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Effect of heat generation or absorption on thermophoretic free convection boundary layer from a vertical flat plate embedded in a porous medium ☆

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Abstract

This paper is focused on the study of coupled heat and mass transfer by boundary-layer free convection over a vertical flat plate embedded in a fluid-saturated porous medium in the presence of thermophoretic particle deposition and heat generation or absorption effects. The governing partial differential equations are transformed into ordinary differential equations by using special transformations. The resulting similarity equations are solved numerically by an efficient implicit tri-diagonal finite-difference method. Comparisons with previously published work are performed and the results are found to be in excellent agreement. Many results are obtained and a representative set is displayed graphically to illustrate the influence of the heat generation or absorption coefficient, buoyancy ratio and the Lewis number on the temperature and concentration profiles and the wall thermophoretic deposition velocity.

Keywords: Coupled heat and mass transfer; Vertical flat plate; Porous medium; Thermophoresis

1. Introduction

Thermophoresis is a phenomenon which causes small particles to be driven away from a hot surface and towards a cold one. Small particles, such as dust, when suspended in a gas with a temperature gradient, experience a force in the direction opposite to the temperature gradient. This phenomenon has many practical applications in removing small particles from gas streams, in determining exhaust gas particle trajectories from combustion devices, and in studying the particulate material deposition on turbine blades. It has also been shown that thermophoresis is the dominant mass transfer mechanism in the modified chemical vapor deposition process used in the fabrication of optical fiber perform and is also important in view of its relevance to postulated accidents by radioactive particle deposition in nuclear reactors. In many industries the composition of processing gases may contain any of an unlimited range of particle, liquid, or gaseous contaminants and may be influenced by uncontrolled factors of temperature and humidity. When

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such an impure gas is bounded by a solid surface, a boundary layer will develop, and energy and momentum transfer gives rise to temperature and velocity gradients. Mass transfer caused by gravitation, molecular diffusion, eddy diffusion, and inertial impact results in deposition of the suspended components onto the surface. In the application of pigments, or chemical coating of metals, or removal of particles from a gas stream by filtration, there can be distinct advantages in exploiting deposition mechanisms to improve efficiency. Goren [1] was one of the first to study the role of thermophoresis in laminar flow of a viscous and incompressible fluid. He used the classical problem of flow over a flat plate to calculate deposition rates and showed that substantial changes in surface depositions can be obtained by increasing the difference between the surface and free stream temperatures. This was later followed by similarity solutions of two dimensional laminar boundary layers and stagnation point flows by Gokoglu and Rosner [2], and Park and Rosner [3]. Similarity solutions were also obtained by Chiou [4] for the problem of a continuously moving surface in a stationary incompressible fluid, including the combined effects of convection, diffusion, wall velocity and thermophoresis. The thermophoretic deposition of small particles in forced convection laminar flow over inclined plates was discussed by Garg and Jayaraj [5]. Epstein et al. [6] have studied the thermophoretic transport of small particles through a free convection boundary layer adjacent to a cold, vertical deposition surface in a viscous and incompressible fluid, while Chiou [7] has considered the particle deposition from natural convection boundary layer flow onto an isothermal vertical cylinder.

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and/or endothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. Vajravelu and Nayfeh [8] and Chamkha [9] have employed temperature-dependent heat generation or absorption effects.

Despite the practical importance of thermophoresis, little work on the subject has been reported especially in porous media. Convective flows in porous media have been extensively investigated during the last several decades due to many practical applications which can be modelled or approximated as transport phenomena in porous media. Comprehensive literature surveys concerning the subject of porous media can be found in the most recent books by Ingham and Pop [10], Nield and Bejan [11], and Pop and Ingham [12]. The purpose of this work is to consider boundary layer free convection thermophoretic deposition of aerosol particles on a vertical isothermal flat plate embedded in a fluid-saturated porous medium in the presence of heat generation or absorption effects. The Darcy and energy equations yield the velocity and temperature distributions in the boundary layer which are then used in the coupled concentration equation to calculate the rates of particle deposition.

2. Governing equations

Consider the steady coupled heat and mass transfer by free convective boundary layer flow over a vertical flat plate of constant temperature T_w and concentration C_w which is embedded in a fluid-saturated porous medium of ambient temperature T_∞ and concentration C_∞, where $T_w > T_\infty$ and $C_w > C_\infty$, respectively. The effect of thermophoresis is usually prescribed by means of the average velocity, which a particle will acquire when exposed to a temperature gradient. In boundary layer flow, the temperature gradient in the horizontal y-direction is very much larger than in the vertical x-direction, and therefore only the thermophoretic velocity in y-direction is considered. In consequence, the thermophoretic velocity v_t can be expressed in the form

$$v_t = -k\frac{\upsilon}{T}\frac{\partial T}{\partial y} \tag{1}$$

where T is the fluid temperature, k is the thermophoretic coefficient and υ is the kinematic coefficient. Allowing for both Brownian motion of particles, thermophoretic transport and heat generation or absorption effects, the governing boundary layer equations can be written in non-dimensional for as, see Nield and Bejan [11] and Chiou [7],

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{2}$$

$$U = \theta + N\phi \tag{3}$$

$$U\frac{\partial\theta}{\partial X}+V\frac{\partial\theta}{\partial Y}=\frac{\partial^2\theta}{\partial Y^2}+\delta\theta \tag{4}$$

$$U\frac{\partial\phi}{\partial X}+V\frac{\partial\phi}{\partial Y}+\frac{\partial(\phi V_t)}{\partial Y}=\frac{1}{Le}\frac{\partial^2\phi}{\partial Y^2} \tag{5}$$

$$V_t=-k\frac{Pr}{N_t+\theta}\frac{\partial\theta}{\partial Y} \tag{6}$$

where the non-dimensional variables are defined as

$$\begin{aligned}&X=x/\ell,Y=Ra^{1/2}(y/\ell),U=u/U_c,V=Ra^{1/2}(v/U_c)\\&V_t=Ra^{1/2}(v_t/U_c),\theta=\frac{T-T_\infty}{T_w-T_\infty},\phi=\frac{C-C_\infty}{C_w-C_\infty}\end{aligned} \tag{7}$$

Here u and v are the velocity components along the x and y axes, C is the fluid concentration, $Pr=\upsilon/\alpha_m$ and $Le=\alpha_m/D_m$ are the Parndtl and Lewis numbers for a porous medium, ℓ is a characteristic length of the plate, $U_c=gK\beta_T(T_w-T_\infty)/\upsilon$ is the characteristic velocity, $Ra=gK\beta_T(T_w-T_\infty)\ell/(\alpha_m\upsilon)$ is the Rayleigh number, $N_t=(T_w-T_\infty)/T_\infty$ is the thermophoresis parameter, $N=\beta_C(C_w-C_\infty)/\beta_T(T_w-T_\infty)$ is the buoyancy ratio and $\delta=Q\ell/(\rho c_p U_c)$ is the dimensionless heat generation or absorption coefficient. Further, g is the gravitational acceleration, K is the permeability of the porous medium, c_p is the specific heat at constant pressure, D_m is the Brownian diffusion coefficient, α_m is the effective thermal diffusivity, Q is the dimensional heat generation or absorption coefficient and β_T and β_C are the thermal expansion coefficients of temperature and concentration, respectively. The boundary conditions of Eqs. (1) (2) (3) (4) (5) and (6) can be written in non-dimensional form as

$$\begin{aligned}&V=0,\quad \theta=1,\quad \phi=1\quad \text{on}\quad Y=0\\&U\to0,\quad \theta\to0,\quad \phi\to0\quad \text{as}\quad Y\to\infty\end{aligned} \tag{8}$$

Eqs. (2) (3) (4) (5) and (6) are transformed into similarity equations by using the following new variables:

$$\begin{aligned}&\xi=X,\qquad \eta=\frac{y}{\xi^{1/2}},\quad \psi=\xi^{1/2}f(\eta)\\&\theta=\theta(\eta),\quad \phi=\phi(\eta)\end{aligned} \tag{9}$$

to yield

$$f'=\theta+N\phi \tag{10}$$

$$\theta''+\frac{1}{2}f\theta'+\delta\theta=0 \tag{11}$$

$$\frac{1}{Le}\phi''+\frac{1}{2}f\theta'+\frac{kPr}{N_t+\theta}\left[\theta'\phi'+\phi\theta''-\frac{\phi}{N_t+\theta}\theta'^2\right]=0 \tag{12}$$

subject to the boundary conditions

$$\begin{aligned}&f(0)=0,\ \theta(0)=1,\ \phi(0)=1\\&f'\to0,\quad \theta\to0,\quad \phi\to0\quad \text{as}\quad \eta\to\infty\end{aligned} \tag{13}$$

where a prime denotes ordinary differentiation with respect to η.

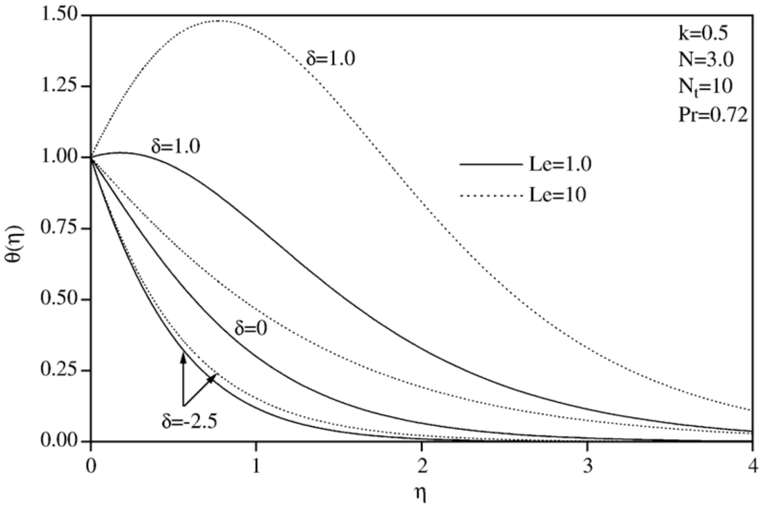

Fig. 1. Effects of *Le* and δ on temperature profiles.

Of interest in this problem are the non-dimensional temperature and concentration profiles $\theta(\eta)$ and $\phi(\eta)$, and the wall thermophoretic deposition velocity V_{tw}, which is given by

$$V_{\mathrm{tw}} = -\frac{kPr}{1+N_{\mathrm{t}}}\theta'(0) \tag{14}$$

It should be noted that for $k=0$ (absence of thermophoresis) and $\delta=0$ (absence of heat generation or absorption), Eqs. (10) (11) and (12) reduce to those of Cheng and Minkowycz [13] when $N=0$ and to those of Bejan and Khair [14] when $N\neq0$, respectively.

3. Results and discussion

The similarity Eqs. (10) (11) and (12) are non-linear, coupled, ordinary differential equations, which possess no closed-form solution. Therefore, they must be solved numerically subject to the boundary conditions given by Eq. (13). The implicit, iterative finite-difference method discussed by Blottner [15] has proven to be adequate for the solution of this type of equations. For this

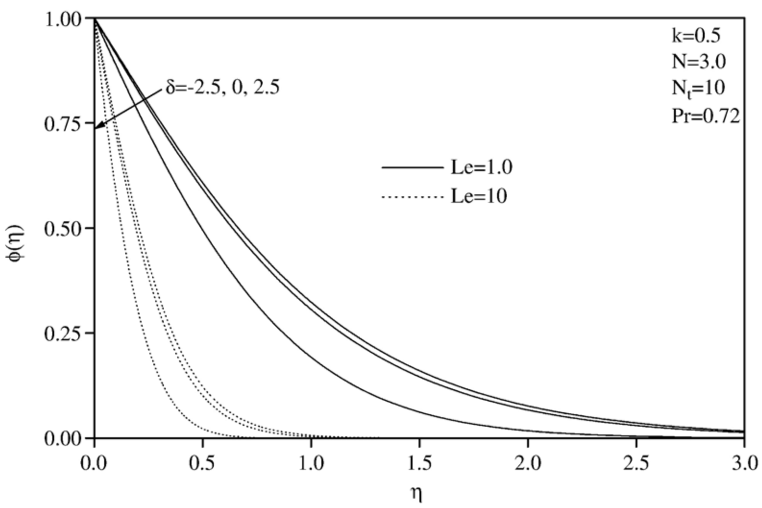

Fig. 2. Effects of *Le* and δ on concentration profiles.

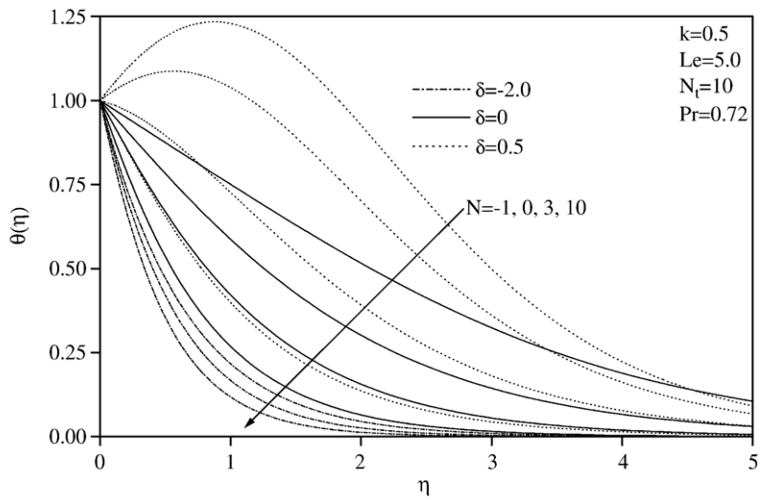

Fig. 3. Effects of N and δ on temperature profiles.

reason, this method is employed in the present work. Eqs. (11) and (12) are discretized using three-point central difference quotients. This converts the differential equations into linear sets of algebraic equations, which can be readily solved by the well-known Thomas algorithm (see Blottner [15]). On the other hand, Eq. (10) is discretized and solved subject to the appropriate boundary condition by the trapezoidal rule. The computational domain in the η-direction was made up of 196 non-uniform grid points. It is expected that most changes in the dependent variables occur in the region close to the plate where viscous effects dominate. However, small changes in the dependent variables are expected far away from the plate surface. For these reasons, variable step sizes in the η-direction are employed. The initial step size $\Delta\eta_1$ and the growth factor K^* employed such that $\Delta\eta_{i+1}=K^*\Delta\eta_i$ (where the subscript i indicates the grid location) were 10^{-3} and 1.03, respectively. These values were found (by performing many numerical experimentations) to give accurate and grid-independent solutions. The solution convergence criterion employed in the present work was based on the difference between the values of the dependent variables at the current and the previous iterations. When this difference reached 10^{-5}, the solution was assumed converged and the iteration process was terminated.

The results are given for several values of the parameters Le, N and δ. However, to check the present numerical results, we calculate the values of the reduced heat transfer, $-\theta'(0)$, and mass transfer, $-\phi'(0)$, from the plate for $k=\delta=0$, $Le=1.0$ and $N=0$ and 1.0. Thus, for $N=0$ we obtained $-\theta'(0)=0.44325$, while the value found by Cheng and Minkowycz [13] is $-\theta'(0)=0.444$. Also, for $k=\delta=0$ and $N=Le=1.0$, we get $-\theta'(0)=-\phi'(0)=0.62783$, while Bejan and Khair [14] obtained $-\theta'(0)=-\phi'(0)=0.628$. It is seen that these results are in excellent agreement and we are, therefore, confident that the present numerical results are very accurate.

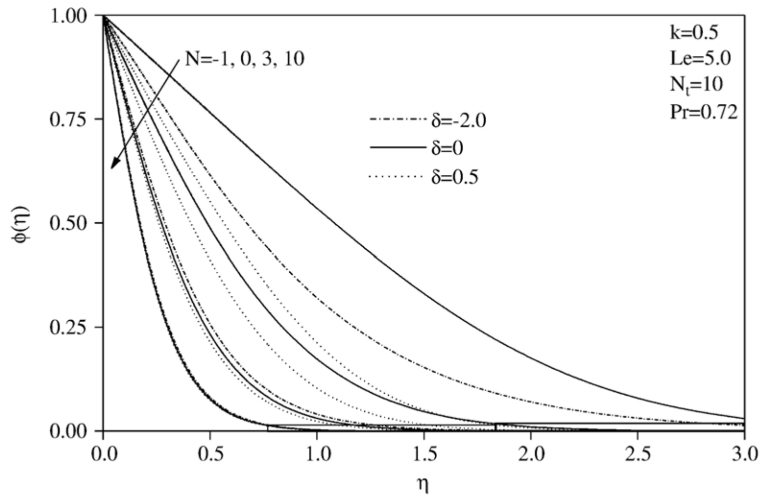

Fig. 4. Effects of N and δ on concentration profiles.

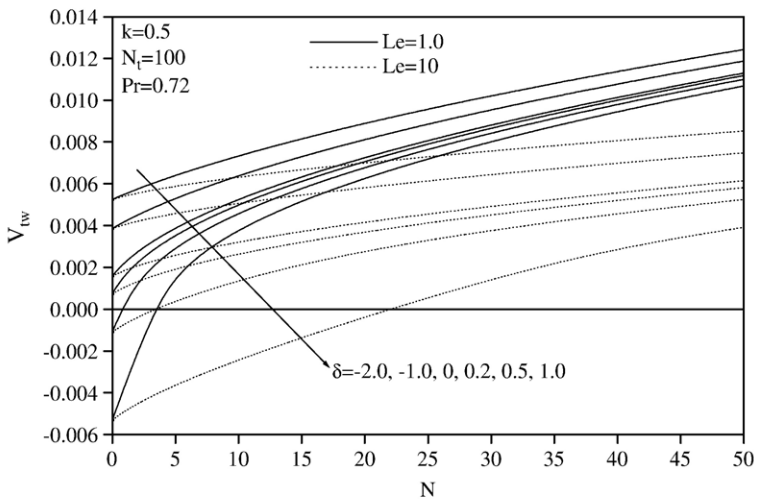

Fig. 5. Effects of δ and N on thermophoretic deposition velocity.

A representative set of numerical results for the temperature and concentration profiles $\theta(\eta)$ and $\phi(\eta)$, and the wall thermophoretic deposition velocity V_{tw} is shown in Figs. 1, 2, 3, 4 and 5 for $Pr=0.72$ and some values of the governing parameters k, Le, N, N_t and δ. These figures illustrate the changes in the temperature and concentration profiles and the wall thermophoretic deposition velocity due to changes in the values of Le, N and δ. The effects of the other governing parameters were studied in a previous paper by Chamkha and Pop [16].

Figs. 1 and 2 present typical temperature and concentration profiles for various values of the heat generation or absorption coefficient δ and two values of the Lewis number ($Le=1.0$ and $Le=10$), respectively. It should be noted here that negative values of δ indicate heat absorption while positive values of δ correspond to heat generation. The presence of heat generation has the tendency to increase the temperature of the fluid in the boundary layer at the expense of reduced particle concentration level there. On the other hand, heat absorption produces the opposite effect; namely, decreases in the temperature and increases in the particle concentration level. In addition, the thermal boundary layer thickness increases for heat generation and decreases for heat absorption while the opposite behavior occurs for the concentration boundary layer thickness. For particular values of $\delta>0$ and the other governing parameters, the maximum temperature does not occur at the wall but rather in the fluid close to the surface. This results in negative wall slopes of the temperature field, which produces negative deposition velocities as seen from the definition given in Eq. (20). In addition, the effect of increasing the Lewis number Le from 1.0 to 10 results in significant increases in the fluid temperature and its boundary layer thickness and decreases in the particle concentration and its boundary layer thickness. It should be noted that the peak value in the temperature profile observed for $\delta=1.0$ for $Le=10$ is much higher than that corresponding to $Le=1.0$.

Figs. 3 and 4 display the effects of the heat generation or absorption coefficient δ and the buoyancy ratio N on the temperature and concentration profiles, respectively. Positive values of N indicate aiding flow while negative values of N indicate opposing flow conditions. Increases in the buoyancy ratio tend to increase the buoyancy-induced flow along the surface at the expense of reduced fluid temperature and particle concentration and their boundary layer thicknesses. These behaviors are clear from Figs. 3 and 4.

Finally, Fig. 5 illustrates the influence of N and δ on the wall thermophoretic deposition velocity V_{tw} for two different values of Lewis number ($Le=1.0$ and $Le=10$). It is observed that as Le increases, the values of V_{tw} decreases. In addition, for the parametric conditions used to obtain this figure, it is predicted that V_{tw} increases as N increases while it decreases as δ increases. This is expected in light of the definition of V_{tw} given in Eq. (14) and the discussion of the previous figures where it is seen that $\theta'(0)$ increases as N increases and decreases as δ increases. It is also predicted that for some positive values of δ, the wall thermophoretic deposition velocity V_{tw} is negative. As discussed before, this related to the fact that the peak temperature value does not occur at the wall but rather in the fluid region close to the plate surface for some heat generation cases. This results in negative wall slopes of the temperature field which produces negative thermophoretic deposition velocities. It is observed that for $Le=1.0$, $\delta=0.5$ and $N>1$, and for $Le=1.0$, $\delta=1.0$ and $N>4$, and for $Le=10$, $\delta=0.5$ and $N>4$, and for $Le=10$, $\delta=1.0$ and $N>22$ all values of V_{tw} are positive.

4. Concluding remarks

The problem of heat and mass transfer by free convective flow of a Newtonian fluid over a vertical flat plate embedded in a porous medium in the presence of thermophoretic particle deposition and heat generation or absorption effects was considered. The governing partial differential equations were transformed into a set similarity equations that was solved numerically by an implicit finite difference method. Favorable comparisons with published results were performed. It was found that the particle concentration level as well as the concentration boundary layer thickness decreased due to increases in either of the heat generation or absorption coefficient, Lewis number or the buoyancy ratio parameter. In addition, the fluid temperature and the thermal boundary layer thickness increased due to increases in either of the heat generation or absorption coefficient or Lewis number and decreased as the buoyancy ratio was increased. Also, the thermophoretic deposition velocity decreased as either of the heat generation or absorption coefficient or the Lewis number was increased and increased as the buoyancy ratio was increased.

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