Unsteady buoyancy driven saline water over a vertical flat plate

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Abstract: The effect of heat and mass transfer on transient laminar free convection flows for saline water over a vertical flat surface is studied. The non-Boussinesq equation to approximate temperature variations with density in buoyancy term is applied. The governing equations are written in their dimensionless form and then solved numerically using an implicit finite difference technique. Two parameters are found to describe the problem; the first one is the density parameter, the second one is the dimensionless Grashof number. Velocity profiles, temperature profiles and Nusselt numbers are discussed.

Keywords: natural convection; mass transfer; saline water; unsteady.

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1 Introduction

The natural convection heat transfer mode in various geometries has received a great deal of attention (Kao, 1976; Na, 1978; Aziz and Na, 1982). There are several transport processes in industry and in nature where buoyancy forces arise from both thermal and mass diffusion caused by temperature gradients and the concentration differences of dissimilar chemical species. The combined effect of heat and mass transfer on different problems can be seen in El-Hakiem and El Amin (2001), Magyari and Keller (1999), Takhar et al. (2000) and Chamkha (2001). Most of the past studies have used the Boussinesq approximation i.e., taking into account that the fluid density varies linearly with temperature. However, this is in applicable for water at low temperatures, where in the local thermodynamic equilibrium a density extreme in pure or saline water at atmospheric pressure may occur. When flows are driven by temperature and salinity gradients around the level of a density extreme, maximum density conditions may strongly influence the resulting motion (Ingham et al., 2001; Duwairi et al., 2006).

The density-temperature relationship is linear for air whereas in water this relationship is linear at high temperatures and non-linear at low temperatures. The density of pure water is maximum at 3.98°C. The density increases as the temperatures decreases approaching 3.98°C, while the density decreases as the temperature decreases from 3.98°C to 0°C. The convective flows generate in pure cold and saline water have been analysed by Gebhart et al. (1979), Carey et al. (1980) and El-Henawy et al. (1986) for different thermally driven flows adjacent to vertical and horizontal surfaces, in these studies the steady state conditions are assumed and a set of similarity solutions have been obtained.

The influence of mass transfer on the natural convection problem in cold and saline water is not addressed yet. In this paper the transient free convection boundary layer over an isothermal vertical flat plate which is generated in both cold and saline water and in the presence of mass transfer is studied. Numerical solutions using MacCormack's technique for different dimensionless velocity, temperature and concentration profiles and different local coefficient of friction, Nusselt numbers and Sherwood number are presented.

2 Basic equations

Consider unsteady laminar free convection boundary layer flow over an isothermal vertical flat plate in both pure and saline water. With respect to an arbitrary origin on this planar wall, the x-axis is taken along plate and the y-axis is taken perpendicular to it into the fluid. Initially, the plates and the fluid are at ambient temperature T_∞ subsequently, the temperature of the plate is impulsively increased to the constant value T_w, where $T_w > T_\infty$. We also assume that the density of the pure or saline water is given by the following equation (Gebhart and Mollendroff, 1977):

$$\rho(s, p, T) = \rho_m(s, p)\left[1 - \beta_T(s, p)\left|T - T_m(s, p)\right|^{q(s,p)}\right] \quad (1)$$

where ρ_m and T_m are the maximum density and temperature for a given pressure p and salinity s and q is the exponent. This equation has very high accuracy to 20°C, salinity of 40% and to a pressure of 100 bars.

If we suppose that another substance is present in water (milk, sugar, dust) it will be diluted in water in a simple, linear way. The variation of concentration with temperature and salinity is unique in nature and it is caused by water special chemical structure. There is no scientific evidence that any other substance will influence water concentration like temperature or salinity. In this study, the variation of concentration with temperature is simply a linear relationship between concentration and temperature and it is given by the following the reasonable relation:

$$C(s, p, T) = C_m(s, p)\left[1 - \beta_c(s, p)\left|T - T_m(s, p)\right|\right] \quad (2)$$

where C_m is the maximum concentration.

The governing equations of this flow can be written in non-dimensional form as:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \left|\theta - R\right|^q - \left|R\right|^q + \mathrm{Gr}^*\left(\left|\phi - R_1\right| - \left|R_1\right|\right) \quad (3)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} \quad (4)$$

$$\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial y^2} \quad (5)$$

where

$$R = \frac{T_m(s,p) - T_\infty}{T_w - T_\infty}, \quad R_1 = \frac{C_m(s,p) - C_\infty}{C_w - C_\infty} \quad (6)$$

is the extreme temperature parameter. The initial and boundary conditions of Equations (3)–(6) are:

$$t \le 0, \quad u = 0, v = 0, \theta = 0, \phi = 0 \quad \text{for all} \quad x \ge 0, y \ge 0$$
$$t > 0, \quad \begin{cases} u = 0, v = 0, \theta = 0, \phi = 0 & \text{for} \quad x = 0, y \ge 0 \\ u = 0, v = 0, \theta = 1, \phi = 1 & \text{for} \quad y = 0, x \ge 0 \\ u = 0, \theta = 0, \phi = 0 & \text{for} \quad y \to \infty. \end{cases} \quad (7)$$

The non-dimensional variables are defined by:

$$t = \mathrm{Gr}_T^{1/2}(\upsilon / l^2)\bar{t}, \quad x = \bar{x}/l, \quad y = \mathrm{Gr}_T^{1/4}(\bar{y}/l)$$
$$u = \mathrm{Gr}_T^{-1/2}(l/\upsilon)\bar{u}, \quad v = \mathrm{Gr}_T^{-1/4}(l/\upsilon)\bar{v}, \quad \theta = T - T_\infty / T_w - T_\infty$$
$$\phi = C - C_\infty / C_w - C_\infty, \quad \mathrm{Gr}_T = g\beta_T l^3 (T_w - T_\infty)^q / \upsilon^2,$$
$$\mathrm{Gr} = \beta_T (T_w - T_\infty)^q / \beta_c (C_w - C_\infty)^q \quad (8)$$

where l is the characteristic length of the plate, $\bar{x}$, $\bar{y}$, $\bar{u}$, $\bar{v}$, $\bar{t}$ represent the dimensional variables, and Gr_T, Gr* is the thermal and modified Grashof number respectively. The values of local coefficient of friction, local Nusselt number and Sherwood number are given by:

$$C_f = \partial u(x, 0, t)/\partial y \quad (9)$$

$$\mathrm{NuGr}_T^{-1/4} = -\partial \theta(x, 0, t)/\partial y \quad (10)$$

$$\mathrm{ShGr}_T^{-1/4} = -\partial \phi(x, 0, t)/\partial y. \quad (11)$$

The boundary layer equations describe the conservation of mass, momentum, energy, and concentration with a non-Boussinesq equation are formulated and solved in their time dependent formulation using the MacCormack's technique, which is an explicit finite difference technique and a second order accuracy in space and time, the details of this solution is clearly explained by Anderson (1995). In order to verify the accuracy of the present method comparison of results with similarity solutions obtained by Pantokratoras (1999) are shown in Table 1 for the steady laminar free convection over a vertical isothermal impermeable plate (zero blowing and suction) with $\mathrm{Gr}^* = 0$ and a linear density-temperature relationship. The comparison is found to be in excellent agreement. As mentioned before the numerical solution used is a time marching technique giving the downstream velocity and temperature profiles using the known upstream profiles. In the present work the above quantities have been calculated by obtaining explicitly the flow field variables at grid point (i, j) at time $\tau + \Delta\tau$ from the known flow field variables at grid points (i, j), $(I + 1, j)$, $(i - 1, j)$, $(i - 1, j)$ and $(i, j + 1)$ at time τ. The flow field variables at all other grid points at time $\tau + \Delta\tau$ are obtained in like fashion. Then the local coefficient of friction, local Nusselt numbers and Sherwood numbers are calculated from Equations (9)–(11).

Table 1 θ'at different time and at $x = 0.5$, $R = 0$, Pr = 11.4, $q = 1$, Gr* = 0. Result in parenthesis is that of similarity solution obtained by Pantokratoras (1999)

T	θ'
3.5	1.0324
4	1.01444
∞	1.06028 [1.06]

3 Results and discussion

In this paper the transient free convection heat and mass transfer effects in saline water from vertical surfaces are investigated. The governing equations are written in their dimensionless from using a set of dimensionless variables then solved using a finite- difference implicit method.

The value of q in these results is taken to be of 1.8; this exponent was found to be in the range of $1 \le q \le 2$ (Gebhart and Mollendroff, 1977). The temperature ratio parameter R is considered to vary between 0.1 and –1.0 while the concentration parameter ratio R_1 given the value –1 in all the predicted results, the concentration buoyancy effect is studied throughout the given values of the modified Grashof number Gr*.

Figures 1–3 depict the dimensionless velocity, temperature and concentration profiles inside the boundary layer at the midpoint of the plate ($x = 0.5$). The steady state conditions are considered for different values of R, the other dimensionless parameters are taken to be Pr = 11, Sc = 800, $q = 1.8$ and Gr* = 1. Obviously, increasing the dimensionless temperature ratio R aids to decrease the velocity due to the important rule that the buoyancy forces played in transferring momentum between fluid layers and increase temperature and concentration of the fluid inside the boundary layer. It is clear from Figures 4 and 5 that Nusselt numbers and Sherwood numbers decreases as R increases, the temperature rise inside the boundary layer at increasing values of time leads to decrease the heat transfer and mass transfer phenomenon. The effect of R on local coefficient of friction is drawn in Figure 6. The coefficient of friction increases as the dimensionless temperature ratio R decreases at all point of the time field. Moreover, it's interesting to note that the steady state time increase as *i R* becomes increasingly more positive.

Figures 7–9 show the effect of modified Grashof number on local coefficient of friction, local Nusselt numbers and local Sherwood numbers. Increasing Gr* had increased coefficient of friction and consequently this will increase velocities inside boundary layer. Increasing Gr* had also increased Nusselt numbers and Sherwood numbers because of higher buoyancy forces. Different Values of local Nusselt numbers against transient time for different dimensionless groups are listed in Table 1.

Figure 1 Steady state dimensionless fluid velocity profiles at the midpoint of the plate as a function of transverse direction y (Pr = 11, Sc = 800, $q = 1.8$, Gr* = 1)

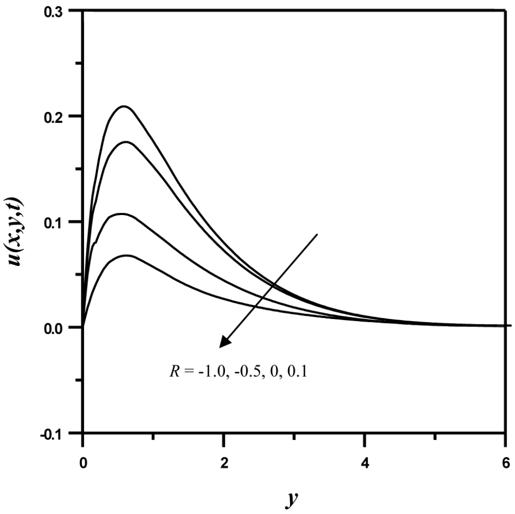


Figure 2 Steady state dimensionless fluid temperature profiles at the midpoint of the plate as a function of transverse direction y (Pr = 11, Sc = 800, $q = 1.8$, Gr* = 1)

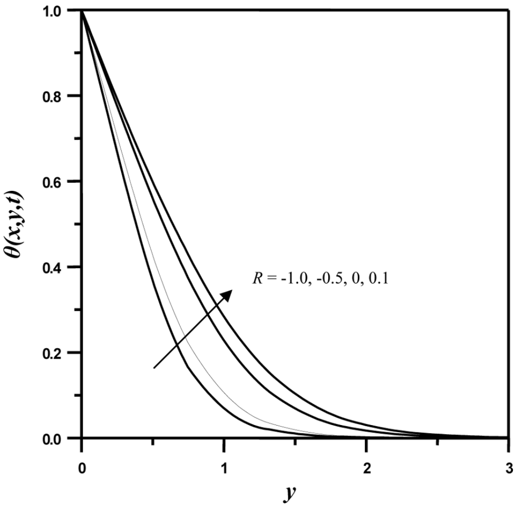


Figure 3 Steady state dimensionless fluid concentration profiles at the midpoint of the plate as a function of transverse direction y (Pr = 11, Sc = 800, $q = 1.8$, Gr* = 1)

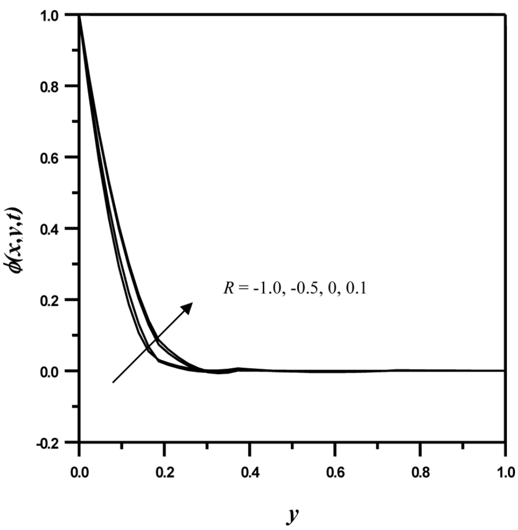


Figure 4 Nusselt number variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, Gr* = 1)

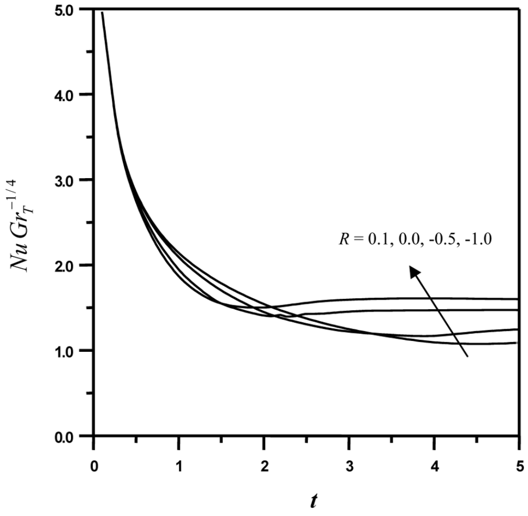


Figure 5 Sherwood number variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, Gr* = 1)

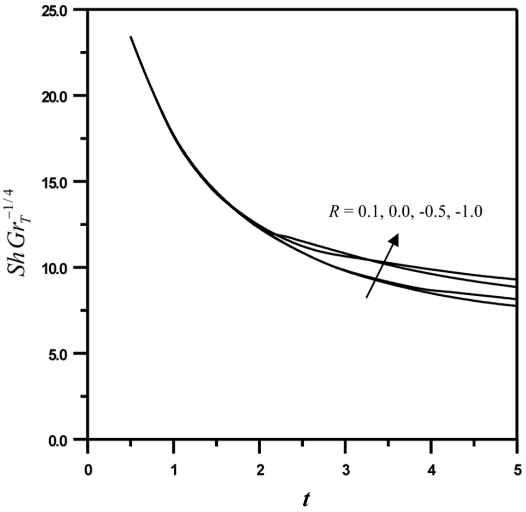


Figure 6 Skin-friction coefficient variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, Gr* = 1)

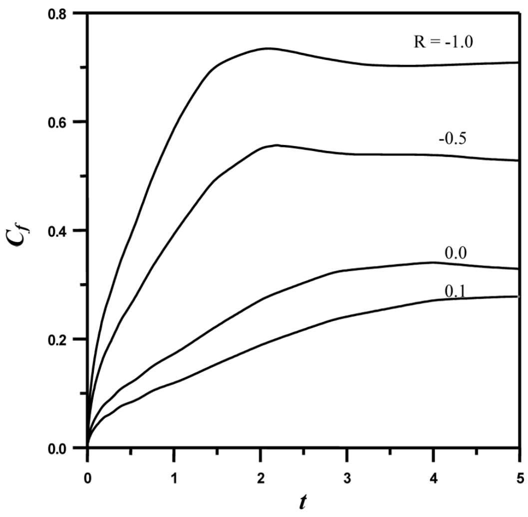


Figure 7 Skin-friction coefficient variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, *R* = 0.1)

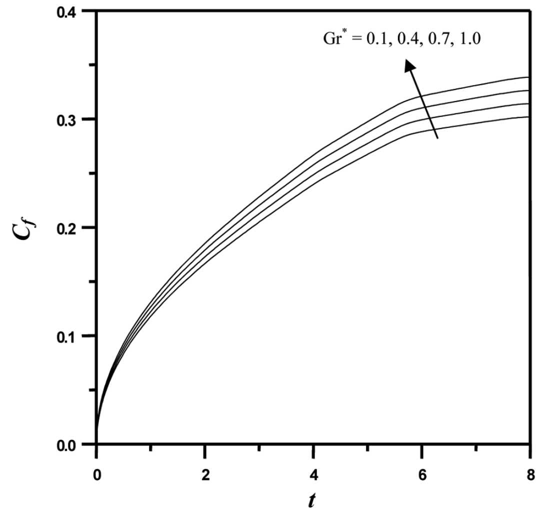


Figure 8 Nusselt number variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, *R* = 0.1)

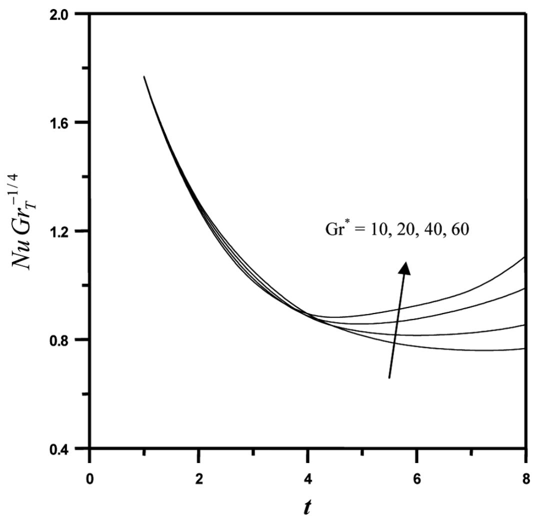
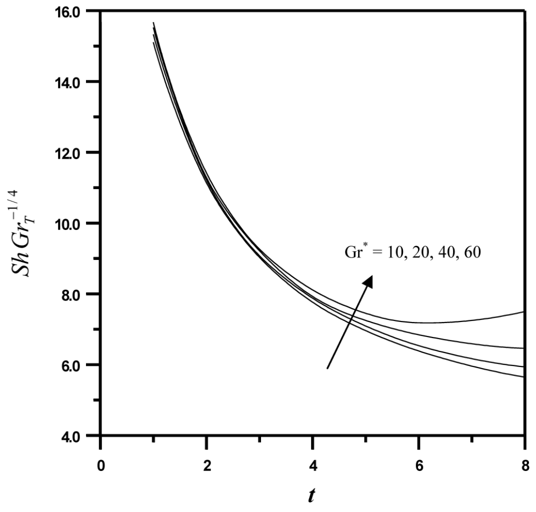


Figure 9 Sherwood number variations at the midpoint of the plate as a function of the dimensionless time *t* (Pr = 11, Sc = 800, *q* = 1.8, *R* = 0.1)



4 Concluding remarks

Numerical solutions for unsteady heat and mass transfer, laminar flow of saline water on vertical flat plate were reported. Based on the obtained graphical results, the following conclusions were deduced:

- The fluid velocity decreased as either of the temperature ratio parameter R increased or the modified Grashof number Gr* was decreased. The coefficient of friction increased as R decreased and or Gr* is increased.
- The temperature and concentration profiles inside the boundary layer are increased as R increased; this triggers the Nusselt number and Sherwood number to decrease.
- The effect of Gr* is to increase the Nusselt number and Sherwood number, this can be noticed for dimensionless time $t > 4$.

References

Anderson, J.D. (1995) *Computational Fluid Dynamics*, Chap. 6, McGraw-Hill, New York.

Aziz, A. and Na, T.Y. (1982) 'Improved perturbation solutions for laminar natural convection on a vertical cylinder', *Wärmeiund–Stoffübertragung*, Vol. 16, No. 2, pp.83–87.

Carey, V.P., Gebhart, B. and Mollendroff, J.C. (1980) 'Buoyancy force reversals in vertical natural convection flows in cold water', *J. Fluid Mech.*, Vol. 97, No. 2, pp.279–297.

Chamkha, A.J. (2001) 'Coupled heat and mass transfer by natural Convection about a truncated cone in the presence of magnetic field and radiation effects', *Numerical Heat Transfer*, *Part A*, Vol. 39, No. 5, pp.511–530.

Duwairi, H.M., Damseh, R.A. and Tashtoush, B. (2006) 'Transient non-boussinesq magnetohydrodynamic free convection flow over a vertical surface', *Int. J. Fluid Mechanics Research*, Vol. 33, No. 2, pp.137–152.

El-Hakiem, M.A. and El Amin, M.F. (2001) 'Mass transfer effects on the non-Newtonian fluids past a vertical plate embedded in a porous medium with non-uniform surface heat flux', *Heat and Mass Transfer*, Vol. 37, No. 3, pp.293–297.

El-Henawy, I., Gebhart, B. and Kazarinoff, N.D. (1986) 'Multiple steady-state solutions for horizontal buoyant flows in cold water', *Int. J. Heat Mass Transfer*, Vol. 29, No. 11, pp.1655–1667.

Gebhart, B., Bendel, M.W. and Shaukatullah, H. (1979) 'Buoyancy-induced flows adjacent to horizontal surfaces in water near its density extremum', *Int. J. Heat Mass Transfer*, Vol. 22, No. 1, pp.137–149.

Gebhart, B. and Mollendroff, J.C. (1977) 'A new density relation for pure and saline water', *Deep Sea Res.*, Vol. 24, No. 7, pp.831–841.

Ingham, D.B., Pop, I. and Wade, A.S. (2001) 'Transient non-boussinesq free convection boundary – layer flow over a vertical flat plate', *Proceeding of the 2nd International Conference on Computational Heat and Mass Transfer, COPPE/EE/UFRJ-Federal University of Rio De Janeiro*, October, Brazil, pp.22–26.

Kao, T.T (1976) 'Local nonsimilar solution for laminar free convection adjacent to a vertical wall', *Transactions of ASME Journal of Heat Transfer*, Vol. 98, No. 2, pp.321–322.

Magyari, E. and Keller, B. (1999) 'Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface', *J. Phys. D: Appl. Phys.*, Vol. 32, Nos. 5, 7, pp.577–585.

Na, T.Y. (1978) 'Numerical solution of natural convection flow past a non- isothermal vertical plate', *Applied Scientific Research*, Vol. 33, Nos. 5, 6, pp.519–543.

Pantokratoras, A. (1999) 'Laminar free convection of pure and saline water along a heated vertical plate', *ASME J. Heat Transfer*, Vol. 121, No. 3, pp.719–722.

Takhar, H.S., Chamkha, A.J. and Nath, G. (2000) 'Combined heat and mass transfer along a vertical moving cylinder with a free stream', *Heat and Mass Transfer*, Vol. 36, No. 3, pp.237–246.

Nomenclature

C	Concentration
C_m	Concentration where density is maximum
C_w	Wall concentration
C_∞	Ambient fluid concentration
g	Magnitude of acceleration due to gravity
Gr_T	Thermal Grashof number
Gr*	Modified Grashof number
K	Thermal conductivity
L	Characteristic length of plate
Pr	Prandtl number, υ/α
Q	Exponent in density and concentration, Equations (1) and (2)
R	Density parameter defined in Equation (6)
R_1	Density parameter defined in Equation (6)
S	Salinity
Sc	Schmidt number, υ/D
t	Non-dimensional time
T	Temperature
T_m	Temperature where the density is maximum
T_w	Wall temperature
T_∞	Ambient fluid temperature
u, v	Non-dimensional velocity components along x- and y-axes
x, y	Non-dimensional coordinates along and normal to the plate

Greek Symbols	
α	Thermal diffusivity
β_c	Coefficient in the concentration, Equation (2)
β_T	Coefficient in the density, Equation (1)
θ	Non-dimensional temperature
μ	Dynamic viscosity
ρ	Fluid density, Equation (1)
ρ_m	Maximum density
ρ_∞	Density of ambient fluid
ν	Kinematic viscosity
ϕ	Non-dimensional temperature
Superscript	
–	Dimensional quantities