International Journal of
Energy & Technology
www.journal-enertech.eu
ISSN 2035-911X
International Journal of Energy & Technology 4 (8) (2012) 1–8

NON-DARCY FORCED CONVECTION THROUGH A WAVY POROUS CHANNEL USING CUO NANOFLUID

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ABSTRACT

Forced convective flow and thermal field characteristics inside a porous wavy walled channel using water-CuO nanofluid have been numerically solved and analyzed. The horizontal channel consists of isothermal wavy walls of temperature T_h. The fluid enters from left with initial velocity U_i and temperature T_i and exits from right. Governing equations are solved using the Penalty Finite Element Method. Simulation is carried out for a range of Reynolds number, Re =10–500; Darcy number, Da = 0.01–∞ and solid volume fraction, ϕ = 0% - 20%. Results are presented in the form of streamlines, isothermal lines, rate of heat transfer and velocity at mid-height of the channel for the above mentioned parameters. It is found that the rate of heat transfer in the channel is increased for both Re and ϕ but it is lessened for higher Da.

Keywords: Water-CuO nanofluid; forced convection; wavy channel; porous medium; finite element method.

1. INTRODUCTION

Nanofluid technology has emerged as a new enhanced heat transfer technique in recent years. Nanofluid is made by adding nanoparticles and a surfactant into a base fluid can greatly enhance thermal conductivity and convective heat transfer. The diameters of nanoparticles are usually less than 100 nm which improves their suspension properties.

The knowledge of free or forced convection heat transfer inside geometries of irregular shape (for example, wavy channel and pipe bend) for porous media has many significant engineering applications; for example, geothermal engineering, solar-collectors, performance of cold storage, and thermal insulation of buildings. A considerable number of published articles are available that deal with flow characteristics, heat transfer, flow and heat transfer instability, transition to turbulence, design aspects, etc. Many researchers [1–6] considered different physical situations which can be classified into two broad groups. The first group problems cover a wide range of applications; for example, flow through blood vessels, food industries, large industrial heat exchangers, etc. In contrast, the relatively new second group deals with the natural convection problem in an enclosed space with wavy walls. Starting from the microelectronic heat transfer cooling device this group covers a wide range of significant engineering applications; for example, microelectro- mechanical device, double-wall thermal insulation, underground cable systems, solar-collectors, electric machinery, etc. Significant contributions have been made by several researchers [7–9] in order to model the problems of this specific group. For wavy cavities filled with porous medium that obeys the Darcy law, Kumar et al. [10–13] reported flow and heat transfer results in a cavity with wavy bottom wall. This work is extended for one vertical wavy surface in Kumar [14]. Misirlioglu et al. [15] was analyzed free convection in a wavy cavity filled with a porous medium. For non-Darcy porous medium, Kumar and Gupta [16, 17] reported the flow and thermal fields' characteristics in wavy cavities.

Kumar et al. [18] found the significant heat transfer enhancement by the dispersion of nanoparticles in the base fluid. Santra et al. [19] modelled the nanofluids as a non-Newtonian fluid and observed a systematic decrease of the heat transfer as the volume fraction of the nanofluids increased. The possible determining factors for the heat transfer reduction in nanofluids include the variations of the size, shape, and distribution of nanoparticles and uncertainties in the thermophysical properties of nanofluids. Most of the published papers are concerned with the analysis of natural convection heat transfer of nanofluids in square or rectangular enclosures; for example, Abu-Nada and Oztop [20], Ghasemi and Aminossadati [21], Muthtamilselvan et al. [22] and Abu-Nada et al. [23].

In reality forced convection in a differentially heated enclosure is a prototype of many industrial applications and has received considerable attention because of its applicability in various fields. The study of convective flow in an irregular geometry is more difficult than that of square or rectangular enclosures.

Many studies have described the larruping behaviors of nanofluids, such as their effective thermal conductivity under static conditions on the convective heat transfer associated with fluid flow phenomena. Dai et al. [24] and Chen et al. [25] investigated convective flow drag and heat transfer of CuO nanofluid in a small tube. Their results showed that the pressure drop of the nanofluid per unit length was greater than that of water. The pressure drop increased with the increasing weight concentration of nanoparticles. In the laminar flow region, the pressure drop had a linear relationship with the *Re* number, while in the turbulent flow region, the pressure drop increased sharply with the increase of the *Re* number. The critical *Re* number became lower while the tube diameter was smaller. The convective heat transfer was obviously enhanced by adding nanoparticles. The nanoparticle weight concentration and flow status were the main factors influencing the heat transfer coefficient: the heat transfer coefficient increased with the increasing weight concentration, and the enhancement of heat transfer in the turbulent flow region was greater than that in the laminar flow region. Pfautsch [26] studied the characteristics, flow development, and heat transfer coefficient of nanofluids under laminar forced convection over a flat plate. He found a significant increase in the heat transfer coefficient: about 16% increase in the heat transfer coefficient for the water based nanofluid and about 100% increase for the ethylene glycol based nanofluid.

Parvin et al. [27] studied MHD Mixed Convection Heat Transfer through Vertical Wavy Isothermal Channels where the rate of heat transfer was more effective in a sinusoidal wavy channel than in a triangular one. In this year, Nasrin and Parvin [28] investigated flow behavior for combined convection in a vertical channel controlled by a heat-generating tube. Their result indicated that the flow and thermal fields in the channel depend appreciably on the heat generating body in different convection regimes. Muneer [29] analyzed partitioning and magnetic field effects on free convection in a square cavity filled with porous medium with uniform heat generation.

Up to now, all the nanofluids in the existing researches used a surfactant to help nanoparticle suspending. Due to the stickiness of the surfactant, the sedimentation on the tube wall became colloid and was difficult to clear. The nanoparticles in the sedimentation could not suspend again. This kind of nanofluid would jam pipes during a prolonged period of operation. In order to overcome this disadvantage, our study used CuO nanoparticle suspensions (consisting of de-ionized water and nanoparticles without surfactant). During the flow process, the good suspension properties remained. So far, there is no research on the analysis of forced convective heat transfer on nanofluid in a channel. Therefore, forced convection also becomes a crucial point of this study. The main issues discussed in this paper are: the convective flow and heat transfer characteristics of water-CuO nanofluid in a wavy channel.

The theoretical prediction in this article is hoped to be a useful guide for the experimentalists to study the effectiveness of the forced convection in a porous channel using nanofluid to increase the rate of heat transfer.

2. MATHEMATICAL FORMULATION

Fig. 1 shows a schematic diagram of the wavy channel. The model describes a channel with two isothermal wavy walls of temperature T_h. Flow enters from left and leaves from the right. The inlet fluid velocity and temperature are U_i and T_i respectively. The working fluid through the channel is water-based nanofluid containing CuO nanoparticles.

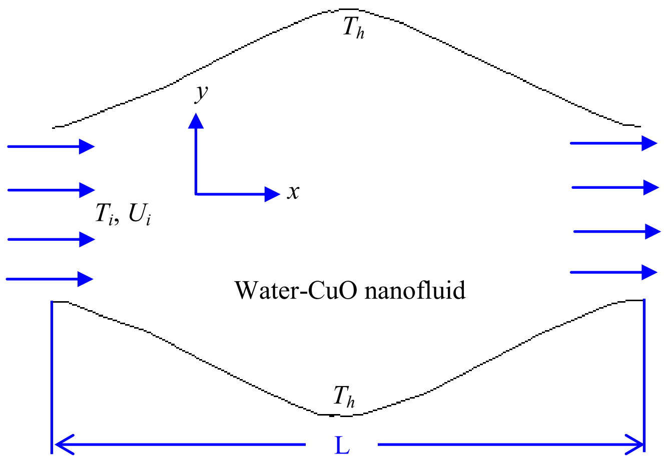


Fig. 1: Depiction of geometry and operation of the channel

In the present problem, it is considered that the flow is steady, two-dimensional, laminar, incompressible, neglected viscous dissipation and radiation effect. Darcy-Brinkman model is assumed to govern the flow through porous medium. The governing equations under Boussinesq approximation are as follows

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu_{nf}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\mu_{nf}}{K}u \tag{2}$$

$$\rho_{nf}\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu_{nf}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu_{nf}}{K}v \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

where, $\rho_{nf}=(1-\phi)\rho_f+\phi\rho_s$ is the density,

$(\rho C_p)_{nf}=(1-\phi)(\rho C_p)_f+\phi(\rho C_p)_s$ is the heat capacitance,

$\beta_{nf}=(1-\phi)\beta_f+\phi\beta_s$ is the thermal expansion coefficient,

$\alpha_{nf}=k_{nf}/(\rho C_p)_{nf}$ is the thermal diffusivity,

$\mu_{nf}=\mu_f(1-\phi)^{-2.5}$ is the dynamic viscosity and

$k_{nf}=k_f\dfrac{k_s+2k_f-2\phi(k_f-k_s)}{k_s+2k_f+\phi(k_f-k_s)}$ is the thermal conductivity of the nanofluid.

The boundary conditions are:

at the upper and lower channel walls: $T=T_h$

at the inlet opening: $T=T_i$, $u=U_i$

at all solid boundaries: $u=v=0$

at the outlet opening convective boundary condition: $p = 0$

The above equations are non-dimensionalized by using the following dimensionless dependent and independent variables

$$X = \frac{x}{L}, Y = \frac{y}{L}, U = \frac{u}{U_i}, V = \frac{v}{U_i}, P = \frac{p}{\rho_f U_i^2}, \theta = \frac{T - T_i}{T_h - T_i}$$

After substitution of the above variables into the equations (1) to (4), we get the following non-dimensional equations as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (5)$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial X} + Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) - \frac{\nu_{nf}}{Da}U \quad (6)$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}}\frac{\partial P}{\partial Y} + Pr\frac{\nu_{nf}}{\nu_f}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) - \frac{\nu_{nf}}{Da}V \quad (7)$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \quad (8)$$

where $Pr = \frac{\nu_f}{\alpha_f}$ is Prandtl number, $Re = \frac{U_i L}{\nu_f}$ is Reynolds number and $Da = \frac{K}{L^2}$ is Darcy number.

The corresponding boundary conditions then take the following form:

at the upper-lower boundaries: $\theta = 1$

at the inlet: $\theta = 0$, $U = 1$

at the outlet: convective boundary condition: $P = 0$

at all solid boundaries: $U = V = 0$

The average Nusselt number on the channel walls takes the following form

$$Nu = -\frac{1}{S}\int_0^S \left(\frac{k_{nf}}{k_f}\right)\frac{\partial \theta}{\partial N}\, dS$$

where $\frac{\partial \theta}{\partial N} = \sqrt{\left(\frac{\partial \theta}{\partial X}\right)^2 + \left(\frac{\partial \theta}{\partial Y}\right)^2}$ and S, N are the non-dimensional length and normal coordinate along the heated surface respectively.

For convenience, a normalized average Nusselt number is defined as the ratio of the average Nusselt number at any volume fraction of nanoparticles to that of the pure water, which is: $Nu^*(\phi) = \frac{Nu(\phi)}{Nu(\phi = 0)}$.

The mean temperature of the fluid is $\theta_{av} = \int \theta\, d\bar{V} / \bar{V}$, where $\bar{V}$ is the volume of the channel.

3. NUMERICAL TECHNIQUE

The Galerkin finite element method [31, 32] is used to solve the non-dimensional governing equations along with boundary conditions for the considered problem. The equation of continuity has been used as a constraint due to mass conservation and this restriction may be used to find the pressure distribution. The penalty finite element method [33] is used to solve the Eqs. (6) - (8), where the pressure P is eliminated by a penalty constraint. The continuity equation is automatically fulfilled for large values of this penalty constraint. Then the velocity components (U, V), and temperature (θ) are expanded using a basis set. The Galerkin finite element technique yields the subsequent nonlinear residual equations. Three points Gaussian quadrature is used to evaluate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson method to determine the coefficients of the expansions. The convergence of solutions is assumed when the relative error for each variable between consecutive iterations is recorded below the convergence criterion ε such that $\left|\psi^{n+1} - \psi^{n}\right| \le 10^{-4}$, where n is the number of iteration and Ψ is a function of U, V and θ.

3.1. GRID REFINEMENT CHECK

In order to determine the proper grid size for this study, a grid independence test is conducted with five types of mesh for Re = 100, Da = 100 and ϕ = 5%. The extreme value of Nu is used as a sensitivity measure of the accuracy of the solution and is selected as the monitoring variable. Considering both the accuracy of numerical value and computational time, the present calculations are performed with 12666 nodes and 8657 elements grid system. This is described in Table 1 and Fig. 2.

Table 1: Grid Test at Re = 100, Da = 100 and ϕ = 5%

Nodes (elements)	Nu	Time (s)
3224 (2569)	1.51465	226.265
5982 (4730)	2.50146	292.594
8538 (6516)	3.30146	388.157
12666 (8657)	3.70146	421.328
20524 (10426)	3.80146	627.375

3.2. MESH GENERATION

In finite element method, the mesh generation is the technique to subdivide a domain into a set of sub-domains, called finite elements, control volume etc. The discrete locations are defined by the numerical grid, at which the variables are to be calculated. It is basically a discrete representation of the geometric domain on which the problem is to be solved. The computational domains with irregular geometries by a collection of finite elements make the method a valuable practical tool for the solution of boundary value problems arising in various fields of engineering. Fig. 3 displays the finite element mesh of the present physical domain.

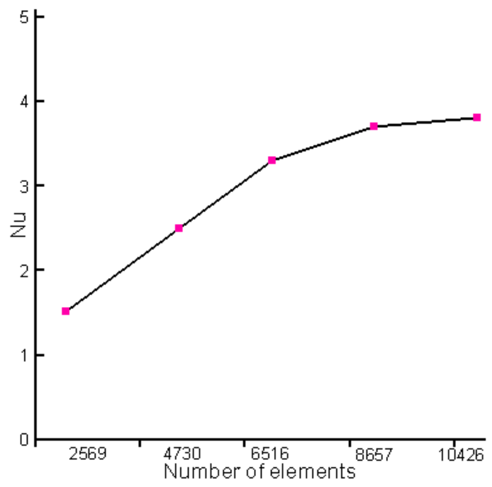


Fig. 2: Grid independency study for *Da* = 100, *Re* = 100 and ϕ = 5%

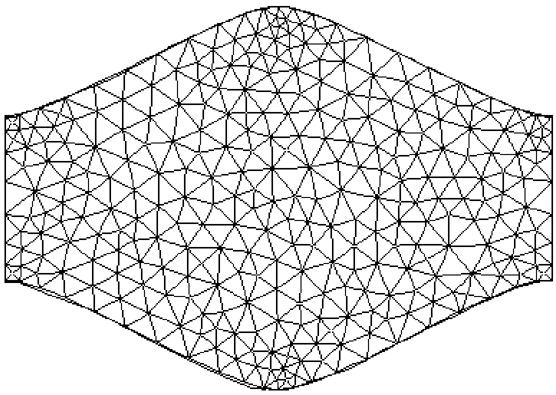


Fig. 3: Mesh generation of the channel

3.3. THERMO-PHYSICAL PROPERTIES

The thermo-physical properties of fluid (water) and solid CuO are tabulated in Table 2. The properties are taken from [30].

Table 2: Thermo-physical properties of water-CuO nanofluid

Physical properties	water	CuO
C_p	4182	540
ρ	998.1	6510
k	0.6	18
β	2.2×10^{-4}	8.5×10^{-6}

4. RESULTS AND DISCUSSION

In this section, numerical results in terms of streamlines and isotherms are displayed for various Reynolds number *Re*, Darcy number *Da* and solid volume fraction ϕ while Prandtl number *Pr* is fixed at 3.77. In addition, the values of the average and normalized Nusselt number, average bulk temperature, mid-height velocity in the channel have been calculated for water-CuO nanofluid.

The velocity (modulus of the velocity vector) and temperature fields are displayed in Fig. 4(i)-(ii) for the effect of Reynolds number (*Re* = 10, 100, 300 and 500). For this Fig. 4, *Da* = 100 and ϕ = 5% are assumed. In the velocity vector, initially the flow covers the whole domain of the channel while it concentrates near the middle of the channel due to increase inertia force from 10 to 500. On the other hand, isothermal lines for different values of Reynolds number shows that at low value of *Re* (= 10), the temperature of the nanofluid rapidly reaches to the temperature of hot walls due to low velocity. With increasing Reynolds number, increment of temperature of water-CuO nonofluid is happened slowly, as a result the isotherms are more compressed along the channel near the hot walls.

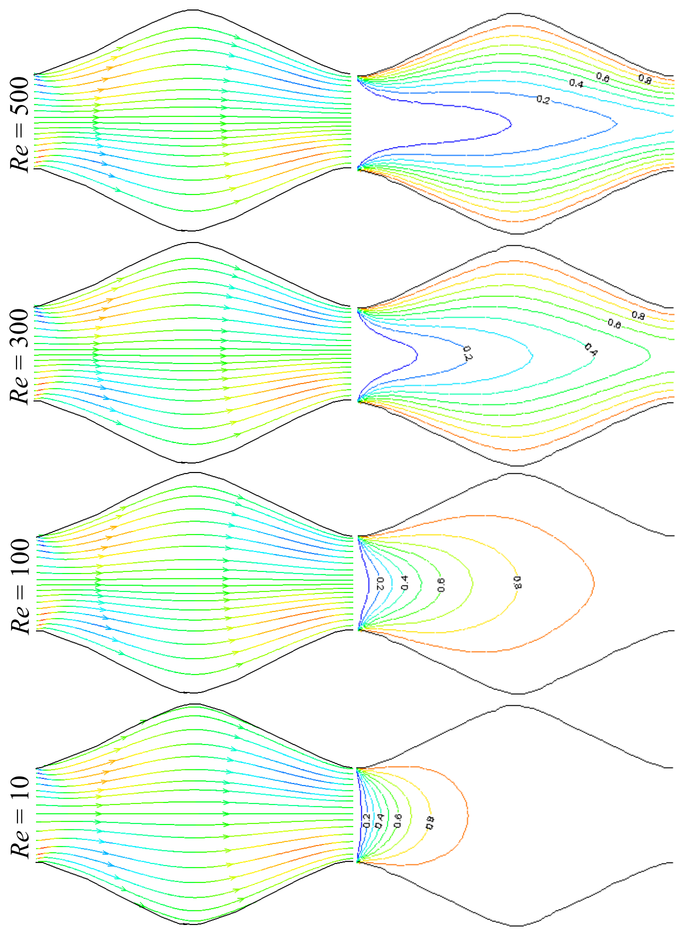
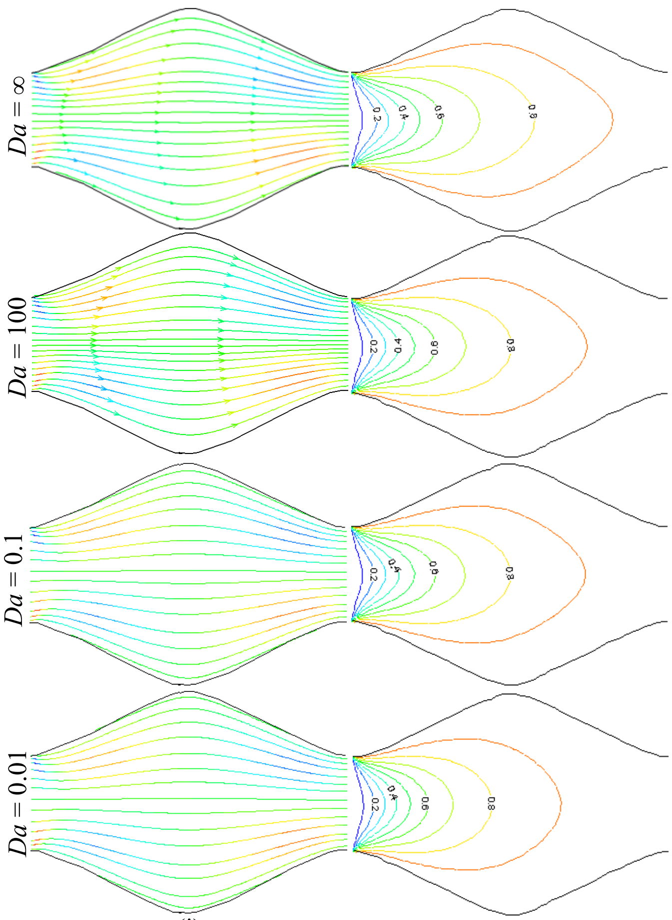


Fig. 4: Effect of *Re* on (i) streamlines and (ii) isotherms at ϕ = 5% and *Da* = 100

Fig. 5(i)-(ii) depicts the variation of Darcy number *Da* on flow and thermal current activities interms of streamlines and isothermal lines respectively at Re = 100 and ϕ = 5%. In the flow field, there is no significant changes except in the mid region of the channel. Streamlines for higher values of Darcy number (Da = 100 - ∞) become more condensed at the middle of the channel due to stronger porosity effects at the regions near the side walls. With the augmentation of Darcy number excels the effect of porous medium which leads to increment of peaks of isothermal lines at the middle of the channel. Increment of *Da* also results in compression of isotherms at the inlet of the channel. Also at lower values of Darcy number the temperature of cold entering water-CuO nanofluid reaches to T_h sooner than that of higher *Da*.

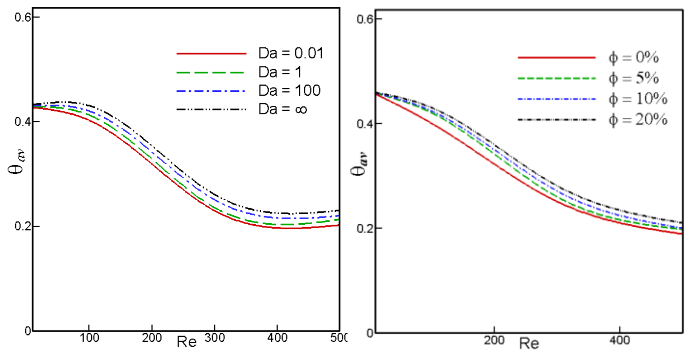


(i) (ii)

Fig. 5: Effect of *Da* on (i) streamlines and (ii) isotherms at Re = 100 and ϕ = 5%

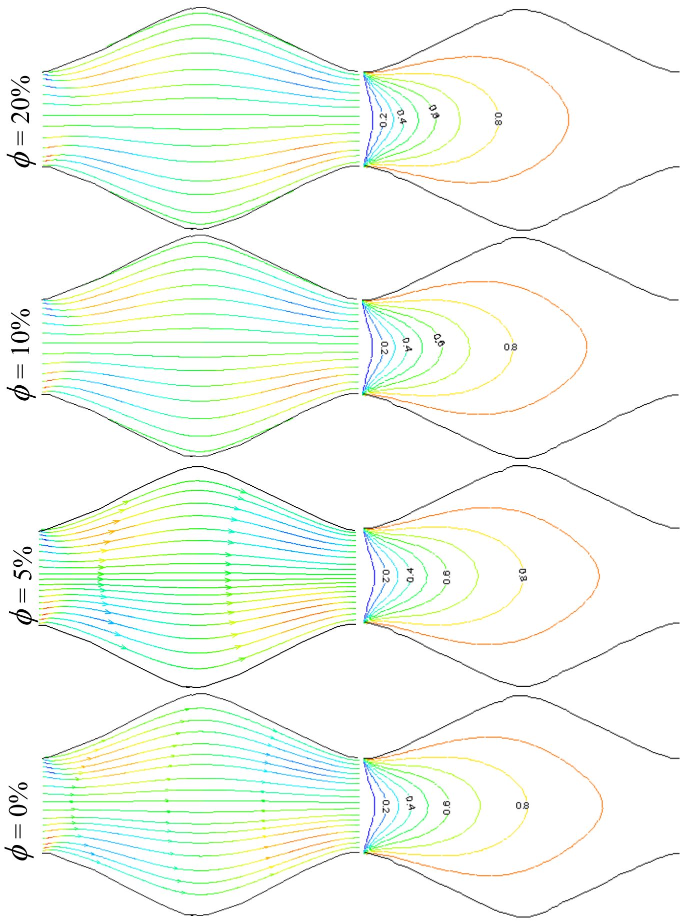
The average bulk temperature (θ_{av}) with *Da*, ϕ and *Re* is expressed in the Fig. 6(i)-(ii). θ_{av} rises for both escalating values of *Da* and solid volume fraction of water-CuO nanofluid but devalues for increasing *Re*.

The effect of solid volume fraction (ϕ) of water-CuO nanoparticles from 0% to 20% on the streamlines and isothermal lines are exposed in Fig. 7 (i)-(ii) while Re = 100 and Da = 100. At first, for ϕ = 0% (clear water), the flow enters in the channel and exits from the right part very rapidly. But, with the rising values of solid volume fraction, the fluid motion is deceased due to higher concentration of nanoparticles. Isotherms slightly compressed near the inlet of the channel for higher values of ϕ. Because increasing solid volume fraction of nanoparticles enhances the thermal conductivity of the fluid. Then this nanofluid takes the wall temperature quickly more than base fluid alone.



(i) (ii)

Fig. 6: Average bulk temperature for the effect of (i) *Da* and (ii) ϕ with various *Re*



(i) (ii)

Fig. 7: Effect of ϕ on (i) streamlines and (ii) isotherms at Re = 100 and Da = 100

Fig. 8(i)-(iii) shows a plot of the average heat transfer rate (Nu) for Da and ϕ and normalized Nusselt number (Nu^*) along with forced convective parameter Re. The rate of heat transfer enhances for both rising Re and ϕ where as it devalues for growing Da. There are some deviations observed in the Nu^*- Re profile for higher ϕ (= 5%, 10% and 20%).

The velocity at the middle of the channel for the influences of Reynolds number, Darcy number and ϕ is illustrated in Fig. 9(i)-(iii). It is seen that the wave amplitude for the highest Re and lowest ϕ is found greater than the remaining values of these two parameters. This is due to the fact that the base fluid (ϕ = 0%) has higher velocity with compared to nanofluid. In addition increase of inertia force leads to more variation in the velocity profile. But this profile becomes almost unchanged due to the variation of Darcy number Da.

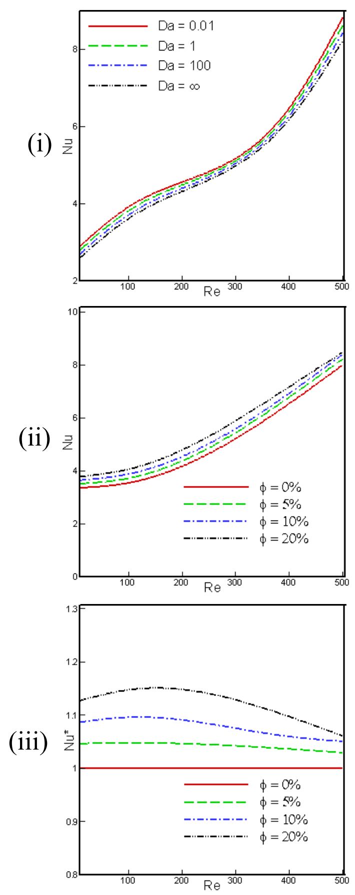


Fig. 8: Effect of Re on (i) Nu and Da, (ii) Nu and ϕ and (iii) Nu^* and ϕ

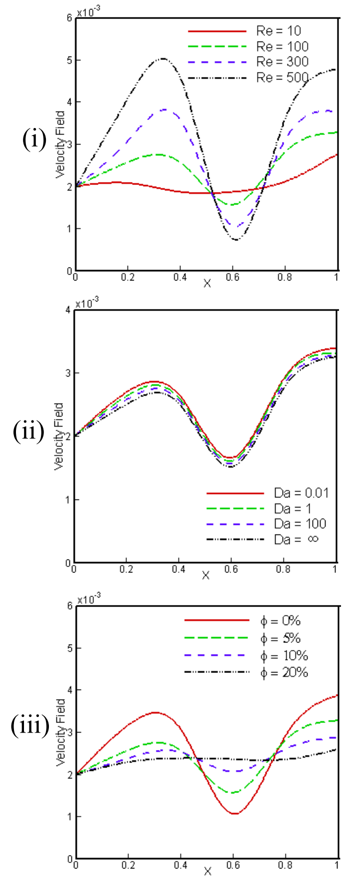


Fig. 9: Mid height velocity for different (i) Re, (ii) Da and (iii) ϕ along X direction

5. CONCLUSION

The problem of non-Darcy effect on forced convection heat transfer in a wavy porous channel using water-CuO nanofluid has been studied numerically. Flow and temperature field in terms of streamlines and isotherms have been considered for various dimensionless parameters. The results of the numerical analysis led to the following conclusions:

- The structure of the fluid flow and temperature field through the channel is found to be significantly dependent upon the Reynolds number.
- The CuO nanofluid with the highest cosidered value of ϕ is found to be the most effective in enhancing performance of heat transfer rate.
- The mean temperature of the fluid in the channel increase with Darcy number and ϕ. In comparison water-CuO nanofluid achieves the higher temperature than the base fluid.
- The mid height velocity profile is perturbed appreciably for considered Re and ϕ values.

Overall the analysis also defines the operating range where water-CuO nanofluid can be assumed effectively in determining the level of heat transfer augmentation.

REFERENCES

[1] I.J. Sobey, The occurrence of separation in oscillatory flow, J. Fluid Mech., Vol. 134, pp. 247–257, 1983.

[2] G. Leneweit, D. Auerbach, Detachment phenomena in low Reynolds number flows through sinusoidally constricted tube, J. Fluid Mech., vol. 387, pp. 129–150, 1999.

[3] S. Mahmud, A.K.M.S. Islam, M.A.H. Mamun, Separation characteristics of fluid flow inside two parallel plates with wavy surface, Int. J. Eng. Sci., vol. 40, pp. 1495–1509, 2002.

[4] G. Wang, S.P. Vanka, Convective heat transfer in periodic wavy passages, Int. J. Heat Mass Transfer, vol. 38, pp. 3219–3230, 1995.

[5] S. Mahmud, A.K.M.S. Islam, C.M. Feroz, Flow and heat transfer characteristics inside a wavy tube, Heat Mass Transfer, vol. 39, pp. 387–393, 2003.

[6] A. Cabal, J. Szumbarski, J.M. Floryan, Stability of flow in a wavy channel, J. Fluid Mech., vol. 457, pp. 191–212, 2002.

[7] L. Adjlout, O. Imine, A. Azzi, M. Belkadi, Laminar natural convection in an inclined cavity with a wavy wall, Int. J. Heat Mass Transfer, vol. 45, pp. 2141–2152, 2002.

[8] S. Mahmud, A.K.M.S. Islam, Natural convection and entropy generation in an inclined wavy enclosure, Int. J. Therm. Sci., vol. 42, pp. 1003–1012, 2003.

[9] P.K. Das, S. Mahmud, Numerical investigation of natural convection in a wavy enclosure, Int. J. Therm. Sci., vol. 42, pp. 397–406, 2003.

[10] B.V.R. Kumar, P.V.S.N. Murthy, P. Singh, Free convection heat transfer from an isothermal wavy surface in a porous enclosure, Int. J. Numer. Method Fluids, vol. 28, pp. 633–661, 1998.

[11] B.V.R. Kumar, Shalini, Natural convection in a thermally stratified wavy vertical porous enclosure, Numer. Heat Transf., Part A, vol. 43, pp. 753–776, 2003.

[12] B.V.R. Kumar, A study of free convection induced by a vertical wavy surface with heat flux in a porous enclosure, Numer. Heat Transf., Part A, vol. 37, pp. 493–510, 2000.

[13] P.V.S.N. Murthy, B.V.R. Kumar, P. Singh, Natural convection heat transfer from a horizontal wavy surface in a porous enclosure, Numer. Heat Transf., Part A, vol. 31, pp. 207–221, 1997.

[14] B.V.R. Kumar, A study of free convection induced by a vertical wavy surface with heat flux in a porous enclosure, Numer. Heat Transf., Part A, vol. 37, pp. 493–510, 2000.

[15] A. Misirlioglu, A.C. Baytas, I. Pop, Free convection in a wavy cavity filled with a porous medium, Int. J. Heat Mass Transfer, vol. 48, pp. 1840–1850, 2005.

[16] B.V.R. Kumar, S. Gupta, Free convection in a non-Darcian wavy porous enclosure, Int. J. Eng. Sci., vol. 41 pp. 1827–1848, 2003.

[17] B.V.R. Kumar, S. Gupta, Free convection in a thermally stratified non-Darcian wavy enclosure, J. Porous Media, vol. 7, pp. 261–277, 2004.

[18] S. Kumar, S.K. Prasad, J. Banerjee, Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model, Applied Mathematical Modelling, vol. 34, pp. 573–592, 2009.

[19] A.K. Santra, S. Sen, N. Chakraborty, Study of heat transfer characteristics of copper–water nanofluid in a differentially heated square cavity with different viscosity models, Journal of Enhanced Heat Transfer, vol. 15, no. 4, pp. 273-287, 2008.

[20] E. Abu-Nada, H.F. Oztop, Effects of inclination angle on natural convection in enclosures filled with Cu–water nanofluid, International Journal for Heat & Fluid Flow, vol. 30, pp. 669-678, 2009.

[21] B. Ghasemi, S.M. Aminossadati, Natural convection heat transfer in an inclined enclosure filled with a water–CuO nanofluid, Numerical Heat Transfer Part A, vol. 55, pp. 807-823, 2009.

[22] M. Muthtamilselvan, P. Kandaswamy, J. Lee, Heat transfer enhancement of copper–water nanofluids in a lid-driven enclosure, Communications in Nonlinear Science and Numerical Simulation, vol. 15, no. 6, pp. 1501-1510, 2009.

[23] E. Abu-Nada, Z. Masoud, H.F. Oztop, A. Campo, Effect of nanofluid variable properties on natural convection in enclosures, International Journal of Thermal Sciences, vol. 49, pp. 479-491, 2010.

[24] W. T. Dai, J. M. Li, X. Chen and B. X. Wang, Experimental investigation on convective heat transfer of copper oxide nonoparticle suspensions inside mini-diameter tubes, J. Eng. Thermophys., vol. 24, no. 4, pp. 633-636, 2003.

[25] X. Chen, , J. M. Li, W. T. Dai and B. X. Wang, Enhancing convection heat transfer in mini tubes with nanoparticle suspensions, J. Eng. Thermophys., vol. 25, no. 4, pp. 643-645, 2004.

[26] E. Pfautsch, Forced convection in nanofluids over a flat plate, Thesis Presented to the Faculty of the Graduate School, University of Missouri, 2008.

[27] Salma Parvin, M.A. Alim and N.F. Hossain, MHD Mixed Convection Heat Transfer through Vertical Wavy Isothermal Channels, International Journal of Energy and Technology, vol. 3, no. 34, pp. 1-9, 2011.

[28] Rehena Nasrin and Salma Parvin, Flow behavior for combined convection in a vertical channel controlled by a heat-generating tube, International Journal of Energy & Technology, vol. 4, no. 1, pp. 1-7, 2012.

[29] Muneer A. Ismael, Partitioning and magnetic field effects on free convection in a square cavity filled with porous medium with uniform heat generation, International

Journal of Energy & Technology, Vol. 4, no. 5, pp. 1-11, 2012.

[30] Zi-Tao Yu, Xu Xu, Ya-Cai Hu, Li-Wu Fan, Ke-Fa Cen, Numerical study of transient buoyancy-driven convective heat transfer of water-based nanofluids in a bottom-heated isosceles triangular enclosure, International Journal of Heat and Mass Transfer, vol. 54, pp. 526-532, 2011.

[31] C. Taylor, P. Hood, A numerical solution of the Navier-Stokes equations using finite element technique, Computer and Fluids, vol. 1, pp. 73-89, 1973.

[32] P. Dechaumphai, Finite Element Method in Engineering, 2nd ed., Chulalongkorn University Press, Bangkok, 1999.

[33] T. Basak, S. Roy, I. Pop, Heat flow analysis for natural convection within trapezoidal enclosures based on heatline concept, Int. J. Heat Mass Transfer, vol. 52, pp. 2471-2483, 2009.

NOMENCLATURE

C_p	specific heat at constant pressure (KJkg^{-1}K^{-1})
k	thermal conductivity (Wm^{-1}K^{-1})
K	medium permeability (m^2)
L	length of the channel (m)
Nu	Nusselt number
Nu^*	normalized Nusselt number
p	dimensional pressure (Nm^{-2})
P	non-dimensional pressure
Pr	Prandtl number
Re	Reynolds number
T	dimensional temperature (K)
u, v	velocity components (ms^{-1}) along x, y direction respectively
U, V	dimensionless velocity components along X, Y direction respectively
x, y	Cartesian coordinates (m)
X, Y	non-dimensional Cartesian coordinates

Greek symbols

α	thermal diffusivity (m^{2}s^{-1})
β	thermal expansion coefficient (K^{-1})
ϕ	solid volume fraction
θ	non-dimensional temperature
μ	dynamic viscosity of the fluid (Kg m^{-1}s^{-1})
ν	kinematic viscosity of the fluid (m^{2}s^{-1})
ρ	density of the fluid (Kg m^{-3})

Subscripts

f	base fluid
h	heated wall
i	inlet state
nf	water-CuO nanofluid
s	solid particle