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Free convection flow over a truncated cone embedded in a porous medium saturated with pure or saline water at low temperatures

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Abstract

Steady free convection boundary layer about a truncated cone embedded in a porous medium saturated with pure or saline water at low temperatures has been studied in this paper. The governing coupled partial differential equations are solved numerically using a very efficient finite-difference method. Several new parameters arise and the results are given for some specific values of these parameters. The obtained results for a Boussinesq fluid are compared with known results from the open literature and it is shown that the agreement between these results is very good.

Keywords: Truncated cone; Boundary layer; Porous medium; Cold or saline water

1. Introduction

Convective flow in porous media has been a subject of great interest for the last several decades due to its numerous thermal engineering applications in various disciplines, such as geophysical thermal insulation, modeling of packed sphere beds, cooling of electronic systems, groundwater hydrology, petroleum reservoirs, coal combustors, ground water pollution, ceramic processes, to name just a few of these applications. Some of the most important analytical, numerical and experimental studies with such applications, which present the current state-of-the-art in the area of convective heat transfer in porous media, have been gathered in the monographs by Nield and Bejan (1999), Ingham and Pop (1998, 2002), Vafai (2000), Pop and Ingham (2001), and Bejan and Kraus (2003).

Studies of convective heat transfer in porous media have been carried out in the past using the Boussinesq approximation, namely the fluid density ρ varies linearly with temperature. However, this is inappropriate for water at low temperatures because of the extremum at about 4 °C in pure water at 1 atm. Such conditions occur commonly in porous medium, such as permeable soils flooded by cold lake or sea water, water–ice

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slurries, etc. A limited number of studies have been devoted in the past to the problem of convective boundary layer adjacent to heated or cooled bodies immersed in a porous medium saturated with cold water wherein a density extremum may arise. It should be mentioned that the buoyancy flow with an extremum may become very complicated, with local flow reversals and convective inversions. Density differences may then not be expressed as a linear function of the temperature. Ramilison and Gebhart (1980) examined the possible similarity solutions for vertical, buoyancy induced flow in a porous medium saturated with cold water. Lin and Gebhart (1986) have considered the corresponding case of a horizontal surface in a porous medium saturated with cold or saline water. Gebhart et al. (1983) obtained multiple steady state solutions for the problem considered by Lin and Gebhart (1986) using two numerical codes. A review of the convective flow in the vicinity of the maximum-density condition in water at low temperatures, along with relevant citations, is available in the survey by Kukula et al. (1987).

The present paper concerns the steady free convection boundary layer adjacent to a heated truncated cone embedded in an extensive porous medium saturated with either pure or saline water under the conditions in which a density extremum might occur. The density state equation used here is that proposed by Gebhart and Mollendorf (1977), which has been shown to be very accurate for both pure and saline water to a pressure level of 1000 bars up to 20 °C, and to 40% salinity. To the best of our knowledge, this problem has not been considered before. However, Yih (1999) made an analysis for free convection boundary layer about a truncated cone in a porous medium saturated with a Boussinesq fluid subjected to the coupled effects of thermal and mass diffusion.

2. Basic equations

Consider the steady free convection over a truncated cone (with half angle γ) embedded in a saturated porous medium filled with pure or saline water. It is assumed that the surface of the truncated cone is maintained at the constant temperature T_w, while the temperature of the ambient fluid is T_∞, where $T_w > T_\infty$. Fig. 1 shows the flow model and physical coordinate system. The governing boundary layer equations are given by, see Chamkha et al. (2004),

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0 \tag{1}$$

$$u = \frac{\rho_m g K}{\mu}[|T - T_m|^q - |T_\infty - T_m|^q]\cos\gamma \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} \tag{3}$$

subject to the boundary conditions

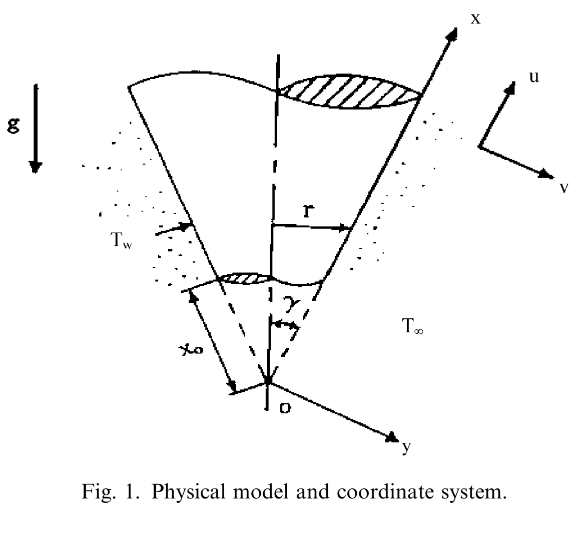

Fig. 1. Physical model and coordinate system.

$$v = 0, \quad T = T_w \quad \text{on } y = 0$$
$$T = T_\infty \quad \text{as } y \to \infty \tag{4}$$

where $r = x\sin\gamma$, x and y are the streamwise and transverse Cartesian coordinates, respectively, u and v are the velocity components in the x and y directions, respectively, T is the fluid temperature, K is the permeability of the porous medium, g is the magnitude of the gravitational acceleration, ρ, μ and α_m are the density, viscosity and effective thermal diffusivity of the porous medium. The new density equation, which applies to both pure and saline water is given by

$$\rho = \rho_m(s,p)[1 - \beta_m(s,p)|T - T_m(s,p)|^q] \tag{5}$$

where p is the pressure, s is the salinity and ρ_m and T_m denote the maximum density and temperature, respectively, for given pressure and salinity levels. The forms and values of q, β_m, ρ_m and T_m are given in the paper by Gebhart and Mollendorf (1977).

We now introduce the following new variables:

$$\xi = x^*/x_0, \quad x^* = (x - x_0)/x_0, \quad \eta = Ra_{x^*}^{1/2}(y/x^*)$$
$$\psi = \alpha_m r Ra_{x^*}^{1/2} f(\xi,\eta), \quad \theta(\xi,\eta) = (T - T_\infty)/(T_w - T_\infty) \tag{6}$$

where ψ is the stream function which is defined in the usual way as $u = (1/r)\partial\psi/\partial y$ and $v = -(1/r)\partial\psi/\partial x$, respectively and $Ra_{x^*} = \rho_m g K \beta_m |T_m - T_\infty|^q x^* \cos\gamma/\mu\alpha_m$ is the modified local Rayleigh number. Substituting (6) into Eqs. (1)–(3) we get

$$f' = |\theta - R|^q - |R|^q \tag{7}$$

$$\theta'' + \left(\frac{1}{2} + \frac{\xi}{1+\xi}\right) f\theta' = \xi\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{8}$$

subject to the boundary conditions (4), which become

$$f(\xi,0) = 0, \quad \theta(\xi,0) = 1, \quad \theta(\xi,\infty) = 0 \tag{9}$$

where primes denote partial differentiation with respect to η and the parameter R is defined as

$$R = \frac{T_m - T_\infty}{T_w - T_\infty} \tag{10}$$

It is worth mentioning that the parameter R places the prescribed temperature T_w and T_∞ with respect to $T_m(s,p)$. It also indicates the local direction of the buoyancy force across the thermal region and thus, also the direction of flow (see Ramilison and Gebhart, 1980).

In terms of the new variables, the velocity components in x- and y-directions are given by

$$u = (\alpha_m Ra_{x^*}/x^*)f'$$
$$v = -\left(\alpha_m Ra_{x^*}^{1/2}/x^*\right)\left[\left(\frac{1}{2} + \frac{\xi}{1+\xi}\right)f + \xi\frac{\partial f}{\partial\xi} - \frac{1}{2}\eta f'\right] \tag{11}$$

We are also interested in the local Nusselt number, which is given by

$$Nu_{x^*}/Ra_{x^*}^{1/2} = -\theta'(\xi,0) \tag{12}$$

It is worth mentioning to this end that Eqs. (7) and (8) become similar for $\xi = 0$ and $\xi = \infty$ and they describe the free convection over a vertical flat plate and, respectively, over a full cone embedded in a porous medium saturated with cold water.

3. Results and discussion

Eqs. (7) and (8) subject to the boundary conditions (9) have been solved numerically for some values of the temperature parameter R in the range between -10 and 0.194 at some upstream coordinates $\xi = 0.0$–1.0 using the finite-difference scheme developed by Blottner (1970). The values of q used are $q = 1$, 1.727147 and

Table 1
Comparison of the values of $-\theta'(\xi,0)$

ξ	Cheng et al. (1985)	Yih (1999)	Present
0	0.4437	0.4439	0.4444
0.5	0.5412	0.5285	0.5294
1.0	0.5991	0.5807	0.5812
2.0	0.6572	0.6373	0.6399
6.0	0.7219	0.7123	0.7130
10.0	0.7391	0.7330	0.7336
20.0	0.7532	0.7500	0.7507
40.0	0.7607	0.7592	0.7596
∞	0.7685	0.7686	0.7690

1.894816 (cold water approximation) also considered by Ramilison and Gebhart (1980). It should be mentioned that close to $R = 0.194$ the convergence of the numerical solution is very slow. This is expected due to the occurrence of the flow reversal across the convective layer. A comparison of the present results for the local Nusselt number, $-\theta'(\xi,0)$, with those reported by Cheng et al. (1985) and Yih (1999) is given in Table 1 for $R = 0$ and $q = 1$ (classical Boussinesq approximation), and some values of the streamwise parameter ξ. It can be seen from Table 1 and Fig. 8 that the present results are in excellent agreement with those of Cheng et al. (1985) and Yih (1999), and we are, therefore, confident that the present numerical results are very accurate.

The non-dimensional velocity $f'(\xi,\eta)$ and non-dimensional temperature $\theta(\xi,\eta)$ profiles are shown in Figs. 2–8. Also, the variation of the local Nusselt number given by Eq. (12) is plotted in Figs. 9 and 10. It is seen from Figs. 2 and 3 that for a fixed value of q and ξ, the velocity profiles increase, while the temperature profiles decrease as the parameter R decreases from zero. The same happens for these profiles when q increases for a fixed value of R and ξ as can be seen from Figs. 4 and 5. However, Figs. 6 and 7 show that both the velocity and temperature profiles decrease as the streamwise coordinate ξ increase from $\xi = 0$ to $\xi = \infty$ (full cone). The present results are also compared in Fig. 8 with those of Ramilison and Gebhart (1980) for the corresponding problem of a vertical flat plate ($\xi = 0$) embedded in a porous medium saturated with cold water. An excellent agreement between these results can be again noticed. We notice that a small flow reversal occurs for the values of R in the range $0.1 < R \leqslant 0.194$ that confirms the findings of Ramilison and Gebhart (1980). Further, Figs. 9 and 10 show that the values of the local Nusselt number increases with a decrease from zero of the temperature parameter R and with the increase of the parameter q. The variation of the heat transfer is very large over the whole range of R for the value of q considered. However, the variation of the local Nusselt number is almost linear with ξ.

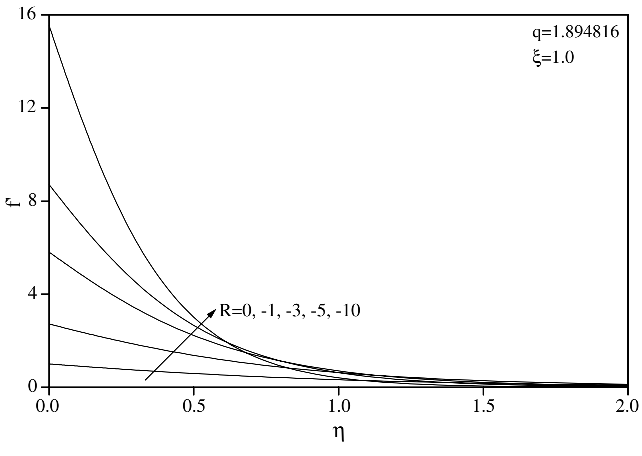

Fig. 2. Effects of R on the tangential velocity profiles.

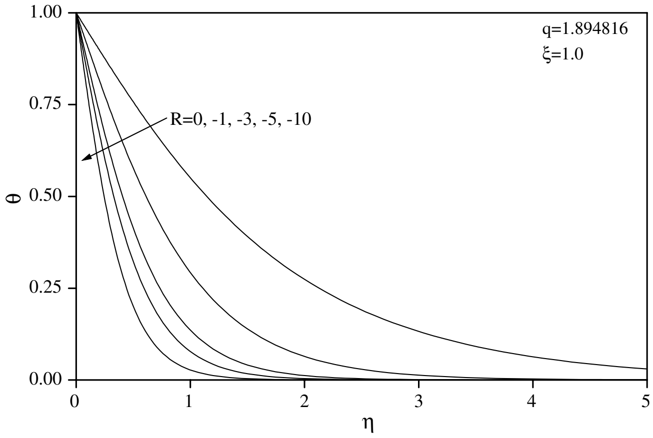

Fig. 3. Effects of R on the temperature profiles.

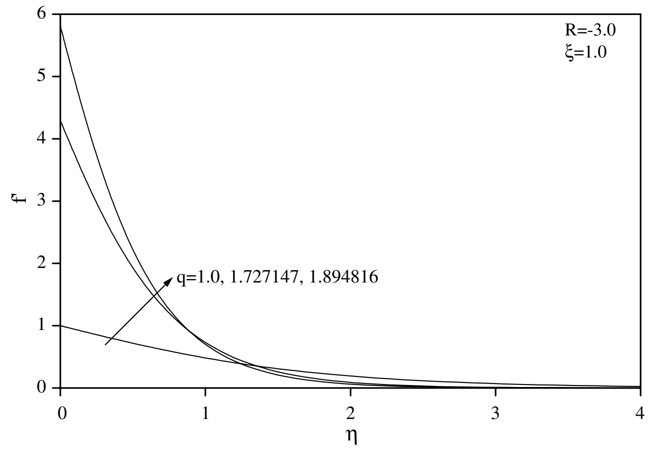

Fig. 4. Effects of q on the tangential velocity profiles.

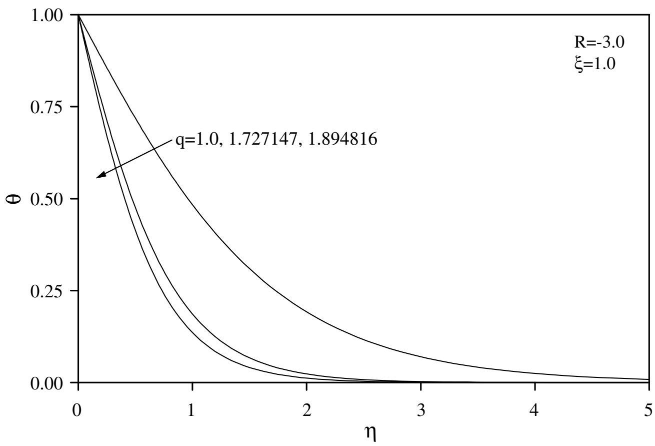

Fig. 5. Effects of q on the temperature profiles.

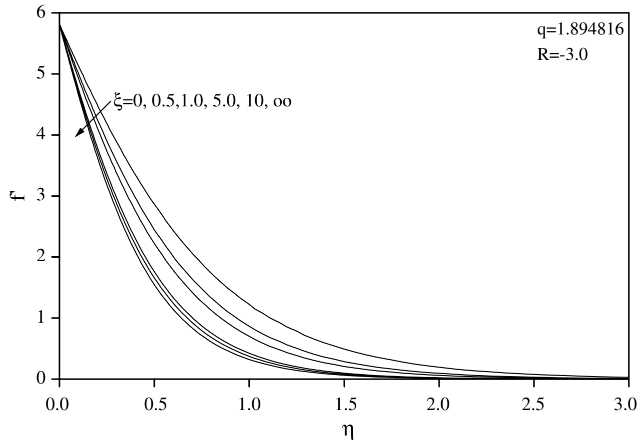

Fig. 6. Development of the tangential velocity profiles.

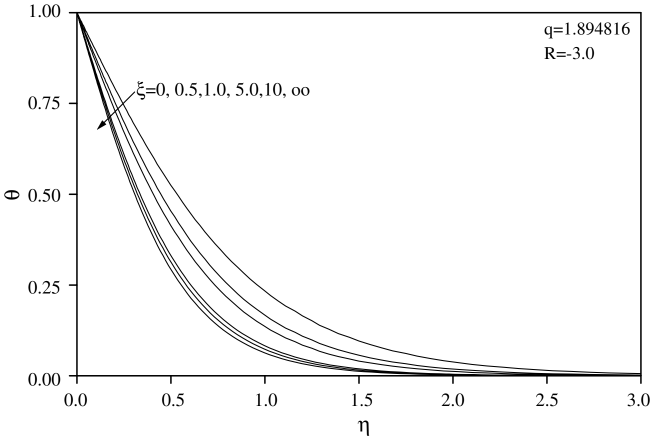

Fig. 7. Development of the temperature profiles.

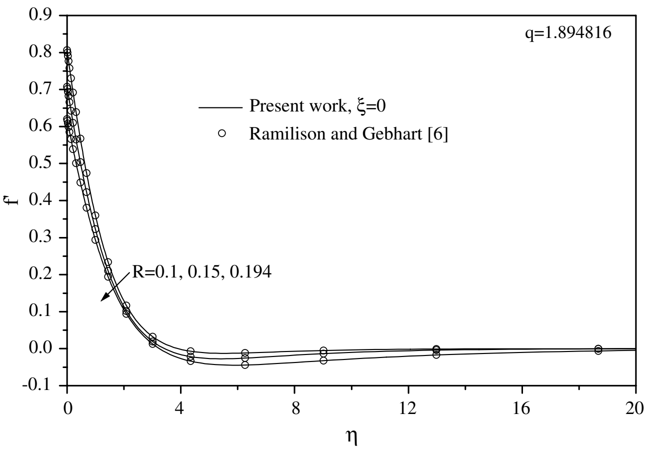

Fig. 8. Reserved flow conditions for various values of R.

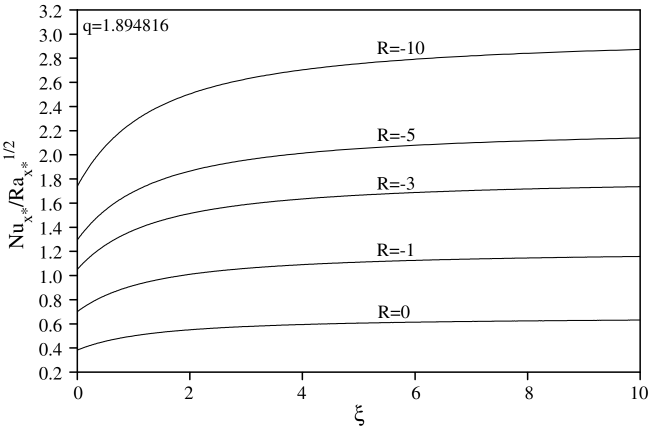

Fig. 9. Effects of R on the local Nusselt number.

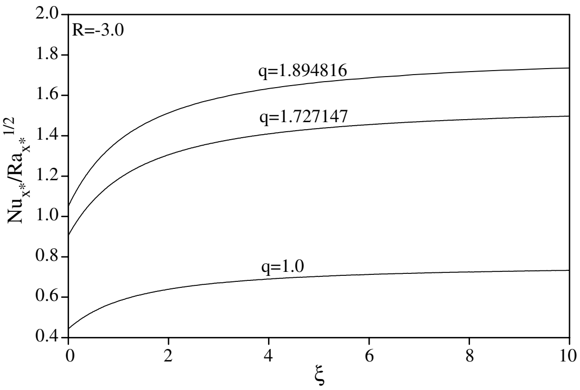

Fig. 10. Effects of q on the local Nusselt number.

4. Conclusions

The free convection boundary layer flow over a heated vertical truncated cone embedded in a porous medium saturated with cold water wherein a density extremum may arise is investigated. Numerical investigations supported by an exact analysis with the finite-difference are made for an isothermal surface and over a wide range of the temperature parameter R and three values of the exponent q in density Eq. (5). Two parameters R, and q arise and they determine the fundamental nature of the density field and the effects of the pressure and salinity levels, respectively. The conventional free convection approximation is also included in the present formulation by choosing $q = 1$ in those flows for which $R = 0$ (classical Boussinesq approximation). It is shown that the effect of q on heat transfer is great when $|R|$ is high, and it increases with an increase of $|R|$.

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