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### SIMILARITY SOLUTION FOR UNSTEADY HEAT AND MASS TRANSFER FROM A STRETCHING SURFACE EMBEDDED IN A POROUS MEDIUM WITH SUCTION/INJECTION AND CHEMICAL REACTION EFFECTS

A. J. Chamkha<sup>a</sup>; A. M. Aly<sup>b</sup>; M. A. Mansour<sup>c</sup>

<sup>a</sup> Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh, Kuwait <sup>b</sup> Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt <sup>c</sup> Department of Mathematics, Faculty of Science, Assuit University, Assuit, Egypt

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# Similarity Solution for Unsteady Heat and Mass Transfer from a Stretching Surface Embedded in a Porous Medium with Suction/Injection and Chemical Reaction Effects

A. J. CHAMKHA,<sup>1</sup> A. M. ALY,<sup>2</sup> AND M. A. MANSOUR<sup>3</sup>

<sup>1</sup>Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh, Kuwait

<sup>2</sup>Department of Mathematics, Faculty of Science, South Valley University, Qena, Egypt

<sup>3</sup>Department of Mathematics, Faculty of Science, Assuit University, Assuit, Egypt

*An analysis is presented to investigate the effects of chemical reaction on unsteady free convective heat and mass transfer on a stretching surface in a porous medium. The governing partial differential equations have been transformed by a similarity transformation into a system of ordinary differential equations, which are solved numerically using an efficient tri-diagonal implicit finite-difference method. The results obtained show that the flow field is influenced appreciably by the presence of unsteadiness parameter, chemical reaction parameter, permeability parameter, and suction/injection parameter.*

**Keywords** Chemical reaction; Porous medium; Similarity solutions; Unsteady flow

## Introduction

Fluid dynamics due to a stretching surface is important since it has many practical applications in manufacturing processes, which include both metal and polymer sheets, for example, the cooling of an infinite metallic plate in a cooling bath, the boundary layer along material handling conveyers, the aerodynamic extrusion of plastic sheets, the boundary layer along a liquid film in condensation processes, paper production, glass blowing, metal spinning, and drawing of plastic films. The quality of the final product depends on the rate of heat transfer at the stretching surface. Since the pioneering study by Crane (1970), who presented an exact analytical solution for steady two-dimensional flow due to a stretching surface in a quiescent fluid, many works on stretched surfaces have been done. The temperature field in the flow over a stretching surface subject to a uniform heat flux was studied by Dutta et al. (1985) and Grubka and Bobba (1985), while Elbashbeshy (1998) considered the

Address correspondence to A. J. Chamkha, Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh 70654, Kuwait. E-mail: achamkha@yahoo.com

case of a stretching surface with a variable surface heat flux. Chen and Char (1988) presented an exact solution of heat transfer from a stretching surface with a variable heat flux. Also, Magyari and Keller (2000) presented an exact solution for self-similar boundary-layer flows induced by permeable stretching walls. Gupta and Gupta (1977) examined the heat and mass transfer for the boundary layer flow over a stretching sheet subject to suction and blowing.

Moreover, coupled heat and mass transfer problems in the presence of chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent times. Possible applications can be found in processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body and energy transfer in a wet cooling tower, and flow in a desert cooler.

Chemical reactions can be modeled as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase, and, a heterogeneous reaction takes place in a restricted area or within the boundary of a phase. Soundalgekar (1977) presented an exact solution to the flow of a viscous fluid past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. The solution was derived by the Laplace transform technique, and the effects of heating or cooling of the plate on the flow field were discussed through the Grashof number. Das et al. (1994) studied the effects of mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Muthucumaraswamy and Ganesan (1998) considered the problem of unsteady flow past an impulsively started isothermal vertical plate with mass transfer by an implicit finite-difference method. Muthucumaraswamy and Ganesan (1999, 2001) solved the problem of unsteady flow past an impulsively started vertical plate with uniform heat and mass flux and variable temperature and mass flux, respectively. Diffusion of a chemically reactive species from a stretching sheet was studied by Andersson et al. (1994). Anjalidevi and Kandasamy (1999, 2000) analyzed the effects of chemical reaction and heat and mass transfer on laminar flow without or with magnetohydrodynamics (MHD) along a semi-infinite horizontal plate. Rahman and Mulolani (2000) studied laminar natural convection flow over a semi-infinite vertical plate at constant species concentration. They found that in the absence of chemical reaction, a similarity transform was possible, while when a chemical reaction occurred, perturbation expansions about an additional similarity variable dependent on reaction rate must be employed. Shateyi et al. (2009) studied laminar natural convection flow from a permeable semi-infinite accelerating vertical surface that is coated with a reacting chemical species. They found that the fluid motion was decelerated by increases in the permeability of the accelerating surface and that the rate of mass transfer increased with Schmidt numbers but reduced with increasing reaction rates and the porosity of the accelerating surface.

Most of the above-mentioned studies dealt with stretching surfaces or chemical reaction where the flows were assumed to be steady. Unsteady flows under the effects of chemical reaction due to stretching surfaces have not received much attention. Dandapat and Maity (2006) studied the flow of a thin liquid film on an unsteady stretching sheet. Abd El-Aziz (2009) studied the effect of radiation on the heat and fluid flow over an unsteady stretching surface by using a similarity transformation to reduce the governing time-dependent boundary layer equations for momentum and

thermal energy to a set of ordinary differential equations. Abo-Eldahab and Azzam (2006) investigated the combined effects of free convective heat and mass transfer on unsteady three-dimensional laminar boundary layer flow over a stretching surface. The stretching rates of the surface were assumed to vary as a reciprocal of a linear function of time. Generation or consumption of the diffusing species due to a homogeneous chemical reaction was considered, and the chemical reaction rate was assumed to vary with time according to a power-law function.

In the present article, we present the results of a study of the effects of chemical reaction parameter on unsteady free convection over a stretching permeable surface embedded in a porous medium. Similarity transformations are used to transform the governing partial differential equations to ordinary differential equations, which are then solved numerically using an accurate implicit finite-difference method.

### Mathematical Analysis

Consider unsteady laminar boundary-layer flow due to a stretching permeable surface embedded in a uniform porous medium. At time  $t = 0$ , the sheet is impulsively stretched with the variable velocity  $U_w(x, t)$ . The fluid properties are assumed to be constant, and a first-order homogeneous chemical reaction is assumed to take place in the flow. Under these assumptions with the usual Boussinesq approximation, the governing boundary-layer equations that are based on the balance laws of mass, linear momentum, energy, and concentration species for this investigation can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta_T(T - T_\infty) + g\beta_C(C - C_\infty) - \frac{\nu}{k_1}u, \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2}, \quad (3)$$

$$\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_c(C - C_\infty), \quad (4)$$

where  $x$  and  $y$  are the Cartesian coordinates,  $t$  represents time,  $u$  and  $v$  are the velocity components along the  $x$  and  $y$  directions, respectively,  $\nu = \mu/\rho$  is the kinematic viscosity, where  $\mu$  is the constant viscosity of the fluid in the boundary layer region,  $\alpha$  is the effective thermal diffusivity,  $k_c$  is the rate of chemical reaction,  $g$  is the acceleration due to gravity,  $\beta_T$  is the volumetric coefficient of thermal expansion,  $\beta_C$  is the volumetric coefficient of concentration expansion,  $k_1$  and  $D$  are permeability of porous media and coefficient of mass diffusivity, respectively, and  $T_\infty$  and  $C_\infty$  are the free stream temperature and concentration, respectively.

The boundary conditions for this problem can be written as:

$$\begin{aligned} u = U_w, \quad v = V_w, \quad T = T_w, \quad C = C_w \quad \text{at } y = 0 \\ u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{as } y \rightarrow \infty \end{aligned} \quad (5)$$

We assume that the stretching velocity  $U_w(x, t)$ , the surface temperature  $T_w(x, t)$ , and the surface concentration  $C_w(x, t)$  are of the form:

$$U_w(x, t) = \frac{ax}{1 - ct}, \quad T_w(x, t) = T_\infty + \frac{bx}{1 - ct}, \quad C_w(x, t) = C_\infty + \frac{bx}{1 - ct} \quad (6)$$

where  $a$ ,  $b$ , and  $c$  are constants with  $ct < 1$ , and both  $a$  and  $c$  have dimension  $\text{time}^{-1}$ .  $V_w$  is the suction/injection parameter;  $V_w > 0$  (injection) and  $V_w < 0$  (suction).

We introduce the following self-similar transformations (see Ishak et al. (2007, 2008, 2009), Devi et al. (1991), and Andersson et al. (2000)):

$$\eta = \left(\frac{U_w}{\nu x}\right)^{1/2} y, \quad \psi = (\nu x U_w)^{1/2} f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty} \quad (7)$$

where  $\psi(x, y, t)$ , a stream function related to the components of the velocity fields, is introduced by the equations  $u = \partial\psi/\partial y$  and  $v = -\partial\psi/\partial x$ ; it can be easily verified that the continuity Equation (1) is identically satisfied. Introducing the relations (7) into Equations (2)–(5), we obtain the following dimensionless ordinary differential equations:

$$f''' + Gr \theta + Gc \phi - Kf' - f'^2 + ff'' - A\left(f' + \frac{1}{2}\eta f''\right) = 0, \quad (8)$$

$$\frac{1}{Pr} \theta'' - f' \theta + f \theta' - A\left(\theta + \frac{1}{2}\eta \theta'\right) = 0, \quad (9)$$

$$\frac{1}{Sc} \phi'' - \gamma \phi - f' \phi + f \phi' - A\left(\phi + \frac{1}{2}\eta \phi'\right) = 0, \quad (10)$$

where a prime denotes ordinary differentiation with respect to  $\eta$  and  $Gr = g\beta_T(T_w - T_\infty)x/U_w^2$  is the Grashof number,  $Gc = g\beta_C(C_w - C_\infty)x/U_w^2$  is the modified Grashof number,  $A = c/a$  is a parameter that measures flow unsteadiness,  $K = \nu x/k_1 U_w$  is the permeability parameter,  $Pr = \nu/\alpha$  is the Prandtl number, and  $Sc = \nu/D$  is the Schmidt number.

The dimensionless boundary conditions (5) now become

$$\begin{aligned} f(0) = fw, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \\ f'(\eta) = 0, \quad \theta(\eta) = 0, \quad \phi(\eta) = 0 \quad \text{as } \eta \rightarrow \infty \end{aligned} \quad (11)$$

with  $fw < 0$  and  $fw > 0$  corresponding to injection and suction, respectively.

The quantities of physical interest are the local skin-friction coefficients  $C_f$ , local Nusselt number  $Nu_x$ , and the local Sherwood number  $Sh_x$ , which are defined as follows:

$$\tau_w = -\mu \left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad C_f = \frac{\tau_w}{\rho U_w^2/2} \quad \text{then} \quad \frac{1}{2} C_f Re_x^{1/2} = -f''(0), \quad (12)$$

$$q_w = -k \left(\frac{\partial T}{\partial y}\right)_{y=0}, \quad Nu_x = \frac{q_w}{(T_w - T_\infty)} \left(\frac{x}{k}\right), \quad \text{then} \quad Nu_x Re_x^{-1/2} = -\theta'(0), \quad (13)$$

$$m_w = -\rho D \left(\frac{\partial C}{\partial y}\right)_{y=0}, \quad Sh_x = \frac{m_w}{(C_w - C_\infty)} \left(\frac{x}{\rho D}\right), \quad \text{then} \quad Sh_x Re_x^{-1/2} = -\phi'(0) \quad (14)$$

**Table I.** Comparison of  $-f''(0)$  with those reported by Sharidan et al. (2006)

$A$	Sharidan et al. (2006) $-f''(0)$	Present work $-f''(0)$
0.8	1.261042	1.261512
1.2	1.377722	1.378052

## Numerical Method and Validation

The heat and mass transfer problem represented by Equations (8)–(10) are nonlinear and, therefore, must be solved numerically. The standard implicit finite-difference method discussed by Blottner (1970) has proven to be adequate and gives accurate results for such equations. For this reason, it is employed in the present work, and graphical and tabular results based on this method will be presented subsequently.

Equations (8)–(10) are discretized using three-point central difference formulae with  $f'$  replaced by another variable  $V$ . The  $\eta$  direction is divided into 196 nodal points, and a variable step size is used to account for the sharp changes in the variables in the region close to the surface where viscous effects dominate. The initial step size used is  $\Delta\eta_1 = 0.001$  and the growth factor  $K^* = 1.0375$  such that  $\Delta\eta_n = K^* \Delta\eta_{n-1}$  (where the subscript  $n$  is the number of nodes minus one). This gives  $\eta_{\max} \approx 35$ , which represents the edge of the boundary layer at infinity. The ordinary differential equations are then converted into linear algebraic equations that are solved by the Thomas algorithm discussed by Blottner (1970). Iteration is employed to deal with the nonlinear nature of the governing equations. The convergence criterion employed in this work was based on the relative difference between the current and the previous iterations. When this difference or error reached  $10^{-5}$ , then the solution was assumed converged and the iteration process was terminated. It is possible to compare the results obtained by this numerical method with the previously published work of Sharidan et al. (2006).

Table I shows a comparison between the numerical results of  $-f''(0)$  for the case of impermeable surface ( $fw = 0$ ), with  $Gr = Gc = K = 0$  and two values of  $A$  reported by Sharidan et al. (2006) and the numerical results obtained in the present work. It is evident from this table that these results are in excellent agreement.

## Results and Discussion

In this section, a representative set of graphical results is presented in Figures 1–9. These figures illustrate the influence of the unsteadiness parameter  $A$ , the chemical reaction parameter  $\gamma$ , and the permeability parameter  $K$  on the velocity, temperature, and the concentration profiles. In addition, Tables I and II summarize the effects of these parameters on the values of the skin-friction coefficient, Nusselt number, and Sherwood number.

Figures 1–3 present typical velocity, temperature, and concentration profiles in the boundary layer for various values of the unsteadiness parameter  $A$ , respectively. Increasing the value of  $A$  leads to decreases in the velocity, temperature, and concentration profiles. In addition, these figures show that the velocity, temperature, and concentration profiles are higher for injection ( $fw = -0.5$ ) than suction ( $fw = 0.5$ ).

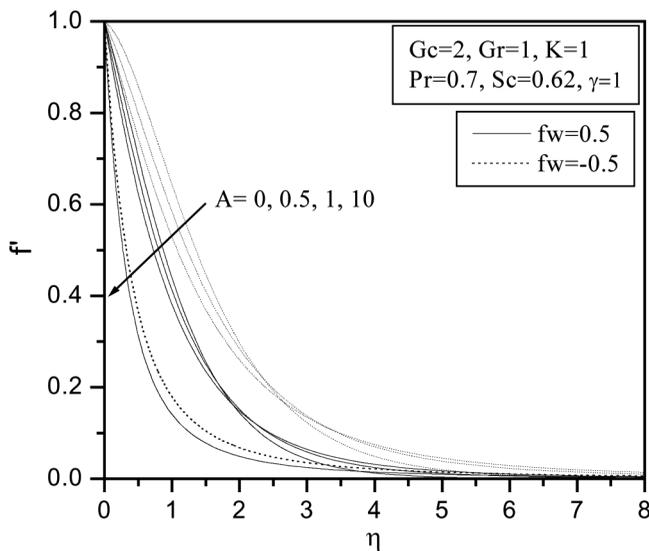


Figure 1. Velocity profiles for different values of the unsteadiness and suction/injection parameters.

Figures 4–6 show the influence of the chemical reaction parameter  $\gamma$  on the velocity, temperature, and concentration profiles in the boundary layer, respectively. Increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as  $\gamma$  increases. Consequently, less flow is induced along the plate, resulting in decreases

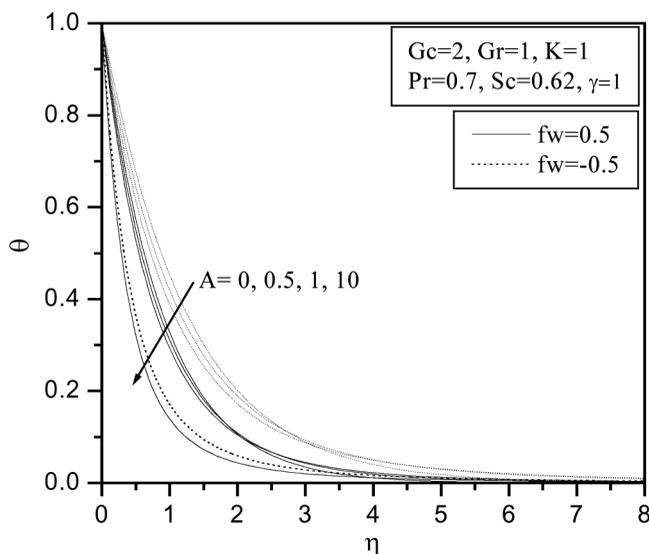
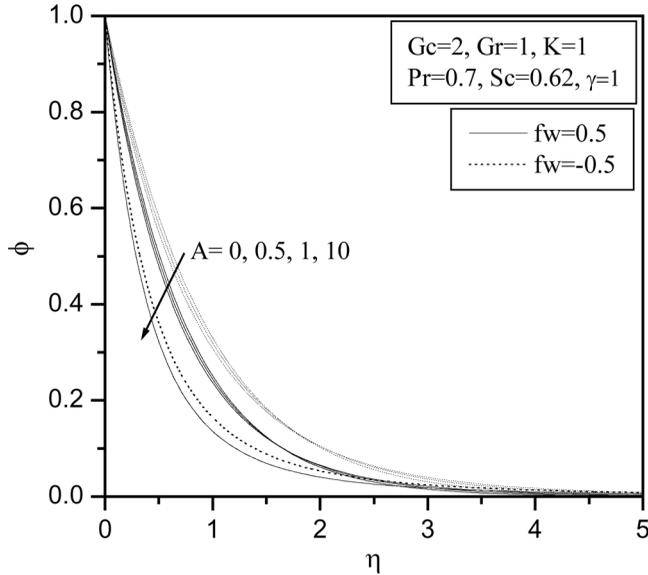


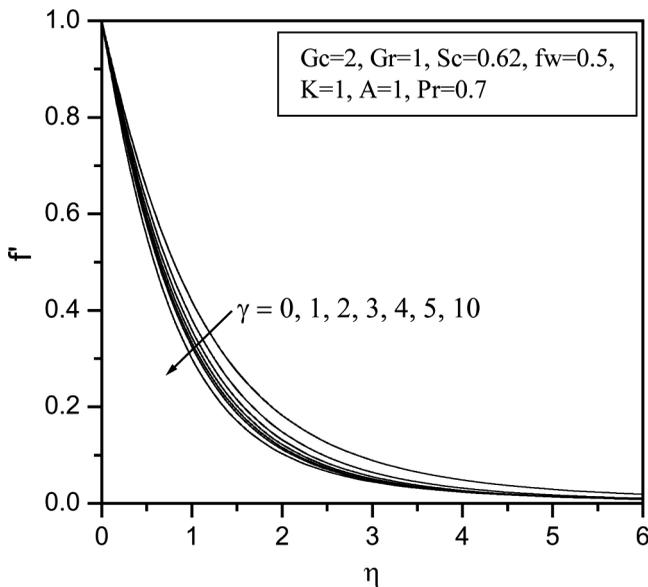
Figure 2. Temperature profiles for different values of the unsteadiness and suction/injection parameters.



**Figure 3.** Concentration profiles for different values of the unsteadiness and suction/injection parameters.

in the fluid velocity in the boundary layer. However, increasing the chemical reaction parameter leads to decreases in the temperature profiles.

Figures 7–9 present typical velocity, temperature, and concentration profiles in the boundary layer for various values of the permeability parameter  $K$  and two values of the suction/injection parameter  $fw$ , respectively. Increasing the value of



**Figure 4.** Velocity profiles for different values of the chemical reaction parameter.

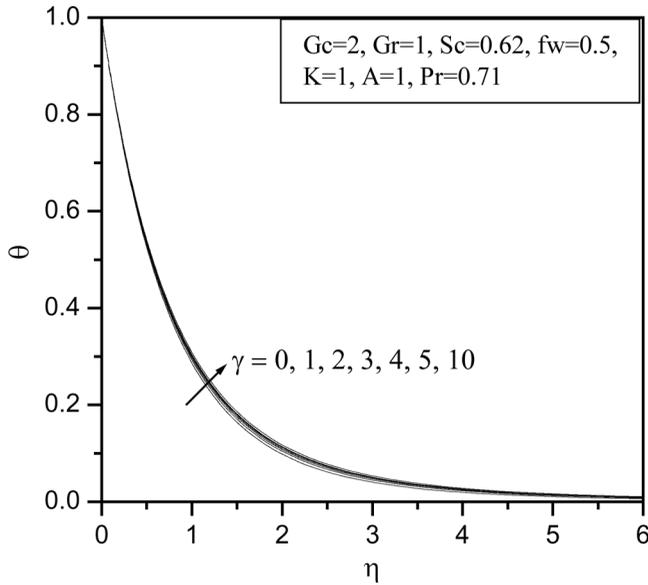


Figure 5. Temperature profiles for different values of the chemical reaction parameter.

$K$  has the tendency to resist the flow causing its velocity to decrease while its temperature and concentration species to increase. These behaviors are clearly depicted in Figs. 7–9.

Table II depicts the effects of some material parameters ( $A, Pr, Sc$ ) on the local skin-friction coefficient and the rates of heat and mass transfer for the fixed values  $\gamma = 1, K = 1, Gr = 1, Gc = 2$ , and  $fw = 0.5$ . It is clearly observed from this table

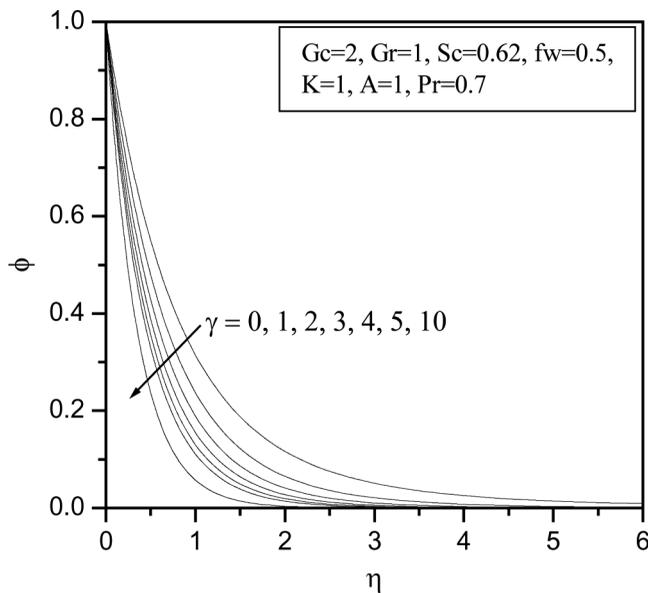
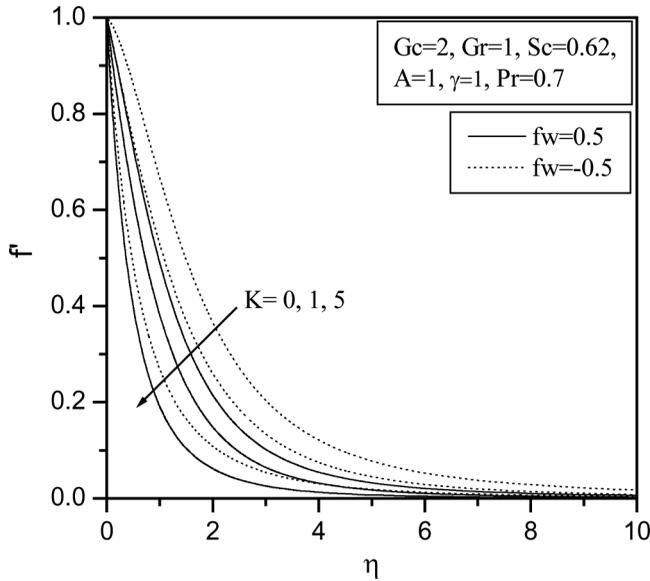
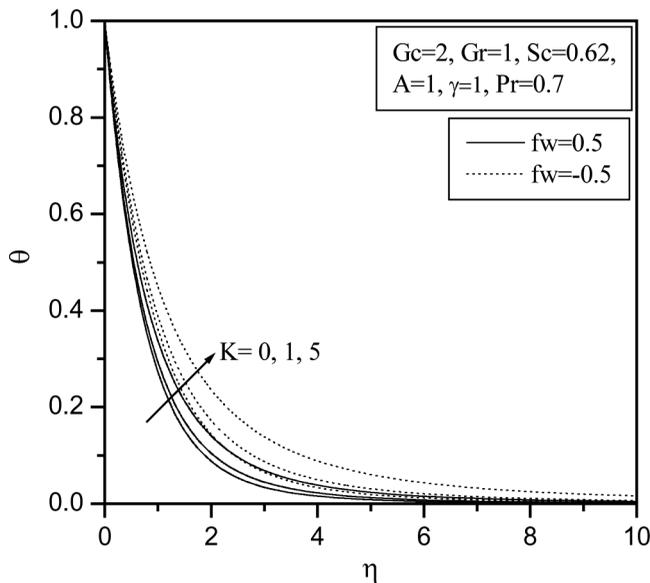


Figure 6. Concentration profiles for different values of the chemical reaction parameter.



**Figure 7.** Velocity profiles for different values of the permeability parameter.

that the skin-friction coefficient increases as the unsteadiness parameter, Prandtl number, and the Schmidt number  $Sc$  increase. Also, the rate of heat transfer or Nusselt number is predicted to increase due to increases in the Prandtl number  $Pr$  and it also increases as the unsteadiness parameter  $A$  increases, while it decreases as the the Schmidt number  $Sc$  increases. In addition, the Sherwood number is predicted to increase as a result of increasing the Schmidt number. It also increases



**Figure 8.** Temperature profiles for different values of the permeability parameter.

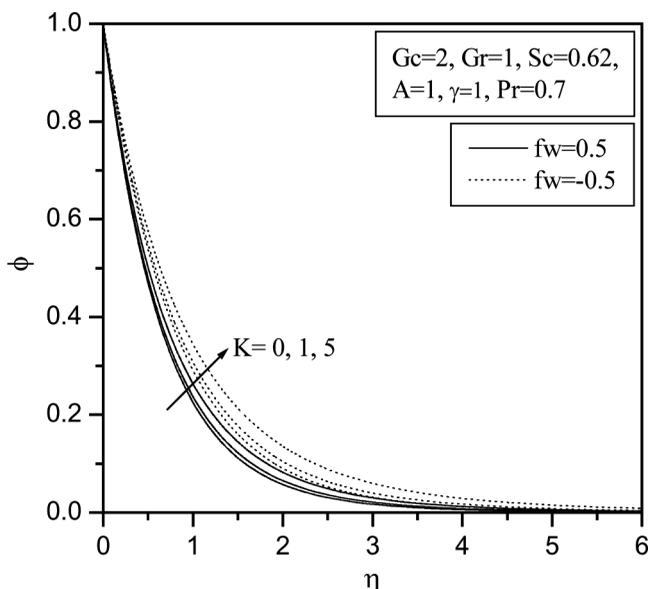


Figure 9. Concentration profiles for different values of the permeability parameter.

as the unsteadiness parameter increases, while it decreases as the Prandtl number is increased.

Table III illustrates the influence of the suction/injection parameter  $f_w$  and the chemical reaction parameter  $\gamma$  on the local skin friction coefficient, heat transfer rate, and mass transfer rate for  $Pr = 0.71$ ,  $Sc = 0.62$ ,  $K = 1$ ,  $Gr = 1$ ,  $Gc = 2$ , and  $A = 0.5$ . It is predicted that the skin-friction coefficient increases while the Nusselt number decreases as the chemical reaction parameter  $\gamma$  increases. On the other hand, the Sherwood number is predicted to increase as the chemical reaction parameter  $\gamma$  increases. In addition, local skin-friction coefficient and the heat and mass transfer

Table II. Local skin-friction coefficient, heat transfer, and mass transfer rates for various values of  $A$ ,  $Pr$ , and  $Sc$  with  $\gamma = 1$ ,  $K = 1$ ,  $Gr = 1$ ,  $Gc = 2$ , and  $f_w = 0.5$

$A$	$Pr$	$Sc$	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$	
0	0.71	0.22	0.27377	1.158393	0.74014	
		0.60	0.49677	1.101087	1.312248	
		0.94	0.59941	1.076931	1.713443	
	0.3	0.62	0.38639	0.62198	1.355045	
		0.71		0.50438	1.099032	1.337571
		1		0.55244	1.384875	1.331027
0	0.71	3	0.69042	2.966999	1.316710	
		0.62	0.50438	1.099032	1.337571	
		1	0.88473	1.324178	1.496687	
		2	1.215097	1.526933	1.648849	
10			2.830195	2.656791	2.599922	

**Table III.** Local skin-friction coefficient, heat transfer, and mass transfer rates for various values of  $f_w$  and  $\gamma$  with  $Pr=0.71$ ,  $Sc=0.62$ ,  $K=1$ ,  $Gr=1$ ,  $Gc=2$ , and  $A=0.5$

$f_w$	$\gamma$	$-f''(0)$	$-\theta'(0)$	$-\phi'(0)$
-0.5	0	0.24058	0.89991	0.84442
	1	0.30663	0.88423	1.126805
	1.5	0.33048	0.87877	1.245332
0	2	0.35067	0.87469	1.354389
	0	0.39918	1.052571	0.97562
	1	0.47544	1.034928	1.264505
0.5	1.5	0.50192	1.029298	1.385144
	2	0.52457	1.024883	1.495473
	0	0.61559	1.230465	1.126619
	1	0.69985	1.212891	1.417797
	1.5	0.72912	1.207257	1.538687
	2	0.75345	1.202608	1.649449

rates are higher for suction ( $f_w > 0$ ) than for injection ( $f_w < 0$ ), that is, they all increase as the suction/injection parameter increases.

## Conclusions

Similarity solutions for the unsteady boundary-layer flow due to a stretching surface embedded in a porous medium with suction or injection were obtained. Both the wall temperature and wall concentration were assumed to be constant. The governing equations for this problem were developed and nondimensionalized, and the resulting equations were then solved numerically using a fourth-order Runge-Kutta scheme with the shooting method. A parametric study illustrating the effects of the various parameters on the flow and heat and mass transfer characteristics was performed. It was found that, in general, the skin-friction coefficient increased as a result of increasing the unsteadiness parameter, suction/injection parameter, Prandtl number, Schmidt number, or the chemical reaction parameter. In addition, the Nusselt number was predicted to increase due to increases in the Prandtl number, suction/injection parameter, or the unsteadiness parameter, while it decreased as either the chemical reaction parameter or the Schmidt number increased. Furthermore, the Sherwood number was predicted to increase as a result of increasing the unsteadiness parameter, chemical reaction parameter, suction/injection parameter, or the Schmidt number, while it decreased as the Prandtl number increased.

## Nomenclature

$A$	unsteadiness parameter
$C$	concentration
$C_p$	specific heat diffusivity
$D$	mass diffusivity
$f'$	free stream velocity

$g$	acceleration due to gravity
$k$	thermal conductivity
$k_1$	permeability of porous medium
$m_w$	mass flux
$Nu_x$	Nusselt number
Pr	Prandtl number
$q_w$	local wall heat flux
$Sc$	Schmidt number
$Sh_x$	Sherwood number
$T$	temperature
$T_w$	wall temperature
$x$	horizontal coordinate
$y$	vertical coordinate

**Greek Letters**

$\beta_C$	coefficient of concentration expansion
$\beta_T$	coefficient of thermal expansion
$\gamma$	chemical reaction parameter
$\theta$	dimensionless temperature
$\nu$	kinematics viscosity
$\rho$	density of the fluid
$\phi$	dimensionless concentration
$\psi$	stream function

**Subscripts**

$w$	refers to wall condition
$\infty$	refers to ambient condition

**Superscript**

'	differentiation with respect to $\eta$
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