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Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel

J. Prathap Kumar [a], J.C. Umavathi [a], Ali J. Chamkha [b,*], Ioan Pop [c]

[a] *Department of Mathematics, Gulbarga University, Gulbarga, 585 106 Karnataka, India*
[b] *Manufacturing Engineering Department, The Public Authority for Applied Education and Training, Shuweikh 70654, Kuwait*
[c] *Faculty of Mathematics, University of Cluj, R-3400 Cluj, CP 253, Romania*

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A B S T R A C T

The problem of fully-developed laminar free-convection flow in a vertical channel is studied analytically with one region filled with micropolar fluid and the other region with a viscous fluid. Using the boundary and interface conditions proposed by previous investigators, analytical expressions for linear velocity, micro-rotation velocity and temperature have been obtained. Numerical results are presented graphically for the distribution of velocity, micro-rotation velocity and temperature fields for varying physical parameters such as the ratio of Grashof number to Reynolds number, viscosity ratio, width ratio, conductivity ratio and micropolar fluid material parameter. It is found that the effect of the micropolar fluid material parameter suppress the velocity whereas it enhances the micro-rotation velocity. The effect of the ratio of Grashof number to Reynolds number is found to enhance both the linear velocity and the micro-rotation velocity. The effects of the width ratio and the conductivity ratio are found to enhance the temperature distribution.

1. Introduction

The research area of micropolar fluids has been of great interest because the Navier–Stokes equations for Newtonian fluids can not successfully describe the characteristics of fluid with suspended particles. There exist several approaches to study the mechanics of fluids with a substructure. Ericksen [1,2] derived field equations which account for the presence of substructures in the fluid. It has been experimentally demonstrated by Hoyt and Fabula [3] and Vogel and Patterson [4] that fluids containing small amount of polymeric additives display a reduction in skin friction. Eringen [5] formulated the theory of micropolar fluids which display the effects of local rotary inertia and couple stresses. This theory can be used to explain the flow of colloidal fluids, liquid crystals, animal blood, etc. Eringen [6] extended the micropolar fluid theory and developed the theory of thermo-micropolar fluids. Extensive reviews of the theory and applications can be found in the review articles by Ariman et al. [7,8] and the recent books by Lukaszewicz [9] and Eringen [10].

Physically, micropolar fluids may be described as non-Newtonian fluids consisting of dumb-bell molecules or short rigid cylindrical element, polymer fluids, fluid suspension, etc. The presence of dust or smoke, particularly in a gas, may also be modeled using micropolar fluid dynamics. The theory of micropolar fluids first proposed by Eringen [5,6] is capable of describing such fluids.

Studies of external convective flows of micropolar fluids have focused mainly on free, forced and mixed convection problems. Natural convection of an enclosed fluid is a long-standing classical subject. Applications are found in a variety

* Corresponding author.
E-mail address: achamkha@yahoo.com (A.J. Chamkha).

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Nomenclature

b	thermal expansion coefficient ratio, β_2/β_1
g	acceleration due to gravity
Gr	Grashof number
GR	Grashof to Reynolds numbers ratio, Gr/Re
h	width ratio, h_2/h_1
h_1	height of Region-I
h_2	height of Region-II
j	micro-inertia density
k	ratio of thermal conductivities, k_1/k_2
k_1	thermal conductivity of the fluid in Region-I
k_2	thermal conductivity of the fluid in Region-II
K	micropolar fluid material parameter
m	ratio of viscosities, μ_1/μ_2
n	micro-rotational velocity
Re	Reynolds number
T_0	average temperature
T	temperature
T_1, T_2	temperature of the boundaries
U_0	average velocity
U	velocity
X, Y	space coordinates

Greek letters

β	coefficient of thermal expansion
γ	spin gradient viscosity
κ	vortex viscosity
μ	viscosity
ρ_1	density of Region-I
ρ_2	density of Region-II
ρ	ratio of densities, ρ_2/ρ_1
ΔT	difference in temperature
θ_i	dimensionless temperature

Subscript

1, 2	reference quantities for Region-I and Region-II, respectively

of engineering problems, such as air conditioning of a room, solar energy collecting devices, material processing and passive cooling of nuclear reactors, to name a few. Studies of the flows of heat convection in micropolar fluids have focused mainly on flat (Ahmadi [11], Jena and Mathur [12] Ÿucel [13], and Rahman et al. [14–18]) or regular surfaces (Balram and Sastry [19], Lien et al. [20,21]). Chamkha et al. [22] analyzed numerical and analytical solutions of the developing laminar free convection of a micropolar fluid in vertical parallel plate channel with asymmetric heating.

The subject of two-fluid flow and heat transfer has been extensively studied due to its importance in chemical and nuclear industries. The design of two-fluid heat transport system for space application requires knowledge of heat and mass transfer processes and fluid mechanics under reduced gravity conditions. Identification of the two-fluid flow region and determination of the pressure drop, void fraction, quality reaction and two-fluid heat transfer coefficient are of great importance for the design of two-fluid systems. Lohrasbi and Sahai [23] studied two-phase MHD flow and heat transfer in a parallel plate channel with the fluid in one phase being electrically conducting. Malashetty and Leela [24] have analyzed the Hartmann flow characteristic of two fluids in horizontal channel. The study of two-phase flow and heat transfer in an inclined channel has been studied by Malashetty and Umavathi [25] and Malashetty et al. [26,27].

The purpose of the present paper is to report analytical results for the problem of fully-developed convective flow of a micropolar and viscous fluid between parallel plate vertical channels with asymmetric wall temperature distribution.

2. Mathematical formulation

The geometry under consideration illustrated in Fig. 1, consists of two infinite parallel plates maintained at different or equal constant temperatures extending in the X and Z directions. The region $-h_1 \leqslant Y \leqslant 0$ is occupied by micropolar fluid of density ρ_1, viscosity μ_1, vortex viscosity κ, thermal conductivity k_1, thermal expansion coefficient β_1 and the region $0 \leqslant Y \leqslant h_2$ is occupied by viscous fluid of density ρ_2, viscosity μ_2, thermal conductivity k_2 and thermal expansion

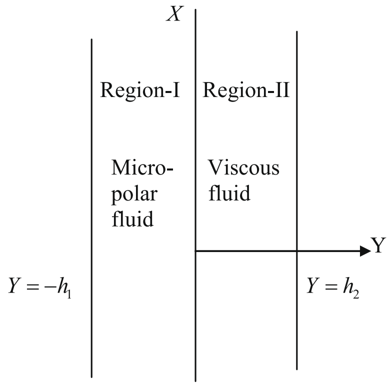

Fig. 1. Physical configuration.

coefficientβ_2. The fluids are assumed to have constant properties except the density in the buoyancy term in momentum equation $\rho_1 = \rho_0[1 - \beta_1(T_1 - T_0)]$ and $\rho_2 = \rho_0[1 - \beta_2(T_2 - T_0)]$. A fluid rises in the channel driven by buoyancy forces. The transport properties of both fluids are assumed to be constant. We consider the fluids to be incompressible and immiscible and the flow is steady, laminar and fully developed. It should be mentioned here that the micropolar and viscous fluids are immiscible (that is, no mixing between the fluids exists) and the constitutive equations for micropolar fluids and viscous fluids are different. Also, the viscosities of both fluids are different. For instance, Synovial fluid which is a clear thixotropic lubrication fluid is a good example of micropolar fluids and water is a good example for viscous fluids and it is well known that a Synovial fluid and water can not be mixed. Since our model is general, one can choose any two different fluids which are immiscible.

It is assumed that the only non-zero components of the velocity $\mathbf{q}$ is the X-component $U_i (i = 1, 2)$. Thus, as a consequence of the mass balance equation, one obtains

$$\frac{\partial U_i}{\partial X} = 0 \tag{1}$$

so that U_i depends only on Y.

The momentum balance equations are

Region-I

$$(\mu_1 + \kappa)\frac{d^2U_1}{dY^2} + \kappa\frac{dn}{dY} + \rho_1 g\beta_1(T_1 - T_0) = 0 \tag{2}$$

$$\gamma\frac{d^2n}{dY^2} - \kappa\left(2n + \frac{dU_1}{dY}\right) = 0 \tag{3}$$

Region-II

$$\rho_2 g\beta_2(T_2 - T_0) + \mu_2\frac{d^2U_2}{dY^2} = 0 \tag{4}$$

where n is the component of micro-rotation vector normal to the plane X—Y, g is the acceleration due to gravity, γ is the spin gradient viscosity and κ is the vortex viscosity. Let us assume that the walls of the channel are isothermal. In particular, the temperature of the boundary at $Y = -h_1$ is T_1, while the temperature at $Y = h_2$ is T_2, with $T_2 \geqslant T_1$.

The energy balance equations are

Region-I

$$\frac{d^2T_1}{dY^2} = 0 \tag{5}$$

Region-II

$$\frac{d^2T_2}{dY^2} = 0. \tag{6}$$

To solve the above system of Eqs. (1)–(6) six boundary conditions are required for velocity and four boundary conditions for temperature. The first two boundary conditions are obtained from the fact that there is no slip near the wall. Next condition is obtained by assuming the continuity of velocity and the last three conditions are obtained from the equality of stresses at the interface and constant cell rotational velocity at the interface as proposed by Ariman et al. [7]. Thus, the appropriate boundary and interface conditions on velocity in the mathematical form are

$$\begin{aligned} &U_1 = 0 \quad \text{at } Y = -h_1, \quad U_2 = 0 \quad \text{at } Y = h_2 \\ &\quad U_1(0) = U_2(0) \\ &(\mu_1 + \kappa)\frac{dU_1}{dY} + \kappa n = \mu_2 \frac{dU_2}{dY}, \quad \frac{dn}{dY} = 0 \quad \text{at } Y = 0 \\ &\quad n = 0 \quad \text{at} \quad Y = -h_2. \end{aligned} \tag{7}$$

For the corresponding temperature boundary conditions it is assumed that the temperatures and heat fluxes are continuous at the interface

$$\begin{aligned} &T = T_1 \quad \text{at} \quad Y = -h_1, \quad T = T_2 \quad \text{at } Y = h_2 \\ &\quad T_1(0) = T_2(0) \\ &k_1 \frac{dT_1}{dY} = k_2 \frac{dT_2}{dY} \quad \text{at } Y = 0. \end{aligned} \tag{8}$$

We assume that

$$\gamma = (\mu_1 + \kappa/2)j = \mu(1 + K/2)j \tag{9}$$

where j is the micro-inertia density and $K = \kappa/\mu_1$ is the micropolar fluid material parameter of Region-I. We notice that $K = 0$ describes the case of a viscous or Newtonian fluid. Relation (9) expresses the fact that the micropolar fluid field can predict the correct behavior in the limiting case when the micro-structure effects become negligible and the total spin reduces to the angular flow velocity or flow vorticity. Relation (9) was established by Ahmadi [11] and Kline [28] and it has been used by many researchers, for example, Rees and Bassom [29], Gorla [30], and Rees and Pop [31]. Further, we introduce the following dimensionless variables

$$\begin{aligned} &y_1 = \frac{Y_1}{h_1}, \quad y_2 = \frac{Y_2}{h_2}, \quad u_1 = \frac{U_1}{U_0}, \quad u_2 = \frac{U_2}{U_0}, \quad \theta_1 = \frac{T_1 - T_0}{\Delta T}, \\ &\theta_2 = \frac{T_2 - T_0}{\Delta T}, \quad N = \frac{h_1}{U_0} n, \quad K = \frac{\mu}{\kappa} \end{aligned} \tag{10}$$

where $j = h_1^2$ is the characteristic length and ΔT is the characteristic temperature which is defined as $\Delta T = T_2 - T_1$ if $T_2 > T_1$.

Using Eq. (10), Eqs. (2)–(6) in dimensionless form become

Region-I

$$\frac{d^2 u_1}{dy^2} + \frac{K}{1+K}\frac{dN}{dy} + GR\frac{\theta_1}{1+K} = 0 \tag{11}$$

$$\frac{d^2 N}{dy^2} - \frac{2K}{2+K}\left(2N + \frac{du_1}{dy}\right) = 0. \tag{12}$$

Region-II

$$\frac{d^2 u_2}{dy^2} + mb\rho h^2 GR\theta_2 = 0. \tag{13}$$

The temperature equations are

Region-I

$$\frac{d^2 \theta_1}{dy^2} = 0. \tag{14}$$

Region-II

$$\frac{d^2 \theta_2}{dy^2} = 0 \tag{15}$$

where Gr is the Grashof number, Re is the Reynolds number and GR is the ratio between the Grashof and Reynolds numbers and are defined as

$$Gr = \frac{g\beta_1 \Delta T h_1^3}{\upsilon_1^2}, \quad Re = \frac{U_0 h_1}{\upsilon_1}, \quad GR = \frac{Gr}{Re}. \tag{16}$$

The dimensionless form of the boundary and interface conditions on velocity become

$$\begin{aligned} &u_1 = 0 \quad \text{at } y = -1, \quad u_2 = 0 \quad \text{at } y = 1 \\ &u_1(0) = u_2(0) \\ &\frac{du_1}{dy} + \frac{K}{1+K} N = \frac{1}{mh(1+K)} \frac{du_2}{dy} \quad \text{at } y = 0. \end{aligned} \tag{17}$$

The dimensionless form of the boundary and interface conditions on micro-rotation velocity become

$$\frac{dN}{dy} = 0 \quad \text{at } y = 0, \quad N = 0 \quad \text{at } y = -1. \tag{18}$$

The boundary and interface conditions on temperature in dimensionless form are

$$\begin{aligned} &\theta_1 = 1 \quad \text{at } y = -1, \quad \theta_2 = 0 \quad \text{at } y = 1 \\ &\theta_1(0) = \theta_2(0) \\ &\frac{d\theta_1}{dy} = \frac{1}{hk} \frac{d\theta_2}{dy} \quad \text{at } y = 0 \end{aligned} \tag{19}$$

where $h = h_1/h_2$, $m = \mu_1/\mu_2$, $k = k_1/k_2$, $\rho = \rho_1/\rho_2$ and $b = \beta_2/\beta_1$ are the channel width ratio, viscosity ratio, thermal conductivity ratio, density ratio and thermal expansion coefficient ratio, respectively.

3. Solutions

Solutions of Eqs. (14) and (15), using boundary and interface conditions (19) are given by

Region-I

$$\theta_1 = C_1 y + C_2. \tag{20}$$

Region-II

$$\theta_2 = C_3 y + C_4. \tag{21}$$

Solutions of the linear velocity and micro-rotation velocity given by Eqs. (11)–(13) using boundary and interface conditions (17) and (18) are

Region-I

$$\begin{aligned} u_1 = &-\frac{K}{(1+K)\sqrt{R}}\left(C_5 sinh\left(\sqrt{R}y\right) + C_6 cosh\left(\sqrt{R}y\right)\right) \\ &- \frac{2GR}{2+K}\left(\frac{C_1}{6}y^3 + \frac{C_2}{2}y^2\right) - C_7 y - C_8 \\ N = &\, C_5 cosh\left(\sqrt{R}y\right) + C_6 sinh\left(\sqrt{R}y\right) + \frac{GR}{2(2+K)}\left(C_1 y^2 + 2C_2 y + \frac{2C_1}{R}\right). \end{aligned} \tag{22}$$

Region-II

$$u_2 = -mbh^2 \rho GR\left(\frac{C_3}{6}y^3 + \frac{C_4}{2}y^2\right) - C_9 y - C_{10} \tag{23}$$

where $C_i, i = 1, \ldots, 10$ are constants of integration and $R = 2K/(1+K)$.

3.1. Limiting case

For a Newtonian fluid ($K = 0$), the solutions of Eqs. (11)–(13) using boundary and interface conditions (17) and (18) are

$$\begin{aligned} u_1 &= -\frac{GR}{6}(C_1 y^3 + 3C_2 y^2) - B_1 y - B_2 \\ u_2 &= -\frac{mbh^2 \rho GR}{6}(C_3 y^3 + 3C_4 y^2) - B_3 y - B_4 \end{aligned} \tag{24}$$

where $B_i, i = 1, \ldots, 4$ are constants of integration.

The solutions for temperature remain the same as given in Eqs. (20) and (21). The constants appeared in the above solutions are listed below

$$C_1 = -\frac{1}{1+kh}, \quad C_2 = C_4, \quad C_3 = -C_4, \quad C_4 = 1 + C_1,$$
$$C_5 = \frac{l_1 + C_6 sinh\sqrt{R}}{cosh\sqrt{R}}, \quad C_6 = -\frac{C_2 GR}{(2+K)\sqrt{R}},$$
$$C_7 = \frac{l_3 + l_5 - l_2 - l_4}{1 + mh(1+K)}, \quad C_8 = l_2 + C_7, \quad C_9 = l_3 - C_{10},$$
$$l_1 = -\frac{GR}{2(2+K)}\left(C_1 - 2C_2 + \frac{2C_1}{R}\right) - \frac{2A}{R(2+K)},$$

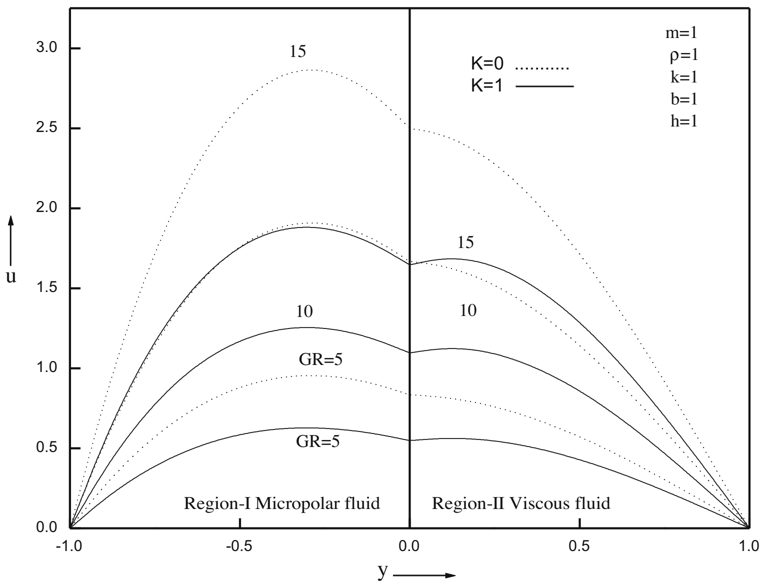

Fig. 2. Velocity profiles for different values of *GR*.

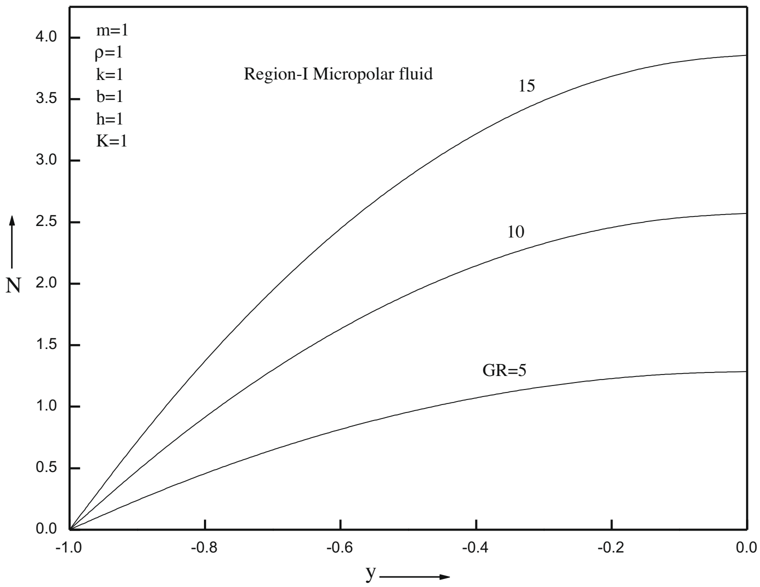

Fig. 3. Micro-rotational velocity profiles for different values of *GR*.

$$l_2 = \frac{K}{1+K}\left(C_5 sinh\sqrt{R} - C_6 cosh\sqrt{R}\right) + \frac{2GR}{2+K}\left(\frac{C_1}{6} - \frac{C_2}{2}\right),$$

$$l_3 = -m\rho bh^2 GR\left(\frac{C_3}{6} + \frac{C_4}{2}\right), \quad l_4 = \frac{C_6 K}{(1+K)\sqrt{R}}, \quad l_5 = mhK(C_1 GR + 2A),$$

$$AN = 2(1+K)R\sqrt{R}\Big(3mhC_1K(2+K) - mnbh^2C_1(2+K)\Big) - C_1 + 3C_2 - m\rho bh^2(2+K) + 2K\sqrt{R}\Big(2RC_2 + C_1(2+R)tanh\sqrt{R}\Big) + 2KC_2\sqrt{R}(1+cosh\sqrt{R}) - C_1(1+K)R\sqrt{R},$$

$$AD = 2\Big((1+K)R\sqrt{R}(3+K-4mhK) - 2Ktanh\sqrt{R}\Big),$$

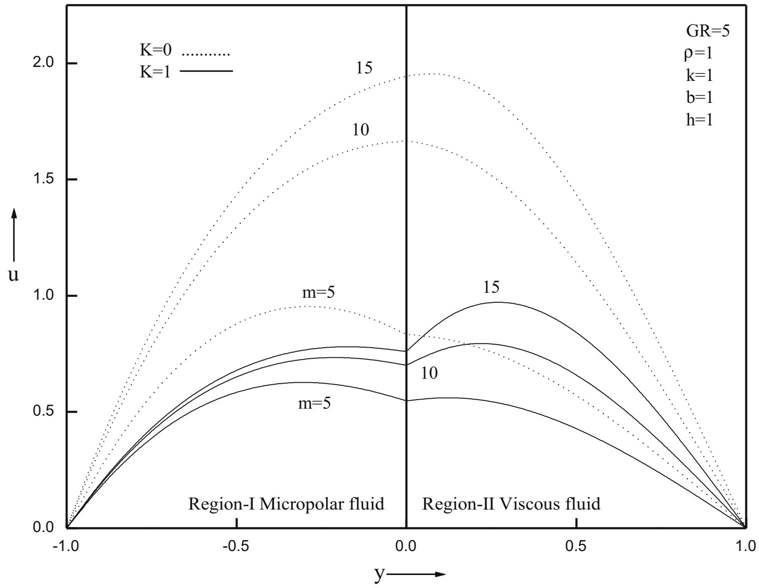

Fig. 4. Velocity profiles for different values of viscosity ratio *m*.

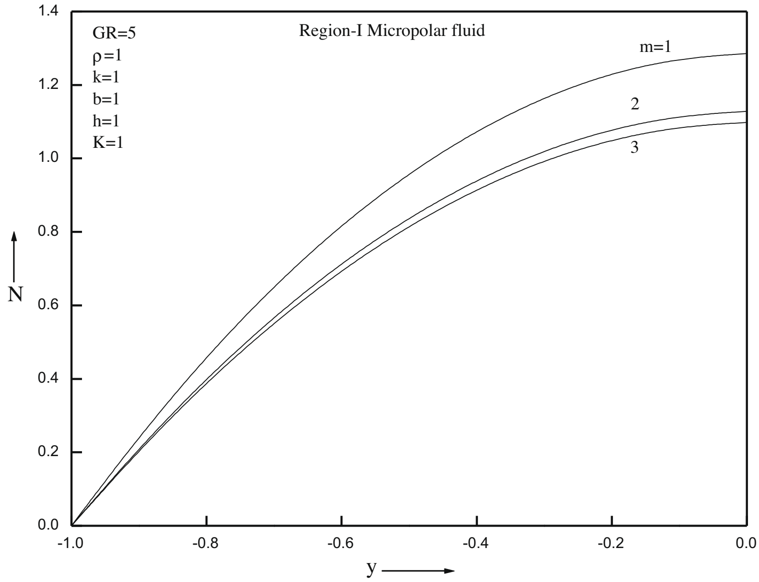

Fig. 5. Micro-rotational velocity profiles for different values of viscosity ratio *m*.

$$A = GR\left(\frac{AN}{AD}\right),$$
$$B_1 = -\frac{1}{1+Kh}; \quad B_2 = 1 + B_1; \quad B_3 = -B_4; \quad B_4 = B_2,$$
$$P_1 = \frac{GR}{6}(3B_2 - B_1), \quad P_2 = -\frac{m\rho bh^2 GR}{6}(B_3 + 3B_4),$$
$$B_5 = \frac{P_1 + P_2}{1 + mh}, \quad B_6 = B_5 - P_1, \quad B_7 = P_2 - B_8, \quad B_8 = B_6.$$

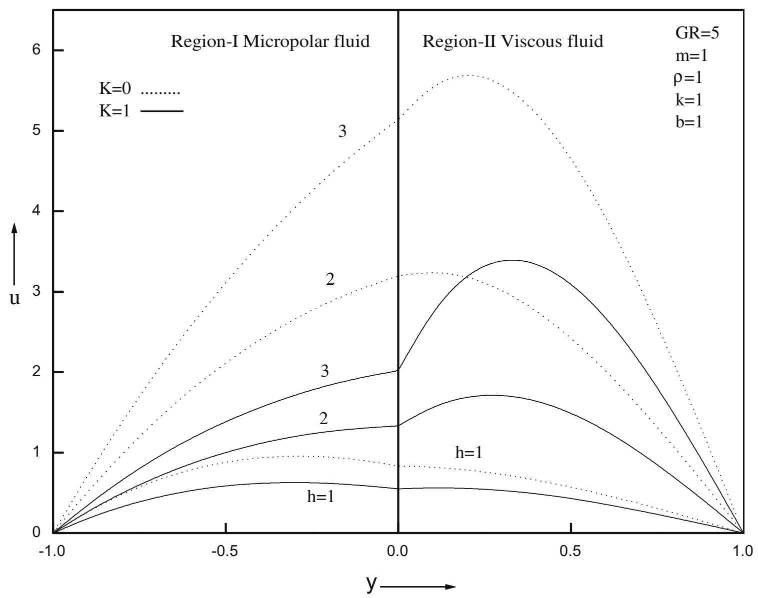

Fig. 6. Velocity profiles for different values of width ratio *h*.

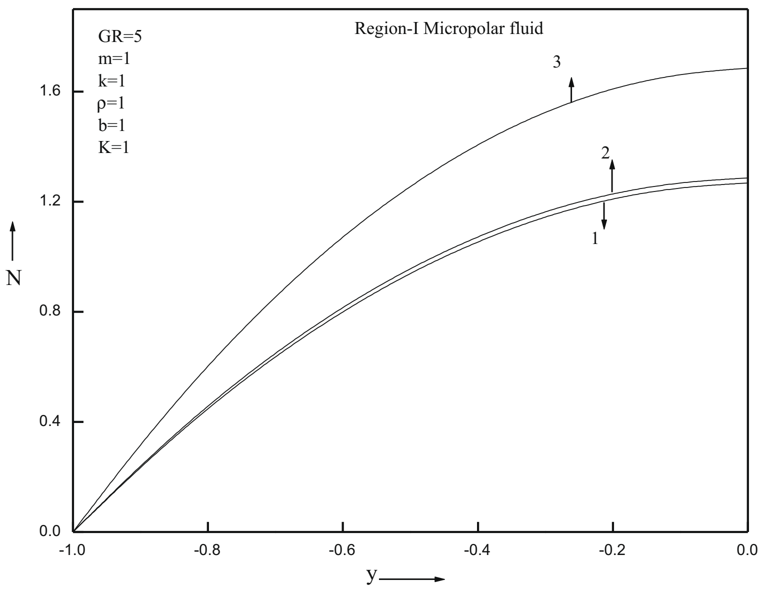

Fig. 7. Micro-rotational velocity profiles for different values width ratio *h*.

4. Results and discussion

An analytical solution for the problem of mixed convective flow and heat transfer of micropolar and viscous fluid in a vertical channel is analyzed. The analytical solutions (10)–(24) are evaluated numerically for different values of governing parameters and the results are presented graphically in Figs. 2–13.

The effect of the mixed convection parameter or Grashof to Reynolds numbers ratio *GR* on the linear velocity and micro-rotation velocity are shown in Figs. 2 and 3. Increase in the mixed convection parameter means an increase of the buoyancy force which supports the motion. It is also observed from Fig. 2 that if the micropolar fluid is replaced by the clear viscous fluid, the effect of mixed convection parameter *GR* is still retained. But the magnitude of promotion is large for viscous–viscous fluids system compared to micropolar–viscous fluids system.

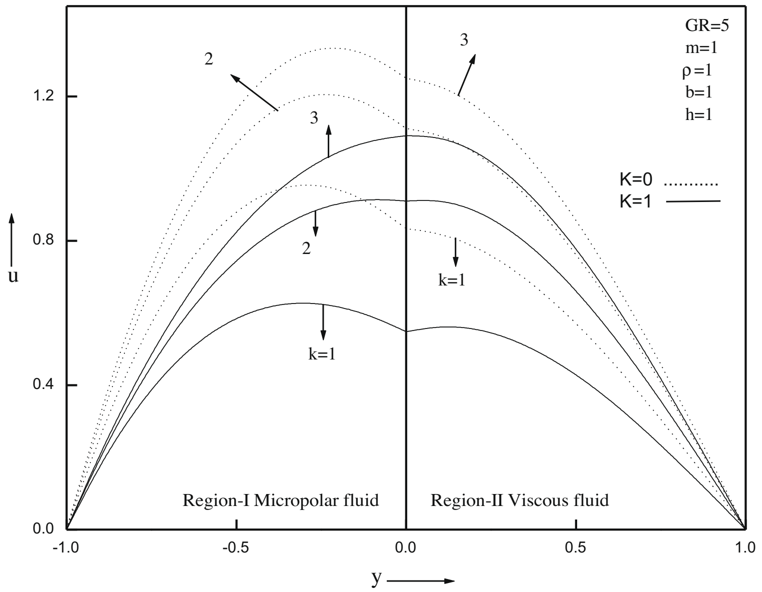

Fig. 8. Velocity profiles for different values of conductivity ratio *k*.

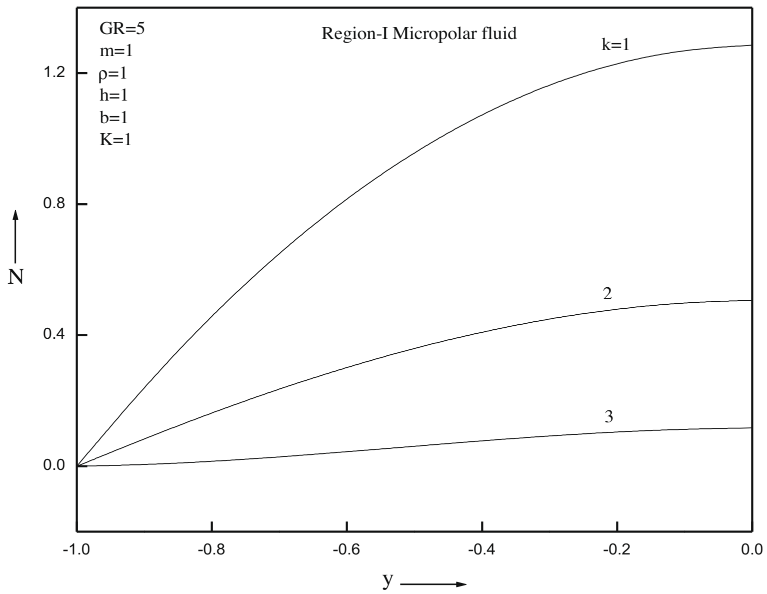

Fig. 9. Micro-rotational velocity profiles for different values of conductivity ratio *k*.

Figs. 4 and 5 display the effect of the viscosity ratio $m(=\mu_1/\mu_2)$ on the linear velocity and micro-rotation velocity. As the viscosity ratio m increases, the linear velocity increases, but the magnitude of promotion is large for $K = 0$ (Newtonian fluid) compared with $K = 1$ (micropolar fluid). The effect of the viscosity ratio m is found to reduce the micro-rotational velocity.

The effect of the channel width ratio $h(=h_2/h_1)$ on the linear velocity and micro-rotation velocity is shown in Figs. 6 and 7, respectively. As the width ratio h increases, both the linear velocity and the micro-rotation velocity increase for viscous–viscous $(K = 0)$ and micropolar–viscous $(K = 1)$ fluids systems. The effect of the width ratio h is also found to promote the temperature field as seen in Fig. 12.

The effects of the conductivity ratio $k(=k_2/k_1)$ on the linear velocity and temperature fields are shown in Figs. 8 and 13, respectively. The effect of the conductivity ratio k is predicted to increase both the velocity and temperature fields. i.e., the larger the conductivity of the micropolar fluid compared to the viscous fluid, the larger the flow nature. But the effect of the conductivity ratio k is found to reduce the micro-rotation velocity as seen in Fig. 9.

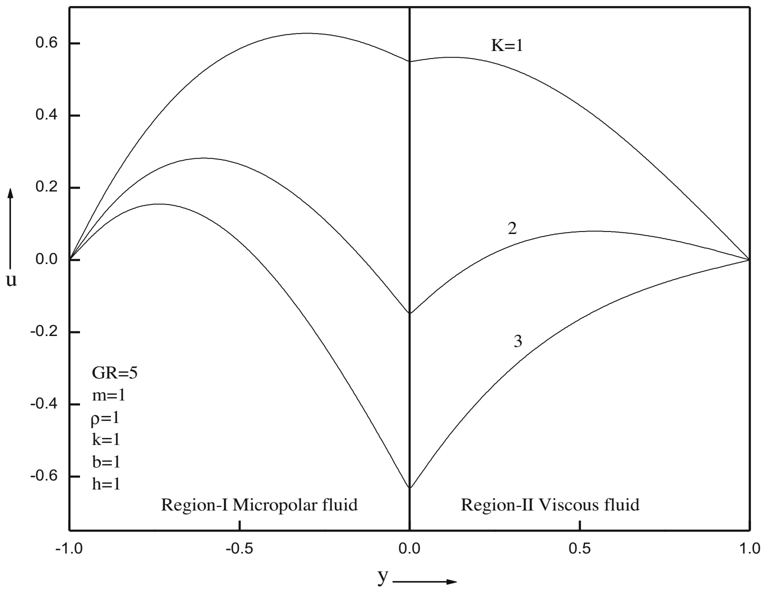

Fig. 10. Velocity profiles for different values of material parameter K.

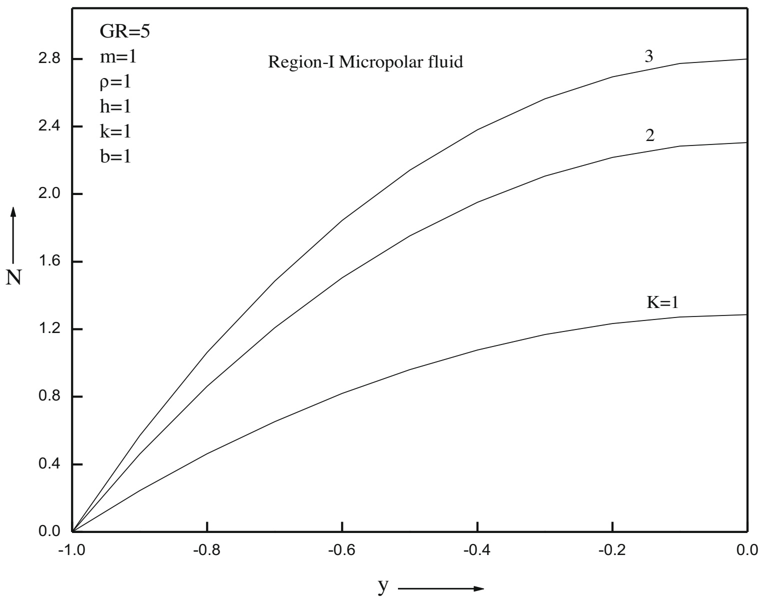

Fig. 11. Micro-rotational velocity profiles for different values of material parameter K.

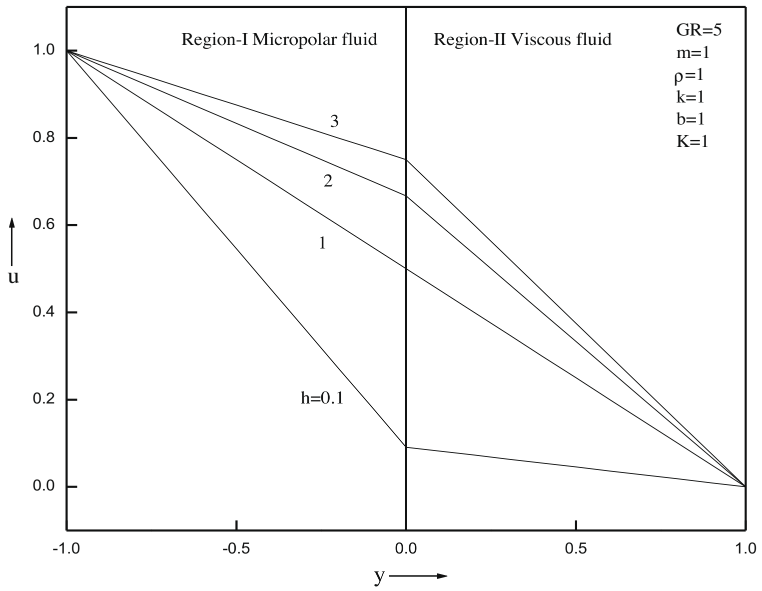

Fig. 12. Temperature profiles for different values of width ratio *h*.

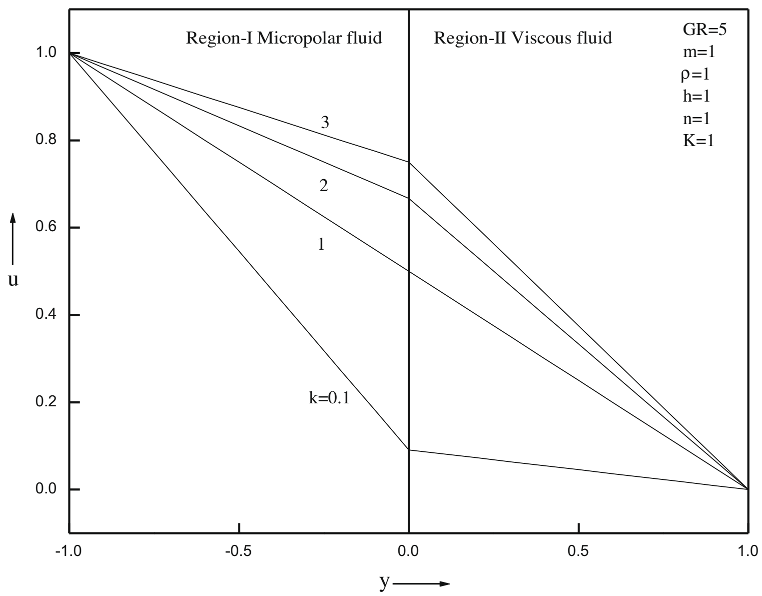

Fig. 13. Temperature profiles for different values of conductivity ratio *k*.

The effect of the micropolar fluid material parameter *K* on the linear velocity field is shown in Fig. 10. It is seen that as *K* increases, the velocity decreases in both regions of the channel. This result is similar to the result obtained by Chamkha et al. [22] for one fluid model considering only micropolar fluid. It is also interesting to note that as *K* increases, the shape of the profiles goes on changing from straight to V shape at the interface. Finally, the effect of the micropolar fluid material parameter *K* is found to increase the micro-rotational velocity as seen in Fig. 11. This is an expected result because an increase in the material parameter implies an increase in the spin gradient viscosity which promotes the rotational velocity.

5. Conclusions

Analytical solutions are obtained for free-convective flow of micropolar–viscous fluids in a vertical channel. Separate solutions are obtained for each fluid and these solutions are matched at the interface using suitable matching conditions.

The solutions are evaluated numerically and the results are shown graphically. It is found that effects of the Grashof to Reynolds numbers ratio, viscosity ratio and width ratio is to promote the linear velocity whereas the micropolar fluid material parameter suppressed the velocity. Also, the effects of the ratio of Grashof number to Reynolds number, width ratio and the micropolar fluid material parameter is found to increase the micro-rotation velocity whereas the viscosity ratio and the conductivity ratio suppress the micro-rotation velocity. The effects of the width and conductivity ratios promote the temperature field.

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