Non-Similar Solutions for Heat and Mass Transfer from a Surface Embedded in a Porous Medium for Two Prescribed Thermal and Solutal Boundary Conditions

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Non-Similar Solutions for Heat and Mass Transfer from a Surface Embedded in a Porous Medium for Two Prescribed Thermal and Solutal Boundary Conditions*

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Abstract

This work studies the effects of homogeneous chemical reaction and thermal radiation on coupled heat and mass transfer by free convection from a surface embedded in a fluid-saturated porous medium. Two different cases of thermal and solutal boundary conditions, namely the prescribed surface temperature and concentration (PSTC) case and the prescribed heat and mass fluxes (PHMF) case are considered. The governing boundary-layer equations are formulated and transformed into a set of non-similar equations. The resulting equations are solved numerically by an accurate and efficient implicit finite-difference method. The obtained results compared with previously published work and found to be in excellent agreement. A representative set of results is displayed graphically to illustrate the influence of the radiation parameter, chemical reaction parameter and the permeability of the porous medium on the velocity, temperature and concentration fields as well as the local skin-friction coefficient, local Nusselt number and the local Sherwood number.

KEYWORDS: chemical reaction, radiation, isothermal surface, heat flux, mass flux, adiabatic surface, plane plume, porous medium

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1. Introduction

Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. Convective flows in porous media have been extensively examined during the last several decades due to many practical applications which can be modeled or approximated as transport phenomena in porous media. Moreover, coupled heat and mass transfer problems in the presence of chemical reactions are of importance in many processes and have, therefore, received considerable amount of attention in recent years. Chemical reactions can occur in processes such as drying, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, evaporation at the surface of a water body, energy transfer in a wet cooling tower and flow in a desert cooler. Chemical reactions are classified as either homogeneous or heterogeneous processes. This depends on whether they occur at an interface or as a single-phase volume reaction. A homogeneous reaction is one that occurs uniformly throughout a given phase. On the other hand, a heterogeneous reaction takes place in a restricted area or within the boundary of a phase.

Soundalgekar (1977) presented an exact solution to the flow of a viscous fluid past an impulsively-started infinite plate with constant heat flux and chemical reaction. The solution was derived by the Laplace transform technique and the effects of heating or cooling of the plate on the flow field were discussed through the Grashof number. Das, et al. (1994) studied the effects of mass transfer on the flow past impulsively-started infinite vertical plate with constant heat flux in the presence of a chemical reaction. Muthucumaraswamy and Ganesan (1998) solved the problem of unsteady flow past an impulsively-started isothermal vertical plate with mass transfer numerically by an implicit finite-difference method. Muthucumaraswamy and Ganesan (2001) and Muthucumaraswamy and Ganesan (1999) also considered the problem of unsteady flow past an impulsively-started vertical plate with uniform heat and mass flux and variable temperature and mass flux, respectively. Diffusion of a chemically-reactive species from a stretching sheet was studied by Andersson, et al. (1994). Anjalidevi and Kandasamy (1999) and Anjalidevi and Kandasamy (2000) analyzed the effects of a chemical reaction and heat and mass transfer on laminar flow along a semi-infinite horizontal plate in the presence or absence of a magnetic field.

The study of the effect of radiation on MHD flow and heat transfer problem has become more important industrially. At high operating temperatures,

radiation effects can be quite significant. Many processes in engineering areas occur at high temperatures. a knowledge of the radiative heat transfer becomes very important for design of the pertinent equipment. Nuclear power plants, gas turbines and the various propulsion devices for aircraft, missiles, satellites and space vehicles are examples of such engineering areas. Bestman (1990) examined the natural convection boundary layer with suction and mass transfer in a porous medium. Makinde (2005) examined the transient free convection interaction with thermal radiation of an absorbing-emitting fluid along a moving vertical permeable plate. Carey and Mollendorf (1978) presented a boundary- layer similarity analysis for the laminar natural convection from a vertical isothermal surface. Their analysis was applicable to liquids wherein the viscosity variations are large compared to other fluid properties. Pantokratoras (2006) discussed the Falkner-Skan flow with constant wall temperature and variable viscosity. The therrmal radiation effect on natural convection in laminar boundary layer flow with constant properties over an isothermal flat plate was investigated by Ali, et al. (1984). Again, the radiative mode of heat transfer becomes important when the temperature difference between the plate surface and ambient is large. Yih (2001) studied the radiation effect on free convection over a vertical cylinder embedded in porous media. El-Hakiem and Rashad (2007) used the Rosseland diffusion approximation in studying the effect of radiation on free convection from a vertical cylinder embedded in a fluid-saturated porous medium. Chamkha and Ben-Nakhi (2008) considered MHD mixed convection-radiation interaction along a permeable surface immersed in a porous medium in the presence of Soret and Dufour effects. El-Hakiem (2009) studied radiative effects on non-Darcy natural convection from a heated vertical plate in saturated porous media with mass transfer for a non-Newtonian fluid. Murthy, et al. (2005) investigated mixed convection heat and mass transfer with thermal radiation in a non-Darcy porous medium. Salem (2006) investigated coupled heat and mass transfer in Darcy-Forchheimer mixed convection from a vertical flat plate embedded in a fluid-saturated porous medium under the effects of radiation and viscous dissipation. El-Amin, et al. (2008) studied the effects of chemical reaction and double dispersion on non-Darcy free convection heat and mass transfer. Mansour, et al. (2008) analyzed the effects of chemical reaction and thermal stratification on MHD free convection heat and mass transfer over a vertical stretching surface embedded in a porous media considering Soret and Dufour numbers. Postelnicu (2007) reported on the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.

The main objective of the present investigation is to study the effects of radiation and chemical reaction on natural convection flows of a Newtonian fluid along a vertical surface embedded in a porous medium for two thermal and solutal

boundary conditions, namely the prescribed surface temperature and concentration (PSTC) condition and the prescribed heat and mass fluxes (PHMF) condition. It is assumed that the temperature differences within the flow are sufficiently small so that T^4 may be expressed as a linear function of temperature. The value of this work lies in the use of different transformations for the PSTC and PHMF cases and obtaining the same non-similar equations for both cases.

2. Mathematical Analysis

Consider steady two-dimensional heat and mass transfer by natural convective flow of a viscous incompressible fluid along a vertical surface of different thermal and solutal boundary conditions embedded in a fluid-saturated porous medium in the presence of a chemical reaction and thermal radiation effects. The flow is taken to be in the direction of x-axis, which is taken along the plate and the y-axis normal to it. The temperature of the quiescent ambient fluid, T_∞ at large values of y, is taken to be constant. The fluid properties are assumed to be constant except the density in the buoyancy term of the momentum equation and the chemical reaction is assumed to be homogeneous, i.e. taking place in the flow and of first order. Under these assumptions and taking the Boussinesq approximation, the governing equations for the problem under consideration can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0, \tag{1}$$

$$u \frac{\partial u}{\partial x} + \upsilon \frac{\partial u}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + g\beta_T (T - T_\infty) + g\beta_C (C - C_\infty) - \frac{v}{k_1} u, \tag{2}$$

$$u \frac{\partial T}{\partial x} + \upsilon \frac{\partial T}{\partial y} = \frac{k}{\rho C_P} \frac{\partial^2 T}{\partial y^2} - \frac{1}{\rho C_P} \frac{\partial q_r}{\partial y}, \tag{3}$$

$$u \frac{\partial C}{\partial x} + \upsilon \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_c (C - C_\infty), \tag{4}$$

where x and y are the coordinate distances along and normal to the surface, respectively. u is the x-component of velocity, υ is the y-component of velocity, T is the fluid temperature, C is the solute concentration, ρ is the fluid density, v is the fluid kinematic viscosity, k_1 is the permeability of the porous medium,

k_c, C_P and g are the rate of chemical reaction, fluid specific heat at constant pressure and the acceleration due to gravity, respectively. β_T is the thermal expansion coefficient, β_c is the concentration expansion coefficient, D is the mass diffusion coefficient, C_∞ and T_∞ are the free stream dimensional concentration and temperature, respectively. q_r is the radiative heat flux.

The radiative heat flux term is simplified by using the Rosseland approximation [see Sparrow and Cess (1962)] as

$$q_r = -\frac{4\sigma_0}{3k^*} \frac{\partial T^4}{\partial y}, \tag{5}$$

where σ_0 and k^* are Stefan-Boltzmann constant and mean absorption coefficient, respectively. For sufficiently small temperature differences and following many previous investigators such as Ali et al. (1984), Mansour (1989), Mansour (1990), Perdikis and Raptis (1996), Raptis and Massalas (1998), Elbashbeshy and Bazid (2000), Ganesan and Loganathan (2002), El-Arabawy (2003) and Siddheshwar and Mahabaleswar (2005), Equation (5) is linearized using Taylor series expansion about T∞. The obtained Taylor series expansion for T^4 neglecting higher order terms is given by

$$T^4 = 4T_\infty^3 T - 3T_\infty^4, \tag{6}$$

Using Equations (5) and (6) in the energy equation (3) we obtain

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \frac{\nu}{\Pr}\left[1 + \frac{4}{3R}\right]\frac{\partial^2 T}{\partial y^2}, \tag{7}$$

where $\Pr = \dfrac{\rho\nu C_P}{k}$ is the Prandtl number and $R = \dfrac{k k^*}{4\sigma_0 T_\infty^3}$ is the dimensionless radiation parameter.

The relevant boundary conditions for each of the cases of the problem under consideration are:

(a) Prescribed Surface Temperature and Concentration (PSTC) case:

$$u = 0,\ \upsilon = 0,\ \mathrm{T} = \mathrm{T_w}(\mathrm{x})\ ,\mathrm{C} = \mathrm{C_w}(\mathrm{x}) \qquad\qquad \text{at } y = 0$$
$$u = 0,\ T = T_\infty,\ C = C_\infty \qquad\qquad\qquad \text{as } y \to \infty$$

(b) Prescribed Heat and Mass Fluxes (PHMF) case:

$$u = 0, \upsilon = 0, \quad \frac{\partial T}{\partial y} = -\frac{q_w(x)}{k}, \quad \frac{\partial C}{\partial y} = -\frac{m_w(x)}{\rho D} \qquad \text{at} \quad y = 0$$

$$u = 0, \ T = T_\infty, \ C = C_\infty \qquad\qquad\qquad \text{as } y \to \infty \qquad (8)$$

The suitable transformations for the PSTC case can be written as

$$\xi = x, \ \eta(x,y) = \frac{y}{x} Gr_x^{1/4}, \ \psi(x,y) = \nu Gr_x^{1/4} f(\eta,x),$$

$$\theta(\eta,x) = \frac{T - T_\infty}{T_w - T_\infty}, \ \phi(\eta,x) = \frac{C - C_\infty}{C_w - C_\infty}, \ T_w - T_\infty = T_0 x, \ C_w - C_\infty = C_0 x$$

$$Gr_x = \frac{g\beta_T x^3(T_w - T_\infty)}{\nu^2}, \ N = \frac{\beta_C(C_w - C_\infty)}{\beta_T(T_w - T_\infty)}, K = \frac{x^2}{2 k_1 Gr_x^{1/2}},$$

$$Sc = \frac{\nu}{D}, \ \gamma = \frac{k_c x^2}{\nu Gr_x^{1/2}} \qquad\qquad\qquad\qquad (9)$$

while the suitable transformations for the PHMF case are given by

$$\xi = x, \ \eta(x,y) = \frac{y}{x} Gr_x^{1/5}, \ \psi(x,y) = \nu Gr_x^{1/5} f(\eta,x),$$

$$T - T_\infty = \frac{q_w x}{k} Gr_x^{-1/5} \theta, \ C - C_\infty = \frac{m_w x}{\rho D} Gc_x^{-1/5} \phi, \ q_w = q_0 x, \ m_w = m_0 x$$

$$Gr_x = \frac{g\beta_T q_w}{k\nu^2} x^4, \ Gc_x = \frac{g\beta_C m_w}{\rho D\nu^2} x^4, \ N = \frac{Gc_x}{Gr_x}, K = \left(\frac{x^2}{k_1}\right)\frac{1}{(Gr_x)^{2/5}},$$

$$Sc = \frac{\nu}{D}, \gamma = \frac{x^2 k_c}{\nu Gr_x^{2/5}} \qquad\qquad\qquad\qquad (10)$$

Substituting Equations (9) or (10) in Equations (1), (2), (4) and (7) leads to the same general equation:

$$f''' + ff'' - f'^2 - Kf' + \theta + N\phi - \xi\left(f'\frac{\partial f'}{\partial \xi} - f''\frac{\partial f}{\partial \xi}\right) = 0 \qquad (11)$$

$$\left(\frac{1}{Pr}\right)\left(1 + \frac{4}{3R}\right)\theta'' + f\theta' - f'\theta - \xi\left(f'\frac{\partial \theta}{\partial \xi} - \theta'\frac{\partial f}{\partial \xi}\right) = 0 \qquad (12)$$

$$\frac{1}{Sc}\phi'' + f\phi' - f'\phi - \gamma\phi - \xi\left(f'\frac{\partial\phi}{\partial\xi} - \phi'\frac{\partial f}{\partial\xi}\right) = 0 \tag{13}$$

where K and γ are the permeability parameter and chemical reaction parameter, respectively, N is the buoyancy ratio and Sc is the Schmidt number.

 The dimensionless boundary conditions for each case become:

(a) Prescribed Surface Temperature and Concentration (PSTC) case:

$$f = \frac{\partial f}{\partial\eta} = 0, \quad \theta = 1, \quad \phi = 1 \qquad \text{on} \qquad \eta = 0$$

$$\frac{\partial f}{\partial\eta} = 0, \quad \theta = 0, \quad \phi = 0 \qquad \text{as} \qquad \eta \to \infty \tag{14a}$$

b) Prescribed Heat and Mass Fluxes (PHMF) case:

$$f = \frac{\partial f}{\partial\eta} = 0, \quad \frac{\partial\theta}{\partial\eta} = -1, \quad \frac{\partial\phi}{\partial\eta} = -1 \quad \text{on} \qquad \eta = 0$$

$$\frac{\partial f}{\partial\eta} = 0, \quad \theta = 0, \quad \phi = 0 \qquad \text{as} \qquad \eta \to \infty \tag{14b}$$

 The above system of non-similar equations (11)-(13) with the boundary conditions (14) have been solved numerically by using an accurate implicit iterative finite-difference method for various values of parameters.

 The local skin-friction coefficient, local Nusselt number and the local Sherwood number are important parameters for this problem. They are defined for each case as follows:

(a) Prescribed Surface Temperature and Concentration (PSTC) case:

The wall shear stress and the local skin-friction coefficient C_{fx} can be written as

$$\tau_w = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \rho\nu^2\frac{Gr_x^{3/4}}{x^2}f''(\xi,0)$$

$$C_{fx} = \frac{\tau_w}{\rho\nu^2/x^2} = Gr_x^{3/4}f''(\xi,0) \text{ or } C_f = C_{fx}Gr_x^{-3/4} = f''(\xi,0) \tag{15}$$

The surface heat flux $q_w(x)$ and the local Nusselt number Nu_x are defined by:

$$q_w = -k \left.\frac{\partial T}{\partial y}\right|_{y=0} - \frac{4\sigma_0}{3k^*}\left(\frac{\partial T^4}{\partial y}\right)_{y=0} = \left[-\theta'(\xi,0)\right](T_w - T_\infty)\frac{k}{x}[Gr_x]^{\frac{1}{4}} \times \left(1 + \frac{4}{3R}\right),$$

$$Nu_x = \frac{q_w}{(T_w - T_\infty)}\left(\frac{x}{k}\right) = -\theta'(\xi,0)(Gr_x)^{1/4}\left(1 + \frac{4}{3R}\right), \text{ or}$$

$$Nu = Nu_x Gr_x^{-1/4} = -\left(1 + \frac{4}{3R}\right)\theta'(\xi,0) \tag{16}$$

The surface mass flux m_w and the local Sherwood number Sh_x are given by:

$$m_w = -\rho D \left.\frac{\partial C}{\partial y}\right|_{y=0} = \left[-\phi'(\xi,0)\right](C_w - C_\infty)\frac{\rho D}{x}(Gr_x)^{1/4},$$

$$Sh_x = \frac{m_w}{(C_w - C_\infty)}\left(\frac{x}{\rho D}\right) = -\phi'(\xi,0)(Gr_x)^{1/4}, \text{ or } Sh = Sh_x Gr_x^{-1/4} = -\phi'(\xi,0) \tag{17}$$

b) Prescribed Heat and Mass Fluxes (PHMF) case:

In a similar way, the local skin-friction coefficient, local Nusselt number and the local Sherwood number for this case take on the respective forms:

$$C_{fx} = Gr_x^{3/5} f''(\xi,0) \quad \text{or} \quad C_f = C_{fx} Gr_x^{-3/5} = f''(\xi,0) \tag{18}$$

$$Nu_x = Gr_x^{1/5}\left(\frac{1}{\theta(\xi,0)}\right), \quad \text{or} \quad Nu = Nu_x Gr_x^{-1/5} = \frac{1}{\theta(\xi,0)} \tag{19}$$

$$Sh_x = Gr_x^{1/5}\left(\frac{1}{\phi(\xi,0)}\right), \quad \textbf{or} \quad Sh = Sh_x Gr_x^{-1/5} = \frac{1}{\phi(\xi,0)} \tag{20}$$

3. Numerical Method and Validation

The heat and mass transfer problem for the surface embedded in a porous medium represented by Equations (11) through (13) are nonlinear and, therefore, must be solved numerically. The standard implicit finite-difference method discussed by Blottner (1970) has proven to be adequate and gives accurate results for such equations. For this reason, it is employed in the present work and graphical results based on this method are presented subsequently.

Equations (11)-(13) are discretized using three-point central difference formulae with f' replaced by another variable V. The η direction is divided into 196 nodal points and a variable step size is used to account for the sharp changes in the variables in the region close to the cylinder surface where viscous effects dominate. The initial step size used is $\Delta\eta_1 = 0.001$ and the growth factor $K = 1.0375$ such that $\Delta\eta_n = K\Delta\eta_{n-1}$ (where the subscript n is the number of nodes minus one). This gives $\eta_{max} \approx 35$ which represents the edge of the boundary layer at infinity. The ordinary differential equations are then converted into linear algebraic equations that are solved by the Thomas algorithm discussed by Blottner (1970). Iteration is employed to deal with the nonlinear nature of the governing equations. The convergence criterion employed in this work was based on the relative difference between the current and the previous iterations. When this difference or error reached 10^{-5}, then the solution was assumed converged and the iteration process was terminated. It is possible to compare the results obtained by this numerical method with the previously published work of Erbas and Ece (1998) and Ece and Erbas (1999) for free convection flow of power-law fluids along a vertical plate with variable wall temperature and heat flux. Comparisons were made for Newtonian fluids. Tables 1 and 2 show the results of these comparisons. It is evident from these comparisons that these results are in excellent agreement. This lends confidence in the numerical results to be reported next.

Table 1. Comparison of values of C_f and Nu with Erbas and Ece (1998) for $Pr = 100$, $K=0$, $N = 0$, $R = \infty$, $\delta = 0$ and $\gamma = 0$ for prescribed surface temperature and concentration (PSTC) case.

ξ	C_f		Nu	
	Erbas and Ece (1998)	Present work	Erbas and Ece (1998)	Present work
0.0	0.285835	0.285809	2.211639	2.211931

Table 2. Comparison of values of C_f and Nu with Ece and Erbas (1999) for $Pr = 100$, $K=0$, $N = 0$, $R = \infty$, $\delta = 0$ and $\gamma = 0$ for prescribed heat and mass fluxes (PHMF) case.

ξ	C_f		1/Nu	
	Ece and Erbas (1999)	Present work	Ece and Erbas (1999)	Present work
0.0	0.177540	0.177514	0.529945	0.529930

4. Results and Discussion

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical illustrations. A representative set of results is

shown in Figs. 1-16. Computations were carried out for various values of physical parameters such as the chemical reaction parameter γ, permeability parameter K and the radiation parameter R.

   Figures 1-3 show the influence of the chemical reaction parameter γ on the velocity, temperature and concentration profiles in the boundary layer for both the Prescribed Surface Temperature and Concentration (PSTC) and the Prescribed Heat and Mass Flux (PHMF) cases, respectively. For both cases of surface boundary conditions, increasing the chemical reaction parameter produces a decrease in the species concentration. In turn, this causes the concentration buoyancy effects to decrease as γ increases. Consequently, less flow is induced along the plate resulting in decreases in the fluid velocity in the boundary layer. Also, increasing the chemical reaction parameter leads to increase in the temperature profiles. In general, it is also observed that away from the free stream conditions, the velocity, temperature and concentration profiles are higher for the PHMF case than those corresponding to the PSTC case. In addition, as expected the wall temperature and concentration change as γ changes for the PHMF case while they are fixed for the PSTC case. These behaviors are clearly depicted in Figs. 1-3.
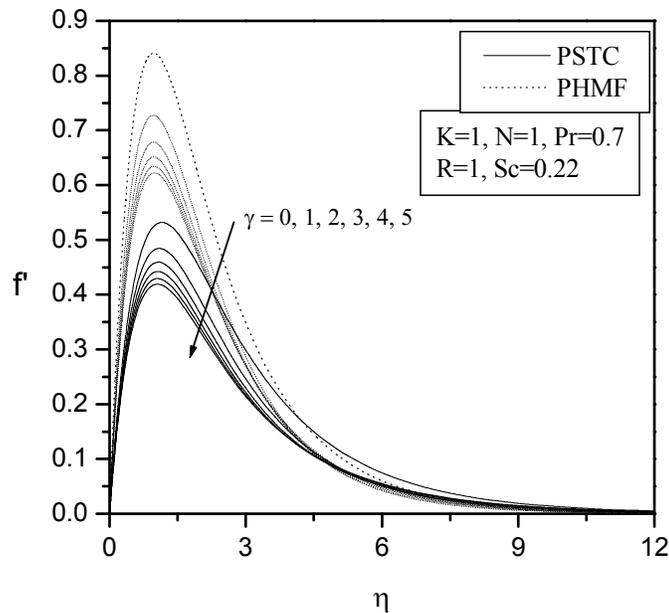
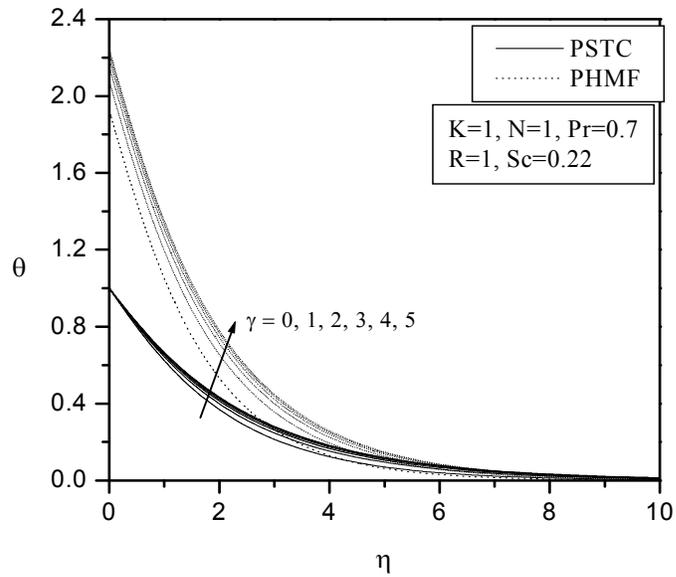Fig. 1. Effects of chemical reaction parameter on the velocity profiles

Fig. 2. Effects of chemical reaction parameter on the temperature profiles
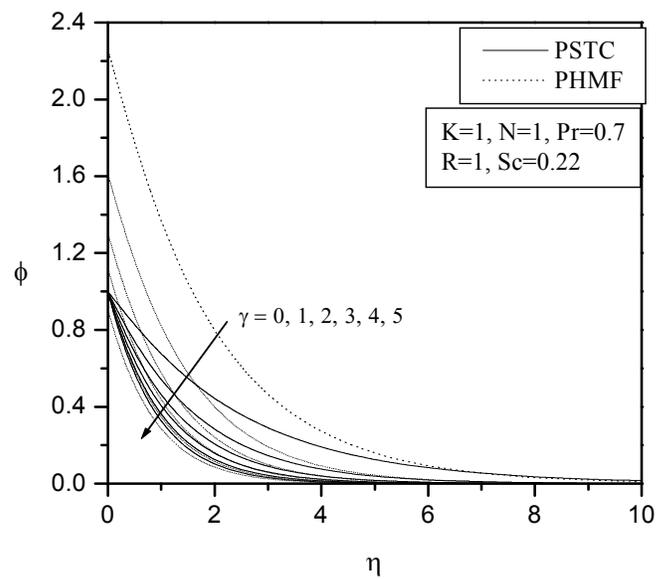

Fig. 3. Effects of chemical reaction parameter on the concentration profiles

Figures 4-6 elucidate the influence of the chemical reaction parameter γ on the axial distributions of the local skin-friction coefficient C_f, local Nusselt number Nu and the local Sherwood number Sh for both the Prescribed Surface Temperature and Concentration (PSTC) and the Prescribed Heat and Mass Flux (PHMF) cases, respectively. As seen from Figs. 1-3, the wall slope of the velocity profile for the PSTC and PHMF cases, which corresponds to the local skin-friction coefficient, decreases as the chemical reaction parameter γ increases. This causes the local skin-friction coefficient C_f to decrease for both the PSTC and the PHMF cases. Similarly, the local Nusselt number (local Sherwood number) which is directly proportional to the wall slope of the temperature (concentration) profile for the PSTC case and inversely proportional to the wall temperature (concentration) for the PHMF case are predicted to decrease (increase) as the chemical reaction parameter γ increases. In addition, it is clear that the dependence of all of C_f, Nu and Sh on the dimensional axial distance ξ for both the PSTC and PHMF cases. It should be noted here that for the parametric conditions employed to obtain the results in Figs. 4-6, the values of C_f and Sh are higher while the values of Nu are lower for the PHMF case than for the PSTC case.
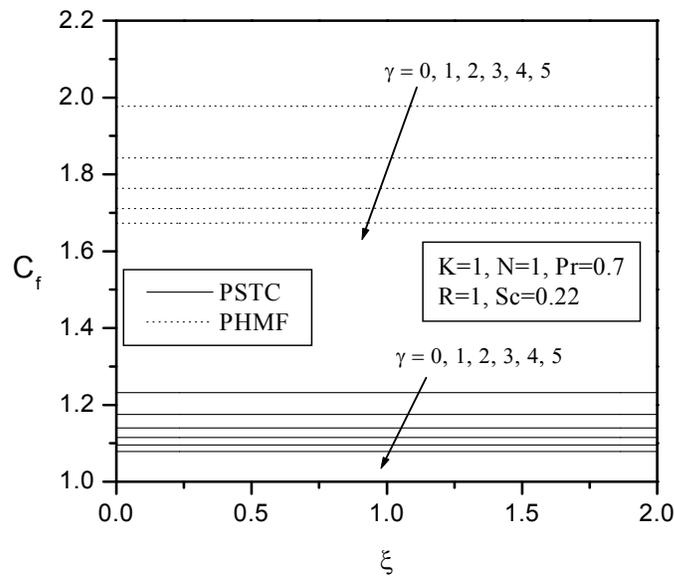

Fig. 4. Effects of chemical reaction parameter on the local skin-friction coefficient
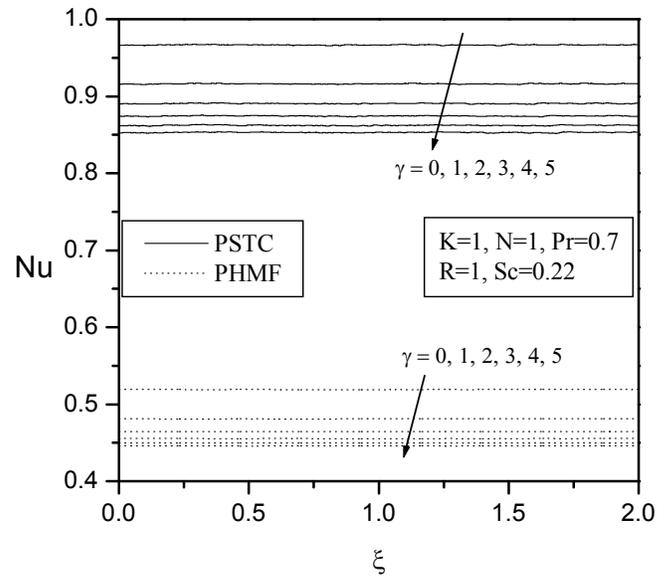
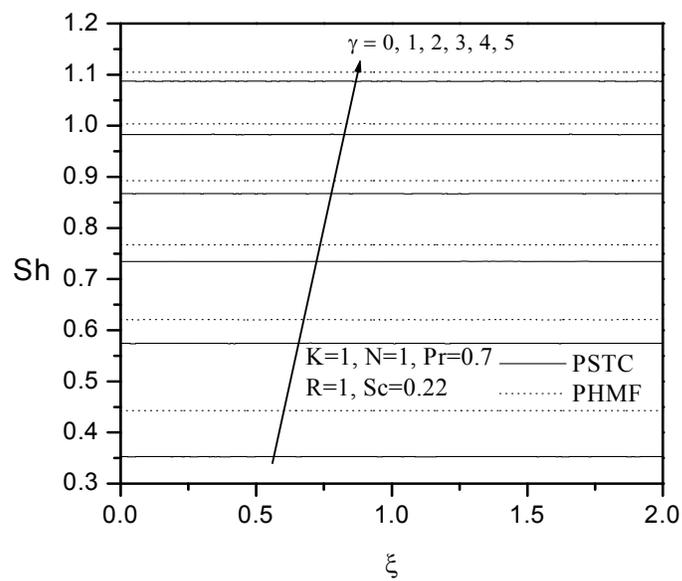Fig. 5. Effects of chemical reaction parameter on the local Nusselt number

Fig. 6. Effects of chemical reaction parameter on the local Sherwood number

Figures 7 and 8 display the effects of the thermal radiation parameter R and the permeability parameter K on the velocity and temperature profiles in the boundary layer for the PSTC and PHMF cases, respectively. Decreasing the thermal radiation parameter R (i.e. increasing radiation effects) produces a significant increase in the thermal state of the fluid causing its temperature to increase. This increase in the fluid temperature induces, through the effect of thermal buoyancy, more flow in the boundary layer causing the velocity of the fluid there to increase. This is true for both the PSTC and PHMF cases. In addition, the presence of a porous medium presents an obstacle to flow resulting in reduced flow. Therefore, as the permeability parameter K increases, the fluid velocity decreases and the fluid temperature increases for both the PSTC and PHMF cases.
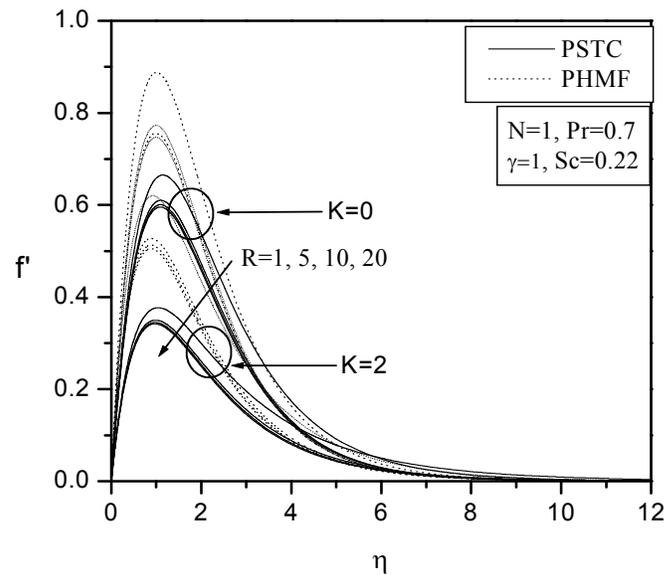

Fig. 7. Effects of radiation and permeability parameters on the velocity profiles
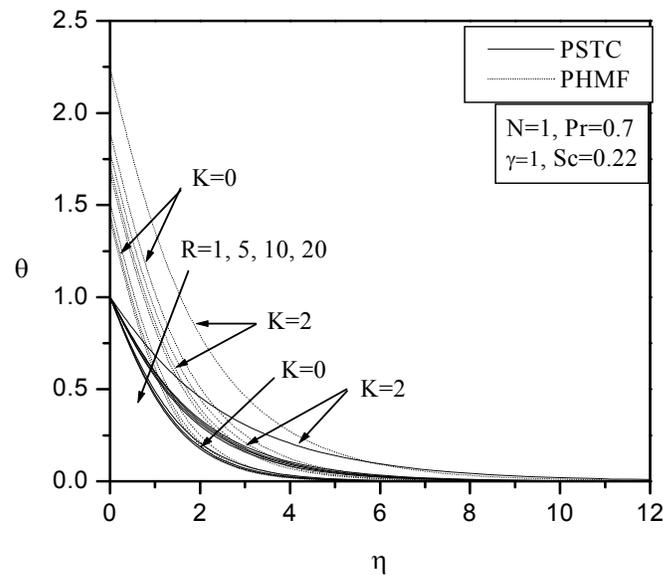
Fig. 8. Effects of radiation and permeability parameters on the temperature profiles

Figures 9 and 10 show the effects of the thermal radiation parameter R and the permeability parameter K on the axial distributions of the local skin-friction coefficient C_f and the local rate of heat transfer or Nusselt number Nu for both the PSTC and PHMF cases, respectively. It is observed that as the thermal radiation parameter R increases, the local skin-friction coefficient and the local Nusselt number decrease for both the PSTC and PHMF cases. In addition, it is predicted that the local skin-friction coefficient decreases while the local Nusselt number increases as the permeability parameter K increases for both the PSTC and PHMF cases. Again, the values of C_f are higher while the values of Nu are lower for the PHMF case than for the PSTC case.
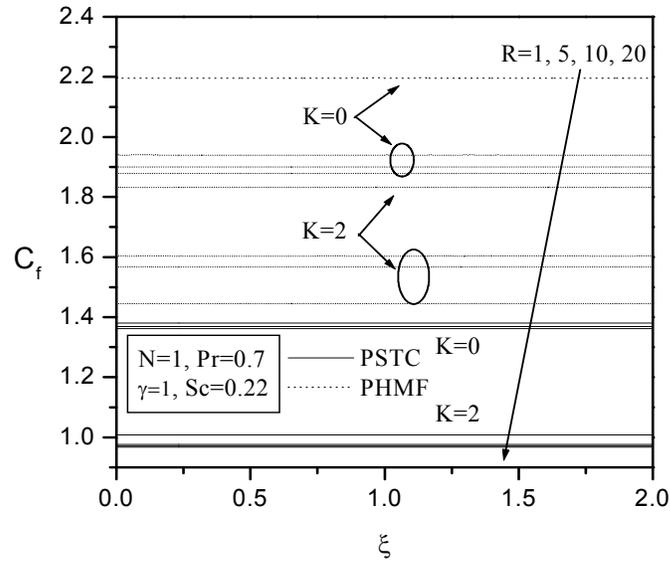
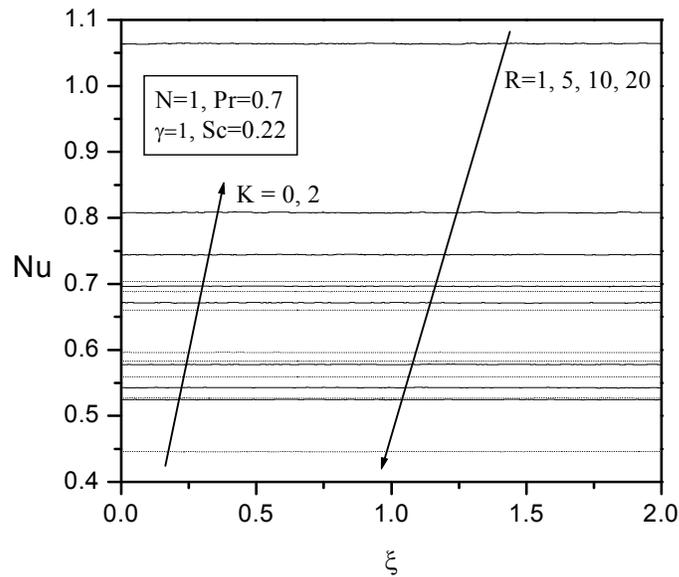Fig. 9. Effects of radiation and permeability parameters on the local skin-friction coefficient

Fig. 10. Effects of radiation and permeability parameters on the local Nusselt number

Figures 11-13 illustrate the effects of the buoyancy ratio N on the velocity, temperature and concentration profiles for both the PSTC and PHMF cases, respectively. For N=0 the flow is induced only by the thermal buoyancy effect due to temperature gradient. However, for N>0 the flow is induced by both the temperature and concentration gradients. In this case, the flow is aided by the concentration buoyancy effect. Therefore, as N increases, the flow along the plate increases at the expense of decreased temperature and concentration values. These decreases in both the temperature and concentration values are accompanied by corresponding decreases in both the thermal and solutal (concentration) boundary layer thicknesses. The increased flow is characterized by the formation of distinctive peaks in the velocity profiles in the immediate vicinity of the plate surface as N increases. However, the velocity appear to decrease far downstream as N increases causing decreases in the hydrodynamic boundary layer thickness. The behaviors are true for both the PSTC and PHMF cases.
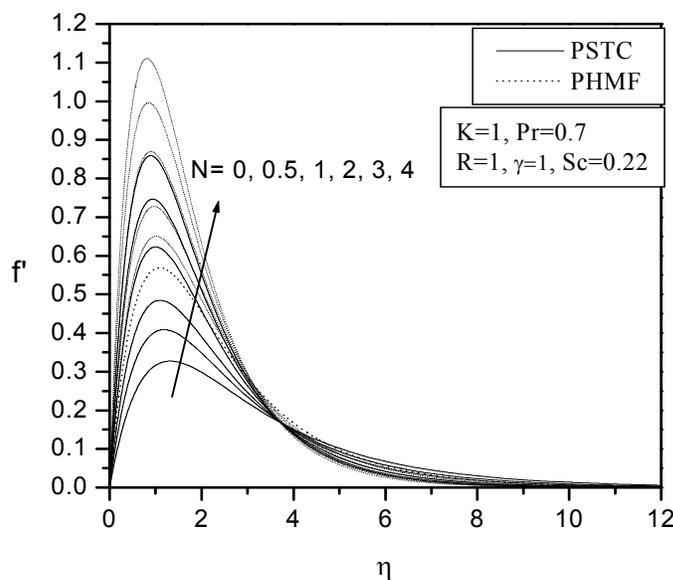

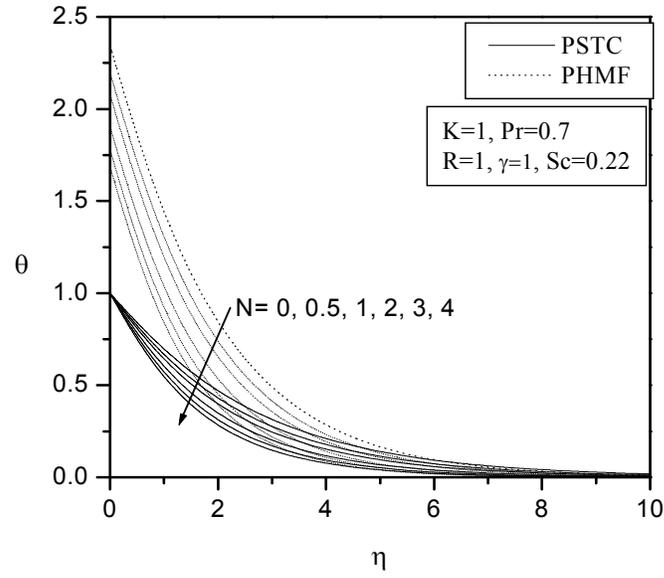Fig. 11. Effects of buoyancy parameter on the velocity profiles

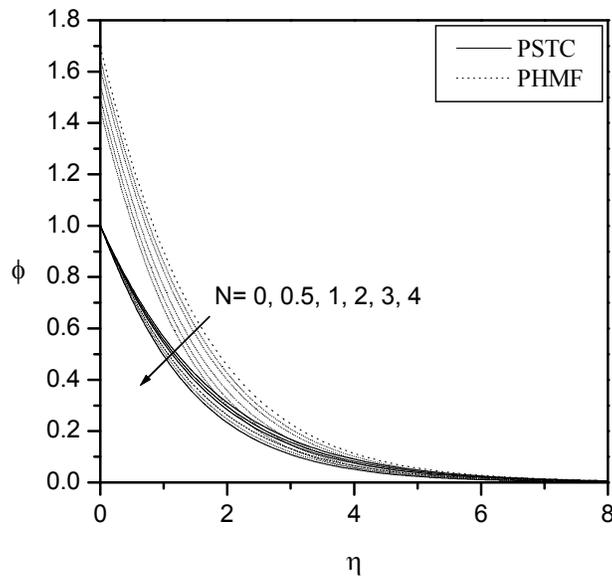Fig. 12. Effects of buoyancy parameter on the temperature profiles

Fig. 13. Effects of buoyancy parameter on the concentration profiles

Finally, Figs. 14-16 illustrate the influence of the buoyancy ratio N on the distributions of the local skin-friction coefficient C_f and the local Nusselt and Sherwood numbers Nu and Sh along the plate for both the PSTC and PHMF cases, respectively. Obviously, increasing the buoyancy ratio has the tendency to increase the flow which causes more friction and increased tendency for heat and mass transfer. This is depicted in the increases in all of C_f, Nu and Sh as N increases in Figs. 14-16 for both the PSTC and PHMF cases. Again, the values of C_f and Sh are higher while the values of Nu are lower for the PHMF case than for the PSTC case.
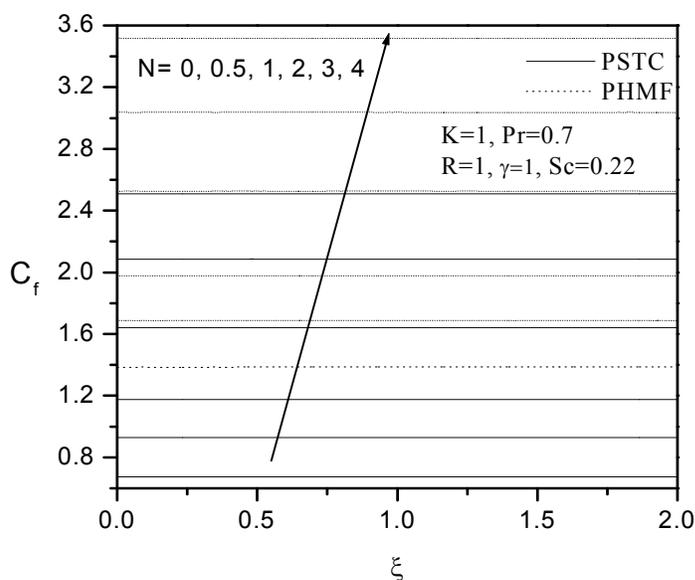

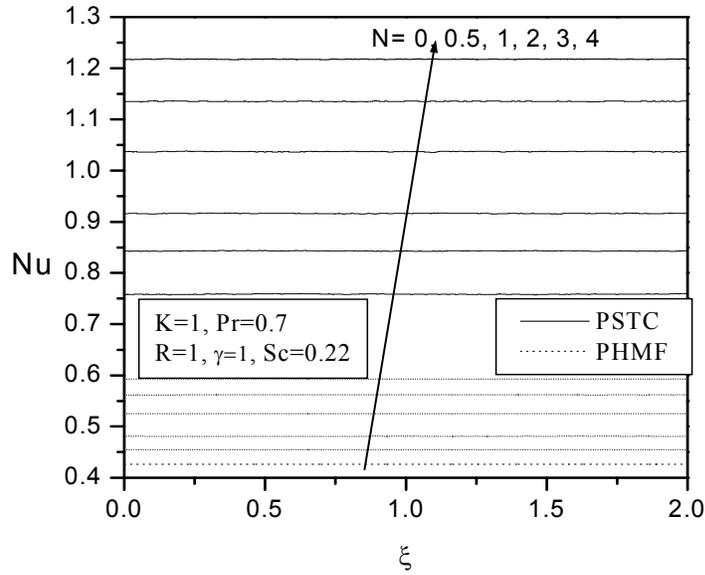Fig. 14. Effects of buoyancy parameter on the local skin-friction coefficient

Fig. 15. Effects of buoyancy parameter on the local Nusselt number
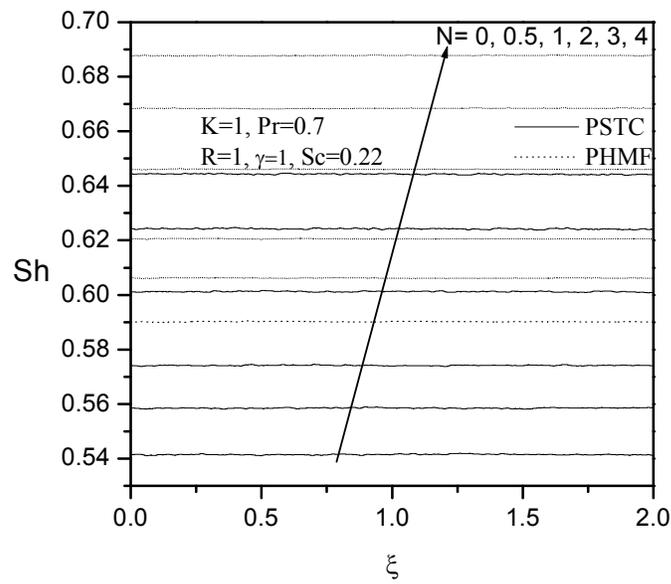

Fig. 16. Effects of buoyancy parameter on the local Sherwood number

5. Conclusions

The problem of natural convective flow along a vertical surface embedded in a porous medium in the presence of homogeneous chemical reaction and thermal radiation effects was investigated. Two different cases of thermal and solutal boundary conditions, namely the prescribed surface temperature and concentration (PSTC) case and the prescribed surface heat and mass fluxes (PHMF) case were studied. The governing boundary-layer equations were formulated, non-dimensionalized and then transformed into a set of non-similarity equations which were solved numerically by an efficient, iterative, tri-diagonal, implicit finite-difference method. It was found that, in general, the fluid velocity, temperature and concentration profiles for the PHMF were predicted to be higher than those for the PSTC case. In addition, the local skin-friction coefficient increased as the buoyancy ratio increased and it decreased as a result of increasing either of the chemical reaction parameter, the permeability parameter or the thermal radiation parameter. Moreover, the local Nusselt number increased due to increases in the either of the buoyancy ratio or the permeability parameter while it decreased as the chemical reaction parameter or the thermal radiation parameter increased. Furthermore, the local Sherwood number was predicted to increase as a result of increasing either of the buoyancy ratio or the chemical reaction parameter.

Nomenclature			
C	Concentration	T_∞	Ambient Temperature
C_p	Specific heat diffusivity	x	Horizontal co-ordinate
D	Mass Diffusivity	y	Vertical co-ordinate
f'	Free stream velocity		
Gr_x	Local Grashof number	**Greek Symbols**	
Gc_x	Local modified Grashof number	β_T	Coefficient of thermal expansion
g	Acceleration due to gravity	β_C	Coefficient of concentration expansion
k_1	Permeability of porous medium	γ	Chemical reaction parameter
k	Thermal conductivity	v	Kinematic viscosity
k^*	Mean absorption coefficient	ϕ	Dimensionless concentration
m_w	Mass flux	θ	Dimensionless temperature
N	Buoyancy ratio	ψ	Stream function
Nu	Nusselt number	ρ	Density of the fluid
Pr	Prandtl number	σ_0	Stefan-Boltzmann constant

q_w	Local wall heat flux	**Subscripts**	
q_r	Radiative heat flux	w	Refers to wall condition
Sc	Schmidt number	∞	Refers to ambient condition
Sh	Sherwood number	**Superscript**	
T	Temperature	'	Differentiation with respect to η
T_w	Wall temperature	x	Local value

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