

## Forced Convection of Blasius Flow of "SECOND-GRADE" Visco-Elastic Fluid

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### ABSTRACT:

The steady state viscoelastic boundary layer flow for the forced convection heat transfer is examined in this paper. The governing equations are formulated and solved numerically using an implicit finite difference technique. The velocity and temperature profiles, boundary layer thicknesses, Nusselt numbers and local skin friction coefficients are shown graphically for different values of viscoelastic parameter. In general, it is found that both the velocity and heat transfer increase inside the boundary layer for the viscoelastic flows as compared to the Newtonian flows due to favorable tensile stresses. Comparison with previous studies shows an excellent agreement.

**Key words:** viscoelastic fluids, forced convection, steady state.

### 1. INTRODUCTION:

Numerous applications of viscoelastic fluids in several manufacturing processes have led to renewed interest among researchers to investigate viscoelastic boundary layer flow over a stretching plastic sheet, Rajagopal et al. [1, 2], Dandapat and Gupta [3], Rollins and Vajravelu [4], Anderson [5], Lawrence and Rao [6], Char [7] and Rao [8]. Some of the physical applications of such study are polymer sheet extrusion from a dye, glass fiber, paper production, and drawing of plastic film etc.

In reality most of the fluids which are considered in industrial applications are more non-Newtonian in nature, especially of viscoelastic type than viscous type; we extend the forced convection heat transfer work to viscoelastic fluid flow and heat transfer. The governing equations for this investigation are written in dimensionless form using a set of dimensionless variables and solved numerically using the finite difference technique. Numerical results for the velocity, and temperature profiles as well as the local coefficient of friction and local Nusselt number under the effect of viscoelastic parameter, are presented.

The viscoelastic fluid model used in this study is called the second-order, or more commonly second-grade model. This rheological model was first introduced by Rivlin and Ericksen [9] and is generally regarded as one of the simplest viscoelastic fluid models available.

For an incompressible homogenous fluid of a second-grade type, the Cauchy stress,  $\Gamma$  is related to the deformation field through:

$$\Gamma = -PI + \alpha_1 A_1 + \alpha_2 A_2 + \alpha_3 A_3 \quad (1)$$

Where  $-PI$  is the isotropic part of the stress tensor,  $\alpha_1, \alpha_2$  and  $\alpha_3$  are the material moduli, and  $A_1, A_2$  and  $A_3$  are kinematical tensors defined by [10, 11]. Based on the response of a second-grade fluid to steady shear flow,  $\alpha_1$  is in fact the same as the coefficient of viscosity,  $\mu$ . Similarly,  $\alpha_2$  and  $\alpha_3$  can be related to the first and second normal stress differences,  $N_1$  and  $N_2$ , respectively. Experimental data available for a large number of viscoelastic fluids suggest that  $N_1$  is positive. On the other hand,  $N_2$  is often found to be either negative or zero. Also when  $N_2$  is measured to be non-zero, it is common to be much smaller than  $N_1$  values. This means that for a second-grade fluid to comply with experimental observations, one should have  $\alpha_2 < 0$  and  $\alpha_3 \leq 0$ . Having this in mind, it should be mentioned that there are some controversies around this rheological model, particularly about the sign of  $\alpha_2$  and the size of  $\alpha_3$ . Fosdick and Rajagopal [12] argue that for a second-grade rheological model to be thermodynamically compatible, the Clausius-Duhem inequality should hold together with the Helmholtz

free energy being at its minimum whenever the fluid is locally at rest. These thermo dynamical constraints put some severe restrictions on the sign and magnitude of the material moduli:

$$\alpha_1 \geq 0, \alpha_2 \geq 0; \text{ and } \alpha_2 + \alpha_3 = 0 \quad (2)$$

The sign proposed above for  $\alpha_2$  is tantamount to saying that  $N_1$  is negative. If this sign is accepted for  $\alpha_2$  then based on equation (2)  $\alpha_1$  should be positive. Both signs are indirect contradiction with experimental data available for viscoelastic fluids. The last relationship in equation (2) also suggests the absolute values of  $N_1$  and  $N_2$  are equal to each other which cannot be confirmed experimentally. Obviously, there certain important issues still unresolved about this controversial rheological model, for a critical review of the second grade model the reader is referred to Dunn [13]. In this study we have decided to take  $\alpha_2 < 0$  and to let  $\alpha_2 + \alpha_3 \neq 0$  in our second grade fluid. We still go one step further and in accordance with the so-called Weissenberg hypothesis [14], assume the second normal stress differences is zero for our fluids; i. e, we set  $\alpha_3 = 0$  in our model.

## 2. PROLEM FORMULATION:

Consider laminar forced convection boundary layer flow of a viscoelastic fluid. The problem is described in a rectangular coordinate system attached to the plate such that the  $x^*$ -axis lies along the plate surface and the  $y^*$ -axis is normal to the plate. It is assumed that at time  $t \leq 0$ , the temperatures of the plate and the viscoelastic fluid are maintained at the constant temperature  $T_\infty$ , and at time  $t > 0$ , the temperature of the plate is impulsively increased to the constant value  $T_w$  such that  $T_w > T_\infty$ . The continuity, momentum and energy equations under the boundary layer can be written as Cortell [15]:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \quad (3)$$

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \nu \frac{\partial^2 u^*}{\partial y^{*2}} - k_0 \left( u^* \frac{\partial^3 u^*}{\partial x^* \partial y^{*2}} + v^* \frac{\partial^3 u^*}{\partial y^{*3}} - \frac{\partial u^*}{\partial y^*} \frac{\partial^2 u^*}{\partial x^* \partial y^*} + \frac{\partial u^*}{\partial x^*} \frac{\partial^2 u^*}{\partial y^{*2}} \right) \quad (4)$$

$$u^* \frac{\partial T}{\partial x^*} + v^* \frac{\partial T}{\partial y^*} = \alpha \frac{\partial^2 T}{\partial y^{*2}} \quad (5)$$

Here  $u^*$  and  $v^*$  are the velocity components in  $x^*$  and  $y^*$  directions respectively,  $\nu$  is the kinematic coefficient of viscosity,  $k_0 = -\alpha_1 / \rho$  is the elastic parameter. Hence in the case of a second order fluid  $k_0$  takes positive values as  $\alpha_1$ , where  $\alpha = k / \rho c_p$  is the thermal diffusivity and other quantities have their usual meanings.

A critical review on the boundary conditions and the existence and uniqueness of the solution has been given by Rajagopal et al. [1]. Most of available literature on boundary layer flow of a viscoelastic over linearly stretching sheets deal with the three boundary conditions on velocity, which are one less than the number required to solve the problem uniquely, Rollins and Vajravelu [4], Anderson [5], Cortell [15] and Mahapatra and Gupta [16]. Troy et al. [17] derived a unique solution of the problem containing exponential terms of similarity variables. Based on the previous discussions on boundary conditions the physical initial and boundary conditions for this problem are given by:

$$\begin{aligned} u^* = U, v^* = 0, T = T_\infty & \quad \text{for } x^* = 0, y^* \geq 0 \\ u^* = 0, v^* = 0, T = T_w & \quad \text{for } y^* = 0, x^* \geq 0 \\ u^* = U, \partial u^* / \partial y^* = 0, T = T_\infty & \quad \text{for } y^* \rightarrow \infty \end{aligned} \quad (6)$$

Where  $(\partial u^* / \partial y^*)_{y^* \rightarrow \infty} = 0$  is taken as a boundary layer condition in order to determine boundary layer thicknesses.

Defining the non-dimensional variables such that

$$\begin{aligned} x = x^* / L, y = (y^* / L) \text{Re}^{1/2}, u = u^* / U, \\ v = (v^* / U) \text{Re}^{1/2}, \Theta = T - T_\infty / T_w - T_\infty \end{aligned} \quad (7)$$

where  $L$  is the characteristic length of the plate and  $\text{Re} = UL/\nu$  is the Reynolds number and then substituting equation (7) into equations. (3-6) yields the following dimensionless equations:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (8)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - k_i \left( u \frac{\partial^3 u}{\partial x \partial y^2} + v \frac{\partial^3 u}{\partial y^3} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial x \partial y} \right) \quad (9)$$

$$u \frac{\partial \Theta}{\partial x} + v \frac{\partial \Theta}{\partial y} = \frac{1}{\text{Pr}} \frac{\partial^2 \Theta}{\partial y^2} \quad (10)$$

Where  $k_i^* = k_0 U / L \nu$  is the modified viscoelastic parameter and  $\text{Pr} = \mu c_p / k$  is the Prandtl number. Note that for the special case of  $k_i^* = 0$  the fluid is again a Newtonian fluid.

The corresponding dimensionless initial and boundary conditions can be written as:

$$\begin{aligned} u = 1, v = 0, \Theta = 1 & \quad \text{for } x = 0, y \geq 0 \\ u = 0, v = 0, \Theta = 1 & \quad \text{for } y = 0, x \geq 0 \\ u = 1, \partial u / \partial y = 0, \Theta = 0 & \quad \text{for } y \rightarrow \infty \end{aligned} \quad (11)$$

The dimensionless skin friction coefficient of friction  $C_f$  and local Nusselt number  $Nu_x$  are important physical parameters for this type of flow and heat transfer situation Khan and Sajayanad [18] and Sadeghy and Sharifi [19]. They can be defined in dimensionless form as:

$$C_f Re^{1/2} = (\partial u / \partial y)_{(x,0)} - 2k_1^* (\partial u / \partial y)_{(x,0)} (\partial v / \partial y)_{(x,0)} \quad (12)$$

$$Nu_x Re^{-1/2} = - (\partial \Theta / \partial y)_{(x,0)} \quad (13)$$

### 3. NUMERICAL SOLUTION:

The steady state boundary layer equations represented by equations (8-10) are solved subject to the boundary conditions given by equation (11) using an implicit finite-difference technique of second-order accuracy in space. The detailed methodology of the solution is clearly explained by Anderson [20]. The employed numerical solution is a space marching technique giving the downstream velocity and temperature profiles using the known upstream profiles. In the present work, the above quantities have been calculated by obtaining explicitly the flow field variables at grid point  $(i, j)$  from the known flow field variables at grid points  $(i, j), (i+1, j), (i-1, j), (i, j-1)$  and  $(i, j+1)$ . Once the velocity and temperature fields are obtained, then the local coefficient of friction and local Nusselt number are calculated from equations (12) and (13). Recently this numerical method is used by Duwairi and Chamkha [21] and Duwairi et al. [22] for the solution of convection heat transfer of micropolar fluids and non-Boussensiq convection heat transfer problems. In order to verify the accuracy of the present method, comparison of results with the similarity solutions obtained by Oosthuizen and Naylor [23] for the steady laminar forced convection over an isothermal impermeable plate of Newtonian fluids is performed as shown in Table 1. In addition, the results from Table 1 show an excellent agreement between the two methodologies. This favorable comparison lends to high confidence in the numerical results that will be reported in the next section.

**Table 1.** Values of Nusselt numbers  $Nu_x$  at  $k_1^* = 0$ ,  $Pr = 0.72$  along stream wise direction

$Nu_x$		
$k_1^* = 0, Pr = 0.72$		
$x$	Present Results	Oosthuizen and Naylor [23]
0.1	1.49421	1.4088
0.2	1.21134	1.32778
0.4	1.13248	1.09997
0.6	1.00032	0.97532
0.8	0.90871	0.91021
1.0	0.887350	0.85678

### 4. RESULTS AND DISCUSSION:

The dimensionless viscoelastic parameter  $k_1^* = (k_0 U / L \nu)$  is found to be proportional directly to the elasticity of the fluid and free stream velocity and inversely to the plate length and kinematic viscosity. On other hand, it is found to be proportional inversely to the plate length and kinematic viscosity.

Figure 1 shows the effect of the modified viscoelastic parameter on the velocity profiles  $u(x, y)$  for different values of modified viscoelastic parameter  $k_1^* = 0, 0.5, 1, 2$  and  $Pr = 7$  and  $x = 0.5$ . It is observed that as the modified viscoelastic parameter increased, the velocity decreases inside the boundary layer. The reason behind that is due to the type of stresses developed in the fluid which are tensile stresses. The tensile stresses tend to decelerate the fluid in the flow direction. However, it is observed that as the modified viscoelastic parameter increased the hydrodynamic boundary layer thicknesses increased.

Figure 2 shows the effect of the modified viscoelastic parameter on the temperature profiles  $\Theta(x, y, t)$  for different values of viscoelastic parameter  $k_1^* = 0, 0.5, 1, 2$  and  $Pr = 7$  and  $x = 0.5$ . It is observed that as the viscoelastic effects are increased the temperatures inside boundary layer are increased and thus temperature gradient near vertical surface are decreased. As a result the heat transfer rates are increased.

Figure 3 shows representative coefficient of friction  $C_f Re^{1/2}$  and local Nusselt numbers profiles  $Nu_x Re^{-1/2}$  for different values of the viscoelastic parameter and  $Pr = 7$  and  $x = 0.5$ . It should be noted that the case where

$k_i^* = 0$  corresponds to Newtonian fluids. Increasing the viscoelastic parameter has the tendency to decrease the local coefficient of friction because of lower velocities due to

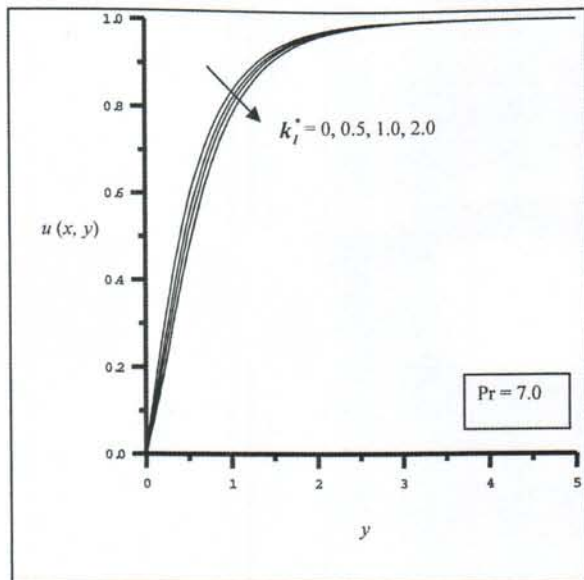


Figure 1. Dimensionless velocity profiles for different values of viscoelastic parameters at midpoint  $x=0.5$

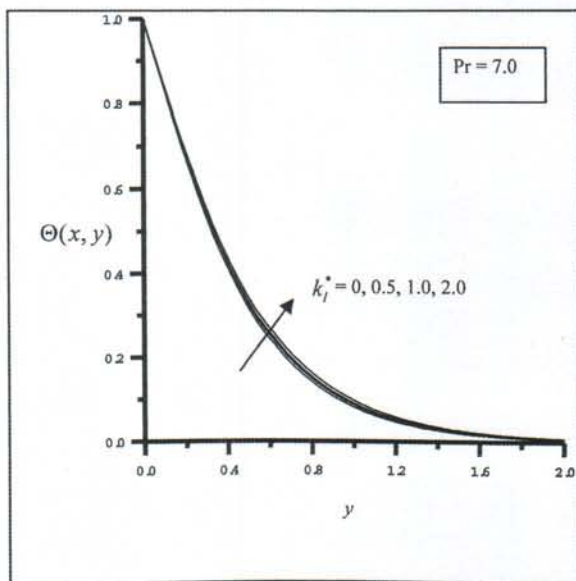


Figure 2. Dimensionless temperature profiles for different values of viscoelastic parameters at midpoint  $x=0.5$

unfavorable tensile stresses between fluid layers and consequently a decrease in local Nusselt numbers.

## 5. CONCLUSIONS:

The steady state laminar forced convection heat transfer effects for a viscoelastic fluid were studied. It was found that as the viscoelastic parameter increased, the local coefficient of friction and local Nusselt numbers decreased. On the other hand, as the velocities are decreased and temperatures are increased due to unfavorable tensile stresses between fluid layers, which had decreased coefficient of heat transfer.

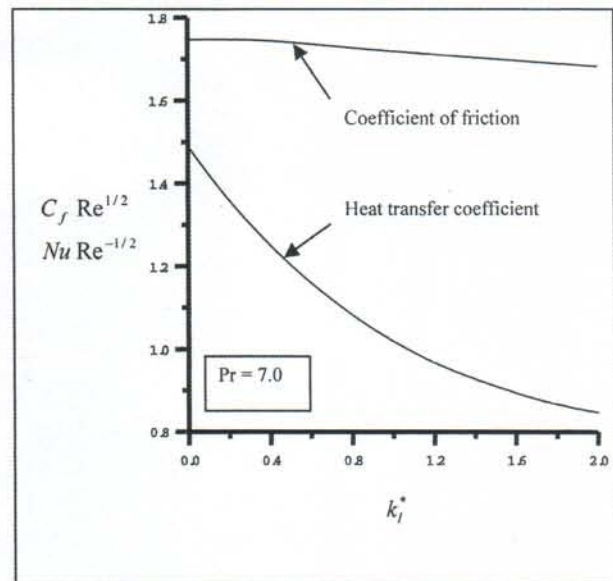


Figure 3. Dimensionless local coefficient of friction and local Nusselt numbers for different values of viscoelastic parameters at midpoint  $x=0.5$

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### Nomenclature

$A_1, A_2, A_3$	Rivlin-Ericksen tensor
$C_f$	local coefficient of friction
$C_p$	specific heat of the fluid at constant pressure
$h$	heat transfer coefficient
$k$	thermal conductivity
$k_0$	elastic parameter
$k_t^*$	dimensionless viscoelastic parameter,
$(k_0 U / L \nu)$	
$L$	characteristic length of plate

$Nu_x$	local Nusselt number
$P_I$	spherical stress
$Pr$	Prandtl number, $\nu/\alpha$
$T$	temperature
$T_w$	wall temperature
$T_\infty$	ambient fluid temperature
$u, v$	dimensionless velocity components along x- and y-axes respectively
$U$	free stream velocity
$x, y$	dimensionless coordinates

### Greek symbols

$\alpha$	thermal diffusivity
$\alpha_1, \alpha_2, \alpha_3$	material moduli
$\Theta$	non-dimensional temperature
$\mu$	dynamic viscosity
$\nu$	kinematic viscosity
$\rho$	fluid density
$\Gamma$	Cauchy stress tensor

### Subscripts

w	surface condition
$\infty$	free stream condition

### Superscripts

*	dimensional variables
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