Compressible Dusty-Gas Boundary-Layer Flow Over a Flat Surface

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Equations governing compressible boundary-layer laminar flow of a two-phase particulate suspension are developed based on a continuum representation of both phases. These equations include such effects as particle-phase viscous stresses, variable position-dependent particle slip coefficient, and general power-law viscosity-temperature and thermal conductivity-temperature relations. The dimensionless form of the equations are applied to the problem of flow over a semi-infinite flat surface. An appropriate transformation is employed to allow proper comparison with previously published results for special cases of this problem. The full coupled system of equations is solved numerically via an implicit finite-difference method. Graphical results for the density, and temperature profiles as well as the displacement thicknesses, skin-friction coefficients, and the wall heat transfer coefficient for both the fluid and particle phases are presented and discussed in detail. In addition, a parametric study is performed to illustrate the influence of the particle to fluid viscosity ratio and the viscosity-temperature power exponent on the flow properties.

Introduction

The problem considered in this paper is that of a steady, compressible, laminar, boundary-layer, two-phase (particle-fluid) flow over a semi-infinite flat surface. This type of flow occurs in many industrial applications. These include fluidized beds and environmental pollutant motions (Rudinger, 1980), gas purification, conveying of powdered materials and transport processes (Soo and Tien, 1960 and Soo, 1961). Special cases of this problem have been considered by Singleton (1965) and Wang and Glass (1988). Singleton (1965) uses a series method and obtains asymptotic solutions for the large-slip region (near the leading edge of the flat plate or surface) and the small-slip region far downstream. Wang and Glass (1988) obtains asymptotic solutions using a series expansion method similar to that of Singleton (1965). In addition, they report finite-difference-based numerical results for the whole computational domain. Their asymptotic large-slip solution provided the initial profiles of flow properties for their numerical finite-difference solution. There have been investigations dealing with the incompressible version of the present problem due to its importance as a fundamental problem in fluid-particle mechanics. Some employed the integral method (Soo, 1967) in obtaining their solutions, others used a series expansion method (see, Soo, 1968; and Datta and Mishra, 1982) and others utilized the finite-difference method (Osiptsov, 1980; Prabha and Jain, 1982; and Chamkha and Peddieson, 1989). Most of these investigators reported that when the dusty-gas model (Marble, 1970) is used to represent the two-phase suspension, a singularity exists in which the particle-phase density distribution at the wall becomes infinite somewhere downstream of the plate's leading edge. This singular behavior in the wall density is believed to be related to the particle-phase tangential velocity at the wall which vanishes at the singularity point. This catastrophic growth in the particle-phase wall density was shown to be of physical nature by imposing fluid-phase suction at the plate's surface and gradually reducing it to approach the impermeable plate solution (Chamkha and Peddieson, 1992). Refinements of the dusty-gas model to include particle-phase diffusive effects were reported and applied to the incompressible problem by Chamkha and Peddieson (1989) which resulted in a singularity-free flow solution. Chamkha (1994) reported the thermal aspects of the problem using the refined model.

It is of special interest in this paper to investigate whether the singular behavior observed in the problem of incompressible two-phase flow over a semi-infinite plate still exists for the compressible version and to understand the effects of particle-phase viscosity on the flow and heat transfer properties. Inclusion of the particle-phase viscous stresses in the dusty-gas model requires the use of additional boundary conditions on the particle phase. The proper boundary conditions to be satisfied by a solid particle phase at a surface are not understood at present. However, there is some experimental evidence that the particles experience some slip at the wall. In the present model, boundary conditions similar to those known from rarefied gas-dynamics with a wall slip coefficient dependent on the axial position of the plate is employed for the particle phase. A numerical method based on the finite-difference algorithm similar to that used by Wang and Glass (1988) is used for the solution of the present problem. Contrary to Wang and Glass (1988), a special modified Blasius transformation is employed which allows exact solutions for the initial profiles which are needed to start the numerical computations.

Problem Formulation

The dusty-gas model discussed by Marble (1970) is widely used in modeling two-phase particulate suspension flow situations. This model is restricted to small particle volume fraction and represents both phases as two interacting continua. Similar continuum models have been reported by other authors (see, for instance Soo, 1967 and Ishii, 1975). Other models based on the Lagrangian modeling approach have also been reported (Berlemont et al., 1990). Both modeling approaches have proven to be successful for modeling two-phase suspensions. However each approach may be more convenient than the other depending on the application. Since the individual motion of each particle is of no interest in the present work, and to allow comparisons between models in the solution of this problem, the continuum or the Eulerian approach will be employed in the present paper.

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Consider steady, compressible, laminar, boundary-layer two-phase flow in a half space bounded by a semi-infinite flat surface. The surface or plate is coincident with the plane $y = 0$ and the flow is a uniform stream in the plane $y > 0$ parallel to the surface. Far from the surface, both phases are in both hydrodynamic and thermal equilibrium. The particles are all assumed to be of one size and spherical in shape and moving with the same velocity. Besides, there is no radiative heat transfer from one particle to another, chemical reaction, coagulation, phase change, and deposition. The fluid phase is assumed to behave as a perfect gas. The fluid and particles motions are coupled only through drag and heat transfer between them. The drag force is modeled using Stokes linear drag theory and the small particle volume fraction assumption inherent in the dusty-gas model (Marble, 1970) is retained in this problem.

The governing equations for this investigation are based on the balance laws of mass, linear momentum, and energy for both phases. These can be written

$$\vec{\nabla}\cdot(\rho\vec{V}) = 0 \tag{1a}$$

$$\vec{\nabla}\cdot(\rho_p\vec{V}_p) = 0 \tag{1b}$$

$$\rho\vec{V}\cdot\vec{\nabla}\vec{V} = \nabla\cdot\underline{\underline{\sigma}} - \vec{f} \tag{1c}$$

$$\rho_p\vec{V}_p\vec{\nabla}\cdot\vec{V}_p = \vec{\nabla}\cdot\underline{\underline{\sigma_p}} + \vec{f} \tag{1d}$$

$$\rho c\vec{V}\cdot\vec{\nabla}T = \vec{\nabla}\cdot k\vec{\nabla}T + \underline{\underline{\sigma}}:\vec{\nabla}\vec{V} + (\vec{V} - \vec{V}_p)\cdot\vec{f} + Q_T \tag{1e}$$

$$\rho_p c_p\vec{V}_p\cdot\vec{\nabla}T_p = \underline{\underline{\sigma_p}}:\vec{\nabla}\vec{V}_p - Q_T \tag{1f}$$

where ρ, $\vec{V}$, $\underline{\underline{\sigma}}$, c, k, and T denote, density, velocity vector, stress tensor, specific heat, thermal conductivity, and temperature for the fluid phase, respectively. Properties and variables with subscript p denote the same thing for the particle phase. $\vec{\nabla}$ is the gradient operator. $\vec{f}$ and Q_T are the interphase drag force per unit volume of suspension, and the interphase heat transfer, respectively.

To completely define the problem, the following equations are used

$$\underline{\underline{\sigma}} = -P\underline{\underline{I}} + \mu(T)(\vec{\nabla}\vec{V} + \vec{\nabla}\vec{V}^T) \tag{2a}$$

$$\underline{\underline{\sigma_p}} = \mu_p(T_p)(\vec{\nabla}\vec{V}_p + \vec{\nabla}\vec{V}_p^T) \tag{2b}$$

$$\vec{f} = \rho_p(\vec{V} - \vec{V}_p)/\tau_v \tag{2c}$$

$$Q_T = \rho_p c_p(T_p - T)/\tau_T \tag{2d}$$

$$P = \rho RT \tag{2e}$$

In Eqs. (2), P is the fluid pressure, $\underline{\underline{I}}$ is the unit tensor, μ and μ_p are the fluid and particle dynamic viscosities, respectively, τ_v, τ_T, and R are the momentum and temperature relaxation times and the gas constant, respectively, and a superposed T denotes the transpose of a second-order tensor.

It is seen from Eqs. (2) that the particle phase is assumed to have viscous effects which are not present in the models reported by Singleton (1965) and Wang and Glass (1988). These effects can model particle-particle interaction and particle-wall interaction. They can also be thought of as a natural consequence of the averaging processes involved in representing a discrete system of particle as a continuum (Drew, 1983; and Drew and Segal, 1971). The particle-phase viscous effects have been investigated by many previous investigators (Gidaspow, 1986; Tsuo and Gidaspow, 1990; and Gadiraju et al., 1991). Also, the particles are assumed to be dragged along by the fluid and, therefore, have no analog of pressure.

Substituting the following dimensionless variables

$$s = x/(\tau_v U_\infty), \quad n = y\,\mathrm{Re}_\infty^{1/2}/(\tau_v U_\infty), \quad \Gamma(H) = \mu/\mu_\infty$$

$$\Gamma_p(H_p) = \mu_p/\mu_{p\infty}, \quad \Pr = \mu c/k, \quad \mathrm{Ec} = U_\infty^2/(cT_\infty)$$

$$\beta = \mu_{p\infty}/\mu_\infty, \quad \gamma = c/c_p$$

$$\vec{V} = U_\infty(u(s,n)\vec{e}_x + v(s,n)/\mathrm{Re}_\infty^{1/2}\vec{e}_y),$$

$$\vec{V}_p = U_\infty(u_p(s,n)\vec{e}_x + v_p(s,n)/\mathrm{Re}_\infty^{1/2}\vec{e}_y)$$

$$H = T/T_\infty, \quad H_p = T_p/T_\infty, \quad Q = \rho/\rho_\infty, \quad Q_p = \rho_p/\rho_\infty \tag{3}$$

(where $\mathrm{Re}_\infty = (\rho_\infty U_\infty^2\tau_v)/\mu_\infty$, e denotes a unit vector, and U_∞, ρ_∞, μ_∞, and T_∞ are the freestream velocity, density, viscosity, and temperature, respectively) along with Eqs. (2) into Eqs.

Nomenclature

- C = fluid-phase skin-friction coefficient
- c = fluid-phase specific heat at constant pressure
- Ec = fluid-phase Eckert number
- e_x, e_y = unit vectors in x and y directions, respectively
- F = nondimensionalized fluid-phase tangential velocity
- f = interphase force per unit volume acting on the particle phase
- G = nondimensionalized fluid-phase transformed normal velocity
- H = nondimensionalized fluid-phase temperature
- $\underline{\underline{I}}$ = unit tensor
- k = fluid-phase thermal conductivity
- P = fluid-phase pressure
- Pr = fluid-phase Prandtl number
- Q = nondimensionalized fluid-phase density
- Q_T = interphase heat transfer rate per unit volume to the particle phase
- r, S_R = constants defined in Eq. (18)
- q_w = wall heat transfer
- R = ideal gas constant
- Re = Reynolds number
- S = particle-phase slip parameter
- t_o = nondimensionalized fluid-phase wall temperature
- T = fluid-phase temperature
- u = x-component of velocity
- U_∞ = free stream velocity
- v = y-component of velocity
- V = fluid-phase velocity vector
- x, y = Cartesian coordinate variables
- β = viscosity ratio
- γ = specific heat ratio
- Δ = fluid-phase displacement thickness
- η = transformed normal coordinate
- Γ = fluid-phase viscosity coefficient
- κ = particle mass loading ratio
- μ = fluid-phase viscosity coefficient
- ρ = fluid-phase density
- $\underline{\underline{\sigma}}$ = fluid-phase stress tensor
- τ_T = temperature relaxation time
- τ_v = momentum relaxation time
- ω = power index for viscosity relation
- ξ = transformed tangential coordinate
- ∇ = gradient operator

Subscripts

- ∞ = free stream
- p = particle phase

Superscripts

- T = transpose of a second-order tensor

(1) and performing the usual boundary-layer order of magnitude analysis give

$$\partial_s(Qu) + \partial_n(Qv) = 0 \quad (4a)$$

$$\partial_s(Q_p u_p) + \partial_n(Q_p v_p) = 0 \quad (4b)$$

$$Q(u\partial_s u + v\partial_n u) = \Gamma\partial_n^2 u + (d_H \Gamma \partial_n H)\partial_n u + Q_p \Gamma(u_p - u) \quad (4c)$$

$$Q_p(u_p\partial_s u_p + v_p\partial_n u_p) = \beta(\Gamma_p \partial_n^2 u_p + (d_{H_p}\Gamma_p\partial_n H_p)\partial_n u_p) + Q_p\Gamma(u - u_p) \quad (4d)$$

$$Q_p(u_p\partial_s v_p + v_p\partial_n v_p) = \beta(\Gamma_p \partial_n^2 v_p + (d_{H_p}\Gamma_p\partial_n H_p)\partial_n v_p) + Q_p\Gamma(v - v_p) \quad (4e)$$

$$\mathrm{Pr}Q(u\partial_s H + v\partial_n H) = \Gamma\partial_n^2 H + d_H\Gamma(\partial_n H)^2 + \mathrm{Ec}\ \mathrm{Pr}\Gamma(\partial_n u)^2 + \mathrm{EcPr}Q_p\Gamma(u_p - u)^2 + 2Q_p\Gamma(H_p - H)/3 \quad (4f)$$

$$Q_p(u_p\partial_s H_p + v_p\partial_n H_p) = \beta\ \mathrm{Ec}\Gamma_p\gamma(\partial_n u_p)^2 + 2\gamma Q_p\Gamma(H - H_p)/(3\ \mathrm{Pr}) \quad (4g)$$

$$QH = 1 \quad (4h)$$

It should be noted in Eqs. (3) that x and y and, hence, s and n are tangential and normal coordinates and that u and v are corresponding velocities.

The physics of the present problem suggests the following boundary conditions:

$$u(s, 0) = 0, \quad u(s, \infty) = 1, \quad v(s, 0) = 0$$
$$H(s, 0) = t_o, \quad H(s, \infty) = 1, \quad u_p(s, \infty) = 1$$
$$v_p(s, 0) = 0, \quad v_p(s, \infty) = v(s, \infty), \quad H_p(s, \infty) = 1$$
$$Q_p(s, \infty) = \kappa, \quad Q(s, \infty) = 1 \quad (5)$$

where t_o is a dimensionless constant and $\kappa = \rho_{p\infty}/\rho_\infty$ is the mass loading ratio of the particles. In all the work to be presented later in this paper, κ is taken to be unity (Wang and Glass, 1988).

In the present work it is assumed that the viscosity-temperature relations for both phases are of the forms:

$$\Gamma = H(s, n)^\omega, \quad \Gamma_p = H_p(s, n)^\omega, \quad 0.5 \le \omega \le 1 \quad (6a, b)$$

where ω is a power index. It is assumed herein that both power indices for the fluid and particle phases are identical, and that the particle phase is treated as a viscous fluid. This would justify the pseudo particle viscosity which behaves as described in Eq. (6b). Singleton (1965) restricted his work to $\omega = 0.5$ while Wang and Glass (1988) allowed for different values of ω as is the case in the present work.

Substituting Eqs. (6) and the following modified Blasius transformations

$$s = \xi/(1 - \xi), \quad n = (2\xi/(1 - \xi))^{1/2}\eta$$
$$u = F(\xi, \eta), \quad v = ((1 - \xi)/(2\xi))^{1/2}(G(\xi, \eta) + \eta F(\xi, \eta))$$
$$u_p = F_p(\xi, \eta),$$
$$v_p = ((1 - \xi)/(2\xi))^{1/2}(G_p(\xi, \eta) + \eta F_p(\xi, \eta)) \quad (7)$$

into Eqs. (4) and (5) and rearranging result

$$\partial_\eta(QG) + QF + 2\xi(1 - \xi)\partial_\xi(QF) = 0 \quad (8)$$

$$\partial_\eta(Q_p G_p) + Q_p F_p + 2\xi(1 - \xi)\partial_\xi(Q_p F_p) = 0 \quad (9)$$

$$H^\omega\partial_\eta^2 F + (\omega H^{\omega-1}\partial_\eta H - QG)\partial_\eta F - 2\xi/(1 - \xi)((1 - \xi)^2 QF\partial_\xi F - Q_p H^\omega(F_p - F)) = 0 \quad (10)$$

$$\beta H_p^\omega \partial_\eta^2 F_p + (\beta\omega H_p^{\omega-1}\partial_\eta H_p - Q_p G_p)\partial_\eta F_p - 2\xi/(1 - \xi)((1 - \xi)^2 Q_p F_p \partial_\xi F_p + Q_p H^\omega(F_p - F)) = 0 \quad (11)$$

$$\beta H_p^\omega \partial_\eta^2 G_p + (\beta\omega H_p^{\omega-1}\partial_\eta H_p - Q_p G_p)\partial_\eta G_p - \beta \mathrm{H}_p^\omega \partial_\eta^2(\eta F_p) + \beta\omega H_p^{\omega-1}\partial_\eta H_p \partial_\eta(\eta F_p) - \eta Q_p G_p \partial_\eta F_p + \eta Q_p F_p^2 - 2\xi(1 - \xi)Q_p F_p \partial_\xi(G_p + \eta F_p) - 2\xi/(1 - \xi)Q_p H^\omega(G_p - G + \eta(F_p - F)) = 0 \quad (12)$$

$$H^\omega\partial_\eta^2 H + (\omega H^{\omega-1}\partial_\eta H - \mathrm{Pr}QG)\partial_\eta H - 2\xi(1 - \xi)\ \mathrm{Pr}QF\partial_\xi H + \mathrm{EcPr}H^\omega(\partial_\eta F)^2 + 2\xi/(1 - \xi)(\mathrm{EcPr}Q_p H^\omega(F_p - F)^2 + 2Q_p H^\omega(H_p - H)/3) = 0 \quad (13)$$

$$Q_p G_p \partial_\eta H_p - \beta\mathrm{Ec}\gamma H_p^\omega(\partial_\eta F_p)^2 + 2\xi(1 - \xi)Q_p F_p \partial_\xi H_p + 4\xi/(1 - \xi)\gamma Q_p H^\omega(H_p - H)/(3\ \mathrm{Pr}) = 0 \quad (14)$$

$$QH = 1 \quad (15)$$

$$F(\xi, 0) = 0, \quad F(\xi, \infty) = 1, \quad G(\xi, 0) = 0$$
$$H(\xi, 0) = t_o, \quad H(\xi, \infty) = 1, \quad F_p(\xi, \infty) = 1$$
$$G_p(\xi, 0) = 0, \quad G_p(\xi, \infty) = G(\xi, \infty), \quad H_p(\xi, \infty) = 1$$
$$Q_p(\xi, \infty) = 1, \quad Q(\xi, \infty) = 1 \quad (16)$$

Equations (8) through (16) represent general equations for steady, compressible, boundary-layer flow of a particle-fluid suspension over a flat surface. They represent a generalization of the dusty-gas model employed by Wang and Glass (1988) to include particle-phase viscous effects. In fact, they reduce to the equations given previously by Wang and Glass (1988) in the absence of the particle-phase viscosity. The advantage of using the modified Blasius transformations (Eq. (7)) is that they allow the closed-form solutions $Q_p = 1$, $H_p = 1$, $F_p = 1$, $G_p = -\eta$ at $\xi = 0$ and they convert the computational region from semi-infinite ($0 \le x < \infty$) to finite ($0 \le \xi \le 1$).

The exact form of boundary conditions to be satisfied by a particle phase at a surface are unknown at present. Since the particle phase may resemble a rarefied gas, a boundary condition borrowed from rarefied-gas dynamics is utilized in the present work. This can be written in the new variables as

$$F_p(\xi, 0) = S((1 - \xi)/(2\xi))^{1/2}\partial_\eta F_p(\xi, 0) \quad (17)$$

where S is a particle-phase slip parameter which is dependent on the coefficient of viscosity and wall slip velocity (difference between particle and fluid velocities). However, no attempt was made to relate S to the internal properties of the suspension.

In reality the particle-phase tangential velocity at the wall is controlled by many physical effects such as sliding friction, the nature of particle/surface collision, etc. It is not possible to model such effects with precision at present. To allow for a variety of particle-phase wall tangential velocity profiles, it is assumed that the wall slip parameter S is a function of the wall position ξ (since the wall slip velocity is related to ξ). A general function of the form

$$S = S_R((1 - \xi)/\xi)^r \quad (18)$$

(where S_R and r are constants) is employed in the present work. It can be seen that the form of Eq. (18) allows perfect particulate slip at $\xi = 0$ followed by approach to no-slip at a rate controlled by the values of S_R and r.

Of practical interest are the fluid- and particle-phase displacement thicknesses, the skin-friction coefficients, and the wall heat transfer coefficient. These are defined, respectively, as

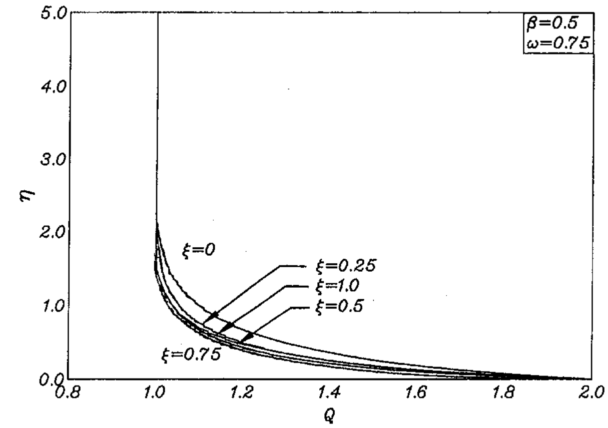

Fig. 1 Fluid-phase density profiles

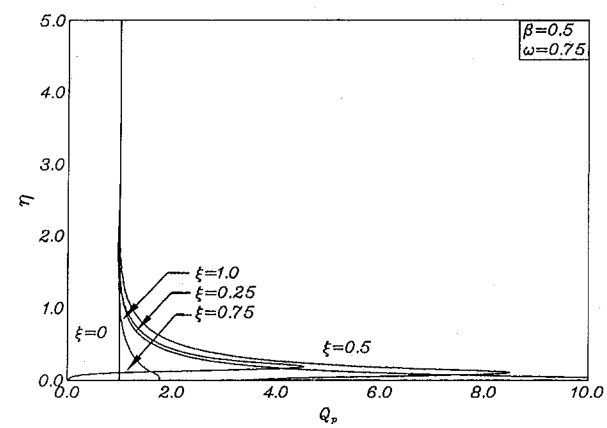

Fig. 2 Particle-phase density profiles

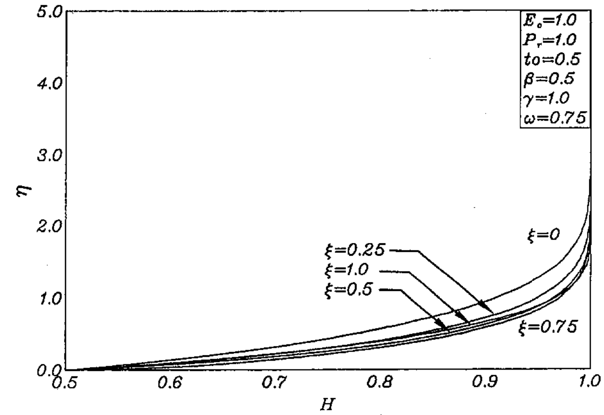

Fig. 3 Fluid-phase temperature profiles

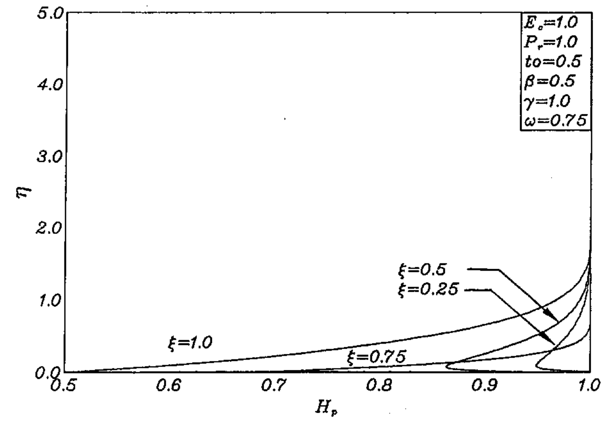

Fig. 4 Particle-phase temperature profiles

$$\Delta(\xi) = \int_0^\infty (1 - QF)d\eta \tag{19a}$$

$$\Delta_p(\xi) = \int_0^\infty (1 - Q_pF_p)d\eta \tag{19b}$$

$$C(\xi) = H(\xi, 0)^\omega \partial_\eta F(\xi, 0) \tag{19c}$$

$$C_p(\xi) = \beta H_p(\xi, 0)^\omega \partial_\eta F_p(\xi, 0) \tag{19d}$$

$$q_w(\xi) = H(\xi, 0)^\omega \partial_\eta H(\xi, 0)/(\mathrm{EcPr}) \tag{19e}$$

Results and Discussion

Equations (8) through (15) are obviously nonlinear and exhibit no closed-form or similar solution. They, therefore, must be solved numerically. The tri-diagonal, implicit, iterative, finite-difference method discussed by Blottner (1970) and similar to that used by Wang and Glass (1988) has proven to be successful in the solution of boundary-layer problems. For this reason, it is adopted in the present work.

All first-order derivatives with respect to ξ are represented by two-point backward difference formulas. All second-order differential equations in η are discretized using a three-point central difference quotients while all first-order differential equations in η are discretized using the trapezoidal rule. The computational domain was divided into 1001 nodes in the ξ direction and 195 nodes in the η direction. Since it is expected that most changes in the boundary layer occur in the vicinity of the wall, variable step sizes in η are utilized with $\Delta\eta_1 = 0.001$ and a growth factor of 1.03. Also, constant small step sizes in ξ with $\Delta\xi = 0.001$ are used. The governing equations are then converted into sets of linear tri-diagonal algebraic equations which are solved by the Thomas Algorithm (Blottner, 1970) at each iteration. The convergence criterion required that the difference between the current and the previous iterations be 10^{-5}. It should be mentioned that many numerical experimentations were performed by altering the step sizes in both directions to ensure accuracy of the results and to assess grid independence. For example, when $\Delta\eta_1$ was set to 0.01 instead 0.001, an average error of about eight percent was observed in the results with the maximum error being close to $\xi = 1$. Also, when $\Delta\eta_1$ was equated to 0.0001 no significant changes of results were observed. For this reason $\Delta\eta_1 = 0.001$ was chosen and employed in producing the numerical results. The flow and heat transfer parameter are not as sensitive to $\Delta\xi$ as they are sensitive to $\Delta\eta_1$. For this reason, a constant step size was used in the ξ direction. The sensitivity analysis of the results to changes in $\Delta\xi$ was also performed. For instance, when $\Delta\xi$ was set to 0.01, an average deviation of five percent from the results with $\Delta\xi = 0.001$. Smaller values of $\Delta\xi$ than 0.01 produced no changes in the results and, therefore, $\Delta\xi$ was set to 0.001 in all the produced results. As far as the convergence criterion is concerned, two types were tried. One was based on the percentage error between the previous and the current iterations and the other was based on their difference. Since we are not dealing with very small numbers, the convergence criterion based on the difference between the previous and current iterations was employed in the present study. No convergence problems were encountered even with the small value of 10^{-5} used in this work. It should be mentioned that when a second-order ξ-derivative is used, a slight enhancement of results (within two percent) was predicted. However, all results to be presented are based on the first-order-accurate approximation of ξ derivatives. Equations

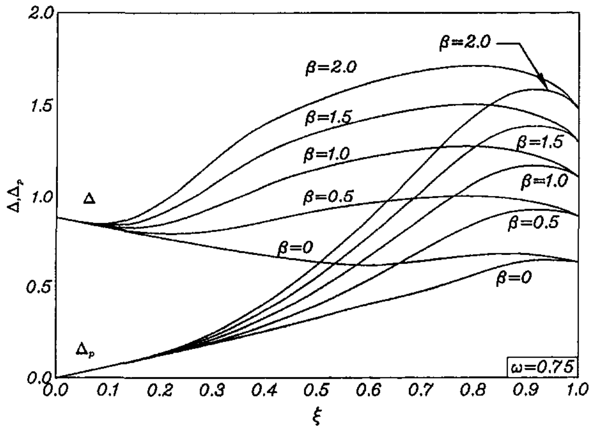

Fig. 5 Fluid and particle-phase displacement thicknesses profiles

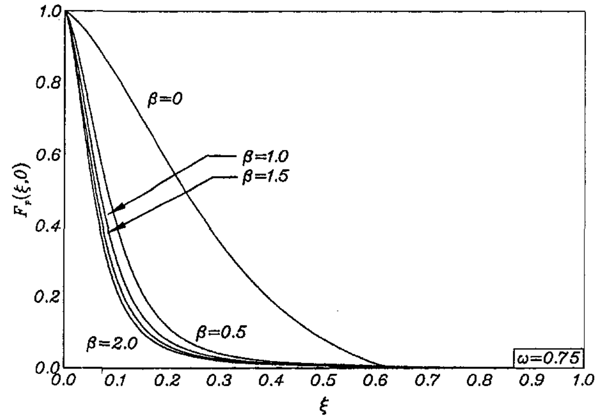

Fig. 8 Wall particle-phase tangential velocity profiles

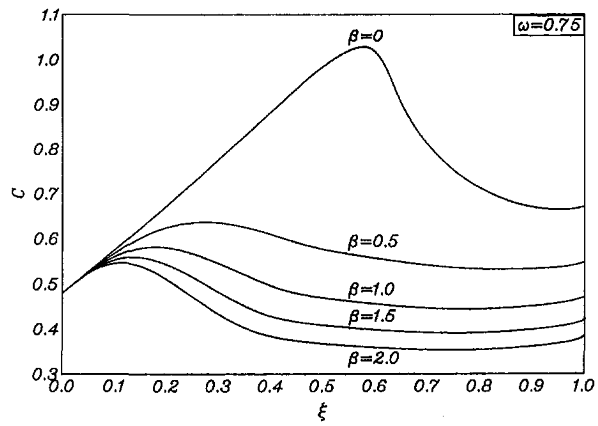

Fig. 6 Fluid-phase skin friction coefficient profiles

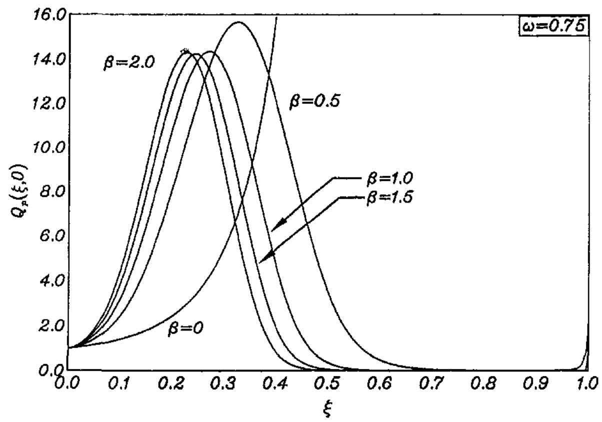

Fig. 9 Wall particle-phase density profiles

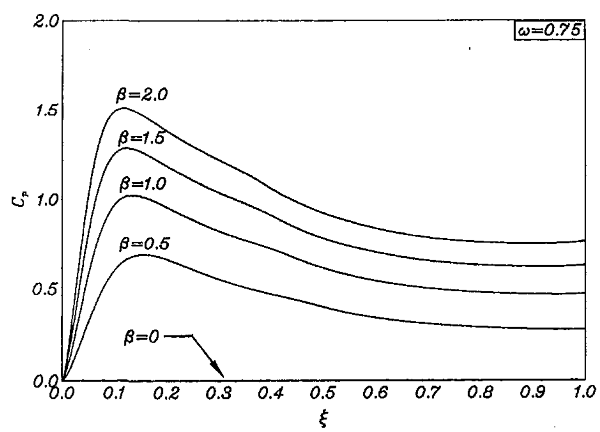

Fig. 7 Particle-phase skin friction coefficient profiles

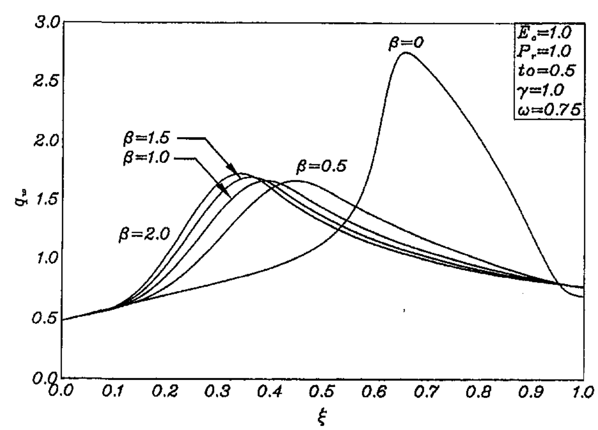

Fig. 10 Wall heat transfer coefficient profiles

(8) through (15) were solved for G, Q_p, F, F_p, G_p, H, H_p, and Q, respectively. Many results were obtained throughout the course of this work. A representative set is presented in Figs. 1 through 10 to show the effects of the viscosity ratio β. In all these figures $S_R = 50$ and $r = 1$.

Figures 1 and 2 present representative fluid- and particle-phase density profiles, respectively. It is seen from these figures that most changes occur in a small region in the vicinity of the wall and uniform conditions exist above it. Since the wall temperature t_O is taken to be 0.5, Eq. (15) implies that the fluid density at the wall must be equal to 2 as is shown in Fig. 5. Figure 6 shows that the particle-phase density at the wall becomes relatively large in the region $0.25 \le \xi \le 0.5$ and then its value vanishes for $\xi = 0.75$ and reaches its expected value at $\xi = 1$. This suggests the existence of a particle-free zone near $\xi = 0.75$. This will be discussed later.

Figures 3 and 4 show temperature profiles for both the fluid and particle phases, respectively. Like the velocity profiles, these figures show a complete transition to equilibrium conditions. It is seen that the particle-phase temperature profiles at $\xi = 0.25$ and $\xi = 0.5$ decrease in value close to the wall and then increase to reach the free-stream value. This behavior probably occurs as these profiles are being adjusted to reach equilibrium conditions with the fluid phase. It should be noted that while the fluid-phase temperature is diffusive and, thus, requires a wall boundary condition, the particulate temperatures do not. In general, if particulate-phase stresses are included in the mathematical model, then a thermal conductivity for particles arises (Soo, 1990). However, this was neglected in the present work compared with other effects and to allow the particle-phase wall temperature to adjust itself through the energy coupling mechanism between the phases.

Figures 5 through 10 are presented to illustrate the effect of the particle-phase viscosity on the flow properties. Figures 5 through 7 show the behavior of the displacement thicknesses of both phases Δ and Δ_p, the fluid-phase skin-friction coefficient C, and the particle-phase skin-friction coefficient C_p along the flat surface for various values of β, respectively. At $\xi = 0$, the drag force between the phases is maximum and it decreases as ξ increases until it vanishes at $\xi = 1$ where equilibrium between the phases exists. This momentum exchange mechanism causes the fluid-phase displacement thickness to decrease (which causes an increase in the fluid-phase skin friction) and the particle-phase displacement thickness to increase until equilibrium is reached. However, as β increases the effective viscosity of the mixture increases and causes a rapid increase in the values of Δ as the flow moves downstream toward equilibrium. It should be noted that the particle-phase streamlines are completely unaffected by the fluid-phase displacement. This is because of the frozen condition (where both phases move independently) existing at $\xi = 0$. These behaviors are evident from Fig. 5. As β increases, the domain of particle-phase viscous effects increases causing an increase in the region close to the wall where significant deviations from uniformity occur. This results in increases in the values of Δ and Δ_p and decreases in the values of C as shown in Figs. 5 and 6. In addition, the values of C_p shown in Fig. 6 increases as β increases since C_p is directly proportional to β (see Eq. (19d)).

Figures 8 and 9 present the wall particle-phase tangential velocity and density profiles for various values of β, respectively. The transition from a perfect slip condition at $\xi = 0$ to a no-slip condition downstream at about $\xi \geq 0.6$ is shown in Fig. 8. Figure 9 shows that for an inviscid particle phase ($\beta = 0$) the particle-phase wall density becomes large in the vicinity of $\xi = 0.5$. This behavior was also observed for the case of incompressible flow (Ospitsov, 1980 and Chamkha and Peddieson, 1989). However, in the present work a continuous solution for $Q_p(\xi, 0)$ is predicted in the entire range $0 \leq \xi \leq 1$ unlike the incompressible case where no continuous solutions existed. The peak value of $Q_p(\xi, 0)$ for $\beta = 0$ is large and falls outside the range of the figure. It has a value of 240 occurring at $\xi = 0.62$ and approaches the equilibrium conditions at $\xi = 1$ without going below unity. This indicates that a particle-free zone does not exist for $\beta = 0$. It is seen that as the particulate slip decreases, lower peak values for $Q_p(\xi, 0)$ are predicted. However, for $\beta > 0$ (viscous particle phase) $Q_p(\xi, 0)$ vanishes over a big region far from the leading edge of the surface and then approaches the equilibrium conditions at $\xi = 1$. The vanishing of $Q_p(\xi, 0)$ is suggestive of the formation of a particle-free zone at the wall somewhere downstream. This phenomenon has been predicted by the work of Young and Hanratty (1991). It is seen from Fig. 9 that as β increases the peaks in the $Q_p(\xi, 0)$ profiles move toward the leading edge of the plate and the particle-free region increases.

If the vanishing of the particle-phase density at the wall represents a physical phenomenon, then the dusty-gas model employed in the present work may be inadequate because the equations of this model are derived under the assumption that the entire space is occupied by both phases. Enhancements to the model which eliminates the existence of the particle-free zone will be discussed in another contribution.

Figure 10 illustrates the changes in the wall heat transfer coefficient q_w as the viscosity ratio β is altered. It is seen that a sharp peak in the values of q_w exits for $\beta = 0$ and this peak moves upstream and its value decreases as β increases. The behavior of q_w with respect to ξ observed in the figure is a property of relaxation type problems.

More graphical results were obtained (and not presented herein for brevity) illustrating the influence of varying ω, S_R, and r. Increasing the power index ω causes the fluid- and particle-phase viscosities to decrease. This reduces the fluid and particle domain of viscous effects which result in decreases in the values of Δ and Δ_p. Both C and C_p are directly proportional to the wall viscosities which are related to the wall temperatures of both phases. The fluid phase has a constant wall temperature ($t_O = 0.5$) while the particle phase has a variable wall temperature. Thus, increasing ω causes $H(\xi, 0)^\omega$ and $H_p(\xi, 0)^\omega$ to decrease and increase, respectively. This causes a decrease in C and an increase in C_p. It is expected that ω has a negligible effect on the particle-phase wall tangential velocity and a slight effect on its wall density. Decreases in the fluid-phase wall viscosity caused by increasing ω, as mentioned earlier, cause decreases in the fluid thermal conductivity at the wall. This causes the wall heat transfer to decrease. All these statements are consistent with predicted results.

In addition, it was observed that as either S_r or r is increased, the values of Δ, Δ_p, and C_p were increased while those of C and q_w decreased. Also, some adjustments were made in the computer program and results for the incompressible case were obtained. These results were in excellent agreement with those reported by Chamkha and Peddieson (1989) and Chamkha (1994). The velocity and temperature fields reported by Wang and Glass (1988) were compared with the present results and were found to be in good agreement. These comparisons gave some confidence in the numerical procedure. No comparisons with experimental data were made since these data are lacking at present.

Conclusion

A continuum dusty-gas model modified to include particle-phase viscous effects was employed in analyzing steady, compressible, laminar, boundary-layer flow of a particulate suspension over a flat surface. The mathematical model included conservation equations for mass, momentum, and energy for each phase where diffusive transport of thermal energy in the particle phase was neglected and where thermal and momentum exchange between the phases was specified in terms of relaxation time constants. Similar to the carrier fluid, the particle phase was assumed to have a general power-law viscosity-temperature relation. The governing equations were solved numerically via an implicit, iterative, finite-difference method and numerical solutions for the flow and heat transfer aspects of the problem were reported and discussed. A parametric study was performed to show the effects of the particle-phase viscosity and the viscosity-temperature power index. In contrast with the incompressible version of the current problem, it was found that continuous solutions existed throughout the computational domain. Another major prediction of the present work was that for a viscous particle phase a particle-free zone was formed at the plate surface. This prediction could not be verified by experimental data due to the absence of such data at present. Favourable comparisons with previously published results on special cases of this problem were made which gave confidence in the accuracy of the numerical method. It is hoped that the present results be of use for environmental agencies in validating computer routines and serve as a stimulus for experimental work on the present problem.

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References

Berlemont, A., et al., 1990, "Particle Lagrangian Simulation in Turbulent Flows," *International Journal of Multiphase Flow*, Vol. 16, pp. 19–34.

Blottner, F. G., 1970, "Finite-Difference Methods of Solution of the Boundary-Layer Equations," *AIAA Journal*, Vol. 8, pp. 193–205.

Chamkha, A. J., and Peddieson, J., 1992, "Singular Behavior in Boundary-Layer Flow of a Dusty Gas," *AIAA Journal*, Vol. 30, pp. 2966–2968.

Chamkha, A. J., and Peddieson, J., 1989, "Boundary-Layer Flow of a Particle-Fluid Suspension Past a Flat Plate," *Developments in Mechanics*, Vol. 15, pp. 315–316.

Chamkha, A. J., 1994, "Effects of Particulate Diffusion on the Thermal Flat Plate Boundary Layer of a Two-Phase Suspension," ASME *Journal of Heat Transfer*, Vol. 116, pp. 236–239.

Datta, N., and Mishra, S. K., 1982, "Boundary Layer Flow if a Dusty Fluid Over a Semi-Infinite Flat Plate," *Acta Mechanica*, Vol. 42, pp. 71–83.

Drew, D. A., 1983, "Mathematical Modeling of Two-phase Flow," *Annual Review of Fluid Mechanics*, Vol. 15, pp. 261–291.

Drew, D. A., and Segal, L. A., 1971, "Analysis of Fluidized Beds and Foams Using Averaged Equations," *Studies in Applied Mathematics*, Vol. 50, pp. 233, 252.

Gadiraju, M., and et al., 1992, "Exact Solutions for Two-Phase Vertical Pipe Flow," *Mechanics Research Communications*, Vol. 19, pp. 7–13.

Gidaspow, D., 1986, "Hydrodynamics of Fluidization and Heat Transfer: Super Computer Modeling," *Applied Mechanics Reviews*, Vol. 39, pp. 1–23.

Ishii, M., 1975, Thermo-Fluid Dynamic Theory of Two-Phase Flow, Eyrolles, Paris.

Marble, F. E., 1970, "Dynamics of Dusty Gases," *Annual Review of Fluid Mechanics*, Vol. 2, pp. 297–446.

Osiptsov, A. N., 1980, "Structure of the Laminar Boundary Layer of a Disperse Medium on a Flat Plate," *Fluid Dynamics*, Vol. 15, pp. 512–517.

Prabha, S., and Jain, A. C., 1980, "On the Use of Compatibility Conditions in the Solution of Gas Particulate Boundary Layer Equations," *Applied Scientific Research*, Vol. 36, pp. 81–91.

Rudinger, G., 1980, *Fundamentals of Gas-Particle Flow*, Elsevier.

Singleton, R. E., 1965, "The Compressible Gas-Solid Particle Flow Over a Semi-Infinite Flat Plate," *Zamp*, Vol. 16, pp. 421–449.

Soo, S. L., 1990, Multiphase Fluid Dynamics, Science Press, Beijing, China.

Soo, S. L., 1961, "Boundary Layer Motion of a Gas-Solid Suspension," University of Illinois, Project Squad Report Ill-3p.

Soo, S. L., 1967, *Fluid Dynamics of Multiphase Systems*, Blaisdell Publishing, Waltham, Mass.

Soo, S. L., 1968, "Non-Equilibrium Fluid Dynamics-Laminar Flow Over a Flat Plate," *ZAMP*, Vol. 19, pp. 545–563.

Soo, S. L., and Tien, C. L., 1960, "Effect of the Wall on Two-Phase Turbulent Motion," *Journal of Fluid Mechanics*, Vol. 27: ASME *Journal of Applied Mechanics*, Vol. 82, Series E, pp. 5–11.

Tsuo, Y. P., and Gidaspow, D., 1990, "Computation of Flow Patterns in Circulating Fluidized Beds," *AIChE Journal*, Vol. 36, pp. 88–896.

Wang, B. Y., and Glass, I. I., 1988, "Compressible Laminar Boundary-Layer Flows of a Dusty Gas Over a Semi-Infinite Flat Plate," *Journal of Fluid Mechanics*, Vol. 186, pp. 223–241.

Young, J. B., and Hanratty, T. J., 1991, "Trapping of Solid Particles at a Wall in a Turbulent Flow," *AIChE Journal*, Vol. 37, pp. 1529–1536.