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FULLY DEVELOPED MIXED CONVECTION OF A MICROPOLAR FLUID IN A VERTICAL CHANNEL WITH BOUNDARY CONDITIONS OF THE THIRD KIND

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ABSTRACT

In this study, we study the flow and heat transfer characteristics of the mixed convection flow of micropolar fluid in a vertical channel with boundary conditions of third kind. The plates exchange heat with an external fluid. Both conditions of equal and of different reference temperatures of the external fluid are considered. The exact solutions are found for the velocity and microrotation velocity. The effects of mixed convection parameter, vortex viscosity on the velocity and microrotation velocity have been discussed for equal and unequal Biot numbers. It is found that the increase in the vortex viscosity parameter decreases the velocity for both assisting and opposing flow. The mirorotation velocity decreases for assisting flow whereas it increases for opposing flow for large values of vortex viscosity parameter for equal Biot numbers. The vortex viscosity parameter reduces the average velocity where as it will not effect the bulk temperature for equal Biot numbers.

Keywords: Mixed convection, micropolar fluid, boundary conditions of third kind.

1. INTRODUCTION

Mixed convection is defined as heat transfer situations where both natural convection and forced convection heat transfer mechanism interact. In the past 20 years, mixed convection in a vertical heated channel has received attention due to its extensive practical applications such as cooling of modern electronic systems, heat exchangers, chemical processing equipments, transport of heated or cooled fluids, and so on. Comprehensive reviews have been conducted by Incropera [1], Aung [2] and Gebhart et al. [3]. Aung and Worku [4] analyzed mixed convection in a vertical channel with boundary conditions of uniform wall temperatures. The cases of either uniform temperature or uniform heat flux at each boundary surface have been studied by Cheng, Kou and Huang [5] and by Hamadah and Wirtz [6]. The boundary condition of linearly varying wall temperatures has been considered by Tao [7]. Aung and Worku [8, 9] have studied the developing flow with asymmetric wall temperatures [8] and with asymmetric wall heat fluxes [9]. The developing flow with asymmetric wall temperatures has been also considered by Ingham, Keen and Heggs [10], with particular reference to situations where reverse flows occur. The laminar mixed convection with viscous dissipation in the fully- developed region of a parallel plate vertical channel has been studied by Barletta [11] Following the work of Aung and Worku [8, 9] and Barletta [11], Umavathi and Malashetty [12], Umavathi et al. [13, 14, 15] and Prathap et al. [16,17,18,19,] analyzed the mixed convection in a vertical channel. The laminar forced convection heat transfer in the thermal entrance region of a rectangular channel has been analyzed either for the temperature boundary condition of the first kind characterized by prescribed wall temperature (Wibulswal [20], Lyczkowski et al. [21] and Javeri, [22]) or for the boundary condition of the second kind, expressed by the prescribed wall heat flux (Hicken, [23] and Sparrow and Siegel, [24]). A more realistic condition in many applications, however, will be temperature boundary condition of third kind: the local wall heat flux is a linear function of the local wall temperature. Heat transfer in laminar region of a flat channel for the temperature boundary condition of third kind was explored by Javeri and Koln [25]. Javeri [26] investigated the influence of the temperature boundary condition of the third kind on the laminar heat transfer in the thermal entrance region of a rectangular channel. Later Zanchini [27] analyzed the effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of third kind. The theory of micropolar fluids developed by Eringen [28, 29, 30] has been a popular area of research in recent years. In this theory, the local effects arising from microstructure and intrinsic motions of the fluid elements were taken into account. It is expected to describe successfully the non-Newtonian behavior of certain fluids, such as liquid crystal, ferro liquids, colloidal fluids and liquid with polymer additives. The similarity solution for the micropolar boundary layer flow over a semi – infinite plate was obtained by Ahmadi [31]. Jena and Mathur [32] employed a finite difference technique to analyze the heat transfer characteristics of mixed convection micropolar fluid flow over an isothermal vertical plate. The conduction effect of wall in micropolar fluids flow has been considered and solved by Hsu and Chen [33] and Hsu and Tsai [34] using the cubic spline collocation method. The problem of mixed convection heat transfer in a micropolar fluid flow has been extensively researched by a number of investigators [35, 36, 37]. Chamkha et al. [38] analyzed numerical and analytical solutions of the developing laminar free convection of a micropolar fluid in a vertical parallel plate channel with asymmetric heating. Kumar et al. [39] examined the finite element solution of mixed convection micropolar fluid flow between two vertical plates with varying temperature. Cheng [40] studied the fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations. Umavathi et al. [41] analyzed the flow and heat transfer of a micropolar fluid sandwiched between viscous fluid layers. Prathap Kumar et al. [42] have studied the problem of fully developed free convective flow of micropolar and viscous fluids in a vertical channel. Most recently, Umavathi and Prathap [43] found the exact solutions for the mixed convection flow of micro-polar fluid in a vertical channel with symmetric and asymmetric wall conditions. Most of the previous studies on mixed convection in channels have been related to Newtonian fluids with boundary conditions of first or second kind. Despite the importance of the micropolar fluids mentioned above, there are only a few research efforts on mixed convection of these fluids in enclosures. The aim of the present article is to analytically study mixed convection heat transfer of micropolar fluid in a vertical channel with boundary conditions of third kind. In the absence of viscous dissipation and vortex viscosity, the solutions agree with Zanchini [27] for Newtonian fluid.

2. MATHEMATICAL FORMULATION

Consider the steady and laminar flow of a micropolar fluid in the fully developed region of a parallel plate vertical channel. The X - axis lies on the axial plates of the channel, and its direction is opposite to the gravitational field. The Y – axis is orthogonal to the walls. The channel occupies the region of space $-L/2 \le Y \le L/2$. The thermal conductivity, the thermal diffusivity, the dynamic viscosity and the thermal expansion coefficient of the fluid are assumed to be constant. As customary, the Boussinesq approximation and the equation of state will be adopted.

$$\rho = \rho_0[1-\beta(T-T_0)] \tag{1}$$

Moreover, it will be assumed that the only nonzero component of the velocity field U is the X-component of the U. Thus, since $\Delta.U = 0$, one has

$$\frac{\partial U}{\partial X} = 0 \tag{2}$$

so that U depends only on Y. It is also assumed that the microrotation distribution is independent of the X – coordinate. With this, the linear, angular momentum balance equation and the energy balance equation, along X and Y directions yield (Chamkha et al. [38])

$$\rho_0\beta g(T-T_0)+(\mu+\kappa)\frac{d^2U}{dY^2}+\kappa\frac{d\bar{N}}{dY}-\frac{\partial P}{\partial X}=0, \tag{3}$$

$$\frac{\partial P}{\partial Y}=0, \tag{4}$$

$$\gamma\frac{d^2\bar{N}}{dY^2}-\kappa\left(2\bar{N}+\frac{dU}{\partial Y}\right)=0, \tag{5}$$

$$\frac{d^2T}{dY^2}=0, \tag{6}$$

where $P = p+\rho_o gX$. Since, on account of equation (4) P depends only on X, equation (3) can be rewritten as

$$(T-T_0)+\frac{(\mu+\kappa)}{\rho_0\beta g}\frac{d^2U}{dY^2}+\frac{\kappa}{\rho_0\beta g}\frac{d\bar{N}}{dY}-\frac{1}{\rho_0\beta g}\frac{\partial P}{\partial X}=0. \tag{7}$$

From equations (7), one obtains,

$$\frac{\partial T}{\partial X}=\frac{1}{\beta g\rho_0}\frac{d^2P}{dX^2}, \tag{8}$$

$$\frac{\partial T}{\partial Y}=-\frac{(\mu+\kappa)}{\rho_0\beta g}\frac{d^3U}{dY^3}-\frac{\kappa}{\rho_0\beta g}\frac{d^2\bar{N}}{dY^2}, \tag{9}$$

$$\frac{\partial^2 T}{\partial Y^2}=-\frac{(\mu+\kappa)}{\rho_0\beta g}\frac{d^4U}{dY^4}-\frac{\kappa}{\rho_0\beta g}\frac{d^3\bar{N}}{dY^3}, \tag{10}$$

Both the walls of the channel will be assumed to have a negligible thickness and to exchange heat by convection with an external fluid. In particular, at $Y=-L/2$ the external convection coefficient will be considered as uniform with the value h_1 and the fluid in the region $Y<-L/2$ will be assumed to have a uniform reference temperature T_1. At $Y=L/2$ the external convection coefficient will be considered as uniform with the value h_2 and the fluid in the region $Y>L/2$ will be assumed to have a uniform reference temperature $T_2 \ge T_1$. Therefore, the boundary conditions on the temperature field can be expressed as

$$-k\left.\frac{\partial T}{\partial Y}\right|_{-L/2}=h_1[T_1-T(X,-L/2)], \tag{11}$$

$$-k\left.\frac{\partial T}{\partial Y}\right|_{L/2}=h_2[T(X,L/2)-T_2]. \tag{12}$$

On account of equation (9), equations (11) and (12) can be rewritten as

$$\left.\frac{(\mu+\kappa)}{\rho_0}\frac{d^3U}{dY^3}+\frac{\kappa}{\rho_0}\frac{d^2\bar{N}}{dY^2}\right|_{Y=-L/2}=\frac{\beta g h_1}{k\nu}\left[T_1-T(X,-L/2)\right] \tag{13}$$

$$\left.\frac{(\mu+\kappa)}{\rho_0}\frac{d^3U}{dY^3}+\frac{\kappa}{\rho_0}\frac{d^2\bar{N}}{dY^2}\right|_{Y=L/2}=\frac{\beta g h_2}{k\nu}\left[T(X,L/2)-T_2\right]. \tag{14}$$

It is easily verified that the equations (13) and (14) imply that $\partial T/\partial X$ is zero both at Y = –L/2 and at Y = L/2. Since equation (8) ensures that $\partial T/\partial X$ does not depend on Y, one is led to the conclusion that $\partial T/\partial X$ does not depend on Y, one is led to the conclusion that $\partial T/\partial X$ is zero everywhere. Therefore, the temperature T depends only on Y, i.e., $T = T(Y)$. Thus, on account of equation (8), there exists a constant A such that

$$\frac{dP}{dX} = A \tag{15}$$

The boundary conditions on U and $\bar{N}$ are given by

$$U(-L/2) = U(L/2) = 0, \tag{16}$$

$$\bar{N}(-L/2) = \bar{N}(L/2) = 0. \tag{17}$$

We use on the following dimensionless parameters:

$$u = \frac{U}{U_0}, \quad N = \frac{D}{U_0}\bar{N}, \quad \theta = \frac{T - T_0}{\Delta T}, \quad y = \frac{Y}{D} \quad \gamma = \left(\mu + \frac{\kappa}{2}\right) j,$$

$$j = D^2, \quad \Omega = \frac{\kappa}{\mu}, \quad Gr = \frac{g\beta \Delta T D^3}{\nu^2}, \quad Re = \frac{U_0 D}{\nu}, \quad \Lambda = \frac{Gr}{Re},$$

$$Bi_1 = \frac{h_1 D}{k}, \; Bi_2 = \frac{h_2 D}{k}, \; R_T = \frac{T_2 - T_1}{\Delta T},$$

$$S = \frac{Bi_1 Bi_2}{Bi_1 Bi_2 + 2Bi_1 + 2Bi_2}, \tag{18}$$

where $D = 2L$ is the hydraulic diameter. The reference velocity U_0 and the reference temperature T_0 are given by

$$U_0 = -\frac{AD^2}{48\mu}, \qquad T_0 = \frac{T_1 + T_2}{2} + S\left(\frac{1}{Bi_1} - \frac{1}{Bi_2}\right)(T_2 - T_1). \tag{19}$$

The reference temperature difference ΔT is given either by

$$\Delta T = T_2 - T_1 \quad \text{if} \quad T_1 < T_2, \tag{20}$$

or by

$$\Delta T = \frac{\nu^2}{C_p D^2} \quad \text{if} \quad T_1 = T_2. \tag{21}$$

Therefore, as in Ref. [27] the value of the dimensionless parameter R_T can be either 0 or 1. More precisely, R_T equals 1 for asymmetric fluid temperatures, $T_1 < T_2$ and equals 0 for symmetric fluid temperatures, $T_1 = T_2$.

Using the non-dimensional variables given in (18), equations (3), (5) and (6) and the boundary conditions given in (11), (12), (16) and (17) take the following form:

$$(1+\Omega)\frac{d^2u}{dy^2} + \Omega\frac{dN}{dy} + \Lambda\theta + 48 = 0, \tag{22}$$

$$\left(1 + \frac{\Omega}{2}\right)\frac{d^2N}{dy^2} - \Omega\left(2N + \frac{du}{dy}\right) = 0, \tag{23}$$

$$\frac{d^2\theta}{dy^2} = 0, \tag{24}$$

$$u(-1/4) = u(1/4) = 0, \tag{25}$$

$$N(-1/4) = N(1/4) = 0, \tag{26}$$

$$\left.\frac{d\theta}{dy}\right|_{y=-1/4} = Bi_1\left[\theta(-1/4) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_1}\right)\right], \tag{27}$$

$$\left.\frac{d\theta}{dy}\right|_{y=1/4} = Bi_2\left[-\theta(-1/4) + \frac{R_T S}{2}\left(1 + \frac{4}{Bi_2}\right)\right]. \tag{28}$$

The dimensionless mean velocity $\bar{u}$ and the dimensionless bulk temperature θ_b are given by

$$\bar{u} = 2\int_{-1/4}^{1/4} u \, dy \tag{29}$$

$$\theta_b = \frac{2}{\bar{u}}\int_{-1/4}^{1/4} u\theta \, dy. \tag{30}$$

On account of equation (15), for upward flow $A < 0$, so that U_0, Re and Λ are positive. For downward flow $A>0$, while U_0, Re and Λ are negative.

Solving equation (24) subjected to boundary conditions (27) and (28) one obtains

$$\theta(y) = 2SR_T y. \tag{31}$$

The exact solution of equations (22) and (23) become

$$u(y) = -\frac{\sqrt{\tau}}{2}\left(C_1 Sinh(\sqrt{\tau}y) + C_2 Cosh(\sqrt{\tau}y) + \frac{L_1}{3}y^3 + \frac{L_2}{2}y^2\right) - \frac{1}{1+\Omega}\left(\frac{SR_T\Lambda y^3}{3} + 24y^2\right) + C_3 y + C_4. \tag{32}$$

$$N(y) = C_1 Cosh(\sqrt{\tau}y) + C_2 Sinh(\sqrt{\tau}y) + L_1 y^2 + L_2 y + L_3 \tag{33}$$

The constants C_1, C_2, $C3$, and C_4 are evaluated using boundary condition (25). The constants appered in the above equations are shown in the Appendix section.

The solution of the equation (22) for $\Omega = 0$ (clear viscous fluid) using boundary condition (25) become,

$$u(y) = -\left(\frac{SR_T\Lambda y^3}{3} + 24y^2\right) + \frac{SR_T\Lambda y}{48} + \frac{3}{2} \tag{34}$$

A Nusselt number can be defined at each boundary, as follow

$$Nu_1 = \frac{D}{R_T\left[T(L/2) - T(-L/2)\right] + (1-R_T)\Delta T}\left.\frac{dT}{dY}\right|_{Y=-L/2}, \qquad Nu_2 = \frac{D}{R_T\left[T(L/2) - T(-L/2)\right] + (1-R_T)\Delta T}\left.\frac{dT}{dY}\right|_{Y=L/2} \tag{35}$$

by employing equation. (18), equation (35) can be written as

$$Nu_1 = \frac{1}{R_T\left[\theta(1/4) - \theta(-1/4)\right] + (1-R_T)}\left.\frac{d\theta}{dy}\right|_{y=-1/4}, \qquad Nu_2 = \frac{1}{R_T\left[\theta(1/4) - \theta(-1/4)\right] + (1-R_T)}\left.\frac{d\theta}{dy}\right|_{y=1/4} \tag{36}$$

Using the solution (31) the Nusselt numbers are obtained as

$$Nu_1 = Nu_2 = 2R_T. \tag{37}$$

3. RESULTS AND DISCUSSION

Equations (22) and (23) subject to the boundary conditions (25) and (26) have been solved exactly and the results are depicted graphically as shown in Figs. (1)-(5). The numerical values for the axial velocity u and microrotation velocity N are presented in Figs. (1a) & (1b) respectively for various values of mixed convection parameter Λ and vortex viscosity parameter Ω for equal Biot numbers. As the vortex viscosity parameter Ω increases axial velocity u decreases for buoyancy assisting flow $(\Lambda > 0)$ and for opposing $(\Lambda < 0)$ flow. It is also observed that from Fig. (1a) that flow reversal occur at the left wall for buoyancy assisting flow and at the right wall for opposing flow. As the vortex viscosity parameter increases, microrotation velocity increases for buoyancy opposing flow and decreases for buoyancy assisting flow. There is a flow reversal at the left wall for $(\Lambda < 0)$ and at the right wall for $(\Lambda > 0)$ as seen in Fig. (1b). The effect of mixed convection parameter Λ and vortex viscosity parameter Ω and microrotation velocity *N* shows the similar results obtained by Chamkha et al. [38].

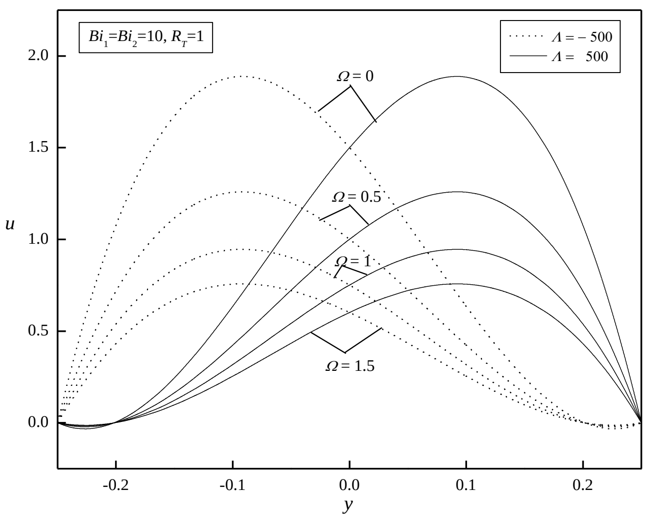

Fig. 1a**:** Plots of velocity vs. *y* for different values of Λ, Ω and $Bi_1 = Bi_2 = 10$.

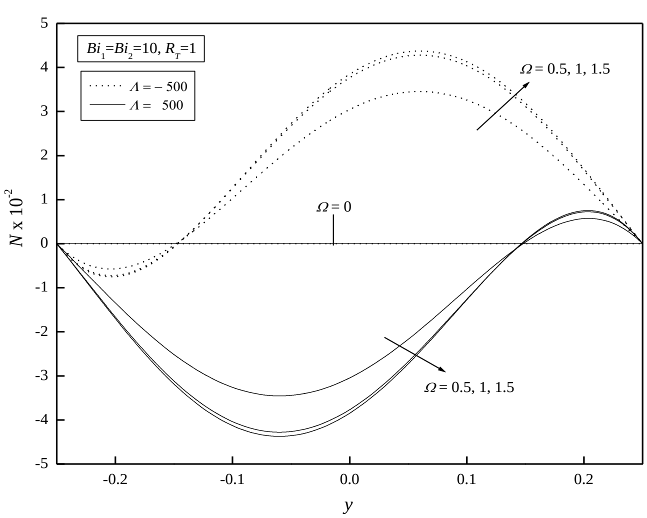

Fig. 1b**:** Plots of microrotation N velocity vs. *y* for different values of Λ, Ω and $Bi_1 = Bi_2 = 10$.

The effect of mixed convection parameter Λ and vortex viscosity parameter Ω on the axial velocity *u* and microrotation velocity *N* for unequal Biot numbers is as shown in Figs. (2a) and (2b) respectively. The velocity decreases as the vortex viscosity parameter Ω increases for both buoyancy assisting and opposing flow but there is no flow reversal as observed for equal Biot numbers (Fig. 1a). The microrotation velocity *N* increases as the vortex viscosity parameter Ω increases for both buoyancy assisting and opposing flow in the channel width from $y = 0$ to 0.25 and reverses its direction in the channel width from $y = -0.25$ to 0 as seen in fig. (2b) for unequal Biot numbers. Figures (1) and (2) are the results obtained for asymmetric wall temperatures $(R_T = 1)$.

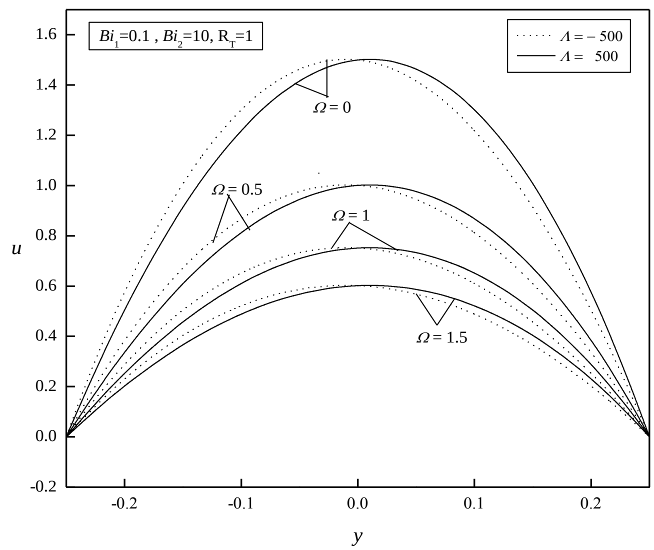

Fig. 2a**:** Plots of velocity vs. *y* for different values of Λ, Ω and $Bi_1 = 0.1$ and $Bi_2 = 10$.

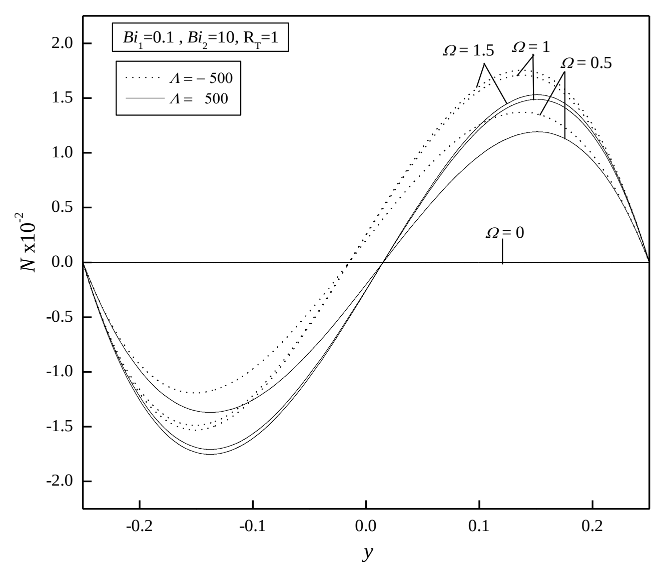

Fig. 2b**:** Plots of microrotation N velocity vs. *y* for different values of Λ, Ω and $Bi_1 = 0.1$ and $Bi_2 = 10$..

Figures (3a) and (3b) shows the velocity and microrotation velocity profiles for various values of vortex viscosity Ω for equal Biot numbers. The velocity decreases as the vortex viscosity parameter increases as shown in fig. (3a). The microrotation velocity N increases at the right wall and decreases at the left wall as the vortex viscosity parameter Ω increases as seen fig. (3b) for equal Biot numbers. Similar results are observed for the effect of vortex viscosity parameter Ω on the velocity and microrotation velocity N for unequal Biot numbers as seen in Figs. (4a) and (4b) respectively. Therefore one can conclude that the nature of flow will not vary for equal or unequal Biot numbers for symmetric wall heat conditions $\left(R_T = 0\right)$. The effect of vortex viscosity parameter Ω on the average velocity also decreases as the vortex viscosity Parameter increases for equal Biot numbers and for asymmetric wall heating as shown in fig. (5a). It is interesting to note that the average velocity remains constant for various values of mixed convection parameter Λ. The bulk temperature θ_b is almost invariant with vortex viscosity parameter Ω as shown in Fig. (5b). this is due to the fact that temperature is a linear function of y.

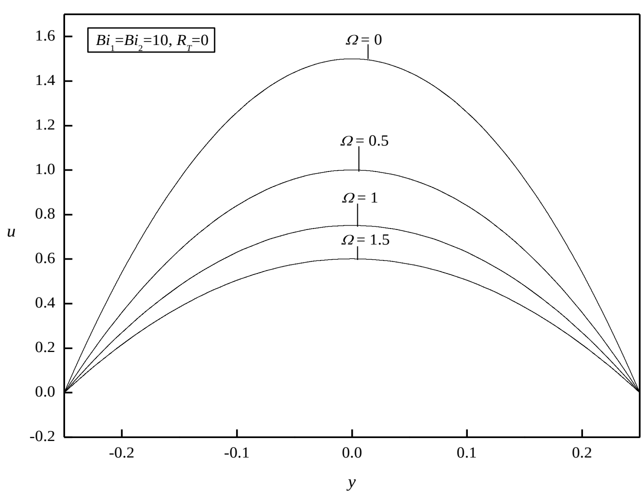

Fig. 3a**:** Plots of velocity vs. *y* for different values of Ω and $Bi_1 = Bi_2 = 10$.

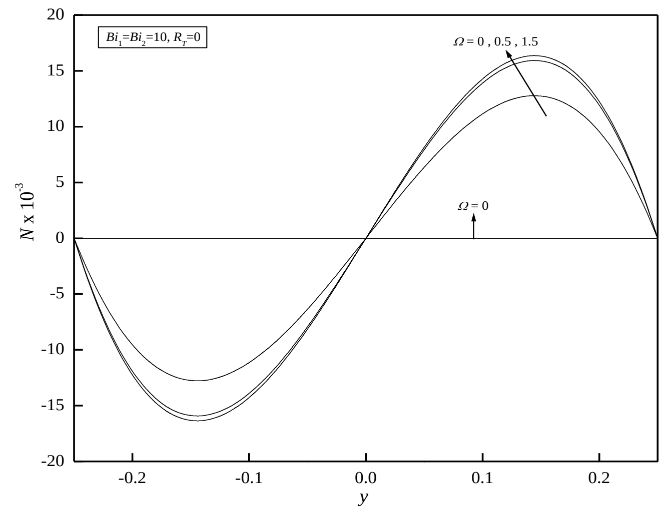

Fig. 3b**:** Plots of microrotation N velocity vs. *y* for different values of Ω and $Bi_1 = Bi_2 = 10$.

Tables (1) and (2) shows the values of velocity for equal and unequal Biot numbers. For values of $\Omega = 0$ (purely viscous fluid) in the present model and for $Br = 0$ in Zanchini [27] the values for velocity agree very well which justify the results of the present model.

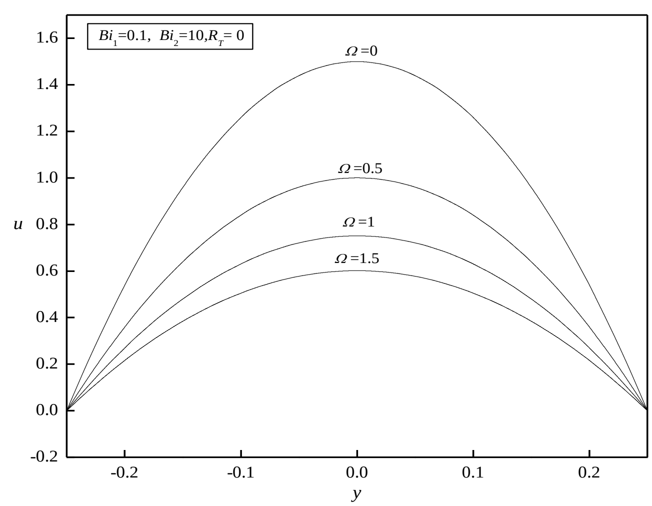

Fig. 4a**:** Plots of velocity vs. *y* for different values of Ω and $Bi_1 = 0.1$ and $Bi_2 = 10$.

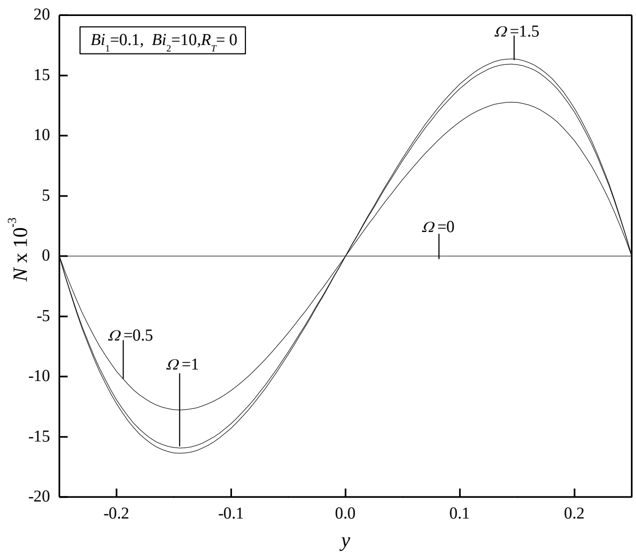

Fig. 4b**:** Plots of microrotation N velocity vs. y for different values of Ω and $Bi_1 = 0.1$ and $Bi_2 = 10$.

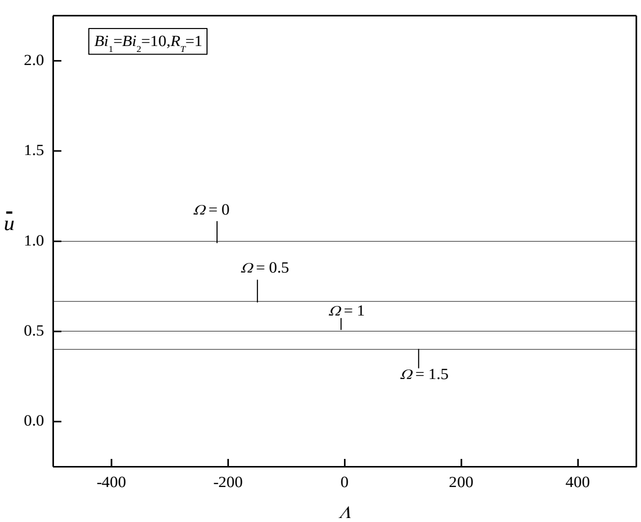

Fig. 5a**:** Plots of Average velocity $\bar{u}$ vs. mixed convection Λ for different values of Ω and $Bi_1 = Bi_2 = 10$.

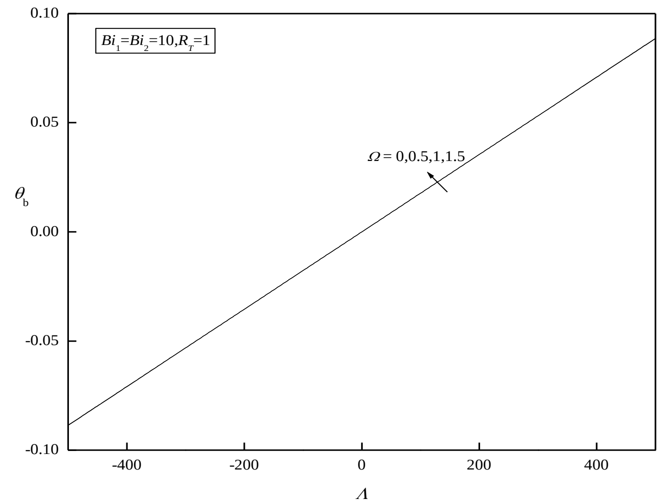

Fig. 5b**:** Plots of Bulk temperature $\bar{q}$ vs. mixed convection Λ for different values of Ω and $Bi_1 = Bi_2 = 10$.

Table 1. Values of velocity for equal Biot number $Bi_1 = Bi_2 = 10$, $\Lambda = 500$ and $R_T = 1$.

y	($\Omega = 0$) present model	($Br = 0$) Zanchini [27]
-0.25	0.	0
-0.20	0.0042857142	0.0042857142
-0.15	0.2457142857	0.2457142857
-0.10	0.6350000000	0.6350000000
-0.05	1.0828571428	1.0828571428
0	1.5	1.5
0.05	1.7971428571	1.7971428571
0.10	1.8849999999	1.885
0.15	1.6742857142	1.6742857142
0.20	1.0757142857	1.0757142857
0.25	0	0

Table 2. Values of velocity for unequal Biot number $Bi_1 = 0.1$, $Bi_2 = 10$, $\Lambda = 500$ and $R_T = 1$

y	($\Omega = 0$) present model	($Br = 0$) Zanchini [27]
-0.25	0	0
-0.20	0.5046226415	0.5046226415
-0.15	0.9128301886	0.9128301886
-0.10	1.2187264150	1.2187264150
-0.05	1.4164150943	1.4164150943
0	1.5	1.5
0.05	1.4635849056	1.4635849056
0.10	1.3012735849	1.3012735849
0.15	1.0071698113	1.0071698113
0.20	0.5753773584	0.5753773584
0.25	0	0

4. CONCLUSION

The problem of mixed convection in a vertical channel was analyzed with boundary conditions of third kind for micropolar fluid. Both conditions of symmetric and asymmetric wall temperature was analyzed for equal and unequal Biot numbers. The velocity decreases for buoyancy assisting and opposing flow as the vortex viscosity parameter increases for both equal and unequal Biot numbers, which were similar results obtained by Chamkha et al. [38]. The microrotation velocity increases for large values of vortex viscosity parameter Ω for buoyancy assisting flow and decreases for buoyancy opposing flow for equal Biot numbers. For unequal Biot numbers microrotation velocity increases at right wall and decreases at the left wall for large values of vortex viscosity parameter. For symmetric wall heat condition the effect of vortex viscosity was to reduce the velocity and increases the microrotation velocity at the right wall and decreases at left wall for both equal and unequal Biot numbers. The average velocity was reduced for larger values of vortex viscosity parameter and remained invariant on bulk temperature. The present model for purely viscous fluid agree with the results of Zanchini [27].

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REFERENCES

[1] F.P. Incropera, Buoyancy effects double-diffusive mixed convection flows, In: Proceedings of the 8th International Heat Transfer Conference, C.L. Tein, V.P. Carey and J.K. Ferrel (eds), vol. 1, pp. 121-130, 1986.

[2] W. Aung, Mixed convection in internal flow. Int: Hand book of Single-Phase Convective Heat transfer, S. Kack, R.K. Shah and W. Aung (eds), John Wiley, New York, 1987.

[3] B. Gebhart, Y Jaluria, R.L. Mahajan and B. Sammarkia, Buoyancy-Induced flows and transport, Hemisphere, D.C. Washington, 1998.

[4] W. Aung and G. Worku, Theory of fully developed combined convection including flow reversal, ASME J. of Heat Transfer, vol. 108, pp. 485-488, 1986.

[5] C-H. Cheng, H-S. Kou and W-H. Huang Flow reversal and heat transfer of fully developed mixed convection in vertical channels, J. Thermophysics and Heat Transfer, vol. 4, pp. 375-383, 1990.

[6] T.T. Hamadah and R.A. Wirtz, Analysis of laminar fully developed mixed convection in a vertical channel with opposing buoyancy, ASME J. Heat Transfer, vol. 113, pp. 507-510,1991

[7] L.N. Tao On combined free and forced convection in channel, ASME J. Heat Transfer, vol. 82, pp. 233–238, 1960.

[8] W. Aung and G. Worku, Developing flow and flow reversal in a vertical channel with asymmetric wall temperatures, ASME J. Heat Transfer, vol. 108, pp. 299–304, 1986.

[9] W. Aung and G. Worku, Mixed convection in ducts with asymmetric wall heat fluxes, ASME J. Heat Transfer, vol. 109, pp. 947-951, 1987.

[10] D.B. Ingham, D.J. Keen, P.J. Heggis, Flows in vertical channels with asymmetric wall temperatures and including situations where reverse flow occur, ASME J. Heat Transfer, vol. 110, pp. 910-917

[11] A. Barletta, Laminar mixed convection with viscous dissipation in a vertical channel, Int. J. of Heat and Mass Transfer, vol. 41, 1998.

[12] J.C. Umavathi and M.S. Malashetty, Magnetohydrodynamic mixed convection in a vertical channel, Int. J. Non-Linear Mechanics, vol. 40, pp. 91-101, 2005a.

[13] J.C. Umavathi and J.P. Kumar, A.J. Chamkha and I Pop, Mixed convection in a vertical porous channel, Transport in Porous Media, vol. 61, pp. 315-335, 2005b.

[14] J.C. Umavathi, Mallikarjun B Patil and I. Pop, On laminar mixed convection flow in a vertical porous stratum with symmetric wall heating conditions, Int. J. Transport Phenomenon, vol. 8, pp. 127-140, 2006.

[15] J.C. Umavathi and J.P. Kumar, A.J. Chamkha and I. Pop, Mixed convection in a vertical porous channel, ERRATUM, Transport in Porous Media, vol. 75, pp.129-132, 2008a.

[16] J. Prathap Kumar, J.C. Umavathi, I. Pop and Basavaraj M. Biradar, Fully developed mixed convection flow in a vertical channel containing porous and fluid layers with isothermal or isoflux boundaries, Tranport in Porous Media, vol. 80, pp. 117-135, 2009.

[17] J. Prathap Kumar, J.C. Umavathi and Basavaraj M. Biradar, Mixed convection of composite porous medium in a vertical channel with asymmetric wall heating conditions, J. Porous Media, vol. 13, pp. 271-285, 2010a.

[18] J. Prathap Kumar, J.C. Umavathi and Basavaraj M. Biradar, Mixed convection of magnetohydrodynamic and viscous fluid in a vertical channel, Int. J. Non-Linear Mechanics, vol. 46, pp.278-285, 2011a.

[19] J. Prathap Kumar, J.C. Umavathi and Basavaraj M. Biradar, Mixed convective flow of immiscible viscous fluids in a vertical channel, Heat Transfer Asian Research, vol. 40, pp. 1-25, 2011b.

[20] P. Wibulswas, Laminar flow heat transfer in non circular ducts, Ph.D. Thesis, London University, 1966. (As reported by Shah and London in 1971).

[21] R.W. Lyczkowski, C.W. Solbrig, D. Gidaspow, Forced convective heat transfer in rectangular ducts-general case of wall resistance and peripheral conduction, Institute of Gas Technology Tech. Info, Center File

3229, 3424S, State, Street, Chicago, I11.60616, 1969 (as reported by Shah and London in 1971).

[22] V. Javeri, Analysis of laminar thermal entrance region of elliptical and rectangular channels with Kantorowich method, Warme-und Stoffuberragung, vol. 9, Pp. 85-98 1976.

[23] E. Hicken, Das temperaturfeld in laminar durchstromten Kanalen mit echteckquerschnitt bei unterschiedlicher Beheizung der Kanalwade, Warme-und Stoffubertragung, vol. 1,Pp. 98-104, 1968.

[24] E.M, Sparrow, R. Siegal, Application of variational methods to the thermal entrance region of ducts, Int. J. Heat Mass Transfer, vol. 1,pp.161-172, 1960.

[25] V. Javeri, Koln, Heat transfer in laminar entrance region of a flat channel for the temperature boundary condition of the third kind, Warme-und Stoffubertragung Thermo- and fluid dynamics, vol. 10, pp.137-144, 1977.

[26] V. Javeri, Laminar heat transfer in a rectangular channel for the temperature boundary condition of the third kind, Int. J. Heat Mass Transfer, vol. 10, pp.1029-1034, 1978.

[27] E. Zanchini, Effect of viscous dissipation on mixed convection in a vertical channel with boundary conditions of the third kind, Int. J. Heat and Mass Transfer, vol. 41, pp. 3949-3959, 1998.

[28] A.C. Eringen, Simple microfluids, Int. J. Engineering Science, vol. 2, pp. 205-217,1964.

[29] A.C. Eringen, Theory of microfluids, J. Mathematical Mechanics, vol. 16, pp. 1-16, 1966.

[30] A.C. Eringen, Theory of thermomicrofluids, J. Mathematical Analysis Applications, vol. 9, pp. 480-496, 1972.

[31] G. Ahmadi, Self-similar solution of incompressible micropolar boundary layer flow over a semi-infinite plate, Int. J. Engineering Science, vol. 14, pp. 639-646, 1976.

[32] S.K. Jena and M. N. Mathur, Mixed convection flow of a micropolar fluid from an isothermal vertical plate, Computer and Mathematics with Applications, vol. 10, pp. 291-304, 1984.

[33] T.H. Hsu and C.K. Chen, Transient analysis of combined forced and free convection-conduction along vertical circular fin in micropolar fluids, Numerical heat Transfer, Part A, vol. 19, pp. 177-185, 1991.

[34] T.H. Hsu and S.Y. Tsai, Natural convection of micropolar fluids in a two-dimensional enclosure with a conductive partition, Numerical Heat Transfer, part A, vol. 28,pp. 69-83, 1995.

[35] P.S.R. Gorla, Mixed convection in a micropolar from a vertical surface with uniform heat flux, Int. J. Engineering Science, vol. 30, pp. 349-358, 1992.

[36] C.Y. Cheng and C.C. Wang, Forced convection in micropolar fluid flow over a wavy surface, Numerical Heat Transfer, Part A, vol. 37, pp. 271-287, 2000.

[37] Y.H. Chu, P.T. Hsu, Y.H. Liu and L.H. Chiu, Mixed convection micropolar fluids along a vertical wavy surface with a discontinuous temperature profile, Numerical Heat Transfer, Part A, vol. 42, pp. 733-755, 2002.

[38] A.J. Chamkha, T. Grosan and I. Pop, Fully developed free convection of a micropolar fluid in a vertical channel, Int. Comm. Heat Mass Tras. vol.29, pp. 1021-1196, 2002.

[39] L. Kumar, R. Bhargava, P. Bhargava and H. S. Takhar, Finite element solution of mixed convection micropolar fluid flow between two vertical plates with varying temperature, Int. J. Archives of mechanics, vol 57, pp. 251-264, 2005.

[40] Cheng Ching-Yang, Fully developed natural convection heat and mass transfer of a micropolar fluid in a vertical channel with asymmetric wall temperatures and concentrations, Int. Comm. Heat Mass Transfer, vol. 33, pp. 627-635, 2006.

[41] J.C. Umavathi, J.P. Kumar, and A.J. Chamkha, Flow and heat transfer of a micropolar fluid sandwiched between viscous fluid layers, Canadian J. Physics, vol. 86, pp. 1-13, 2008b.

[42] J. Prathap Kumar, J.C. Umavathi, A.J. Chamkha and I Pop, Fully developed free convective flow of micropolar and viscous fluids in a vertical channel, App. Mathematical Modeling, vol. 34, pp. 1175-1186, 2010b.

[43] J.C. Umavathi and J.P. Kumar, Mixed convection flow of micropolar fluid in a vertical channel with symmetric and asymmetric wall heating conditions, Int. J. Appl. Mech. Eng. vol. 16,pp. 141-159, 2011.

NOMENCLATURE

A	constant
Bi_1 Bi_2	Biot numbers
Br	Brinkman number, $\mu U_0^2 / k\Delta T$
C_P	specific heat at constant pressure [$kJ.kg^{-1}.K^{-1}$]
D	hydraulic diameter, $2L$
g	acceleration due to gravity [ms^{-2}]
Gr	Grashof number, $g\beta\Delta TD^3/\nu^2$
h_1, h_2	external heat transfer coefficients [$Wm^{-2}k^{-1}$]
j	microinertia density
k	thermal conductivity [$Wm^{-1}k^{-1}$]
L	channel width [m]
Nu_1, Nu_2	nusselt numbers
$D = 2L$	hydraulic diameter
$\bar{N}$	angular velocity of micropolar fluid
N	microrotation velocity
p	non-dimensional pressure gradient [Nm^{-2}]
P	difference between the pressure and the hydrostatic pressure, $p + \rho_0 gX$
Re	Reynolds number, $U_0 D/\nu$
R_T	temperature difference ratio [K]
T	temperature [K]
T_1, T_2	reference temperatures of the external fluid [K]
T_0	reference temperature [K]

u	dimensionless velocity in the X -direction
U_0	reference velocity [ms^{-1}]
U	velocity component in the X -direction [ms^{-1}]
x	dimensionless stream wise coordinate
X	stream wise coordinate [m]
y	dimensionless transverse coordinate
Y	transverse coordinate [m]

<u>Greek Symbols</u>

ΔT	reference temperature difference
β	thermal expansion coefficient
Ω	vortex viscosity
κ	vortex viscosity parameter
ν	kinematic viscosity
Λ	mixed convection parameter, Gr/Re
γ	spin gradient viscosity
θ	dimensionless temperature $\left(T - T_0\right)/\Delta T$
θ_b	dimensionless bulk temperature
μ	viscosity [kgm^{-1}s^{-1}]
ρ	mass density [Kg m^{-3}]
ρ_0	value of the mass density when $T = T_0$ [Kg m^{-3}]

APPENDIX

$$L_1 = \frac{SR_T\Lambda}{2+\Omega},\ L_2 = \frac{48}{2+\Omega},\ L_3 = \frac{2SR_T\Lambda}{\tau\left(2+\Omega\right)} + \frac{I_1}{2+\Omega},\ C_1 = \frac{-\left(\frac{L_1}{16} + L_3\right)}{Cosh\left(\frac{\sqrt{\tau}}{4}\right)},\ C_2 = \frac{-L_2}{4Sinh\left(\frac{\sqrt{\tau}}{4}\right)}.$$

$$C_3 = \frac{-1}{96}\left(2\Omega L_1 + 2SR_T\Lambda + \frac{384\Omega Sinh\left(\frac{\sqrt{\tau}}{4}\right)}{\sqrt{\tau}}C_1\right), C_4 = \frac{-1}{32}\left(48 + \Omega L_2 + \frac{32\Omega Cosh\left(\frac{\sqrt{\tau}}{4}\right)}{\sqrt{\tau}}C_2\right)$$