

# Coupled Heat and Mass Transfer in Mixed Convective Flow of a Non-Newtonian Fluid over a Permeable Surface Embedded in a Non-Darcian Porous Medium

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*Abstract:* - This work considers steady, laminar, heat and mass transfer by mixed convection flow of a non-Newtonian power-law fluid over a semi-infinite, isothermal, vertical and permeable surface embedded in a porous medium in the presence of temperature-dependent heat generation or absorption. A mixed convection parameter for the entire range of free-forced-mixed convection is employed and a set of non-similar equations are obtained. These equations are solved numerically by an efficient implicit, iterative, finite-difference method. The obtained results are checked against previously published work for special cases of the problem and are found to be in good agreement. A parametric study illustrating the influence of the concentration to thermal buoyancy ratio, power-law fluid viscosity index, heat generation or absorption parameter, the Lewis number and the porous medium inertia parameter on the local Nusselt and the Sherwood numbers is conducted. The obtained results are shown graphically and the physical aspects of the problem are discussed.

*Key-Words:* - Mixed convection, non-Darcy porous media, suction or injection, heat and mass transfer, numerical solution, non-Newtonian fluid.

## 1 Introduction

Buoyancy-induced flows from vertical surfaces embedded in porous media have been the subject of many investigations. This is due fact that these flows have many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, groundwater pollution, and underground energy transport. Cheng and Minkowycz [1] have presented similarity solutions for free thermal convection from a vertical plate in a fluid-saturated porous medium. Ranganathan and Viskanta [2] have considered mixed convection boundary layer flow along a vertical surface in a porous medium. Nakayama and Koyama [3] have suggested similarity transformations for pure, combined and forced convection in Darcian and non-Darcian porous media. Lai [4] has investigated coupled heat and mass transfer by mixed convection from an isothermal vertical plate in a porous medium. Hsieh et al. [5] have presented non-similar solutions for combined convection in porous media. All of the above references considered Newtonian fluids.

A number of industrially important fluids such as molten plastics, polymers, pulps, foods and slurries and fossil fuels which may saturate underground beds display non-Newtonian fluid behavior. Non-Newtonian fluids exhibit a non-linear relationship between shear stress and shear rate. Chen and Chen [6] have presented similarity solutions for free convection of non-Newtonian fluids over vertical surfaces in porous media. Mehta and Rao [7] have investigated buoyancy-induced flow of non-Newtonian fluids over a non-isothermal horizontal plate embedded in a porous medium. Also, Mehta and Rao [8] have analyzed buoyancy-induced flow of non-Newtonian fluids in a porous medium past a vertical plate with non-uniform surface heat flux. In a series of papers, Gorla and co-workers [9-13] have studied mixed convection in non-Newtonian fluids along horizontal and vertical plates in porous media. Jumah and Mujumdar [14] have considered free convection heat and mass transfer of non-Newtonian power-law fluids with yield stress from a vertical flat plate in saturated porous media.

Most studies on porous media referenced above have employed the Darcy model. It is known

by now, that the Darcy model becomes inadequate for high velocity flows for which the porous medium inertia effects are comparable to viscous resistance. The porous medium inertia effect term was first called the Forchheimer term by Muskat [15]. The modified form of the Darcy-Forchheimer equation for non-Newtonian power-law fluids was developed by Shenoy [16]. Nakayama and Shenoy [17] have used the Forchheimer extended Darcy model for studying the flow confined within parallel walls subjected to uniform heat flux and immersed in a porous medium saturated with a non-Newtonian power-law fluid. Ibrahim et al. [18] have studied non-Darcy mixed convection flow along a vertical plate embedded in a non-Newtonian fluid saturated porous medium with wall suction or blowing.

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic chemical reactions and dissociating fluids in packed-bed reactors, the working fluid heat generation or absorption effects are important. Simulation of such situations requires the addition of a heat source or sink term in the energy equation. This term has been assumed to be either a constant (Acharya and Goldstein [19] or temperature-dependent (Vajravelu and Nayfeh [20]).

The effects of fluid wall suction or injection the flow and heat transfer characteristics along vertical semi-infinite plates have been investigated by several authors (Cheng [21], Lai and Kulacki [22,23], Minkowycz et al. [24] and Hooper et al. [25]). Some of these studies have reported similarity solutions (Cheng [21] and Lai and Kulacki [22,23]) while others have obtained non-similar solutions (Minkowycz, et al. [24] and Hooper et al. [25]). Lai and Kulacki [22,23] have reported similarity solutions for mixed convection flow over horizontal and inclined plates embedded in fluid-saturated porous media in the presence of surface mass flux. On the other hand, Minkowycz et al. [24] have discussed the effect of surface mass transfer on buoyancy-induced Darcian flow adjacent to a horizontal surface using non-similarity solutions. Also, Hooper et al. [25] have considered the problem of non-similar mixed convection flow along an isothermal vertical plate in porous media with uniform surface suction or injection and introduced a single parameter for the entire regime of free-forced-mixed convection. Their non-similar variable represented the effect of suction or injection at the wall.

The objective of this paper is consider coupled heat and mass transfer by mixed convection for a

non-Newtonian power-law fluid from a permeable vertical plate embedded in a fluid-saturated non-Darcian porous medium in the presence of suction or injection and heat generation or absorption effects. This will be done for constant temperature and concentration wall conditions in the entire range of free-forced-mixed convection regime.

## 2 Problem Formulation

Consider steady mixed convection flow of a non-Newtonian power-law fluid over a permeable semi-infinite vertical surface embedded in a porous medium in the presence of temperature difference-dependent heat generation or absorption. The power-law model of Ostwald-de-Waele which is adequate for many non-Newtonian fluids is considered in the present work. Uniform suction or injection with speed  $v_0$  is imposed at the surface boundary. The porous medium is assumed to be uniform, isotropic and in thermal equilibrium with the fluid. All fluid properties are assumed constant. Under the Boussinesq and boundary-layer approximations, the governing equations for this problem can be written as (see Ibrahim et al. [18]):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\frac{\partial u^n}{\partial y} + \rho \frac{KF}{\mu} \frac{\partial u^2}{\partial y} = \frac{K}{\mu} \rho g \left[ \beta_T \frac{\partial T}{\partial y} + \beta_c \frac{\partial c}{\partial y} \right] \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_e \frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{\rho c_p} (T - T_\infty) \quad (3)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \quad (4)$$

where  $x$  and  $y$  denote the vertical and horizontal directions, respectively.  $u$ ,  $v$ ,  $T$  and  $c$  are the  $x$ - and  $y$ -components of velocity, temperature and concentration, respectively.  $\rho$ ,  $\mu$ ,  $n$ ,  $c_p$  and  $D$  are the fluid density, consistency index for viscosity, power-law fluid viscosity index, specific heat at constant pressure, and mass diffusion coefficient, respectively.  $K$ ,  $F$  and  $\alpha_e$  are the porous medium modified permeability, empirical constant associated with the Forchheimer porous medium inertia term and the effective thermal diffusivity, respectively.  $\beta_T$ ,  $\beta_c$ ,  $Q_0$  and  $T_\infty$  are the thermal expansion coefficient, concentration expansion coefficient, heat generation (>0) or absorption (<0) coefficient and the free stream temperature, respectively.

The modified permeability of the porous medium  $K$  for flows of non-Newtonian power-law fluids is given by:

$$K = \frac{1}{2C_t} \left( \frac{n\varepsilon}{3n+1} \right)^n \left( \frac{50k^*}{3\varepsilon} \right)^{(n+1)/2} \quad (5)$$

where

$$k^* = \frac{\varepsilon^3 d^2}{150(1-\varepsilon)^2} \quad (6)$$

$$C_t = \begin{cases} \frac{25}{12}, \text{ Christopher and Middleman [26]} \\ \frac{2}{3} \left( \frac{8n}{9n+3} \right) \left( \frac{10n-3}{6n+1} \right) \left( \frac{75}{16} \right)^{3(10n-3)/(10n+11)} \\ \text{Dharmadhikari and Kale [27]} \end{cases}, \quad (7)$$

where  $\varepsilon$  and  $d$  are the porosity and the particle diameter of the packed-bed porous medium.

The boundary conditions suggested by the physics of the problem are

$$v(x,0) = v_o, T(x,0) = T_w, c(x,0) = c_w \quad (8)$$

$$u(x,\infty) = U_\infty, T(x,\infty) = T_\infty, c(x,\infty) = c_\infty$$

where  $v_o$ ,  $T_w$  and  $c_w$  are the wall normal velocity, temperature and concentration, respectively.

It is convenient to transform the governing equations into a non-similar dimensionless form which can be suitable for solution as an initial-value problem with  $\xi$  playing the role of time. This can be done by introducing the stream function such that

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (9)$$

and using

$$\eta = \frac{y}{x} (Pe_x^{1/2} + Ra_x^{1/2}), \xi = \frac{v_o x}{\alpha_e} (Pe_x^{1/2} + Ra_x^{1/2})^{-1} \quad (10)$$

$$\psi = \alpha_e (Pe_x^{1/2} + Ra_x^{1/2}) f(\xi, \eta), \quad (11)$$

$$\theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty}, C(\xi, \eta) = \frac{c - c_\infty}{c_w - c_\infty}$$

where  $Pe_x = U_\infty x / \alpha_e$  and  $Ra_x = x / \alpha_e [\rho g \beta_T |T_w - T_\infty| K / \mu]^{1/n}$  are the local Peclet and modified Rayleigh numbers, respectively.

Substituting Eqs. (6) through (8) into Eqs. (1) through (5) produces

$$nf^{n-1} f'' + 2Re^* f' f'' = (1-\chi)^{2n} (\theta' + NC') \quad (12)$$

$$\theta'' + \frac{1}{2} f \theta' + \xi^2 \phi \theta = \frac{1}{2} \xi (f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi}) \quad (13)$$

$$Le^{-1} C'' + \frac{1}{2} f C' = \frac{1}{2} \xi (f' \frac{\partial C}{\partial \xi} - C' \frac{\partial f}{\partial \xi}) \quad (14)$$

$$f(\xi, 0) + \xi \frac{\partial f}{\partial \xi}(\xi, 0) = -2\xi, \theta(\xi, 0) = 1, C(\xi, 0) = 1$$

$$f(\xi, \infty) = \chi^2, \theta(\xi, \infty) = 0, C(\xi, \infty) = 0 \quad (15)$$

where

$$Le = \frac{\alpha_e}{D}, N = \frac{\beta_c (c_w - c_\infty)}{\beta_T (T_w - T_\infty)}, \phi = \frac{Q_o}{\rho c_p v_o},$$

$$Re^* = \frac{\rho F K}{\mu} \left( \frac{\alpha_e}{x} \right)^{2-n} [Pe_x^{1/2} + Ra_x^{1/2}]^{2(2-n)}, \quad (16)$$

$$\chi = [1 + (Ra_x / Pe_x)^{1/2}]^{-1}$$

are the Lewis number, concentration to thermal buoyancy ratio, dimensionless heat generation (>0) or absorption (<0) coefficient, the non-Darcy porous medium or inertia parameter and the mixed convection parameter, respectively. It should be noted that  $\chi = 0$  ( $Pe_x = 0$ ) corresponds to pure free convection while  $\chi = 1$  ( $Ra_x = 0$ ) corresponds to pure forced convection. The entire regime of mixed convection corresponds to values of  $\chi$  between 0 and 1.

Of special significance for this problem are the local Nusselt and Sherwood numbers. These physical quantities can be defined as

$$Nu_x = \frac{hx}{k_e} = -(Pe_x^{1/2} + Ra_x^{1/2}) \theta'(\xi, 0); \quad (17)$$

$$h = \frac{q_w}{T_w - T_\infty}; q_w = -k_e \left( \frac{\partial T}{\partial y} \right)_{y=0}$$

$$Sh_x = \frac{h_m x}{D} = -(Pe_x^{1/2} + Ra_x^{1/2}) C'(\xi, 0); \quad (18)$$

$$h_m = \frac{m_w}{c_w - c_\infty}; m_w = -D \left( \frac{\partial c}{\partial y} \right)_{y=0}$$

where  $k_e$  is the porous medium effective thermal conductivity and  $q_w$  and  $m_w$  are respectively the wall heat transfer and wall mass transfer.

### 3 Numerical Method and Validation

Equations (12) through (15) represent an initial-value problem with  $\xi$  playing the role of time. This non-linear problem can not be solved in closed form and, therefore, a numerical solution is necessary to describe the physics of the problem. The implicit, tri-diagonal finite-difference method similar to that

discussed by Blottner [28] has proven to be adequate and sufficiently accurate for the solution of this kind of problems. Therefore, it is adopted in the present work.

All first-order derivatives with respect to  $\xi$  are replaced by two-point backward-difference formulae when marching in the positive  $\xi$  direction and by two-point forward-difference formulae when marching in the negative  $\xi$  direction. Then, all second-order differential equations in  $\eta$  are discretized using three-point central difference quotients. This discretization process produces a tri-diagonal set of algebraic equations at each line of constant  $\xi$  which is readily solved by the well known Thomas algorithm (see Blottner [28]). During the solution, iteration is employed to deal with the non-linearities of the governing differential equations. The problem is solved line by line starting with line  $\xi=0$  where similarity equations are solved to obtain the initial profiles of velocity, temperature and concentration and marching forward (or backward) in  $\xi$  until the desired line of constant  $\xi$  is reached. Variable step sizes in the  $\eta$  direction with  $\Delta\eta_1 = 0.001$  and a growth factor  $G = 1.04$  such that  $\Delta\eta_n = G\Delta\eta_{n-1}$  and constant step sizes in the  $\xi$  direction with  $\Delta\xi = 0.01$  are employed. In this case,  $\eta_\infty=52$  was employed. These step sizes are arrived at after many numerical experimentations performed to assess grid independence on the values of local Nusslet and Sherwood numbers. For example, when  $\Delta\eta_1 = 0.0001$  was used instead of  $\Delta\eta_1 = 0.001$  with all other numerical values kept the same as above, both  $-\theta'(\xi,0)$  and  $-C'(\xi,0)$  changed less than 0.5%. Similarly, using  $\Delta\xi = 0.001$  instead of  $\Delta\xi = 0.01$  yielded less than 1%. The convergence criterion employed in the present work is based on the difference between the current and the previous iterations. When this difference reached  $10^{-6}$  for all points in the  $\eta$  directions, the solution was assumed converged and the iteration process was terminated.

Tables 1 and 2 present a comparison of  $-\theta'(\xi,0)$  at selected values of  $\xi$  and  $\chi$  between the results of the present work and those reported earlier by Hooper et al. [25] for  $n=1$ ,  $N=0$  and  $\phi=0$ . It is clear from this comparison that a good agreement between the results exists. This lends confidence in the correctness of the numerical results to be reported subsequently. It should be noted that in Table 2, the value of  $-\theta'(\xi,0)$  at  $\xi = -2$  and  $\chi = 1$  seems to be in error or a typo as this value cannot be 1.0502.

Table 1. Values of  $-\theta'(\xi,0)$  at selected values of  $\xi$  and  $\chi$  for  $n=1$ ,  $N=0$  and  $\phi=0$ . (Present work)

	$\xi=-2.0$	$\xi=0.0$	$\xi=1.0$	$\xi=2.0$
$\chi=0.0$	1.99894	0.44401	0.14240	0.03408
$\chi=0.2$	1.99762	0.37339	0.09140	0.01431
$\chi=0.4$	1.99757	0.35071	0.06997	0.00754
$\chi=0.5$	1.99824	0.36045	0.07097	0.00725
$\chi=0.6$	2.00066	0.38338	0.08029	0.00884
$\chi=0.8$	2.01485	0.46044	0.12471	0.02055
$\chi=1.0$	2.04971	0.56433	0.19979	0.05036

Table 2. Values of  $-\theta'(\xi,0)$  at selected values of  $\xi$  and  $\chi$  for  $n=1$ ,  $N=0$  and  $\phi=0$ . (Hooper et al. [25])

	$\xi=-2.0$	$\xi=0$	$\xi=1.0$	$\xi=2.0$
$\chi=0.0$	2.0015	0.4437	0.1417	0.0335
$\chi=0.2$	2.0003	0.3732	0.0907	0.0139
$\chi=0.4$	2.0005	0.3504	0.0693	0.0072
$\chi=0.5$	2.0016	0.3603	0.0704	0.0069
$\chi=0.6$	2.0042	0.3832	0.0797	0.0085
$\chi=0.8$	2.0185	0.4602	0.1242	0.0201
$\chi=1.0$	1.0502	0.5642	0.1996	0.0502

## 4 Results and Discussion

Figures 1 through 6 illustrate the effects of the buoyancy ratio  $N$  and the transformed suction or injection parameter  $\xi$  in the range of the mixed convection parameter  $0 \leq \chi \leq 1$  on the local Nusselt number  $[\text{Nu}_x / (\text{Pe}_x^{1/2} + \text{Ra}_x^{1/2}) = -\theta'(\xi,0)]$  and the local Sherwood number  $[\text{Sh}_x / (\text{Pe}_x^{1/2} + \text{Ra}_x^{1/2}) = -C'(\xi,0)]$  for power-law fluid viscosity indices  $n=0.5$  (shear-thinning or pseudo-plastic fluid),  $n=1.0$  (Newtonian fluid) and  $n=1.5$  (shear-thickening or dilatant fluid), respectively. In general, increases in the value of  $N$  have the tendency to increase the buoyancy effect causing more induced flow along the surface in the vertical direction. This enhancement in the flow velocity is achieved at the expense of reduced fluid temperature and concentration as well as reduced thermal and concentration boundary layers. This

causes the negative wall slope of the temperature and concentration profiles to increase. This yields enhancements in both the local Nusselt and Sherwood numbers. Also, it is noted that as the transformed suction or injection parameter,  $\xi$  increases for fixed values of  $N > 0$  and  $\chi \neq 1$ , the temperature and concentration as well as their boundary layers increase. This produces higher flow velocities due to increases in the buoyancy effects. As a result of increasing the value of  $\xi$ , the local Nusselt and Sherwood numbers increase. From the definition of  $\chi$ , it is seen that increases in the value of the parameter  $Ra_x/Pe_x$  causes the mixed convection parameter  $\chi$  to decrease. Thus, small values of  $Ra_x/Pe_x$  correspond to values of  $\chi$  close to unity which indicate almost pure forced convection regime. On the other hand, high values of  $Ra_x/Pe_x$  correspond to values of  $\chi$  close to zero which indicate almost pure free convection regime. Furthermore, moderate values of  $Ra_x/Pe_x$  represent values of  $\chi$  between 0 and 1 which correspond to the mixed convection regime. For the forced convection limit ( $\chi = 1$ ) it is clear from Eq. (12) that the velocity in the boundary layer  $f'$  is uniform irregardless of the value of  $n$ . However, for smaller values of  $\chi$  (higher values of  $Ra_x/Pe_x$ ) at a fixed value of  $N$  and  $n=1.0$ , the buoyancy effect increases. As this occurs, the fluid velocity close to the wall increases for values of  $\chi \leq 0.5$  due to the buoyancy effect which becomes maximum for  $\chi = 0$  (free convection limit). This decrease and increase in the fluid velocity  $f'$  as  $\chi$  is decreased from unity to zero is accompanied by a respective increase and a decrease in the fluid temperature and concentration. As a result, the local Nusselt and Sherwood numbers tend to decrease and then increase as  $\chi$  is increased from 0 to unity forming slight dips close to  $\chi=0.4$  for  $n=1$  (Figs. 3 and 4) and  $\chi=0.5$  for  $n=1.5$  (Figs. 5 and 6) for almost all values of  $\xi$  and  $N$  considered.

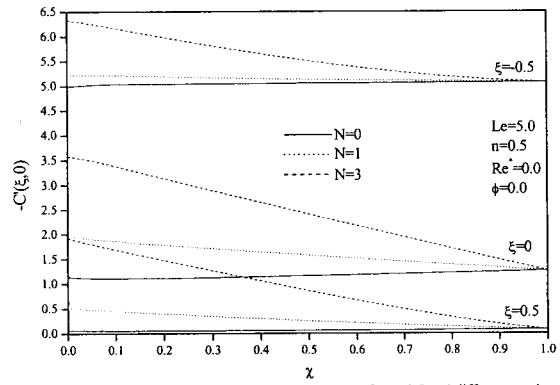


Fig. 2. Effects of  $N$  and  $\chi$  on local Sherwood number for  $n=0.5$  and different  $\xi$  values

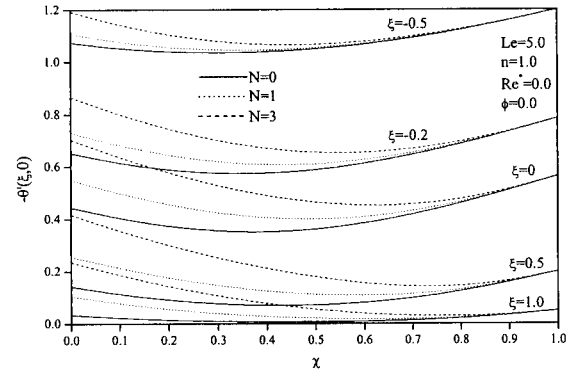


Fig. 3. Effects of  $N$  and  $\chi$  on local Nusselt number for  $n=1.0$  and different  $\xi$  values

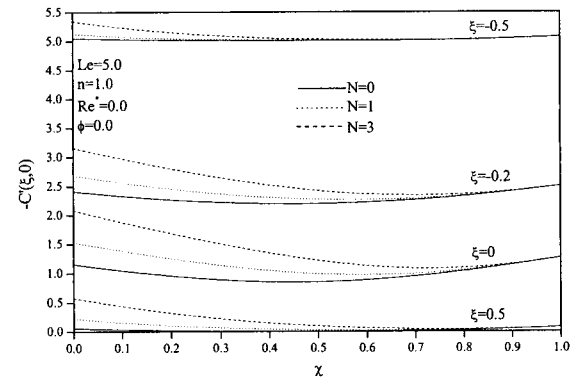


Fig. 4. Effects of  $N$  and  $\chi$  on local Sherwood number for  $n=1.0$  and different  $\xi$  values

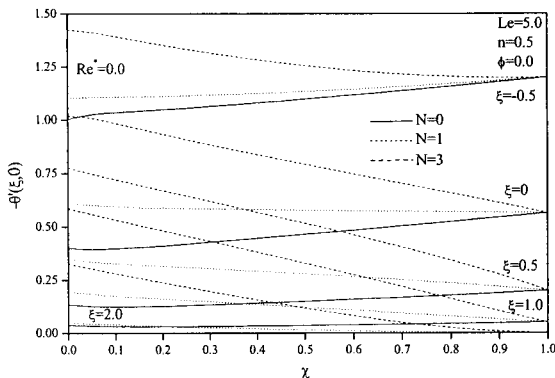


Fig. 1. Effects of  $N$  and  $\chi$  on local Nusselt number for  $n=0.5$  and different  $\xi$  values

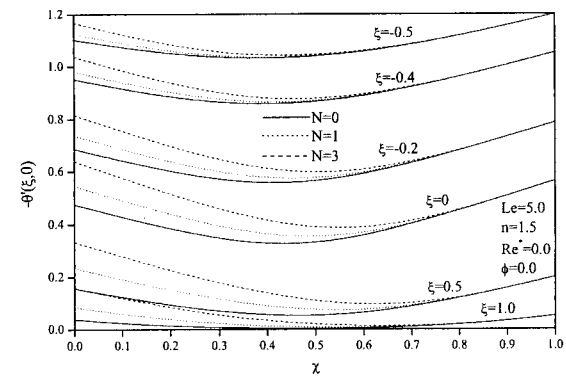


Fig. 5. Effects of  $N$  and  $\chi$  on local Nusselt number for  $n=1.5$  and different  $\xi$  values

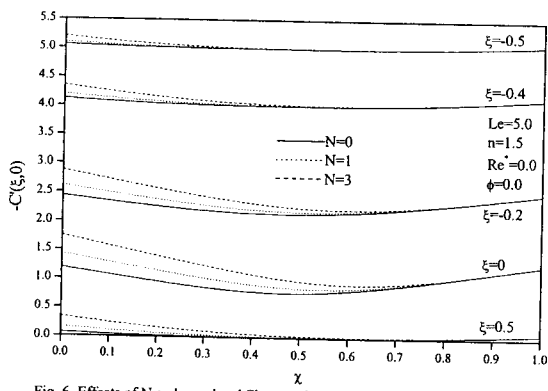


Fig. 6. Effects of  $N$  and  $\chi$  on local Sherwood number for  $n=1.5$  and different  $\xi$  values

However, for  $n=0.5$  and  $N \leq 1.0$ , the local Nusselt and Sherwood numbers are predicted to increase as  $\chi$  is increased from 0 to unity. On the other hand, for  $N=3$ , the added buoyancy due to concentration gradient causes a different trend in which the local Nusselt and Sherwood numbers increase with increasing values of  $\chi$ . Furthermore, by comparison of Figs. 1-6, one can easily conclude that the local Nusselt and Sherwood numbers decrease as the power-law fluid index  $n$  increases. It is also observed that while the local Nusselt and Sherwood numbers change in the whole range of free and mixed convection regime, they remain constant for the forced-convection regime. This is obvious since for  $\chi=1$  and fixed values of  $Le$  and  $\phi$ , the equations are the same and do not depend on  $n$  and  $N$ . All of the above trends are clearly displayed in Figs. 1-6.

The effect of the heat generation or absorption coefficient  $\phi$  on the local Nusselt number  $[Nu_x / (Pe_x^{1/2} + Ra_x^{1/2}) = -\theta'(\xi, 0)]$  for different values of  $n$  (0.5, 1.0, 1.5) in the range  $-2 \leq \xi \leq 2$  is displayed in Fig. 7. The presence of a heat generation effects in the flow represented by positive values of  $\phi$  enhances the thermal state of the fluid causing its temperature to increase. This, in turn, increases the thermal buoyancy effect which produces higher induced flow. On the contrary, the presence of a heat absorption effects in the flow represented by negative values of  $\phi$  reduces the fluid temperature which, in turn, decreases the induced flow due to thermal buoyancy effects. Thus, the wall slope of the temperature profile increases as  $\phi$  increases causing the local Nusselt number which is directly proportional to  $-\theta'(\xi, 0)$  to decrease for all values of  $\xi$  except  $\xi=0$  since  $\phi$  does not appear in Eq. (13) at  $\xi=0$ . For the heat generation case ( $\phi = 1.0$ ) shown, the maximum fluid temperature does not occur at the wall but rather in the fluid layer adjacent to the surface for some values of  $\xi > 0$ . This causes the slope of the temperature profile at the

wall  $\theta'(\xi, 0)$  to become positive and thus, the local Nusselt number is negative. All these behaviors are clear from Fig. 7. Also, it is observed that as  $n$  increases, the local Nusselt number decreases and that the amount of reduction depends on the values of  $\xi$  and  $\phi$ .

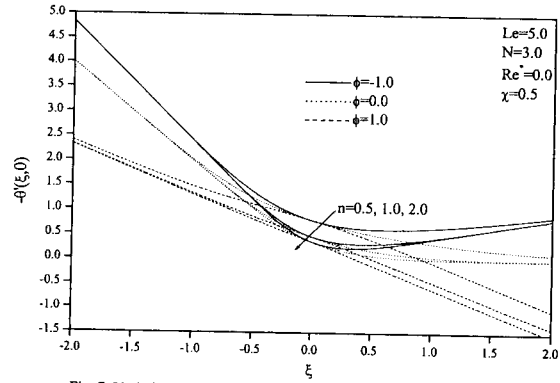


Fig. 7. Variation of local Nusselt number with  $\xi$  for different values of  $n$  and  $\phi$

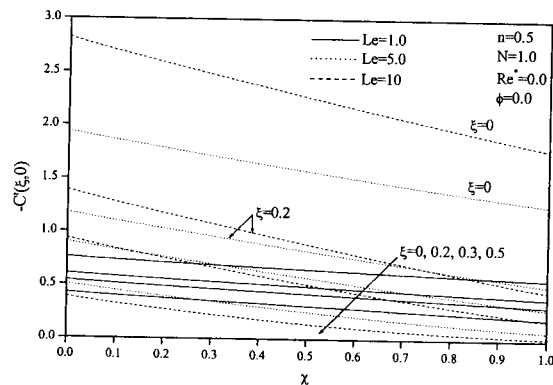


Fig. 8. Effects of  $Le$  and  $\chi$  on local Sherwood number for  $n=0.5$  and different  $\xi$  values

Figure 8 depicts the influence of the Lewis number  $Le$  on the local Sherwood number in the whole mixed convection range  $0 \leq \chi \leq 1$  for values of  $\xi > 0$ . Increasing the values of the Lewis number in the range  $0 \leq \xi \leq 0.2$  results in increasing the concentration level close to the wall while decreasing its distribution within the boundary layer away from the surface. This causes the local Sherwood number to increase. However, in the range  $\xi > 0.2$ , increasing  $Le$  reduces the concentration everywhere in the boundary layer causing the local Sherwood number to decrease. These behaviors are shown clearly in Fig. 8.

Figures 9 through 14 illustrate the variation of local Nusselt and Sherwood numbers in the whole mixed convection range  $0 \leq \chi \leq 1$  due to changes in the porous medium inertia parameter  $Re^*$  for surface suction and injection conditions and different values of the power-law fluid index  $n$  corresponding to shear-thinning or pseudo-plastic fluid ( $n=0.5$ ), Newtonian fluid ( $n=1.0$ ) and shear-thickening or

dilatant fluid ( $n=2.0$ ), respectively. In general, the presence of the porous medium inertia effect has the tendency to increase resistance to flow through the porous medium. This results in lower flow velocities and higher temperature and solute concentration levels as observed from other results not presented here for brevity. As a result, the local Nusselt and Sherwood numbers  $[-\theta'(\xi,0), -C'(\xi,0)]$  decrease as the porous medium inertia parameter  $Re^*$  increases. This is true for all considered values of  $n$ . As evident from Figures 9-14, the porous medium inertia effect ( $Re^*>0$ ) has a pronounced effect on the heat and mass transfer rates. From these figures it is also seen as mentioned before, that for increasing values of either the suction/injection parameter  $\xi$  or the power-law fluid viscosity index  $n$ , the local Nusselt and Sherwood numbers decrease.

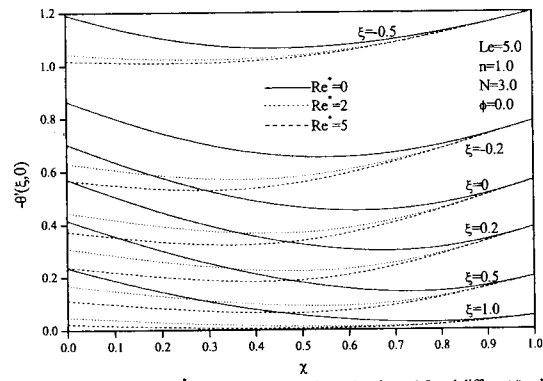


Figure 11. Effects of  $Re^*$  and  $\chi$  on local Nusselt number for  $n=1.0$  and different  $\xi$  values

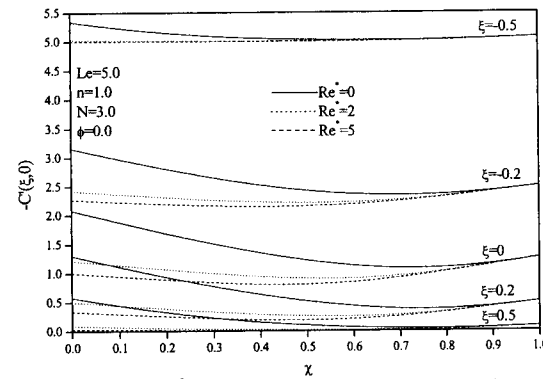


Figure 12. Effects of  $Re^*$  and  $\chi$  on local Sherwood number for  $n=1.0$  and different  $\xi$  values

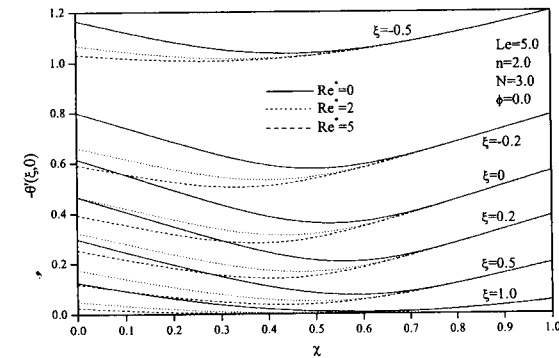


Figure 13. Effects of  $Re^*$  and  $\chi$  on local Nusselt number for  $n=2.0$  and different  $\xi$  values

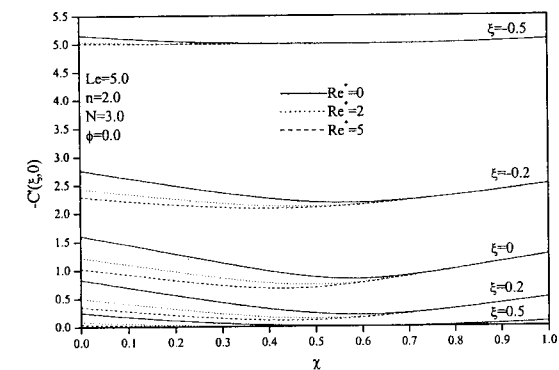


Figure 14. Effects of  $Re^*$  and  $\chi$  on local Sherwood number for  $n=2.0$  and different  $\xi$  values

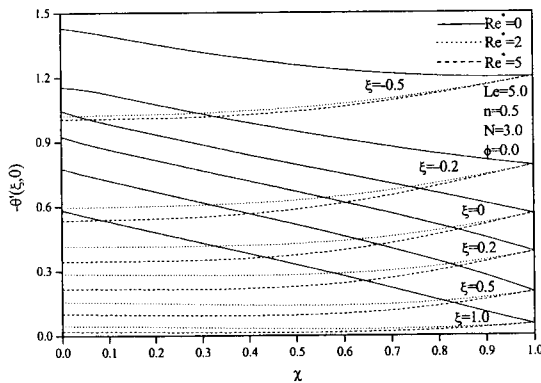


Figure 9. Effects of  $Re^*$  and  $\chi$  on local Nusselt number for  $n=0.5$  and different  $\xi$  values

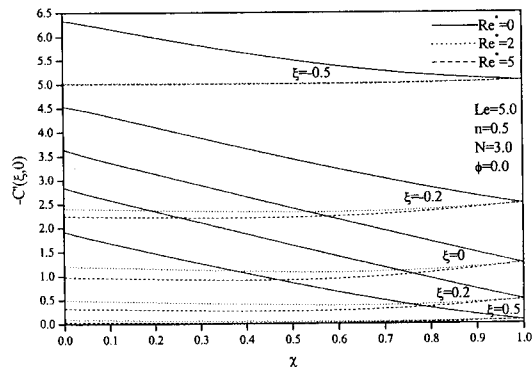


Figure 10. Effects of  $Re^*$  and  $\chi$  on local Sherwood number for  $n=0.5$  and different  $\xi$  values

## 5 Conclusions

This study considered coupled heat and mass transfer by mixed convection from a vertical permeable surface embedded in a non-Darcian porous medium for a non-Newtonian power-law fluid in the presence of temperature-dependent heat generation or absorption effects. A single parameter for the entire range of free-forced-mixed convection regime was employed. The obtained non-similar differential equations were solved numerically by an efficient implicit finite-difference method. The results focused on the effects of the buoyancy ratio, power-law fluid viscosity index, mixed convection parameter, suction or injection parameter, heat generation or absorption parameter, Lewis number and the porous medium inertia parameter on the local Nusselt and Sherwood numbers. It was found that as the buoyancy ratio was increased, both the local Nusselt and Sherwood numbers increased in the whole range of free and mixed convection regime while they remained constant for the forced-convection regime. However, they decreased and then increased forming dips as the mixed-convection parameter was increased from the free-convection limit to the forced-convection limit for both Newtonian and dilatant fluid situations. On the other hand, in general, for pseudo-plastic fluids the local Nusselt and Sherwood numbers decreased with increasing values of the mixed-convection parameter for smaller values of the buoyancy ratio (less than or equal unity) while they increase with it for larger values of the buoyancy ratio. Furthermore, it was concluded that the local Nusselt and Sherwood numbers decreased as either the power-law fluid viscosity index or the porous medium inertia parameter was increased. The effect of heat generation was found to decrease the local Nusselt number while the opposite was predicted for heat absorption conditions. In general, the local Sherwood number was increased with increases in the Lewis number.

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