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Heat and Mass Transfer by MHD Stagnation-Point Flow of a Power-Law Fluid towards a Stretching Surface with Radiation, Chemical Reaction and Soret and Dufour Effects

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Heat and Mass Transfer by MHD Stagnation-Point Flow of a Power-Law Fluid towards a Stretching Surface with Radiation, Chemical Reaction and Soret and Dufour Effects

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Abstract

The thermal-diffusion and diffusion-thermo effects on heat and mass transfer by magnetohydrodynamic (MHD) mixed convection stagnation-point flow of a power-law non-Newtonian fluid towards a stretching surface in the presence of a magnetic field, thermal radiation and homogenous chemical reaction effects have been studied. A suitable set of dimensionless variables is used and similar equations governing the problem are obtained. The resulting equations have the property that they reduce to various special cases previously considered in the literature. An adequate implicit tri-diagonal finite-difference scheme is employed for the numerical solution of the obtained equations. Various comparisons with previously published work are performed and the results are found to be in excellent agreement. Representative results for the velocity, temperature, and concentration profiles as well as the local skin-friction coefficient, the local Nusselt number and the local Sherwood number illustrating the influence of the magnetic parameter, power-law fluid index, mixed convection parameter, concentration to thermal buoyancy ratio, thermal radiation, chemical reaction, and Dufour and Soret numbers are presented and discussed.

KEYWORDS: heat and mass transfer, non-Newtonian fluid, stagnation-point flow, Dufour and Soret effects, radiation, chemical reaction

1. Introduction

A number of important practical fluids such as molten plastic, food stuff, polymers, slurries, etc., are non-Newtonian in their flow characteristics. Due to the growing use of these non-Newtonian substances in various manufacturing and processing industries, considerable efforts have been directed towards understanding their friction and heat transfer characteristics. Many of the inelastic non-Newtonian fluids encountered in chemical engineering processes and biochemical industries are known to follow the empirical Ostwald–de Waele model (see, Metzner (1965)), or the so-called power-law viscosity model in which the shear stress varies according to a power function of the strain rate. However, because such fluids have more complicated equations that relate the shear stress to the velocity field than Newtonian fluids have, additional factors must be considered in examining various fluid mechanics and heat transfer phenomena.

Furthermore, the study of magnetohydrodynamic (MHD) flow of an electrically-conducting fluid is of considerable interest in modern metallurgical and metal-working processes. There has been a great interest in the study of magnetohydrodynamic flow and heat transfer in any medium due to the effect of a magnetic field on the boundary-layer flow control and on the performance of many systems using electrically-conducting fluids. This type of flow has attracted the interest of many researchers due to its application in many engineering problems such as MHD generators, plasma studies, nuclear reactors, and geothermal energy extractions. By the application of a magnetic field, hydromagnetic techniques are used for the purification of molten metals from non-metallic inclusions. Therefore, the type of problem that we are dealing with is very useful to polymer technology and metallurgy. The corresponding problem of two-dimensional stagnation-point flow of a power-law fluid towards a rigid surface was investigated by Kapur and Srivastava (1963). The extension of the same problem to the axi-symmetric case was studied by Maiti (1965) and later on by Koneru and Manohar (1968). Sapunkov (1967) investigated two-dimensional orthogonal stagnation-point flow of a power-law fluid towards a rigid surface in the presence of a uniform transverse magnetic field. Djukic (1974) studied hydromagnetic Hiemenz flow of a power-law fluid towards a rigid plate. Mahapatra and Gupta (2001) analyzed steady orthogonal stagnation-point flow of an electrically-conducting fluid towards a stretching surface. Mahapatra, et al. (2009) employed the homotopy analysis method to find analytical solutions for magnetohydrodynamic viscous stagnation-point flow of a power-law fluid over a stretching surface.

On other hand, thermal radiation effects with chemical reactions on free and forced convection flow and mass transfer have public importance such as the combustion of fossil fuels, atmospheric re-entry with suborbital velocities, plasma wind tunnels, electric spacecraft propulsion, hypersonic flight through planetary atmosphere photo-dissociation, photo ionization, and geophysics. Analytical solutions for the study of heat and mass transfer on MHD flow of a stretched vertical surface with the effects of heat generation and chemical reaction were presented by Chamkha (2003). The effects of radiation and chemical reaction on MHD free convective flow and mass transfer past a vertical cone surface were investigated by Afify (2004). The effects of chemical reaction and thermal radiation on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media in the presence of magnetic field were studied by Seddeek, et al. (2005). Ibrahim, et al. (2008) studied the effect of chemical reaction and radiation absorption on unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate. Pal and Talukdar (2010) investigated the combined effects of thermal radiation and first-order chemical reaction on heat and mass transfer by MHD mixed convection flow past a vertical plate saturated a porous medium. Rashad, et al. (2010) studied coupled heat and mass transfer by mixed convection about solid sphere saturated porous medium in the presence of chemical reaction effect.

Finally, in industrial and chemical engineering processes which involve multi-component fluid, concentrations vary from point to point resulting in mass transfer. Energy flux can be generated not only by temperature gradient but also by concentration gradient as well. The energy flux caused by concentration gradient is called the Dufour effect and the same by temperature gradient is called the Soret effect. These effects are very significant when temperature and concentration gradients are high. The importance of these effects in convective transport of clear fluids has been studied by Bergaman and Srinivasan (1989) and Zimmerman, et al. (1992). Heated jets or diffusion flames created by blowing combustible gas from a vertical pipe are controlled by forced convection in the initial region and by buoyancy forces far from the jet or pipe exit. Industrial smoke stacks usually have a significant momentum flux to assist the initial rise of contaminant plume. The simplest physical model of such a flow is two-dimensional laminar flow along a vertical flat plate. Recent applications of this model can be found in the area of reactor safety, combustion flames and solar collectors as well as building energy conservation. Baron (1963) has studied thermal diffusion effects in mass transfer. Sparrow, et al. (1964) have considered diffusion-thermo effects in stagnation-point flow of air with injection of gases of various molecular weights into the boundary layer. Li, et al. (2006) used an implicit finite-volume method to investigate thermal-diffusion (Soret) and diffusion-themo (Dufour) effects in a strongly endothermic chemically-reacting flow in a porous medium. Postelnicu (2007) has discussed the influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces embedded in fluid-saturated porous medium considering Soret and

Dufour effects. Chamkha and Ben-Nakhi (2008) studied the Dufour and Soret effects on heat and mass transfer by mixed convection from a vertical permeable plate embedded in porous media in the presence of thermal radiation and magnetic field. Bég, et al. (2009) have analyzed free convection magnetohydrodynamic heat and mass transfer from a stretching surface to a saturated porous medium with Soret and Dufour effects.

Hence, based on the above discussion, the purpose of the present problem is to generalize the work of Mahapatra, et al. (2009) and consider the effects of chemical reaction and thermal radiation on coupled heat and mass transfer by MHD mixed convection stagnation-point flow of a non-Newtonian fluid towards a stretching surface. The order of the chemical reaction in this work is taken as first-order reaction.

2. Governing Equations

Consider steady, laminar, heat and mass transfer by mixed convection, boundary layer stagnation-point flow of an electrically-conducting, optically dense and non-Newtonian power-law fluid obeying the Ostwald-de Waele model (see, Metzner (1965)) past a heated or cooled stretching vertical surface in the presence of thermal radiation, chemical reaction and thermal-diffusion and the diffusion-thermo effects. It is assumed that the stretching velocity is given by $U_w(x) = ax$, and the velocity distribution in frictionless potential flow in the neighborhood of the stagnation point at $x = y = 0$ is given by $U(x)=cx$. The stretching surface is maintained at a constant temperature T_w and a constant concentration C_w, and the ambient temperature and concentration far away from the surface of the surface T_∞ and C_∞ are assumed to be uniform. For $T_w \succ T_\infty$ and $C_w \succ C_\infty$ an upward flow is induced as a result of the thermal and concentration buoyancy effects. A uniform magnetic field is applied in the y-direction normal to the flow direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. In addition, the Hall effect and the electric field are assumed negligible. The small magnetic Reynolds number assumption uncouples the Navier-Stokes equations from Maxwell's equations. A first-order homogeneous chemical reaction is assumed to take place in the flow. All physical properties are assumed constant except the density in the buoyancy force term. By invoking all of the boundary layer, Boussineq and Rosseland diffusion approximations, the governing equations for this investigation can be written as;

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\mathrm{d}U}{\mathrm{d}x}+\frac{1}{\rho}\frac{\partial \tau_{xy}}{\partial y}+g\beta(T-T_\infty)+g\beta^*(C-C_\infty)-\frac{\sigma B_0^2}{\rho}(u-U)\,, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}+\frac{Dk_T}{C_sC_p}\frac{\partial^2 C}{\partial y^2}\,, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}+\frac{Dk_T}{T_m}\frac{\partial^2 T}{\partial y^2}-K_1(C-C_\infty)\,, \tag{4}$$

and the relevant boundary conditions are given by

$$\begin{aligned}&u=u_w(x)=cx\,,\ v=0\,,\ T=T_w\,,\ C=C_w\ \text{ at }\ y=0\,,\\ &u=U(x)=ax\,,\ v=-ay\,,\ T=T_\infty\,,\ C=C_\infty\ \text{ at }\ y\to\infty\,,\end{aligned} \tag{5}$$

where u, v, T, C are the fluid x-component of velocity, y-component of velocity, temperature, and concentration, respectively. g, ρ, α, D, β_T, and β_C are the gravitational acceleration, kinematic viscosity, thermal diffusivity, mass diffusivity, coefficient of thermal expansion, and coefficient of concentration of expansion, respectively. σ, B_0, q_r, and K_1 are the electrical conductivity, magnetic induction, mass diffusivity, radiative heat flux, and the dimensional of chemical reaction, respectively. C_p, T_m, k_T and C_s are the specific heat at constant pressure, mean fluid temperature, thermal diffusion ratio and concentration susceptibility. In the present problem, we have $\partial u/\partial y > 0$ when $a/c > 1$ (the ratio of free stream velocity and stretching velocity) which gives the shear stress as:

$$\tau_{xy}=K\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n$$

where K is the consistency coefficient and n is the power-law fluid. If $n<1$, the fluid is called pseudo-plastic power-law fluid and if $n>1$, it is called dilatant power-law fluid since the apparent viscosity decreases(shear-thinning) or increases with the increase in shear rate(shear-thickening) accordingly as $n<1$ or $n>1$.

In addition, the radiative heat flux q^r is described according to the Rosseland approximation such that:

$$\frac{\partial q^r}{\partial y}=-\frac{4\sigma_1}{3\chi}\frac{\partial T^4}{\partial y}\quad, \tag{6}$$

where σ_1 and χ are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. As done by Raptis (1998a,1998b), the fluid-phase temperature differences within the flow are assumed to be sufficiently small so that T^4 may be expressed as a linear function of temperature. This is done by expanding T^4 in a Taylor series about the free-stream temperature T_∞ and neglecting higher-order terms to yield

$$T^4 = 4T_\infty^4 T - 3T_\infty^4 . \tag{7}$$

By using Eqs. (6) and (7) in the last term of Eq. (3), we obtain

$$\frac{\partial q^r}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3\chi}\frac{\partial^2 T}{\partial y^2}, \tag{8}$$

Applying the following transformations

$$\psi = \left(\frac{K/\rho}{c^{1-2n}}\right)^{1/(n+1)} x^{2n/(n+1)} f(\eta),\ \theta = \frac{T - T_\infty}{T_w - T_\infty},\ \phi = \frac{C - C_\infty}{C_w - C_\infty},$$
$$\eta = y\left(\frac{c^{2-n}}{K/\rho}\right)^{1/(n+1)} x^{(1-n)/(1+n)}, \tag{9}$$

where $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ define the stream function ψ, the governing equations (2)-(4) (taking Eq. (8) into account) become

$$n(f'')^{(n-1)} f''' + \left(\frac{2n}{n+1}\right) ff'' - f'^2 - Mf' + M\frac{a}{c} + \frac{a^2}{c^2} + \Lambda(\theta + N\phi) = 0, \tag{10}$$

$$\frac{1}{\Pr}\left(1 + \frac{4R_d}{3}\right)\theta'' + \left(\frac{2n}{n+1}\right) f\theta' + Df\phi'' = 0, \tag{11}$$

$$\frac{1}{Sc}\phi'' + \left(\frac{2n}{n+1}\right) f\phi' + Sr\theta'' - \gamma\phi = 0, \tag{12}$$

and the transformed dimensionless boundary conditions become

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1,\ \phi(0) = 1,$$
$$f'(\infty) \to a/c,\ \theta(\infty) \to 0,\ \phi(\infty) \to 0, \tag{13}$$

where M is the magnetic parameter, Λ is the dimensionless mixed convection parameter, N is the concentration to thermal buoyancy ratio, Pr is the generalized Prandtl number, Sc is the generalized Schmidt number, R_d is the radiation parameter, γ is the dimensionless of chemical reaction parameter, *Df* is the Dufour number and *Sr* is the Soret number. These parameters are given by

$$M = \frac{\sigma B_0^2}{\rho c}, \ \Lambda = \frac{g\beta(T_w - T_\infty)x^3/\upsilon^2}{u_w^2 x^2/\upsilon^2} = \frac{Gr_x}{\mathrm{Re}_x^2}, \ N = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)},$$
$$\mathrm{Pr} = \frac{\rho C_p c x^2}{k}\mathrm{Re}_x^{-2/(n+1)}, \ Sc = \frac{cx^2}{D}\mathrm{Re}_x^{-2/(n+1)}, \ R_d = \sigma_1 T_\infty^3/3k\chi, \ \gamma = K_1/c,$$
$$Du = \frac{Dk_T(C_w - C_\infty)cx^2}{C_s C_p (T_w - T_\infty)}\mathrm{Re}_x^{-2/(n+1)}, \ Sr = \frac{Dk_T(T_w - T_\infty)cx^2}{T_m(C_w - C_\infty)}\mathrm{Re}_x^{-2/(n+1)}. \quad (14)$$

with $Gr_x = g\beta(T_w - T_\infty)x^3/\upsilon^2$ being the local Grashof number, $\mathrm{Re}_x = u_w x/\upsilon$ is the local Reynolds number and *k* is the thermal conductivity. It should be noted that Λ> 0 corresponds to an assisting flow (heated plate), Λ< 0 corresponds to an opposing flow (cooled plate) and Λ= 0 yields forced convection flow.

The skin-friction coefficient C_f at the wall is given by:

$$C_f = 2[f''(0)]^n \left(\frac{(cx)^{2-n} x^n}{K/\rho}\right)^{-1/(1+n)}, \quad (15)$$

the local Nusselt number Nu_x is given by:

$$Nu_x = -K\left(\frac{u_w^{2-n}}{K/\rho}\right)^{1/n+1}\left(1 + \frac{4R_d}{3}\right)\theta'(0), \quad (16)$$

and the local Sherwood number Sh_x is given by:

$$Sh_x = -D\left(\frac{u_w^{2-n}}{K/\rho}\right)^{1/n+1}\phi'(0). \quad (17)$$

3. Results and Discussions

In order to get a clear insight of the physical problem, numerical results are displayed with the help of graphical and tabulated illustrations. The system of equations (10)-(12) with the boundary conditions (13) were solved numerically by means of an efficient, iterative, tri-diagonal implicit finite-difference method discussed previously by Blottner (1970). It is possible to compare the results obtained by this numerical method with the previously published work of Mahapatra, et al. (2009). They employed the homotopy analysis method to find the analytical solutions of the magnetohydrodynamic viscous stagnation-point flow of a power-law fluid over a stretching surface in the absence of heat and mass transfer. Table 1 shows that excellent agreement between the results exists. This lends confidence in the numerical results to be reported subsequently. Computations were carried out for various values of parameters, the value of Prandtl number Pr = 0.78 which corresponds physically to metal ammonia and the value of Schmidt number Sc = 0.6 which represents water vapor. The value of the Schmidt number (Sc) is chosen to represent the most common diffusing chemical species which are of interest and the values of Dufour and Soret numbers are chosen in such a way that their product is constant provided that the mean temperature T_m is kept constant as well. The results of this parametric study are shown in Figs. 1-8.

Table 1. Comparison of $f''(0)$ for various values of M at n=1, between analytical solutions obtained by homotopy analysis method and numerical method in the present work in the absence of heat and mass transfer.

M	Mahapatra, et al. (2009) Analytical results	Present results
0.0	2.0175	2.017798
0.5	2.1363	2.136279
1.0	2.2491	2.249317
1.5	2.3567	2.356882
2.0	2.4597	2.459712
3.0	2.6540	2.653975
5.0	3.0058	3.005763
10.0	3.7447	3.744703
20.0	4.9004	4.901177
1000.0	31.6858	31.69113

Figures 1(a) through 1(c) illustrate the effects of the fluid power-law index n on the velocity f', temperature θ and concentration ϕ profiles at two different values of the magnetic parameter M, respectively. It can be noted that the velocity profile for dilatant fluids (1<n<2) is larger than that for pseudo-plastic fluids (0<n<1), i.e., the effect of increasing the value of the power-law index parameter n is to reduce all of the velocity, temperature and concentration profiles. Also, the effect of increasing the values of power-law index n leads to thinning of the thermal and concentration boundary thickness and thereby reducing the boundary-layer thickness i.e. the thickness is much larger for thermal and concentration thinning (pseudo plastic) fluids (0<n<1) than those of Newtonian (n = 1) and thermal and concentration thickening (dilatant) fluids (1<n<2), as clearly seen from Fig. 1. On the other hand, application of a uniform magnetic field normal to the flow direction produces a force which acts in the negative direction of flow. This force is called the Lorentz force which tends to slow down the movement of the electrically-conducting fluid in the vertical direction. This retardation effect is accompanied by an appreciable increase in the fluid temperature and concentration. These behaviors are clearly depicted in Figs. 1(a) through 1(c).

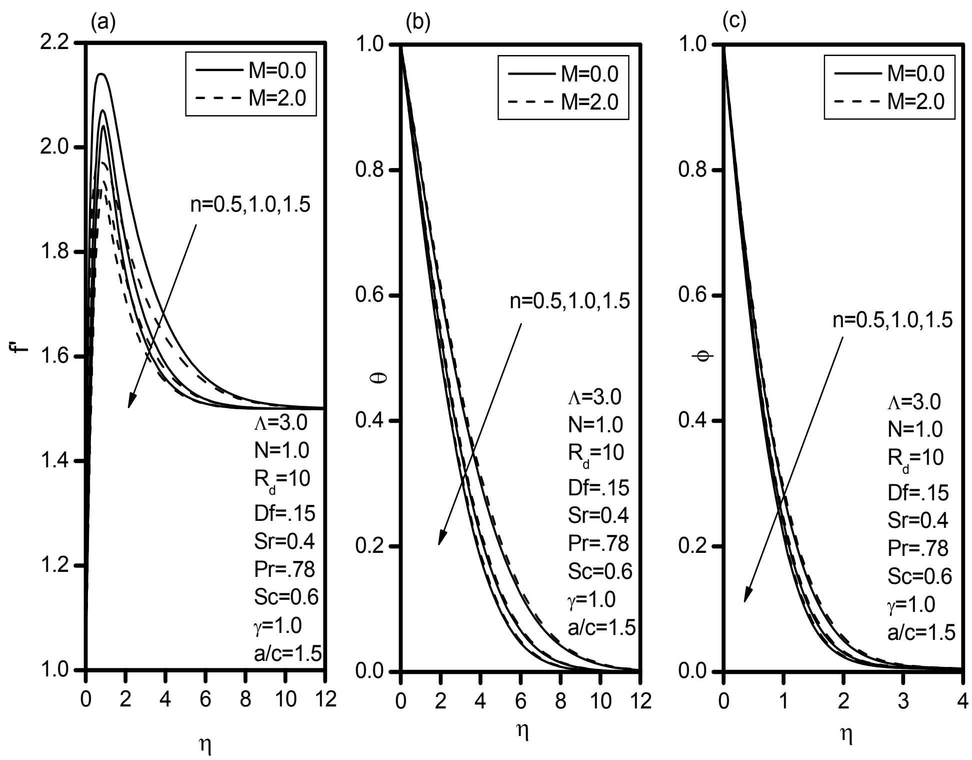

Figure 1. Effects of M and n on the (a) velocity, (b) temperature, (c) concentration profiles.

Figures 2(a)-2(c) illustrate the effect of increasing the strength of the magnetic field *M*, and power-law index *n* on the local skin-friction coefficient C_f, local Nusselt number Nu_x and the local Sherwood number Sh_x, respectively. As seen from the definitions of C_f, Nu_x and Sh_x, they are directly proportional to $f''(0)$, $-(1+4R_d/3)\theta'(0)$ and $-\phi'(0)$, respectively. For this reason, they are shown in figures 2-8. It can be seen that for a fixed value of *n*, as the magnetic parameter *M* increases, the resistance to flow and the wall slopes of the temperature and concentration profiles increase as shown earlier in Figs. 1(a)-1(c), this produces reductions in all the skin-friction coefficient and the Nusselt and Sherwood numbers. In addition, as seen from Figs. 2(a)-2(c), increases in the value of the power-law index *n* cause enhancement in all of the skin-friction coefficient and local Nusselt and Sherwood numbers.

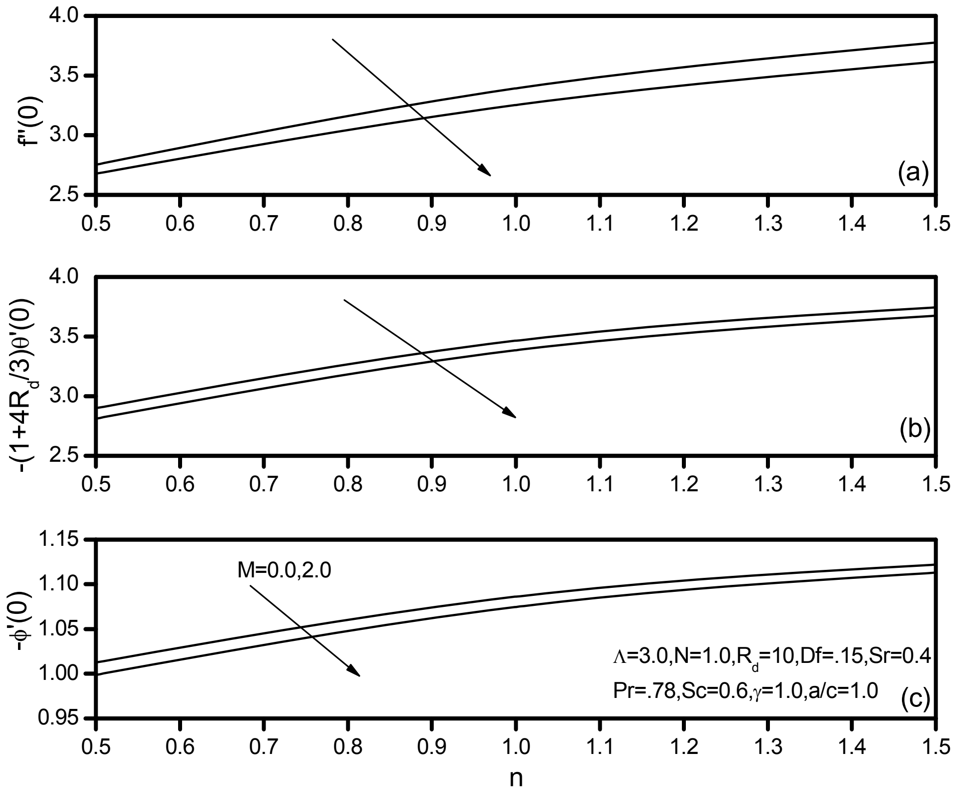

Figure 2. Effects of M and n on the (a) local skin-friction coefficient, (b) local Nusselt number, (c) local Sherwood number.

The effects of the ratio of velocity parameter a/c and the mixed parameter Λ for buoyancy aiding/opposing flow on the velocity and temperature and concentration profiles are presented in Figs. 3(a)-3(c), respectively. For a given value of Λ, increasing the ratio of velocity parameter a/c has a tendency to accelerate the flow. This, in turn, produces increases in the maximum velocity and decreases in both of the fluid temperature and concentration. Similarly, the presence of the thermal buoyancy effects represented by finite values of the mixed parameter has the tendency to induce more flow along the surface at the expense of small reductions in the temperature and concentration. This is reflected in the increases in velocity and slight decreases in both of the fluid temperature and concentration as Λ increases shown in Figs. 3(a)-3(c), respectively. Distinctive peaks in the velocity profiles which are characteristics of free-convection flows are also observed as Λ increases.

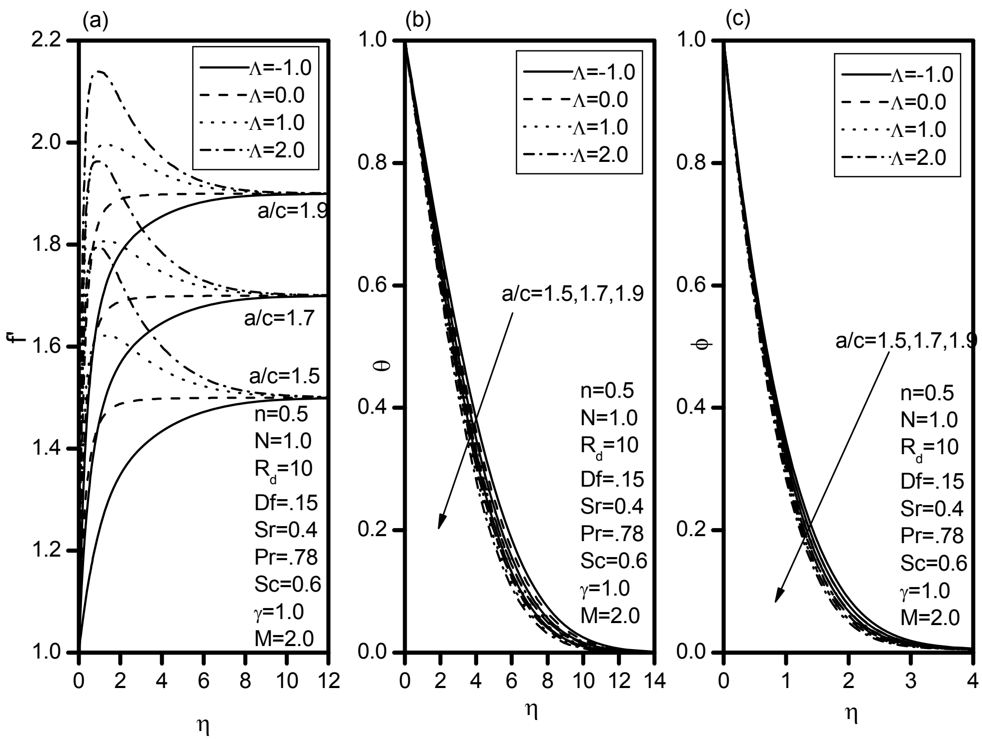

Figure 3. Effects of a/c and Λ on the (a) velocity, (b) temperature, (c) concentration profiles.

Figures 4(a)-4(c) show the variations of the local skin-friction coefficient and local Nusselt and Sherwood numbers against the mixed convection parameter Λ for different values of the ratio of velocity parameter a/c, respectively. For $\Lambda>0$ (assisting flow), there is a favorable pressure gradient due to the buoyancy forces which result in flow acceleration and consequently, there is a larger skin-friction coefficient than in the non-buoyant case ($\Lambda= 0$) or opposing flow case ($\Lambda< 0$). Therefore, increasing the mixed convection parameter Λ enhances the local skin-friction coefficient as well as the local the Nusselt and Sherwood numbers. On the other hand, it is found that increasing the ratio of velocity parameter a/c leads to increases in all of the local skin-friction coefficient and the local Nusselt and Sherwood numbers for both aiding and opposing flows.

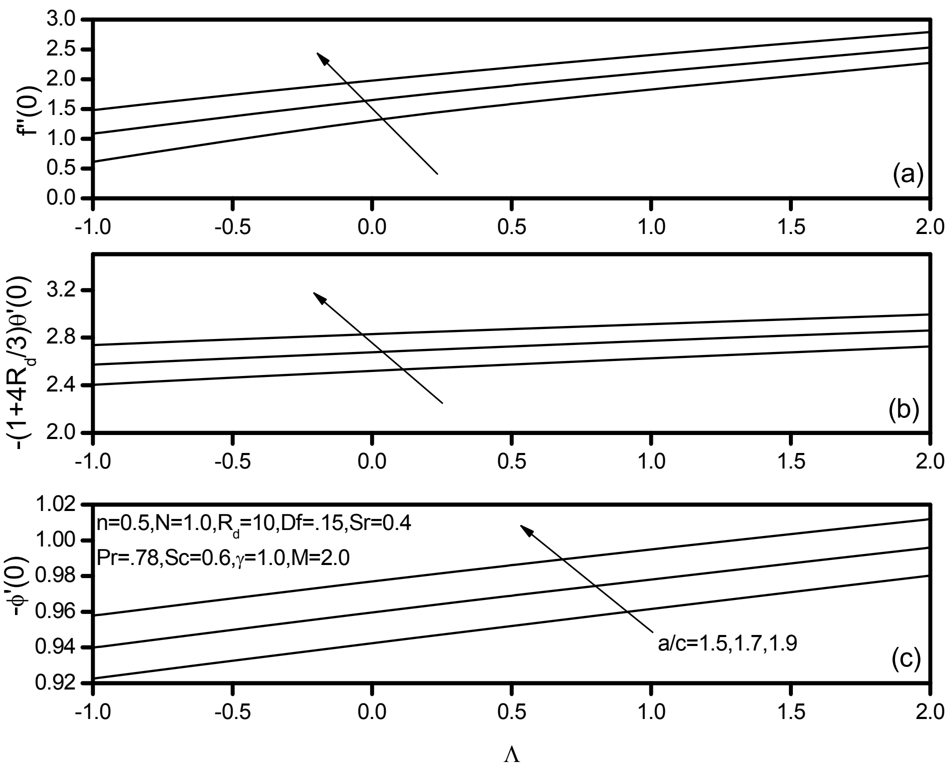

Figure 4. Effects of a/c and Λ on the (a) local skin-friction coefficient, (b) local Nusselt number, (c) local Sherwood number.

The effects of the chemical reaction γ on the velocity, temperature and concentration profiles are shown in Figs. 5(a)-5(c), respectively. It is seen that both the velocity and concentration decrease with the increase of the chemical reaction parameter γ, while the temperature profiles show insignificant changes as γ increases. It is evident that the concentration boundary-layer thickness gradually reduces from the boundary to the centre of the model (see Fig. 5(c)) when the chemical reaction parameter γ increases. In addition, the condition $R_d = 0$ corresponds to the case where no thermal radiation is present. The presence of thermal radiation enhances the thermal state of the fluid causing its temperature to increase and the solute concentration to decrease as seen from Figs. 5(b)-5(c). This, in turn, increases the thermal buoyancy effect and decreases the concentration buoyancy effect. However, since the increase in the thermal buoyancy effect is much higher than that of the decrease in the concentration buoyancy effect, higher induced flow along the plate in produced. This is depicted in the increases in the fluid velocity as R_d increases as seen from 5(a).

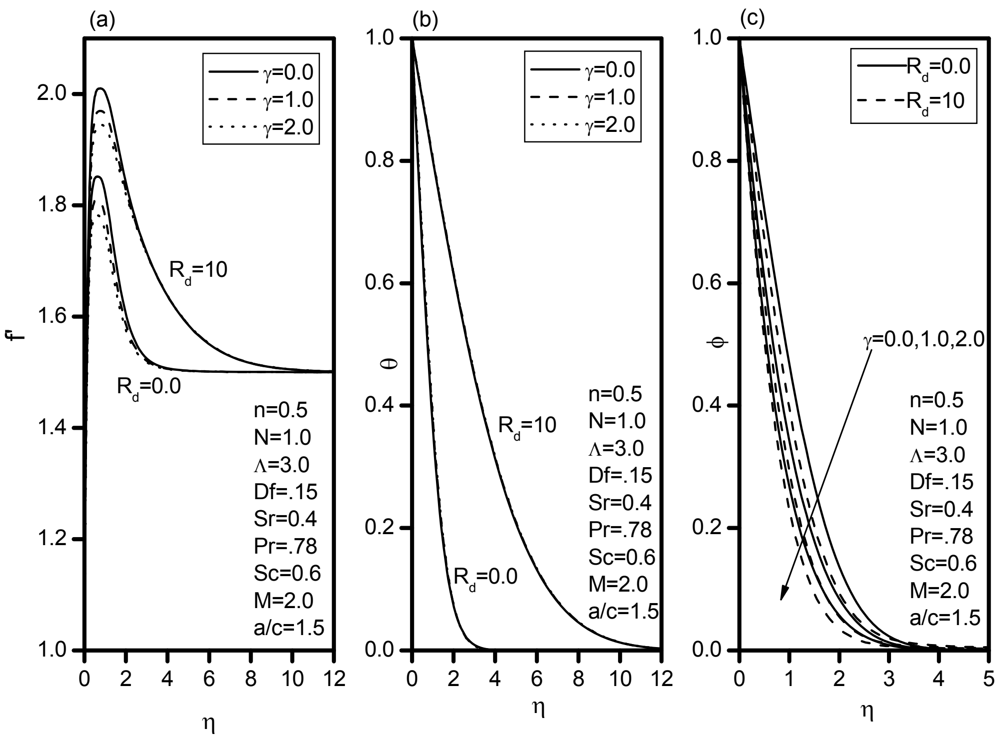

Figure 5. Effects of R_d and γ on the (a) velocity, (b) temperature, (c) concentration profiles.

The variations of the local skin-friction coefficient and the local Nusselt and Sherwood numbers against the chemical reaction parameter γ in the absence or presence of thermal radiation are shown in Figs. 6(a)-6(c), respectively. It can be seen that as γ increases, the local Sherwood number increases while the opposite effect is found for both of the local skin-friction coefficient and the local

Nusselt number. This is because as γ increases, the concentration difference between the surface and the fluid decreases and so the rate of mass transfer at the surface must increase while both of the wall local skin-friction coefficient and the rate of heat transfer decreases as a result of the decrease in the flow velocity and fluid temperature, respectively. Moreover, as seen earlier from Figs. 5(b)-5(c), both of the wall slope of the velocity and temperature profiles increase while the wall slope of the concentration profile decreases as R_d increases. This causes enhancements in the local skin-friction coefficient as well as the local Nusselt number and the local Sherwood number with increasing values of R_d as obvious from Figs. 6(a)-6(c). This result is expected because the presence of thermal radiation works as a heat source and so the quantity of heat added to the flow causes the motion of the fluid to accelerate.

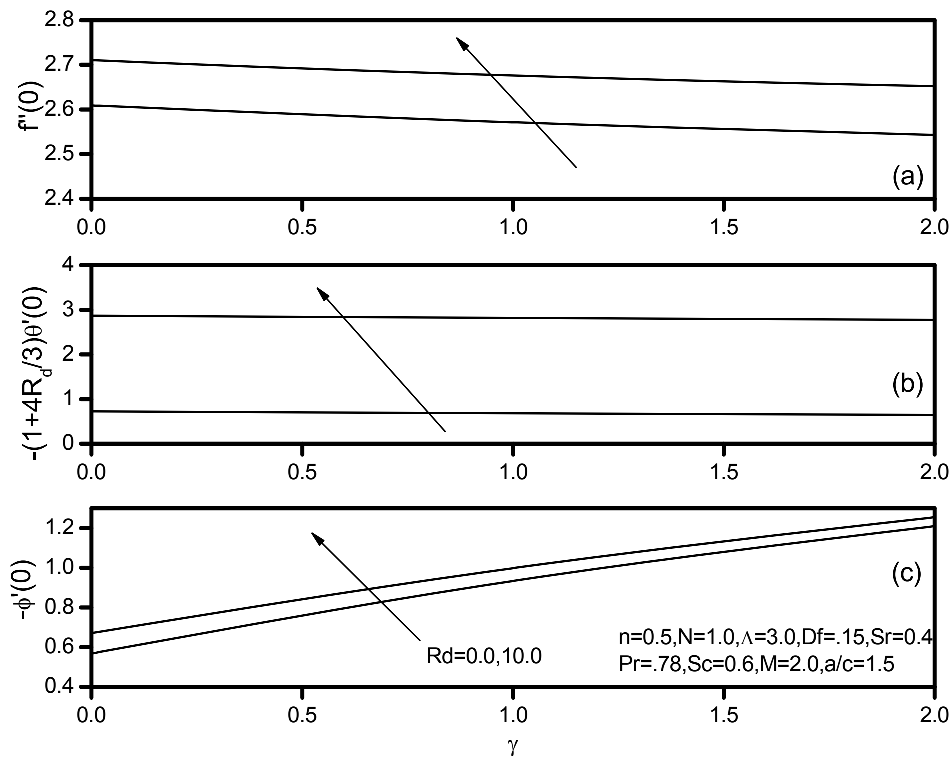

Figure 6. Effects of R_d and γ on the (a) local skin-friction coefficient, (b) local Nusselt number, (c) local Sherwood number.

Figures 7(a)-7(c) present the changes in the velocity, temperature and concentration profiles that are brought about by changes in the buoyancy ratio N for various values of the Dufour number D_f and Soret number S_r, respectively. For fixed values of D_f and S_r, increases in the value of N have the tendency to increase the buoyancy effects causing more induced flow along the stretching sheet in the vertical direction reflected by the increases in the fluid velocity. This enhancement in the flow velocity is achieved at the expense of reduced fluid temperature and concentration as well as slight reduction in the thermal and concentration boundary layers as seen from 7(b)-7(c). Also, the concentration profiles increase with increasing the value of Sr (or decreasing D_f), but the fluid temperature decreases as *Sr* increases (or D_f decreases). We notice that, this behavior is a direct consequence of the Soret effect, which produces a mass flux from lower to higher solute concentration driven by the temperature gradient.

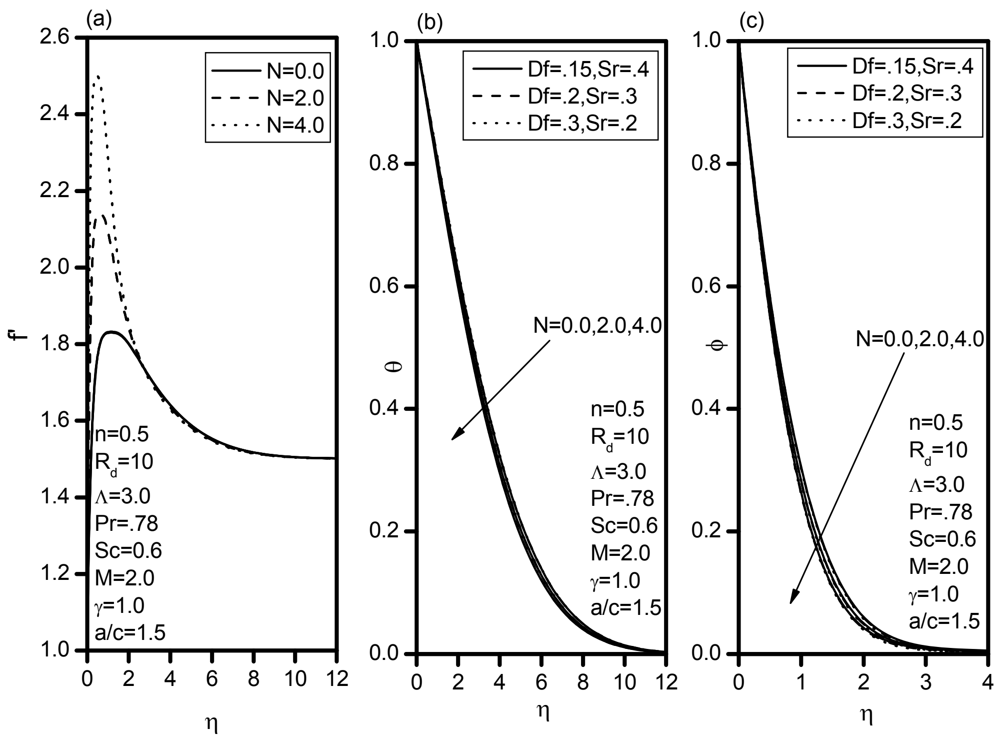

Figure 7. Effects of Df, Sr and N on the (a) velocity, (b) temperature, (c) concentration profiles.

Figures 8(a)-8(c) illustrate the effects of the Dufour number D_f, Soret number S_r and the buoyancy ratio *N* on the local skin-friction coefficient, local Nusselt number, and the local Sherwood number, respectively. Obviously, increasing the buoyancy ratio *N* increases the flow along the stretching surface while its temperature and concentration decrease causing the negative wall slope of the temperature and concentration profiles to increase. This yields enhancements in all the local skin-friction coefficient, wall heat and mass transfer represented by Nu_x and Sh_x. These behaviors are evident from Figs. 8(a)-8(c). On the other hand, increasing the value of S_r (or decreasing D_f) causes increases in the fluid temperature and decreases in the solute concentration. Since the Soret effect appears in the concentration equation (9), the changes in the local Sherwood number that are brought about by changing *Sr* are much more significant than those in the local Nusselt number. These behaviors are evident from Figs. 8(b)-8(c).

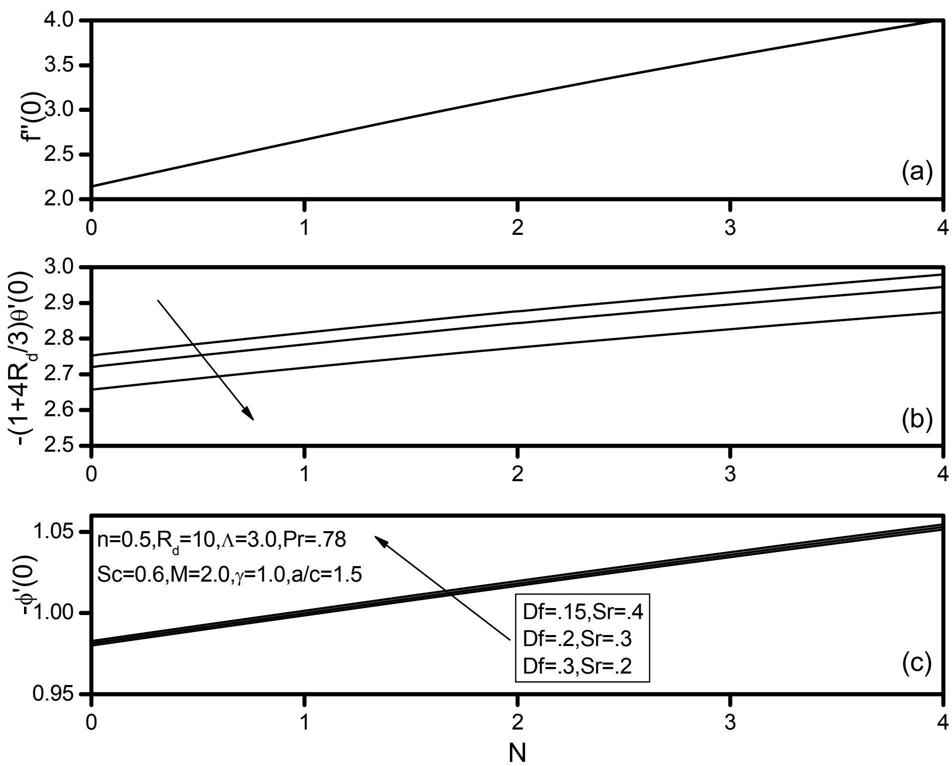

Figure 8. Effects of Df, Sr and N on the (a) local skin-friction coefficient, (b) local Nusselt number, (c) local Sherwood number.

4. Conclusions

This work considered coupled heat and mass transfer by mixed convection stagnation-point flow of a power-law non-Newtonian fluid towards a stretching surface in the presence of a magnetic field, thermal radiation, homogenous chemical reaction, thermal-diffusion and diffusion-thermo effects. The stretching surface was maintained at a constant temperature and a constant concentration.

The governing equations were formulated and transformed into a set of similar equations. These equations were solved numerically by an implicit, iterative, tri-diagonal, finite-difference method. The obtained results were checked against previously published work and were found to be in excellent agreement. Numerical results for the velocity, temperature, and concentration profiles as well as the local skin-friction coefficient, local Nusselt number, and local Sherwood number were reported graphically. It was found that both the Nusselt and Sherwood numbers decreased due to increases in magnetic field parameter. However, they both increased due to increases in either of the power-law fluid index, buoyancy ratio, mixed convection parameter, velocity ratio or the thermal radiation effects (reciprocal of the radiation parameter). Also, increases in the values of either of the chemical reaction parameter or the Dufour number produced decreases in the local Nusselt number and increases in the local Sherwood number. However, the opposite behavior was predicted as the Soret number was increased for which the local Nusselt number was increased while the local Sherwood number was decreased. Finally, the skin-friction coefficient was increased as either of the power-law fluid index, buoyancy ratio, mixed convection parameter, velocity ratio or the thermal radiation effects increased, and it was decreased due to increases in either of the magnetic field parameter or the chemical reaction parameter.

5. References

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