Heat and mass transfer by non-Darcy free convection from a vertical cylinder embedded in porous media with a temperature-dependent viscosity

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A.J. Chamkha
*Manufacturing Engineering Department,
The Public Authority for Applied Education and Training,
Shuweikh, Kuwait*

S.M.M. EL-Kabeir
*Department of Mathematics,
College of Science and Humanitarian Studies in Al-Kharj,
Al-Kharj University, Al-Kharj, Saudi Arabia and
Department of Mathematics, Faculty of Science, South Valley University,
Aswan, Egypt, and*

A.M. Rashad
*Department of Mathematics, Faculty of Science, South Valley University,
Aswan, Egypt*

Abstract

Purpose – The purpose of this paper is to consider heat and mass transfer by natural convection from a vertical cylinder in porous media for a temperature-dependent fluid viscosity in the presence of radiation and chemical reaction effects.

Design/methodology/approach – The governing equations are transformed into non-similar differential equations and then solved numerically by an efficient finite-difference method.

Findings – It is found that there are significant effects on the heat and mass transfer characteristics of the problem due to the variation of viscosity and radiation and chemical reaction effects.

Originality/value – The paper combines the effects of radiation, chemical reaction, non-Darcy porous media effects along with the variation of viscosity with temperature.

Keywords Free convection, Porous medium, Heat transfer, Mass transfer, Non-Darcy effect, Conduction-radiation interaction, Variable viscosity, Chemical reactions

Paper type Research paper

Nomenclature

A	= constant
C	= fluid concentration
C_p	= specific heat at a constant pressure
D	= pore diameter
D	= mass diffusivity
Er	= Ergun number
f, f'	= the dimensionless free modified stream function and free stream velocity
G	= acceleration due to gravity

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H = surface temperature excess ratio
K = thermal conductivity
K = permeability of medium
K^* = inertial coefficient
K_1 = dimensional chemical reaction parameter
Le = Lewis number
m_w = local chemical species transfer
N = ratio of the buoyancy forces due to the temperature and concentration
Nu_x = local Nusselt number
q_w = heat transfer
R = coordinate measured normal to the surface
r_0 = radius of the vertical cylinder
Ra_x = modified local Rayleigh number
Ra = Rayleigh number based on pore diameter
R_d = conduction-radiation parameter
Sh_x = local Sherwood number
T = fluid temperature
T_r = viscosity reference temperature
u, v = velocity components along x- and r-directions, respectively
X = coordinate measured along to the surface

Greek symbols

α = thermal diffusivity
β = thermal expansion coefficient
β^* = concentration expansion coefficient
β_R = Rosseland extinction coefficient
ϕ = dimensionless concentration
γ = dimensionless of chemical reaction parameter
γ^* = thermal property of the fluid
η = pseudo-similarity variable
μ = dynamic viscosity
ν = kinematic viscosity
θ = dimensionless temperature
θ_r = dimensionless reference temperature
ρ = fluid density
σ = Stefan-Boltzmann constant
Ξ = transverse curvature parameter
ψ = modified stream function

Subscripts

w = condition at surface
∞ = condition at ambient medium

1. Introduction

The corresponding problem of convective heat and mass transfer in a saturated porous medium has many important applications in geothermal and geophysical engineering such as the extraction of geothermal energy, the migration of moisture in fibrous insulation, under ground disposal of nuclear waste, and the spreading of chemical pollutants in saturated soil. Recent books by Nield and Bejan (2006) and Ingham and Pop (1998, 2002) present a comprehensive account of the available information in the field. Also, many studies have been carried out to include various physical aspects for the problem of natural convection flow over a vertical cylinder in a porous medium. Minkowycz (1976) was the first author to present free convection about a vertical cylinder embedded in a porous medium with a variable wall temperature. Yücel (1984) employed an implicit finite-difference method to examine free convection about a vertical cylinder in a porous medium. Bassom and Rees (1996) investigated the variable wall temperature case on free convection from an isothermal vertical cylinder embedded in a saturated porous medium. Yih (1999) examined the effect of radiation on the heat transfer characteristics in natural convection over an isothermal vertical cylinder embedded in a porous medium. Chamkha *et al.* (2004) studied free convection from a vertical cylinder embedded in a porous medium. EL-Hakiem and Rashad (2007) investigated the effect of temperature-dependent viscosity on non-Darcy natural convection flow over a vertical cylinder in a saturated porous medium. EL-Kabeir *et al.* (2008) studied the effect of thermal radiation on combined heat and mass transfer in MHD non-Darcy free convection about a horizontal cylinder embedded in a saturated porous medium.

On the other hand, in many engineering applications such as nuclear reactor safety, combustion systems, solar collectors, metallurgy, and chemical engineering, there are many transport processes that are governed by the joint action of the buoyancy forces from both thermal and mass diffusions in the presence of chemical reaction effect. Representative applications of interest include: solidification of binary alloy and crystal growth, dispersion of dissolved materials or particulate water in flows, drying and dehydration operations in chemical and food processing plants, and combustion of atomized liquid fuels. Also, the effects of thermal radiation with chemical reaction on free convection flow and mass transfer have a publication importance such as the combustion of fossil fuels, atmospheric re-entry with suborbital velocities, plasma wind tunnels, electric spacecraft propulsion, hypersonic flight through planetary atmosphere photo-dissociation, photo ionization, and geophysics. Diffusion and chemical reaction in an isothermal laminar flow along a soluble flat plate were studied by Fairbanks and Wike (1950). The effects of chemical reaction and mass transfer on flow past an impulsively infinite vertical plate with constant heat flux were studied by Das *et al.* (1994). Andersson *et al.* (1994) have studied the flow and mass diffusion of a chemical species with first-order and higher order reactions over a linearly stretching surface. Anjalidevi and Kandasamy (1999) have analyzed the steady laminar flow along a semi-infinite horizontal plate in the presence of a species concentration and chemical reaction. Takhar *et al.* (2000) have investigated the flow and mass diffusion of chemical species with first-order and higher order reactions over a continuously stretching sheet with the magnetic field effect. Muthucumaraswamy (2002) has studied the effects of a chemical reaction on a moving isothermal vertical infinitely long surface with suction. Analytical solutions for the overall heat and mass transfer on MHD flow of a uniformly stretched vertical permeable surface with the effects of heat generation/absorption and chemical reaction were presented by Chamkha (2003). The effects of radiation and chemical reaction on MHD free convective flow and mass transfer past a vertical isothermal cone surface were investigated by Afify (2004). Abo-Eldahab and Salem (2005) have analyzed the MHD flow and heat transfer with the diffusion and chemical reaction effects over a moving cylinder. The influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces embedded in fluid-saturated porous medium subjected to a chemical reaction was analyzed by Postelnicu (2007). Rashad and EL-Kabeir (2010) have studied the effects of thermal/mass diffusions and chemical reaction on the heat and mass transfer by unsteady mixed convection boundary layer past an impermeable vertical stretching sheet embedded in a porous medium.

In this investigation, it was proposed to study the effect of temperature-dependent viscosity on the combined heat and mass transfer by non-Darcy natural convection flow over an isothermal vertical cylinder embedded in a fluid-saturated porous medium in the presences of thermal radiation and chemical reaction effects (Figure 1). In formulating the equations governing the flow, a formula for viscosity proportional to an inverse linear function of temperature has been used; following Ling and Dybbs (1987) and Lai and Kulacki (1990). The second-level local non-similarity method is used to convert the non-similar equations into a system of ordinary differential equations. The effects of Ergun number (non-Darcy parameter), viscosity/temperature, transverse curvature parameter, conduction radiation, surface temperature excess ratio parameters on the non-Darcy flow, combined heat and mass transfer have been presented graphically.

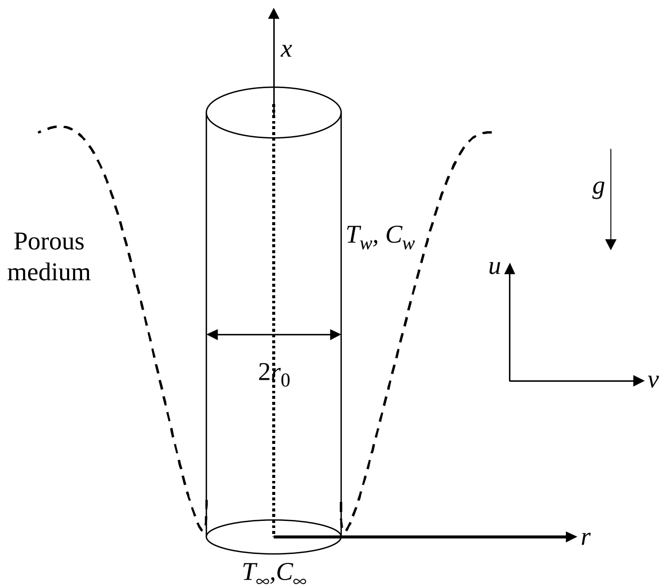

Figure 1.
Physical model and coordinate system

2. Mathematical formulation

Consider coupled heat and mass transfer by non-Darcy natural convection from a vertical cylinder embedded in a fluid-saturated porous medium in the presence of thermal radiation and chemical reaction effects. The temperature of the fluid and the concentration of the diffusing species at the surface of the cylinder T_w and C_w, are assumed to be higher than the corresponding values T_∞ and C_∞ far away from the surface. The variations of fluid properties are limited to density variations which affect the buoyancy force term only and the viscosity of the fluid is assumed to be an inverse linear function of temperature. Viscous dissipation has been neglected. For the flow in the porous medium, we adopt the non-Darcy model proposed by Ergun (1952). The fluid is considered to be a gray, absorbing-emitting radiation but non-scattering medium and the Rosseland approximation is used to describe the radioactive heat flux in the energy equation. Under the above assumptions, the governing boundary-layer equations for the steady-state non-Darcy free convection flow over vertical cylinder, the governing equations for the flow, heat, and mass transfer are (Yih, 1999; EL-Hakiem and Rashad, 2007):

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \tag{1}$$

$$\frac{\partial u}{\partial r} + \frac{\rho_\infty K^*}{\mu}\,\frac{\partial u^2}{\partial r} = \mu u \frac{\mathrm{d}}{\mathrm{dr}}\left(\frac{1}{\mu}\right) + \frac{\rho_\infty g K}{\mu}\left(\beta\frac{\partial T}{\partial r} + \beta^*\frac{\partial C}{\partial r}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{\alpha}{r}\,\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) - \frac{1}{\rho_\infty C_p}\,\frac{1}{r}\,\frac{\partial}{\partial r}(rq_r), \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial r}=\frac{D}{r}\frac{\partial}{\partial r}\left(r\frac{\partial C}{\partial r}\right)-K_1(C-C_\infty). \quad (4)$$

The boundary conditions are defined as follows:

$$r=r_0: \quad v=0, \ \ T=T_w, \ \ C=C_w,$$

$$r\to\infty: \quad u=0, \ \ T=T_\infty, \ \ C=C_\infty, \quad (5)$$

where x and r are coordinates measured along and normal to the surface, respectively. u, v are the velocity components in the x- and r-directions, T and C are the fluid temperature and concentration, respectively, ρ_∞ is the density away from the hot wall, C_p and K_1 are the specific heat at constant pressure and the dimensional chemical reaction parameter, respectively, q_r is the radiative heat flux in the r-direction, r_0 is the radius of the vertical cylinder, K is the permeability of porous medium, μ is the viscosity of the fluid, g is the gravity vector, α and D are the equivalent thermal and mass diffusivities specially of the saturated porous medium, β and β^* are coefficient of thermal expansion, and coefficient of concentration expansion, respectively, K^* is the inertial coefficient, and Darcy' s law is recovered when $K^*=0$.

By using Rosseland approximation (Siegel and Howell, 1972), we take:

$$q_r=-\frac{4\sigma}{3\beta_R}\frac{\partial T^4}{\partial r}=-\frac{16T^3}{3\beta_R}\frac{\partial T}{\partial r}, \quad (6)$$

with σ the Stefan-Boltzmann constant, β_R the Rosseland extinction coefficient. The term $16\sigma T^3/3\beta_R$ can be considered as the radiative conductivity.

The viscosity of the fluid is assumed to be an inverse linear function of temperature and it can be expressed as:

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma^*(T-T_\infty)\right] \ \text{or} \ \frac{1}{\mu}=a(T-T_r), \quad (7)$$

where $a=\gamma^*/\mu_\infty$ and $T_r=T_\infty-1/\gamma^*$.

For liquids such as water and crude oil, it is a reasonably good approximation. Here, both a and T_r are constants, γ^* is the thermal property of the fluid. The viscosity of the liquids usually decreases with an increasing temperature and it increases for gases such as air. So for liquids (water) $a>0$ and for gases (air) $a<0$.

The governing equations and boundary conditions can be made dimensionless by introducing the modified stream function such that:

$$ru=\frac{\partial\psi}{\partial r}, \quad rv=-\frac{\partial\psi}{\partial x}, \quad (8)$$

and using the following dimensionless variables:

$$\xi = \frac{2x}{r_0 Ra_x^{1/2}}, \quad \eta = \frac{r_0 Ra_x^{1/2}}{2x}\left(\frac{r^2}{r_0^2} - 1\right), \quad f(\xi,\eta) = \frac{\psi}{\alpha r_0 Ra_x^{1/2}},$$

$$\theta(\xi,\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\xi,\eta) = \frac{C - C_\infty}{C_w - C_\infty}, \quad u = \frac{\alpha Ra_x}{x} f',$$

$$v = -\frac{\alpha Ra_x^{1/2}}{x}\frac{r}{r_0}\left(\frac{1}{2}f + \frac{1}{2}\xi\frac{\partial f}{\partial \xi} - \frac{1}{2}\eta f'\right), \quad Er = \frac{K^*\alpha}{\upsilon_\infty d},$$

$$N = \frac{\beta^*(C_w - C_\infty)}{\beta(T_w - T_\infty)}, \quad \upsilon_\infty = \frac{\mu_\infty}{\rho_\infty}, \quad R_d = \frac{k\beta_R}{4\sigma T_\infty^3} \quad Ra = \frac{g\beta K(T_w - T_\infty)d}{\upsilon_\infty \alpha}, \tag{9}$$

$$Ra_x = \frac{g\beta K(T_w - T_\infty)x}{\upsilon_\infty \alpha}, \quad H = \frac{T_w}{T_\infty}, \quad Le = \frac{\alpha}{D},$$

$$\theta_r = \frac{(T_r - T_\infty)}{(T_w - T_\infty)} = \frac{-1}{\gamma(T_w - T_\infty)}, \quad \gamma = \frac{K_1 r_0}{\alpha}.$$

Substituting equations (8) and (9) into equations (1) thorough (5) yields the following non-similar dimensionless equations:

$$f'' - \frac{f'\theta'}{(\theta - \theta_r)} - 2ErRa\frac{(\theta - \theta_r)}{\theta_r}f'f'' + \frac{(\theta - \theta_r)}{\theta_r}(\theta' + N\phi') = 0, \tag{10}$$

$$(1 + \eta\xi)\theta'' + \frac{1}{2}(2\xi + f)\theta' + \frac{4}{3R_d}\left\{(1 + \eta\xi)\theta'[(H - 1)\theta + 1]^3\right\}' = \frac{\xi}{2}\left(f'\frac{\partial\theta}{\partial\xi} - \theta'\frac{\partial f}{\partial\xi}\right) \tag{11}$$

$$\frac{1}{Le}(1 + \eta\xi)\phi'' + \frac{1}{2}(2\xi + f)\phi' - \frac{\xi^2}{4}\gamma\phi = \frac{\xi}{2}\left(f'\frac{\partial\phi}{\partial\xi} - \phi'\frac{\partial f}{\partial\xi}\right) \tag{12}$$

and the boundary conditions become as follows:

$$\eta = 0: \quad f = 0, \ \theta = 1, \ \phi = 1,$$
$$\eta \to \infty: \quad f' = 0, \ \theta = 0, \ \phi = 0. \tag{13}$$

In the above equations, the primes denote the differential with respect to η, f and f' are the dimensionless free modified stream function and free stream velocity, d is the pore diameter, υ_∞ is the kinematic viscosity at ambient medium, k is the thermal conductivity, Ra_x the modified local Rayleigh number for the flow through the porous medium, Er and Ra are the Ergun number and Rayleigh number, respectively, based on pore diameter, N is the ratio of the buoyancy forces due to the temperature and concentration, ξ is the transverse curvature parameter, R_d is the conduction-radiation parameter, H is the surface temperature excess ratio, θ, ϕ are the dimensionless temperature and concentration, Le is the Lewis number, γ is the dimensionless of chemical reaction parameter. Here, θ_r is constant which represents the effect of variable viscosity of the fluid. It is negative for liquids (water) and positive for gases (air). If θ_r is large, then the effect of variable viscosity can be neglected. It is also necessary to note that $N = 0$ corresponds to the no species diffusion present, $N > 0$ corresponds to the aiding flow effects and $N < 0$ corresponds to the opposing flow effects.

It may be remarked that the system of equations (10)-(12) with the boundary conditions (15) reduce to the equations of a Darcy free convection flow over an isothermal vertical cylinder saturated porous medium, when $R_d, \theta_r \rightarrow \infty$ and $Er = 0$, this case has been studied by Minkowycz (1976), Yücel (1984) and Bassom and Rees (1996). It may also be remarked that the non-dimensional equations (13)-(14) with the boundary conditions (12) when $\theta_r \rightarrow \infty$ and $Er = 0$ reduce to the equations of a Darcy free convection flow over an isothermal vertical cylinder saturated porous medium in the presence of radiation interaction which has been investigated by Yih (1999) and when $N = \gamma = 0$, this case has been studied by EL-Hakiem and Rashad (2007).

For the flow adjacent to the surface of the cylinder, the heat transfer q_w and the local chemical species transfer m_w at the surface of the cylinder are given, respectively, by:

$$q_w = \left[\left(k + \frac{16\sigma T^3}{3\beta_R} \right) \frac{\partial T}{\partial r} \right]_{r=r_0}, \quad m_w = D \left(\frac{\partial C}{\partial r} \right)_{r=r_0}, \tag{14}$$

For practical applications, it is usually the local Nusselt number and local Sherwood number that is of interest. These can be expressed as:

$$Nu_x = \frac{q_w x}{k(T_w - T_\infty)}, \quad Sh_x = \frac{m_w x}{D(C_w - C_\infty)}, \tag{15}$$

Substituting equations (14) into equation (15), we obtain:

$$\frac{Nu_x}{Ra_x^{1/2}} = -\left(1 + \frac{4H^3}{3R_d} \right) \theta'(\xi, 0), \quad \frac{Sh_x}{Ra_x^{1/2}} = -\phi'(\xi, 0) \tag{16}$$

3. Numerical method

The non-similar equations (10) through (12) are non-linear and possess no analytical solution and must be solved numerically. The efficient, iterative, tri-diagonal, implicit finite-difference method discussed by Blottner (1970) has proven to be adequate for the solution of such equations. The equations are linearized and then descritized using three points central difference quotients with variable step sizes in the η direction and using two-point backward difference formulae in the ξ direction with a constant step size. The resulting equations form a tri-diagonal system of algebraic equations that can be solved by the well-known Thomas algorithm (Blottner, 1970). The solution process starts at $\xi = 0$ where equations (10) through (12) are solved and then marches forward using the solution at the previous line of constant ξ until it reaches the desired value of ξ. Owing to the non-linearities of the equations, an iterative solution with successive over or under relaxation techniques is required. The convergence criterion required that the maximum absolute error between two successive iterations be 10^{-6}. The computational domain was made of 196 grids in the η direction and 1,001 grids in the ξ direction. A starting step size of 0.01 in the η direction with an increase of 1.036 times the previous step size and a constant step size in the ξ direction of 0.01 were found to give very accurate results. The maximum value of η (η_∞) which represented the ambient conditions was assumed to be 275. The step sizes employed were arrived at after performing numerical experimentations to assess grid independence and ensure accuracy of the results. The accuracy of the aforementioned numerical method was validated by direct comparisons with the numerical results reported earlier by Minkowycz (1976), Yücel (1984), Bassom and Rees (1996), Yih (1999) and EL-Hakiem and Rashad (2007) in the absence of inertial coefficient, constant viscosity, thermal radiation and concentration buoyancy effects. Table I presents the results of these various comparisons. It can be seen from this table that excellent agreement between the results exists. This favorable comparison lends confidence in the numerical results to be reported in the next section.

4. Results and discussion

Numerical results are presented graphically for the Ergun number Er ranging from 0.0 to 0.2, the transverse curvature parameter ξ ranging from 0 to 10, the conduction-radiation parameter R_d ranging from 1 to 10^{10}, the surface temperature excess ratio H ranging from 1.1 to 2.5, the buoyancy ratio N ranging from -1.0 to 3.0, viscosity/temperature parameter $\theta_r = 2$, 4 for air case (gases), γ ranging from 0 to 3 and Lewis number $Le = 0.1$ to 10.0. The values of Lewis number (Le) are chosen to represent the most common diffusing chemical species which are of interest.

Figures 2 and 3 show typical velocity and temperature profiles in the boundary layer for various values of the surface temperature excess ratio H and the viscosity/temperature parameter θ_r, respectively. Increasing the values of θ_r increases, the velocity profiles increases and the temperature reduces. It is seen that the velocity increases with the increase in the viscosity parameter θ_r while the thermal boundary-layer thickness decreases as the viscosity increases. Thus, the increase of viscosities accelerates the fluid motion and reduces the temperature of the fluid along the wall. It is also seen that the momentum boundary-layer thickness gradually increases and the thermal boundary layer reduces from the boundary to the centre of the model when the viscosity parameter θ_r increases. The local Nusselt number for various values of viscosity/temperature parameter θ_r is shown in Figure 4. It is found that the local Nusselt number reduces with an increasing in the values of θ_r. This is an expected result because, in the case under consideration, an increase in the values of θ_r

Table I.
Values of $-\theta'(\xi,0)$ for $\theta_r, R_d = 10^{10}$ (i.e. $R_d, \theta_r \to \infty$) for a Darcy flow model ($Er = 0$)

ξ	Minkowycz (1976)	Yücel (1984)	Bassom and Rees (1996)	Yih (1999)	EL-Hakiem and Rashad (2007)	Present results
0	–	–	0.4438	0.4437	0.4438	0.4439
1	0.6149	0.6192	0.6191	0.6192	0.6149	0.6192
2	0.7668	0.7753	0.7750	0.7750	0.7668	0.7751
3	0.9085	0.9201	0.9191	0.9192	0.9085	0.9193
4	1.044	1.0571	1.055	1.0554	1.0443	1.0557
5	1.176	1.1884	1.185	1.1855	1.1764	1.1858
6	1.305	–	1.310	1.3110	1.3052	1.3112
7	1.435	–	1.431	1.4325	1.4354	1.4328
8	1.565	–	1.549	1.5509	1.5653	1.5512
9	1.696	–	1.665	1.6665	1.6960	1.6667
10	1.830	–	1.777	1.7797	1.8304	1.7781
20	–	–	2.8249	2.8245	2.8976	2.8247

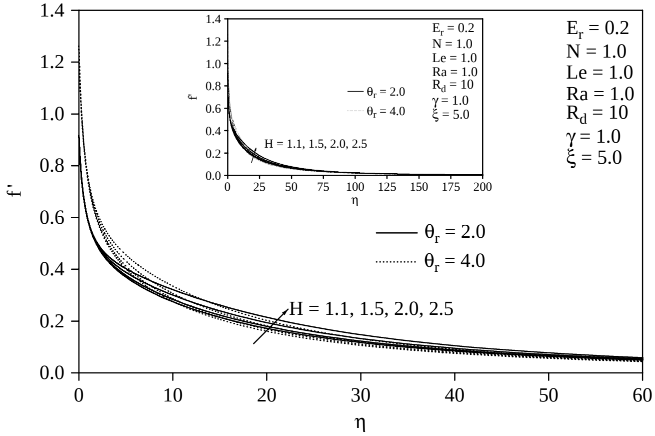

Figure 2.
Effect of H and θ_r on velocity profiles

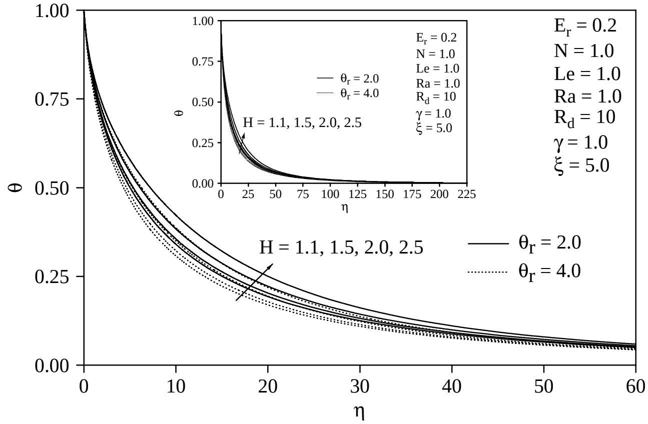

Figure 3.
Effects of H and θ_r on temperature profiles

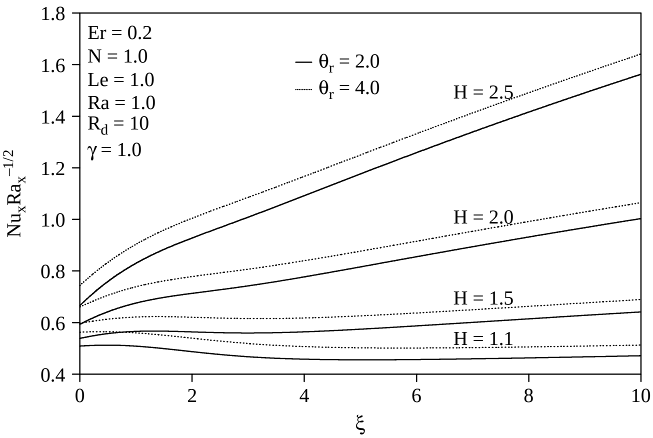

Figure 4.
Effects of H and θ_r on local Nusselt number

means a decrease in the temperature difference $\Delta T = (T_w - T_\infty)$, where the temperature difference ΔT between the hot cylinder and the free stream is maintained constant. Also, it is observed that the concentration of the fluid is almost not affected when the viscosity increases. Moreover, it is also seen that both the velocity and temperature profiles enhances with an increasing of the values of the surface temperature excess ratio H and the local Nusselt number increases with increasing in the values of H. The reason for this trend can be explained as follows, a higher value of H implies higher values of wall temperature. Consequently, the temperature gradient and hence Nusselt number increase is shown in Figure 4. Further, it is obvious that from the governing equations (10)-(12) are uncoupled. Therefore, changes in the values of H will cause no changes in the distributions of concentration of fluid, and also, the concentration of the fluid are not significant with increase of viscosity parameter θ_r for this reason, no figures for these variables are presented herein.

Figures 5-7 show typical velocity, temperature and concentration profiles in the boundary layer for various values of the chemical reaction parameter γ and the Lewis

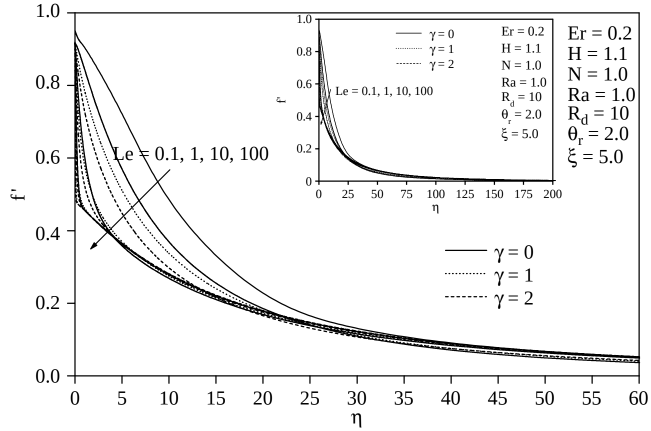

Figure 5.
Effects of Le and γ on velocity profiles

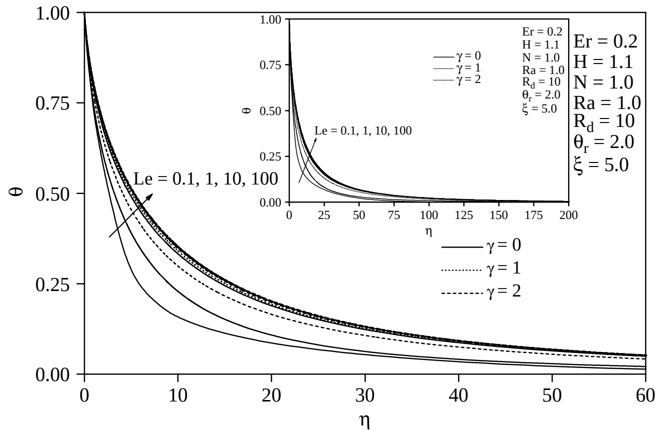

Figure 6.
Effects of Le and γ on temperature profiles

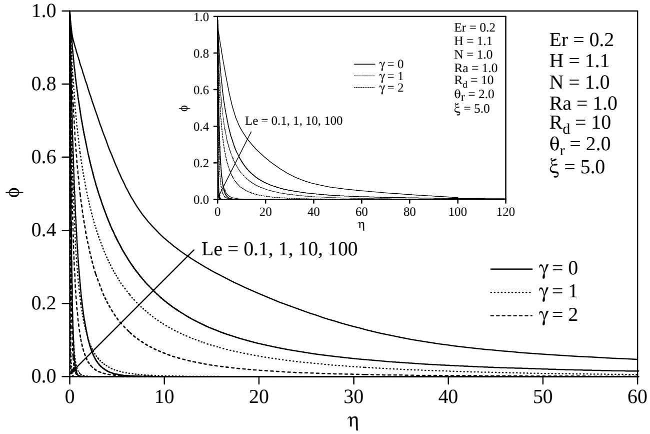

Figure 7.
Effects of Le and γ on concentration profiles

number *Le*, respectively. It is seen from these figures that the velocity and concentration of the fluid reduces significantly with increase of destructive reaction ($\gamma > 0$) of chemical reaction whereas also for increasing values of γ the values of temperature distribution slightly enhance. In turn, this causes the concentration buoyancy effects to decrease as γ increases. Consequently, less flow is induced along the cylinder resulting in decreases in the fluid velocity in the boundary layer. In addition, the concentration boundary-layer thickness decreases as γ increases. This causes from Figures 8 and 9, that the local Nusselt number decrease and the corresponding local Sherwood number increases owing to increase the values of chemical reaction parameter γ. It is also seen that the Lewis number is an important parameter in heat and mass transfer in porous medium processes as it leads to a decrease in the concentration profiles with an increase in *Le*, because the smaller values of *Le* are equivalent to an increase in the chemical molecular diffusivity. That is, the thickness of the concentration boundary layer decreases as the Lewis number increases, therefore, it is evident from figures that the

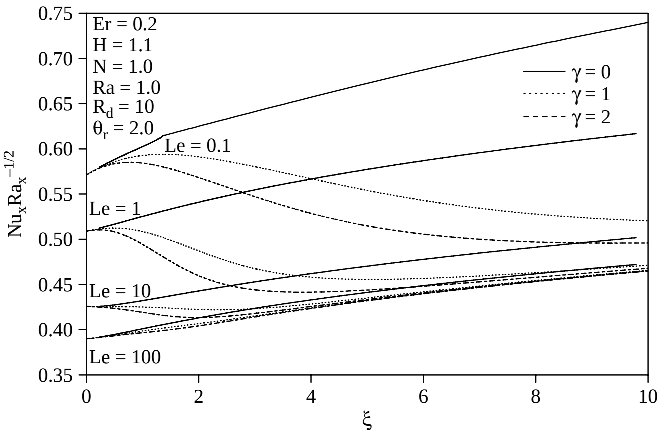

Figure 8.
Effects of Le and γ on local Nusselt number

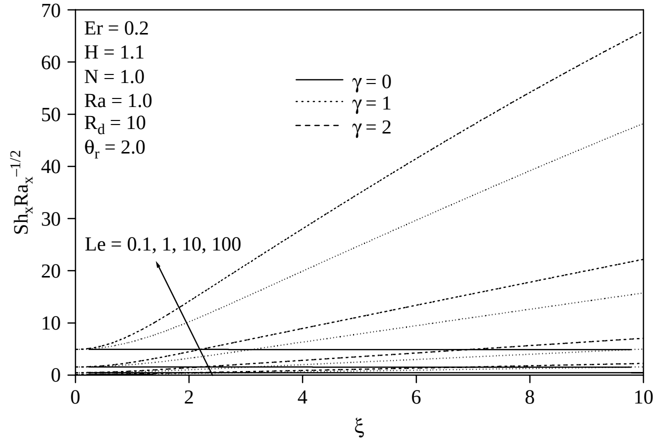

Figure 9.
Effects of Le and γ on local Sherwood number

increase in the Lewis number decreases the heat transfer coefficient, where as it increases the mass transfer coefficient.

Figures 10 and 11 show the velocity, temperature and concentration profiles in the boundary layer for various values of the Ergun number *Er* and buoyancy ratio *N*. An increasing in the values of the inertia parameter (Ergun number) implies more resistance to the flow which results in an increase in the momentum boundary layer and hence in the thermal boundary layer. Consequently, the velocity decreases whereas the temperature increases as *Er* increases. The local Nusselt number and local Sherwood number for various values of Ergun number *Er* are shown in Figures 12 and 13. It is found that the values of the local Nusselt and Sherwood numbers decrease due to increase in the value of *Er*, i.e. the increase in *Er* implies that the medium is offering more resistance to the fluid motion. Hence, the fluid motion is decelerated which results hence the inertia effect tends to in thicker momentum, thermal and concentration boundary layers. On other hand, positive values of *N* indicate aiding flow while negative values of *N* correspond to opposing flow. Increases in the value of

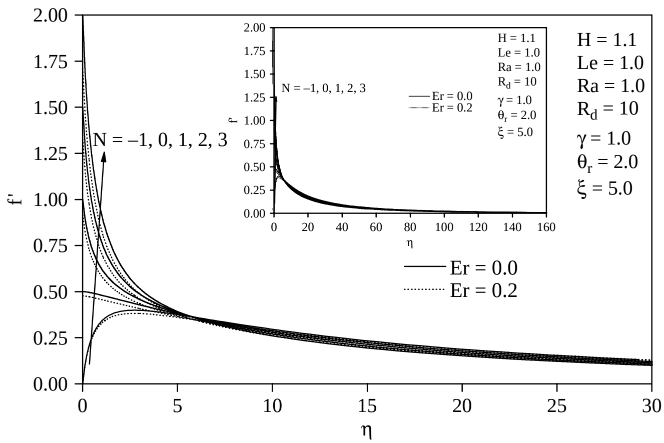

Figure 10.
Effects of Er and N on velocity profiles

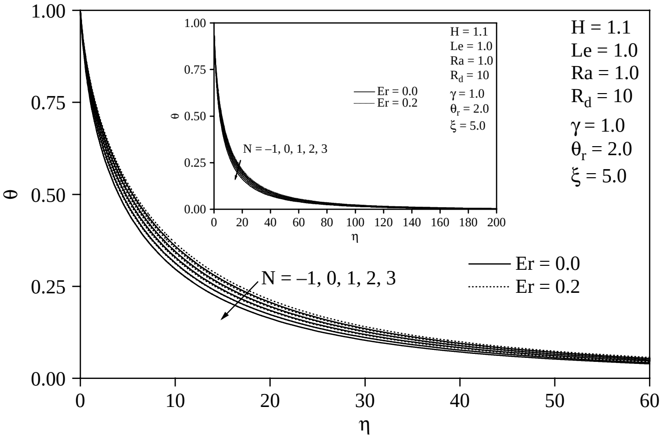

Figure 11.
Effects of Er and N on temperature profiles

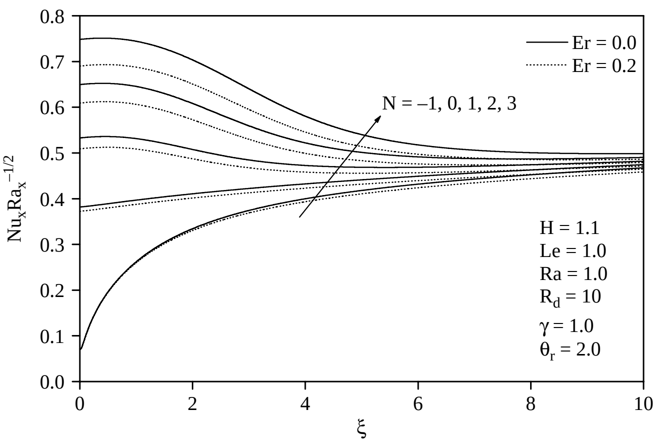

Figure 12.
Effects of Er and N on local Nusselt number

N have a tendency to increase the buoyancy effects due to concentration difference. This induces more flow along the cylinder surface causing the velocity of the fluid to increase as evident from Figure 10. This increase in the flow velocity occurs at the expense of both the temperature and concentration, which decrease as N increases as evident from Figure 11. In addition, the variations in the values of the local Nusselt and Sherwood numbers as a result of changing the value of N, respectively, as shown in Figures 12 and 13. It is clearly observed from these figures that that the effect of buoyancy ratio N is to increase the surface heat and mass transfer rates. This can be attributed to the fact that increasing N increases the vertical velocity and decreases the thickness of the temperature and concentration boundary layers. Therefore, the temperature and concentration gradients are increased and, hence, so are heat and mass transfer rates.

Figures 14 and 15 show the effects of the conduction-radiation parameter R_d on the velocity and the temperature profiles in the boundary layer, respectively. Decreasing

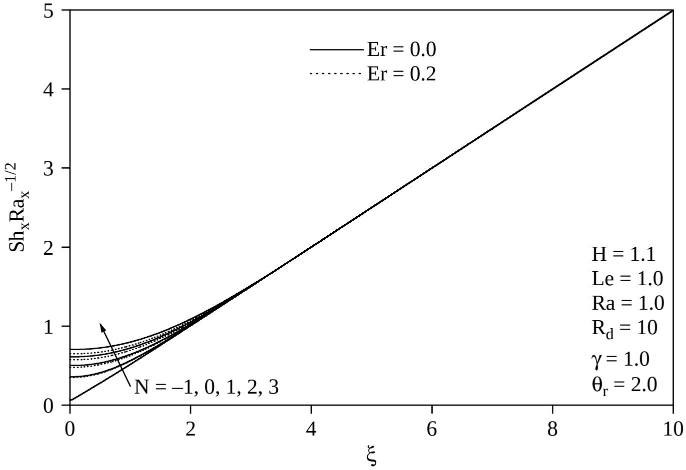

Figure 13.
Effects of Er and N on local Sherwood number

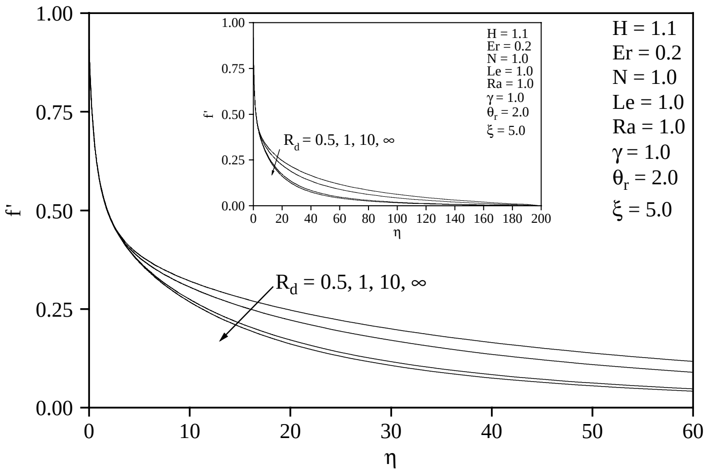

Figure 14.
Effects of R_d on velocity profiles

the thermal radiation parameter R_d produces significant increase in the thermal state of the fluid causing its temperature to increase. This increase in the fluid temperature induces by virtue of the thermal buoyancy effect more flow in the boundary layer causing the velocity of the fluid there to increase. These trends are clearly depicted by the increases in the velocity and temperature profiles as R_d decreases. Figure 16 shows the local Nusselt number for various values of the conduction-radiation parameter R_d. Obviously, the local Nusselt number increases with decreasing in the value of R_d. The reason for this trend can be explained as follows, smaller values of R_d implies higher values of wall temperature, the presence of thermal radiation works as a heat source and so the quantity of heat added to the fluid increases. Consequently, the temperature gradient and hence Nusselt number increase. Again, the profiles of concentration are unchanged by changes in R_d for the same reason mentioned above, thus no figures for

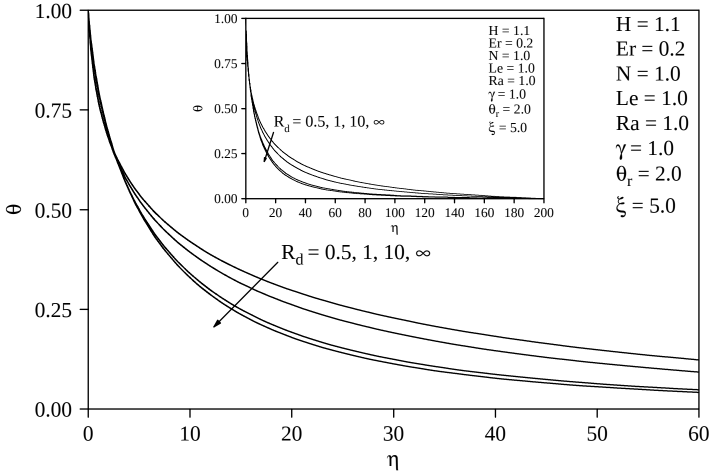

Figure 15. Effects of R_d on temperature profiles

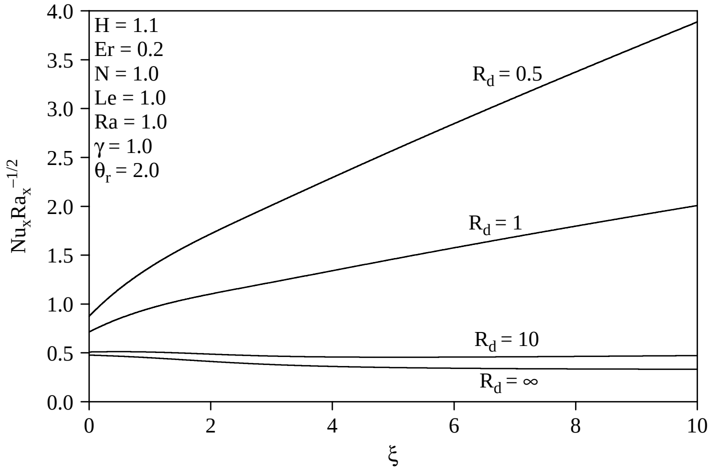

Figure 16. Effects of R_d on local Nusselt number

this parameter is presented herein. Finally, the local Nusselt and Sherwood numbers increase with increasing the transverse curvature parameter ξ due to the thickening of thermal and concentration boundary layers. The effect of ξ is more pronounced for large ξ, as shown in Figures 12, 13 and 16.

5. Conclusions

The problem of steady state, laminar heat and mass transfer by non-Darcy free convection boundary-layer flow adjacent to an isothermal vertical cylinder embedded in a saturated porous medium in the presence of thermal radiation and chemical reaction effects with temperature-dependent viscosity was considered. The fluid viscosity was assumed to vary as an inverse linear function of temperature and the Rosseland diffusion approximation was considered. A set of non-similar governing differential equations was obtained and solved numerically by an implicit finite-difference methodology. Comparisons with previously published work on various special cases of the general problem were performed and the results were found to be in excellent agreement. A representative set of numerical results for the velocity, temperature and concentration profiles as well as the local Nusselt number and the local Sherwood number was presented graphically and discussed. It was found that, in general, the local Nusselt number reduced as both of the conduction-radiation parameter and the Lewis number were increased. Also, while this physical parameter enhanced as the transverse curvature parameter and the surface temperature excess ratio increased. Furthermore, it was also found that the local Nusselt number decreased while the corresponding local Sherwood number increased owing to the increases in the value of chemical reaction parameter. On other hand, the parameter characterizing the variable viscosity strongly affected the velocity and temperature. The increase of the viscosity accelerated the fluid motion and reduced the temperature of the fluid along the wall. The results presented demonstrated quite clearly that viscosity/temperature parameter, which is an indicator of the variation of viscosity with temperature had a substantial effect on the drag and heat transfer characteristics, and therefore, the local Nusselt number increased as the viscosity/temperature parameter was increased. Finally, the effects of inertia parameter (Ergun number) at the cylinder surface implied that the medium was offering more resistance to the fluid motion, thus it was found to decrease both the local Nusselt and Sherwood numbers, while increasing the buoyancy ratio through the cylinder surface produced the opposite effect, namely increases in both the local Nusselt and Sherwood numbers.

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Corresponding author
A.J. Chamkha can be contacted at: achamkha@yahoo.com