_1 y}(k_{23}\cos B_1 y + k_{24}\sin B_1 y)\end{bmatrix}\end{aligned} \tag{37}$$

$$
\begin{aligned}
\theta_{21} = {} & \mathrm{e}^{A_8 y}(XC_{15}\cos B_8 y + XC_{16}\sin B_8 y) + A_9\mathrm{e}^{m_2 y} + A_{10}\mathrm{e}^{2m_1 y} + K_{43}\mathrm{e}^{m_1 y} \\
& + K_{29}\mathrm{e}^{m_4 y}(K_{39}\cos B_2 y + K_{40}\sin B_2 y) + K_{32}\mathrm{e}^{A_2 y}(K_{41}\cos B_2 y + K_{42}\sin B_2 y) \\
& + i\left[\begin{array}{l} \mathrm{e}^{A_8 y}(YC_{15}\cos B_8 y + YC_{16}\sin B_8 y) + k_3 B_9\mathrm{e}^{m_2 y} + B_{10}\mathrm{e}^{2m_1 y} + k_{43}\mathrm{e}^{m_1 y} \\ + K_{29}\mathrm{e}^{m_4 y}(k_{39}\cos B_2 y + k_{40}\sin B_2 y) + K_{32}\mathrm{e}^{A_2 y}(k_{41}\cos B_2 y + k_{42}\sin B_2 y) \end{array}\right]
\end{aligned} \tag{38}
$$

It should be noted that all of the constants appearing in the above solutions are defined in the Appendix section.

4 Results and discussion

In this section, flow and heat transfer results for unsteady oscillatory flow of two immiscible fluids through a horizontal channel with time-dependent transpiration velocity are presented and discussed for various parametric conditions. Exact solutions are obtained for steady state conditions with constant transpiration velocity while analytical solutions for variable transpi-

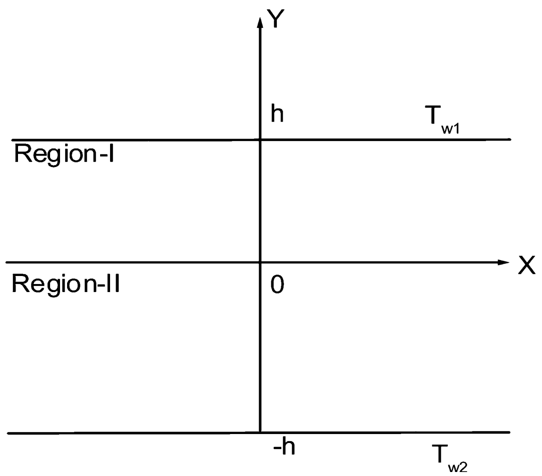

Fig. 1 Physical configuration

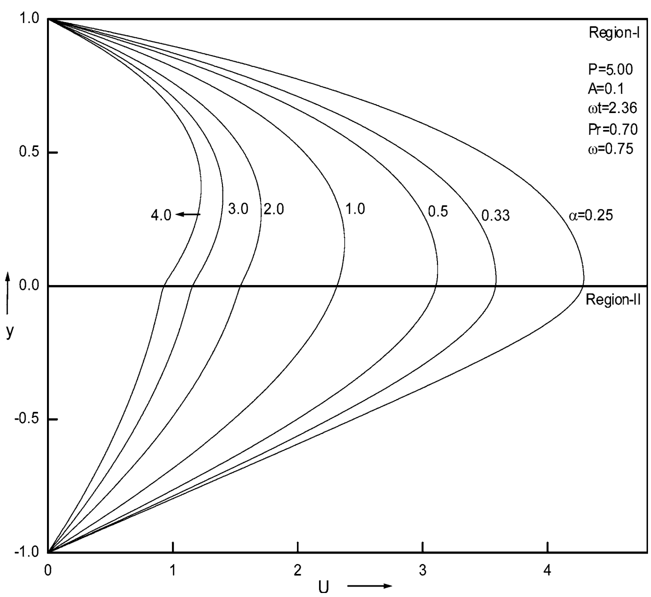

Fig. 2 Velocity profiles for different values of ratio of viscosity α

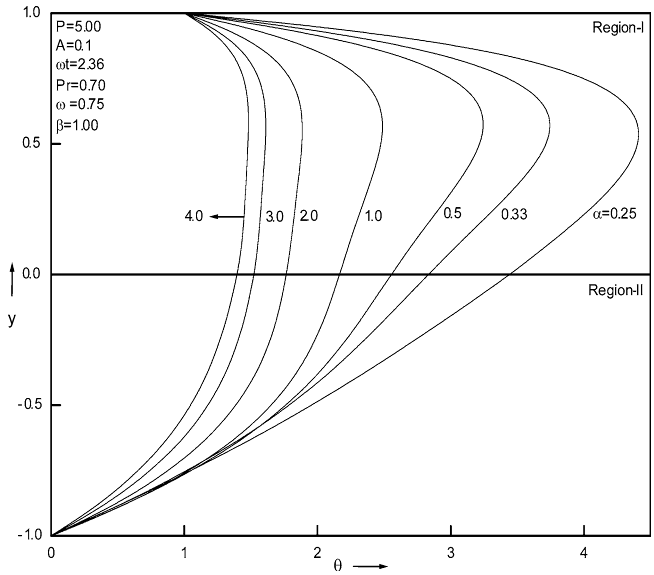

Fig. 3 Temperature profiles for different values of ratio of viscosity α

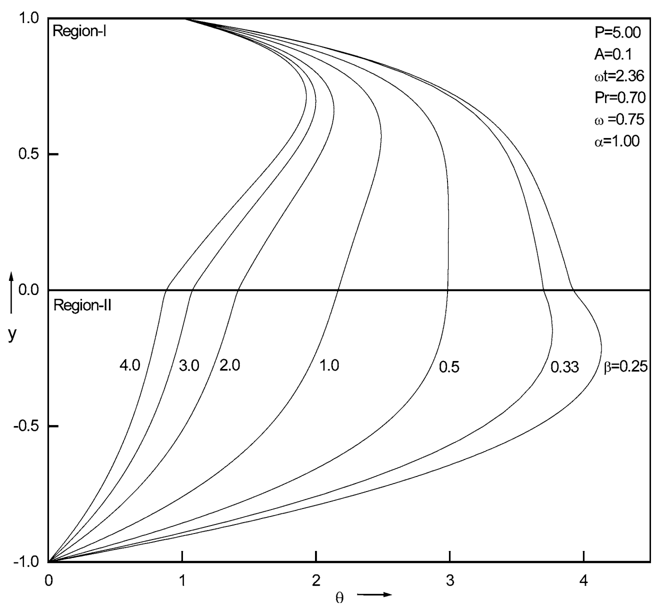

Fig. 4 Temperature profiles for different values of thermal conductivity ratio β

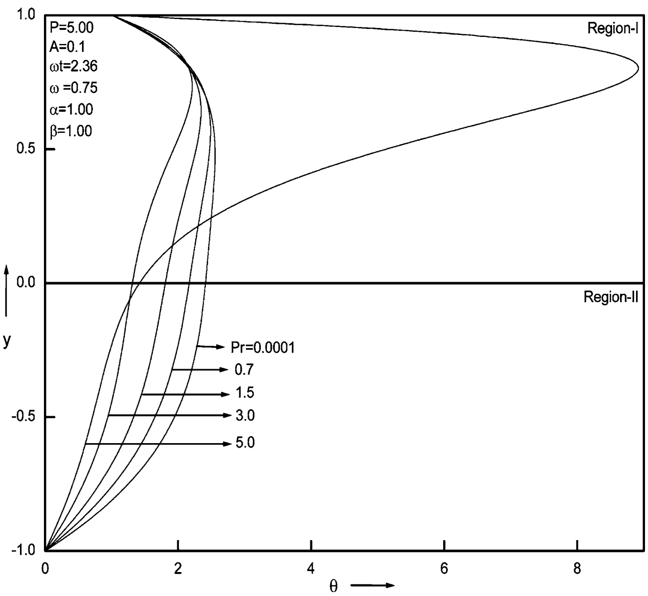

Fig. 5 Temperature profiles for different values of Prandtl number Pr

ration velocity are obtained under the constraint that the coefficient of exponent of periodic frequency parameter ε is small as shown in Eqs. 17 and 18. Numerical evaluations of these solutions are performed and presented graphically in Figs. 1, 2, 3, 4, 5, 6 and 7 for different parameters such as the viscosity ratio α, conductivity ratio β, Prandtl number Pr and the frequency parameter ε *A*. The effect of periodic frequency parameter ω *t* on the velocity and temperature profiles is shown in Table 1.

Figure 2 shows the immiscible fluids velocity profiles in both regions (Region-I and Region-II) of the channel for different values of the viscosity ratio α. As the viscosity ratio α increases, the velocity decreases in both fluid regions. As α increases the viscosity of the fluid in the lower region becomes thick and hence, the velocity decreases. Figure 3 shows a similar effect of the viscosity ratio on the temperature profiles. The magnitude of reduction of temperature as α increases is less in Region-II compared to Region-I. This is due to the fact that the wall temperature in Region-I is larger than that in Region-II.

Figure 4 displays the influence of the thermal conductivity ratio β on the temperature profiles in both fluid regions of the channel. Increases in the thermal conductivity ratio have the tendency to cool down the thermal state in the channel. This is depicted in the reductions in the fluids temperatures as β increases shown in Fig. 4.

The effect of the Prandtl number Pr on the temperature profiles is shown in Fig. 5. The Prandtl number is a measure of momentum diffusion to heat diffusion. It is a measure of the relative importance of viscosity and heat conduction in a flow field. Thus, as the Prandtl number increases, the viscous forces dominate over the heat conduction effects and hence, the temperature decreases. However, for the parametric values used to obtain Fig. 5, there appear to be a critical value of Pr for which a distinctive peak in temperature occurs in Region-I. This critical value of Pr for the conditions of Fig. 5 is between 3 and 5. As Pr increases beyond this value, the peak temperature in Region-I becomes more distinctive as evident from Fig. 5. It should be mentioned that when α was changed, a different temperature profile behavior was observed. This suggests that the existence of the peak temperature is related to the viscosity ratio of the two fluids.

Figures 6 and 7 show the effect of the frequency parameter ε *A* on the velocity and temperature profiles in the channel. It should be reminded that the frequency parameter was assumed to be small in evaluating the analytical solutions i.e., $\varepsilon A < 1$. The condition $\varepsilon A = 0$ is for steady state solutions. As the frequency parameter increases, both the velocity and temperature values increase.

Finally, the effect of the periodic frequency parameter ω *t* on the velocity and temperature profiles is shown in Table 1. As the periodic frequency parameter ω *t* in-

Table 1 Velocity and temperature profiles for different values of periodic frequency parameter ω *t*

y	ω *t*									
	0.0000		0.7857		1.5714		2.3571		3.1428	
	u	θ	*u*	θ	*u*	θ	*u*	θ	*u*	θ
1.0	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000
0.8	1.1009	2.2139	1.0996	2.2078	1.0964	2.1929	1.0932	2.1779	1.0919	2.1718
0.6	1.8196	2.4976	1.8176	2.4863	1.8128	2.4589	1.8080	2.4316	1.8060	2.4203
0.4	2.2256	2.4410	2.2234	2.4263	2.2181	2.3908	2.2127	2.3554	2.2105	2.3407
0.2	2.3759	2.3090	2.3738	2.2927	2.3686	2.2532	2.3634	2.2138	2.3613	2.1975
0.0	2.3171	2.1823	2.3152	2.1656	2.3106	2.1253	2.3060	2.0850	2.3041	2.0683
−0.2	2.0871	2.0412	2.0855	2.0259	2.0818	1.9891	2.0781	1.9523	2.0765	1.9371
−0.4	1.7172	1.8233	1.7160	1.8113	1.7132	1.7822	1.7105	1.7531	1.7093	1.7411
−0.6	1.2328	1.4561	1.2320	1.4480	1.2302	1.4285	1.2284	1.4090	1.2277	1.4010
−0.8	0.6547	0.8684	0.6543	0.8645	0.6535	0.8550	0.6526	0.8455	0.6522	0.8416
−1.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

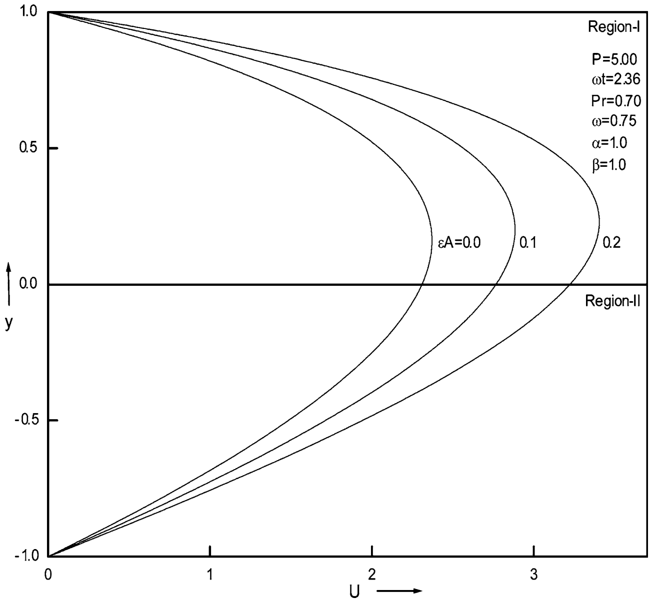

Fig. 6 Velocity profiles for different values of frequency parameter εA

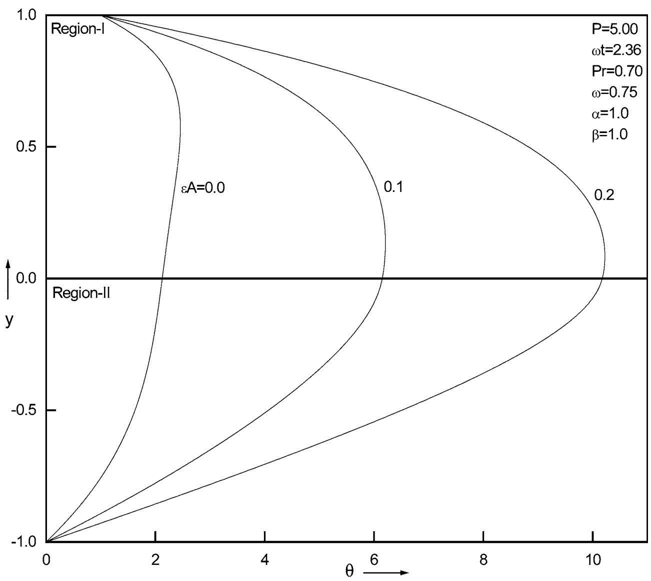

Fig. 7 Temperature profiles for different values of frequency parameter εA

creases, both the velocity and temperature profiles decrease and the magnitude of suppression is predicted to be very small.

5 Conclusions

The problem of unsteady flow of two immiscible fluids through a horizontal channel with time-dependent oscillatory wall transpiration velocity was investigated analytically. Both fluids were assumed to be Newtonian and have constant physical properties. Separate closed-form solutions for each fluid were obtained taking into consideration suitable interface matching conditions. The closed-form results were numerically evaluated and presented graphically for various values of the frequency parameter, periodic frequency parameter, viscosity and conductivity ratios and the Prandtl number. It was predicted that both the velocity and temperature profiles decreased as the periodic frequency parameter was increased. Furthermore, it was concluded that the flow and heat transfer aspects in a horizontal channel with permeable walls can be controlled by considering different fluids having different viscosities, conductivities and also by varying the amplitude of the transpiration velocity at the boundary.

Acknowledgements The first author thanks Professor M.S. Malashetty, Department of Mathematics, Gulbarga University, Gulbarga for his constant encouragement and the University Grant Commission for providing financial support under Special Assistance Programme.

Appendix

$$A_1 = \frac{1+\sqrt{r_1}\cos\left(\frac{\theta_1}{2}\right)}{2}; \quad A_2 = \frac{1+\sqrt{r_1}\cos\left(\frac{\theta_1}{2}\right)}{2\alpha}$$

$$A_3 = \frac{\mathrm{Pr}+\sqrt{r_2}\cos\left(\frac{\theta_2}{2}\right)}{2}; \quad A_4 = \frac{2\,\mathrm{Pr}\,AK_1(4-2\,\mathrm{Pr})}{(4-2\,\mathrm{Pr})^2+\omega^2\,\mathrm{Pr}^2}$$

$$A_5 = \frac{\mathrm{Pr}\,AK_2(1-\mathrm{Pr})}{(1-\mathrm{Pr})^2+\omega^2\,\mathrm{Pr}^2}; \quad A_6 = \frac{-2k_3C_2(4-2\,\mathrm{Pr})}{(4-2\,\mathrm{Pr})^2+\omega^2\,\mathrm{Pr}^2}$$

$$A_7 = \frac{-2k_3P(1-\mathrm{Pr})}{(1-\mathrm{Pr})^2+\omega^2\,\mathrm{Pr}^2}; \quad A_8 = \frac{\mathrm{Pr}+\sqrt{r_3}\cos\left(\frac{\theta_3}{2}\right)}{2\beta}$$

$$A_9 = \frac{\mathrm{Pr}\,Am_2C_8\left(\beta m_2^2-m_2\,\mathrm{Pr}\right)}{\left(\beta m_2^2-m_2\,\mathrm{Pr}\right)^2+\omega^2\,\mathrm{Pr}^2};$$

$$A_{10} = \frac{2\,\mathrm{Pr}\,Am_1K_4\left(4\beta m_1^2-2m_1\,\mathrm{Pr}\right)}{\left(4\beta m_1^2-2m_1\,\mathrm{Pr}\right)^2+\omega^2\,\mathrm{Pr}^2}$$

$$A_{11} = \frac{\mathrm{Pr}\,Am_1K_5\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)}{\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)^2+\omega^2\,\mathrm{Pr}^2};$$

$$A_{12} = \frac{-2\alpha Pm_1K_7E_1\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)}{\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)^2+\omega^2\,\mathrm{Pr}^2}$$

$$A_{13} = \frac{-2\alpha Pm_1K_7\omega\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)}{\left(\beta m_1^2-m_1\,\mathrm{Pr}\right)^2+\omega^2\,\mathrm{Pr}^2}$$

$$B_1 = \frac{\sqrt{r_1}\sin\left(\frac{\theta_1}{2}\right)}{2}; \quad B_2 = \frac{\sqrt{r_1}\sin\left(\frac{\theta_1}{2}\right)}{2\alpha}$$

$$B_3 = \frac{\sqrt{r_2}\sin\left(\frac{\theta_2}{2}\right)}{2}; \quad B_4 = \frac{2\,\mathrm{Pr}^2\,AK_1\omega}{(4-2\,\mathrm{Pr})^2+\omega^2\,\mathrm{Pr}^2}$$

$$B_5 = \frac{\Pr^2 AK_2\omega}{(1-\Pr)^2+\omega^2\Pr^2}; \quad B_6 = \frac{-2k_3C_2\omega\Pr}{(4-2\Pr)^2+\omega^2\Pr^2}$$

$$B_7 = \frac{-2k_3P\omega\Pr}{(1-\Pr)^2+\omega^2\Pr^2}; \quad B_8 = \frac{\sqrt{r_3}\sin\left(\frac{\theta_3}{2}\right)}{2\beta}$$

$$B_9 = \frac{\Pr^2 Am_2C_8\omega}{\left(\beta m_2^2 - m_2\Pr\right)^2+\omega^2\Pr^2};$$

$$B_{10} = \frac{2\Pr^2 Am_1K_4\omega}{\left(4\beta m_1^2 - 2m_1\Pr\right)^2+\omega^2\Pr^2}$$

$$B_{11} = \frac{\Pr^2 Am_1K_5\omega}{\left(\beta m_1^2 - m_1\Pr\right)^2+\omega^2\Pr^2};$$

$$B_{12} = \frac{-2\alpha Pm_1K_7E_1\omega\Pr}{\left(\beta m_1^2 - m_1\Pr\right)^2+\omega^2\Pr^2}$$

$$B_{13} = \frac{-2\alpha Pm_1K_7\omega^2\Pr}{\left(\beta m_1^2 - m_1\Pr\right)^2+\omega^2\Pr^2}$$

$$E_1 = \alpha m_1^2 - m_1; \quad E_2 = 2m_3 - \Pr; \quad E_3 = m_3^2 - m_3\Pr$$

$$E_4 = E_3 - B_1^2; \quad E_5 = E_4^2 + \omega^2 \overset{2}{\Pr} - E_2^2B_1^2; \quad E_6 = 2A_1 - \Pr$$

$$E_7 = A_1^2 - A_1\Pr; \quad E_8 = E_7 - B_1^2;$$

$$E_9 = E_8^2 + \omega^2 \overset{2}{\Pr} - E_6^2B_1^2$$

$$E_{10} = 2\beta m_4 - \Pr; \quad E_{11} = \beta m_4^2 - m_4\Pr; \quad E_{12} = E_{11} - \beta B_2^2$$

$$E_{13} = E_{12}^2 + \omega^2 \overset{2}{\Pr} - E_{10}^2B_2^2; \quad E_{14} = 2\beta A_2 - \Pr;$$

$$E_{15} = \beta A_2^2 - A_2\Pr$$

$$E_{16} = E_{15} - \beta B_2^2; \quad E_{17} = E_{16}^2 + \omega^2 \overset{2}{\Pr} - E_{14}^2B_2^2$$

$$F_1 = 2E_2^2E_4; \quad F_2 = 2E_2E_4^2 - E_2E_5; \quad F_3 = -E_4E_5$$

$$F_4 = 2E_2E_4\omega\Pr; \quad F_5 = -E_5\omega\Pr; \quad F_6 = 2E_6^2E_8$$

$$F_7 = 2E_6E_8^2 - E_6E_9; \quad F_8 = -E_8E_9; \quad F_9 = 2E_6E_8\omega\Pr$$

$$F_{10} = -E_9\omega\Pr; \quad F_{11} = -2E_{10}^2E_{12};$$

$$F_{12} = -2E_{10}E_{12}^2 + E_{10}E_{13}$$

$$F_{13} = E_{13}E_{12}; \quad F_{14} = -2E_{10}E_{12}\omega\Pr; \quad F_{15} = E_{13}\omega\Pr$$

$$F_{16} = -2E_{14}^2E_{16}; \quad F_{17} = -2E_{14}E_{16}^2 + E_{14}E_{17};$$

$$F_{18} = E_{17}E_{16}$$

$$F_{19} = -2E_{14}E_{16}\omega\Pr; \quad F_{20} = E_{17}\omega\Pr$$

$$K_1 = \frac{-C_2^2}{4-2\Pr}; \quad K_2 = \frac{-2C_2P}{1-\Pr}; \quad K_3 = \frac{P^2}{\Pr}$$

$$K_4 = \frac{-\alpha m_1^2C_4^2}{4\beta m_1^2 - 2\Pr m_1}; \quad K_5 = \frac{-2\alpha m_1C_4P}{m_1^2\beta - \Pr m_1}; \quad K_6 = \frac{\alpha P^2}{\Pr}$$

$$K_7 = \frac{Am_1C_4}{E_1^2+\omega^2}; \quad K_8 = \frac{AP}{\omega}; \quad K_9 = A_1XC_9 + B_1XC_{10}$$

$$K_{10} = A_1XC_{10} - B_1XC_9; \quad K_{11} = \frac{2C_2}{4E_2^2E_4^2B_1^2 + E_5^2}$$

$$K_{12} = -F_1B_1^2K_9 + F_2B_1K_{10} + F_3K_9;$$

$$K_{13} = -F_1B_1^2K_{10} - F_2B_1K_9 + F_3K_{10}$$

$$K_{14} = \frac{2P}{4E_6^2E_8^2B_1^2 + E_9^2}; \quad K_{15} = -F_6B_1^2K_9 + F_7B_1K_{10} + F_8K_9$$

$$K_{16} = -F_6B_1^2K_{10} - F_7B_1K_9 + F_8K_{10};$$

$$K_{17} = -F_1B_1^2k_9 + F_2B_1k_{10} + F_3k_9$$

$$K_{18} = -F_1B_1^2k_{10} - F_2B_1k_9 + F_3k_{10};$$

$$K_{19} = -F_6B_1^2k_9 + F_7B_1k_{10} + F_8k_9$$

$$K_{20} = -F_6B_1^2k_{10} - F_7B_1k_9 + F_8k_{10}; \quad K_{21} = K_{12} - k_{17}$$

$$K_{22} = K_{13} - k_{18}; \quad K_{23} = K_{15} - k_{19}; \quad K_{24} = K_{16} - k_{20}$$

$$K_{25} = A_4 - B_6; \quad K_{26} = A_5 - B_7; \quad K_{27} = A_2XC_{11} + B_2XC_{12}$$

$$K_{28} = A_2XC_{12} - B_2XC_{11}; \quad K_{29} = \frac{-2\alpha m_1C_4}{E_{13}^2 - 4E_{10}^2E_{12}^2B_2^2}$$

$$K_{29} = \frac{-2\alpha m_1C_4}{E_{13}^2 - 4E_{10}^2E_{12}^2B_2^2};$$

$$K_{31} = -F_{11}B_2^2K_{28} - F_{12}B_2K_{27} + F_{13}K_{28}$$

$$K_{32} = \frac{-2\alpha P}{E_{17}^2 - 4E_{14}^2E_{16}^2B_2^2};$$

$$K_{33} = -F_{16}B_2^2K_{27} + F_{17}B_2K_{28} + F_{18}K_{27}$$

$$K_{34} = -F_{16}B_2^2K_{28} - F_{17}B_2K_{27} + F_{18}K_{28};$$

$$K_{35} = -F_{11}B_2^2k_{27} + F_{12}B_2k_{28} + F_{13}k_{27}$$

$$K_{36} = -F_{11}B_2^2k_{28} - F_{12}B_2k_{27} + F_{13}k_{28};$$

$$K_{37} = -F_{16}B_2^2k_{27} + F_{17}B_2k_{28} + F_{18}k_{27}$$

$$K_{38} = -F_{16}B_2^2k_{28} - F_{17}B_2k_{27} + F_{18}k_{28}; \quad K_{39} = K_{30} - k_{35}$$

$$K_{40} = K_{31} - k_{36}; \quad K_{41} = K_{33} - k_{37}; \quad K_{42} = K_{34} - k_{38}$$

$$K_{43} = A_{11} + A_{12} - B_{13}$$

$$k_1 = \frac{A\Pr C_6}{\omega}; \quad k_2 = \frac{AK_3}{\omega}; \quad k_3 = \frac{AC_2}{\omega}$$

$$k_4 = \frac{-AK_6}{\omega}; \quad k_{12} = F_4B_1K_{10} + F_5K_9;$$

$$k_{13} = -F_4B_1K_9 + F_5K_{10}$$

$$k_{15} = F_9B_1K_{10} + F_{10}K_9; \quad k_{16} = -F_9B_1K_9 + F_{10}K_{10};$$

$$k_{17} = -F_4B_1k_{10} + F_5k_9$$

$$k_{18} = -F_4B_1k_9 + F_5k_{10}; \quad k_{19} = F_9B_1k_{10} + F_{10}k_9;$$

$$k_{20} = -F_9B_1k_9 + F_{10}k_{10}$$

$$k_{21} = k_{12} + K_{17}; \quad k_{22} = k_{13} + K_{18}; \quad k_{23} = k_{15} + K_{19}$$

$$k_{24} = k_{16} + K_{20}; \quad k_{25} = A_6 + B_4; \quad k_{26} = A_7 + B_5$$

$$k_{27} = A_2YC_{11} + B_2YC_{12}; \quad k_{28} = A_2YC_{12} - B_2YC_{11};$$

$$k_{30} = F_{14}B_2K_{28} + F_{15}K_{27}$$

$$k_{31} = -F_{14}B_2K_{27} + F_{15}k_{28}; \quad k_{33} = F_{19}B_2K_{28} + F_{20}K_{27};$$

$$k_{34} = -F_{19}B_2K_{27} + F_{20}K_{28}$$

$k_{35} = F_{14}B_2k_{28} + F_{15}k_{27};\quad k_{36} = -F_{14}B_2k_{27} + F_{15}k_{28};$

$k_{37} = F_{19}B_2k_{28} + F_{20}k_{27}$

$k_{38} = -F_{19}B_2k_{27} + F_{20}k_{28};\quad k_{39} = k_{30} - K_{35};$

$k_{40} = k_{31} + K_{36}$

$k_{41} = k_{33} + K_{37};\quad k_{42} = k_{34} - K_{38};$

$k_{43} = B_{11} + B_{12} + A_{13}$

$l_1 = k_1e^2 + k_2e^1 + k_3;\quad l_2 = k_4e^{-2m_1} + k_5e^{-m_1} - k_6;$

$l_3 = k_1 + k_2 - k_4 - k_5$

$$l_4 = 2k_1 + k_2 + k_3 - 2\beta m_1k_4;\quad l_5 = 1 + \frac{\Pr(e^{-m_2} - 1)}{m_2\beta};$$

$$l_6 = l_2 + l_3 + \frac{l_4(e^{-m_2} - 1)}{m_2\beta}$$

$$l_7 = e^{A_1}\cos B_1;\quad l_8 = e^{A_1}\sin B_1;\quad l_9 = \frac{A\left(C_2e^1 + P\right)}{\omega}$$

$l_{10} = e^{-A_2}\cos B_2;\quad l_{11} = -e^{-A_2}\sin B_2;\quad l_{12} = K_7E_1e^{-m_1}$

$l_{13} = K_7\omega e^{-m_1} + K_8;\quad l_{14} = K_7E_1$

$$l_{15} = \frac{A(C_2 + P)}{\omega} - K_7\omega - K_8;\quad l_{16} = -\alpha m_1K_7E_1$$

$$l_{17} = \frac{AC_2}{\omega} - \alpha m_1K_7\omega;\quad l_{18} = \frac{\alpha B_2l_{10}}{l_{11}} - \alpha A_2;$$

$$l_{19} = \frac{\alpha B_2l_{12}}{l_{11}} + l_{16}$$

$$l_{20} = \frac{\alpha B_2l_{13}}{l_{11}} + l_{17};\quad l_{21} = A_1 + l_{18};\quad l_{22} = l_{18}l_{14} + l_{19}$$

$l_{23} = l_{18}l_{15} + l_{20};\quad l_{24} = e^{A_3}\cos B_3;\quad l_{25} = e^{A_3}\sin B_3$

$$l_{26} = K_{25}e^2 + K_{26}e^1 + K_{11}e^{m_3}(K_{21}\cos B_1 + K_{22}\sin B_1) + K_{14}e^{A1}(K_{23}\cos B_1 + K_{24}\sin B_1)$$

$$l_{27} = k_{25}e^2 + k_{26}e^1 + k_1e^{\Pr} + k_2 + K_{11}e^{m_3} (k_{21}\cos B_1 + k_{22}\sin B_1) + K_{14}e^{A_1}(k_{23}\cos B_1 + k_{24}\sin B_1)$$

$l_{28} = e^{-A_8}\cos B_8;\quad l_{29} = -e^{-A_8}\sin B_8$

$$l_{30} = A_9e^{-m_2} + A_{10}e^{-2m_1} + K_{43}e^{-m_1} + K_{29}e^{-m_4}(K_{39}\cos B_2 - K_{40}\sin B_2) + K_{32}e^{-A_2}(K_{41}\cos B_2 - K_{42}\sin B_2)$$

$$l_{31} = k_3 + B_9e^{-m_2} + B_{10}e^{-2m_1} + k_{43}e^{-m_1} + K_{29}e^{-m_4} (k_{39}\cos B_2 - k_{40}\sin B_2) + K_{32}e^{-A_2}(k_{41}\cos B_2 - k_{42}\sin B_2)$$

$$l_{32} = K_{25} + K_{26} + K_{11}K_{21} + K_{14}K_{23} - A_9 - A_{10} - K_{43} - K_{29}K_{39} - K_{32}K_{41}$$

$$l_{33} = k_{25} + k_{26} + k_1 + k_2 - k_3 + K_{11}k_{21} + K_{14}k_{23} - B_9 - B_{10} - k_{43} - K_{29}k_{39} - K_{32}k_{41}$$

$$l_{34} = 2K_{25} + K_{26} + K_{11}(m_3K_{21} + B_1K_{22}) + K_{14}(A_1K_{23} + B_1K_{24}) - \beta\left[m_2A_9 + 2m_1A_{10} + m_1K_{43} + K_{29}(m_4K_{39} + B_2K_{40}) + K_{32}(A_2K_{41} + B_2K_{42})\right]$$

$$l_{35} = 2k_{25} + k_{26} + \Pr k_1 + K_{11}(m_3k_{21} + B_1k_{22}) + K_{14}(A_1k_{23} + B_1k_{24}) - \beta[m_2B_9 + 2m_1B_{10} + m_1k_{43} + K_{29}(m_4k_{39} + B_2k_{40}) + K_{32}(A_2k_{41} + B_2k_{42})]$$

$$l_{36} = \frac{\beta B_8l_{28}}{l_{29}} - \beta A_8;\quad l_{37} = \frac{\beta B_8l_{30}}{l_{29}} + l_{34};$$

$$l_{38} = \frac{\beta B_8l_{31}}{l_{29}} + l_{35}$$

$l_{39} = A_3 + l_{36};\quad l_{40} = l_{36}l_{32} + l_{37};\quad l_{41} = l_{36}l_{33} + l_{38}$

$$XC_9 = \frac{l_8l_{23}}{B_1l_7 - l_8l_{22}};\quad XC_{10} = \frac{-l_7XC_9}{l_8};$$

$XC_{11} = XC_9 + l_{14}$

$$XC_{12} = \frac{-(l_{10}XC_{11} + l_{12})}{l_{11}};\quad XC_{13} = \frac{l_{25}l_{40} - B_3l_{26}}{B_3l_{24} - l_{25}l_{39}};$$

$$XC_{14} = \frac{-(l_{24}XC_{13} + l_{26})}{l_{25}}$$

$$XC_{15} = XC_{13} + l_{32};\quad XC_{16} = \frac{-(l_{28}XC_{15} + l_{30})}{l_{29}}$$

$$YC_9 = \frac{l_8l_{24} - B_1l_9}{B_1l_7 - l_8l_{22}};\quad YC_{10} = \frac{-(l_7YC_9 + l_9)}{l_8};$$

$YC_{11} = YC_9 + l_{15}$

$$YC_{12} = \frac{-(l_{10}YC_{11} + l_{13})}{l_{11}};\quad YC_{13} = \frac{l_{25}l_{41} - B_3l_{27}}{B_3l_{24} - l_{25}l_{39}};$$

$$YC_{14} = \frac{-(l_{24}YC_{13} + l_{27})}{l_{25}}$$

$$YC_{15} = YC_{13} + l_{33};\quad YC_{16} = \frac{-(l_{28}YC_{15} + l_{31})}{l_{29}}$$

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