and become more uniform. This is to be expected because the net radiative heat flux from the wall decreases as the wall surfaces become better reflectors and conduction becomes the dominant mode of heat transfer in the walls.

Conclusions

Combined heat transfer from a radiating and convecting flow of an absorbing, emitting, and scattering medium in a reflecting channel with conducting walls was numerically investigated. Our numerical results clearly indicate that in many high temperature applications, if the effects of scattering and wall reflection are ignored, the position and magnitude of the maximum wall temperature and the behavior of the convective Nusselt number can be grossly misrepresented.

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Convective Heat Transfer of a Particulate Suspension

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Introduction

HEAT transfer between a particulate suspension and a solid body is a problem whose solution involves the consideration of the equations of motion of a two-phase system. Both Soo[1] and Marble[2] have developed the conservation laws of mass, linear momentum, and energy for two-phase flow in general. These equations are sufficiently complex to preclude the possibility of exact solutions except in very idealized cases. Most closed-form solutions presently known are discussed by Soo[1] and Marble.[2]

The problem of steady single-phase flow of a Newtonian fluid past an infinite flat plate with uniform suction (the asymptotic suction profile) was given sometime ago by Schlichting.[3] Chamkha and Peddieson[4] reported exact solutions for the two-phase asymptotic suction profile. In their work, Chamkha and Peddieson[4] did not consider the thermal aspects of the problem.

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The purpose of this note is to report exact solutions for the temperature fields and the wall heat transfer for flow of a particle/fluid mixture past an infinite porous flat plate. The fluid phase is assumed to be incompressible and the volume fraction of suspended particles is assumed to be small. It is assumed, further, that the particles are sufficiently dilute and they do not interact with each other. It is also assumed that there is no radiative heat transfer from one particle to another. Numerical computations of the exact solutions are performed and a representative set of graphical results is presented and discussed.

Governing Equations

Consider the two-dimensional, steady, laminar, two-phase flow that takes place in a half-space bounded by an infinite porous flat plate. Let the flow be a uniform stream parallel to the x, y plane with the plate being coincident with the plane $y = 0$. Far from the plate, both phases are in equilibrium moving with a velocity V_∞ in the x direction. Uniform fluid-phase suction with velocity v_0 is imposed at the plate surface. Let the freestream suspension temperature be denoted by T_∞ and assume that the particle-phase density is constant. Because the plate is infinitely long, the fluid and particle velocity parallel to the plate, as well as the fluid and particle temperature, are independent of the x coordinate.

The dimensional form of the governing equations (which are based on the balance laws of mass, linear momentum, and energy for both the fluid and particulate phases) for the problem under consideration can be shown to reduce to

$$-\rho v_0 u' = \mu u'' + \rho_p(u_p - u)/\tau_v$$

$$-\rho c v_0 T' = kT'' + \mu(u')^2 + c_p\rho_p(T_p - T)/\tau_T + \rho_p(u_p - u)^2/\tau_v$$

$$-\rho_p v_0 u_p' = -\rho_p(u_p - u)/\tau_v$$

$$-\rho_p c_p v_0 T_p' = -\rho_p c_p(T_p - T)/\tau_T \tag{1}$$

where u is the fluid-phase velocity in the x direction, u_p is the particle-phase velocity in the x direction, T is the fluid-phase temperature, T_p is the particle-phase temperature, ρ is the fluid-phase density, ρ_p is the particle-phase density, μ is the fluid-phase dynamic viscosity, k is the fluid phase thermal conductivity, c is the fluid-phase specific heat at constant pressure, c_p is the particle-phase specific heat, τ_v is the velocity relaxation time (the time required by a particle to reduce its velocity relative to the fluid by e^{-1} of its original value in the unaccelerated state), τ_T is the temperature relaxation time (the time required for the temperature difference between a particle and fluid to be reduced to e^{-1} of its initial value), and a prime denotes ordinary differentiation with respect to y. It should be pointed out that the negative sign on the left-hand side of Eq. (1) is due to the fact that the suction velocity v_0 is in the direction opposite to the positive y direction and v_0 is the absolute value of the suction velocity. It can be seen from Eq. (1) that both phases are coupled through drag and heat transfer between them.

Substituting

$$y = \nu\eta/V_\infty, \qquad u = V_\infty F(\eta), \qquad u_p = V_\infty F_p(\eta)$$

$$T = T_\infty G(\eta), \qquad T_p = T_\infty G_p(\eta) \tag{2}$$

(where ν is the fluid kinematic viscosity $\nu = \mu/\rho$) into Eq. (1) and rearranging yield

$$F'' + r_v F' + \kappa\alpha(F_p - F) = 0 \tag{3a}$$

$$G'' + r_v P_r G' + \kappa P_r \gamma\varepsilon(G_p - G) + E_c P_r (F')^2 + E_c P_r \kappa\alpha(F_p - F)^2 = 0 \tag{3b}$$

$$r_v F_p' + \alpha(F - F_p) = 0 \tag{3c}$$

$$r_v G_p' + \varepsilon(G - G_p) = 0 \tag{3d}$$

where a prime denotes ordinary differentiation with respect to η and

$$r_v = v_0/V_\infty, \quad \kappa = \rho_p/\rho, \quad \alpha = \nu/(\tau_v V_\infty^2), \quad P_r = \mu c/k$$

$$\gamma = c_p/c, \quad \varepsilon = \nu/(\tau_T V_\infty^2), \quad E_c = V_\infty^2/(cT_\infty) \tag{4}$$

are the suction parameter, the particle loading, the velocity inverse Stokes number, the fluid-phase Prandtl number, specific heats ratio, the temperature inverse Stokes number, and the Eckert number, respectively. If it is desired to interpret the results to be reported subsequently far downstream of flow past a semi-infinite flat plate, then v_0 must be very small compared to V_∞ (i.e., $r_v << 1$). However, this restriction is not employed in the present work.

The boundary and matching conditions under which Eq. (3) are solved are

$$F(0) = 0 \tag{5a}$$

$$F(\infty) = 1 \tag{5b}$$

$$F_p(\infty) = 1 \tag{5c}$$

$$G(0) = G_0 \tag{5d}$$

$$G(\infty) = 1 \tag{5e}$$

$$G_p(\infty) = 1 \tag{5f}$$

(where G_0 is a constant nondimensional wall temperature.)

Of interest is the wall heat transfer coefficient, which can be defined as

$$c_q = -kT'(0) \tag{6}$$

where the prime denotes ordinary differentiation with respect to y. Equation (6) can be nondimensionalized by using Eq. (2) and substituting $c_q = \rho V_\infty^3 \mathring{q}_w$ to give

$$\mathring{q}_w = -G'(0)/(P_r E_c) \tag{7}$$

where the negative sign indicates the direction of the heat flux.

Results

It can be seen from Eq. (3) that the solution for the velocity fields is independent from the temperature fields. Equations (3a) and (3c) were solved in closed form subject to Eqs. (5a–5c) by Chamkha and Peddieson.[4] The corresponding results were shown to be

$$F = 1 - \exp(-\lambda\eta), \; F_p = 1 - \alpha/(\alpha + r_v\lambda)$$
$$\exp(-\lambda\eta)$$
$$\lambda = (A + (A^2 + 4B)^{1/2})/2 > 0$$
$$A = (r_v^2 - \alpha)/r_v, \qquad B = \alpha(1 + \kappa) \tag{8}$$

Equations (3b) and (3d) can be combined into a third-order differential equation in terms of G. This can be shown to be

$$G''' + (r_v^2 P_r - \varepsilon)/r_v G'' - P_r\varepsilon(1 + \kappa\gamma)G' + E_c P_r((F')^2)'$$
$$+ E_c P_r \kappa\alpha((F_p - F)^2)' - E_c P_r \varepsilon/r_v (F')^2$$
$$- E_c P_r \kappa\alpha\varepsilon/r_v (F_p - F)^2 = 0 \tag{9}$$

Using the expressions for F and F_p given in Eq. (8), it can be shown that the solution of Eq. (9), subject to Eqs. (5d–5f) is

$$G = 1 + C_2 \exp(-m_2\eta) + c_3 \exp(-2\lambda\eta) \tag{10}$$

where

$$c_2 = G_0 - 1 - c_3$$
$$c_3 = E_c P_r \lambda/(2r_v(\alpha + r_v\lambda)^2(-4\lambda^2 + 2\lambda c_1 - D))$$
$$((2\lambda r_v + \varepsilon)((\alpha + r_v\lambda)^2 + \kappa\alpha r_v^2))$$
$$c_1 = (r_v^2 P_r - \varepsilon)/r_v, \; D = P_r\varepsilon(1 - \kappa\gamma)$$
$$m_2 = (c_1 + (c_1^2 - 4D))^{1/2}/2 > 0 \tag{11}$$

Substituting Eq. (10) into Eq. (3d) and solving for G_p give

$$G_p = 1 + \varepsilon c_2/(r_v m_2 + \varepsilon)\exp(-m_2\eta)$$
$$+ \varepsilon c_3/(2\lambda r_v + \varepsilon)\exp(-2\lambda\eta) \tag{12}$$

The wall heat transfer coefficient $\mathring{q}_w$ can be found by differentiating Eq. (10) once, evaluating it at $\eta = 0$, and then substituting the result into Eq. (7). If this is done

$$\mathring{q}_w = (c_2 m_2 + 2\lambda c_3)/(P_r E_c) \tag{13}$$

Numerical evaluations of Eqs. (10), (12), and (13) are performed and a typical set of results is presented in Figs. 1–3. These results show the thermal aspect for flow of a two-phase (particle-fluid) suspension past an infinite flat plate with uniform suction.

Figures 1 and 2 show the influence of the particle loading κ on the temperature profiles for the fluid and particle phases, respectively. Increases in the values of κ have the tendency to increase the fluid-phase temperature, which is decreased due to the presence of the plate at a temperature G_0 that is

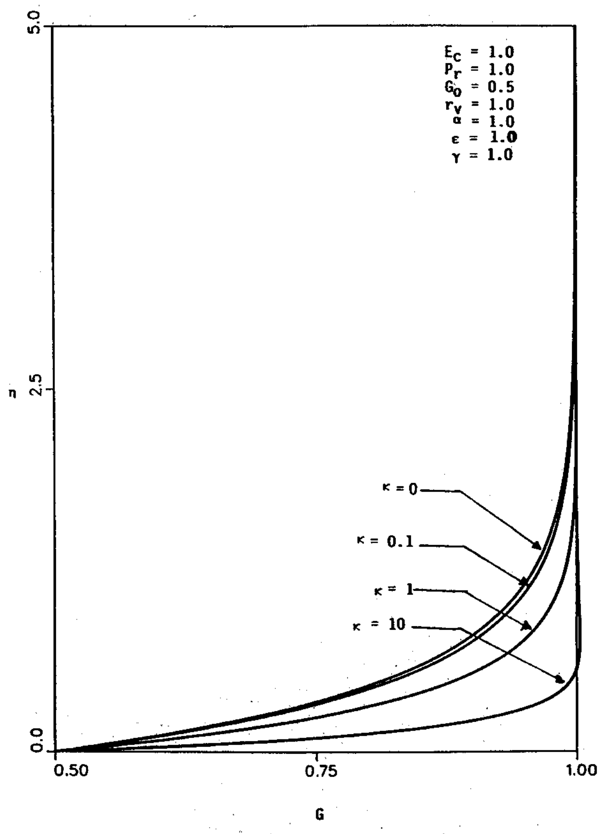

Fig. 1 Fluid-phase temperature profiles.

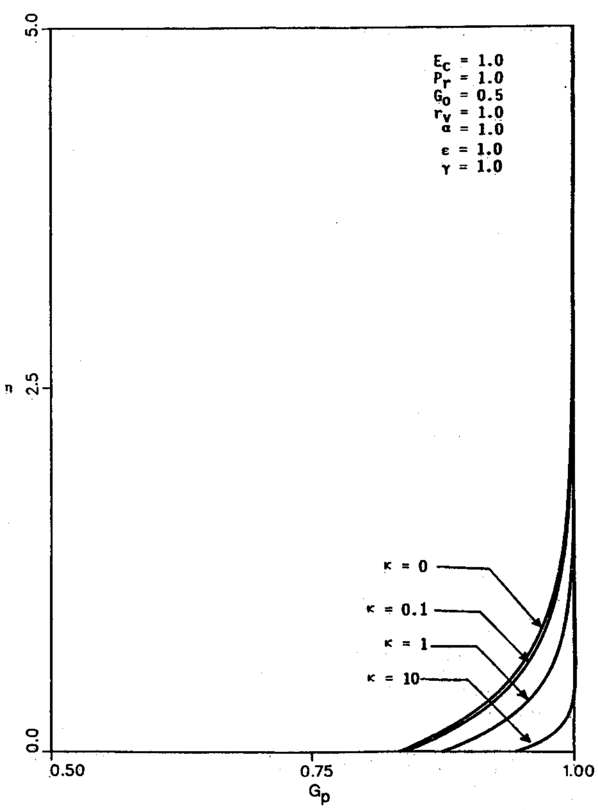

Fig. 2 Particle-phase temperature profiles.

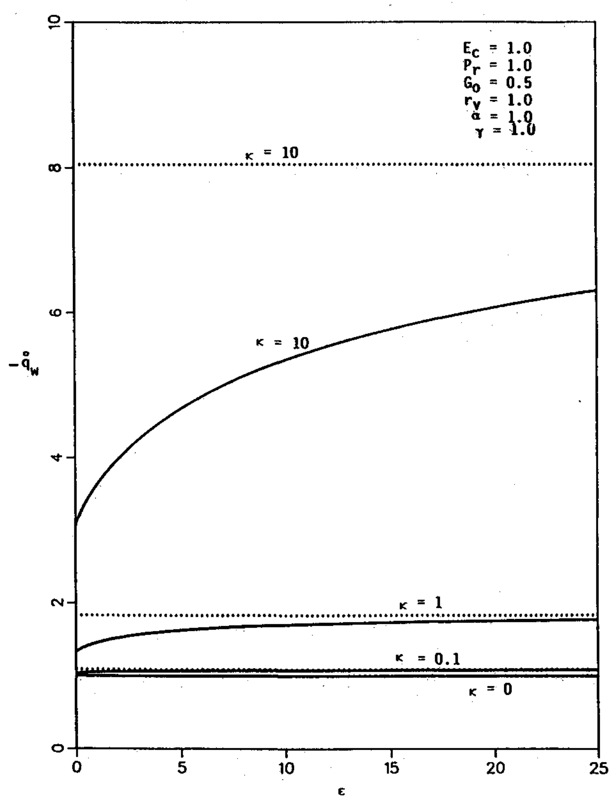

Fig. 3 Wall heat transfer coefficient vs ε.

less than unity, the temperature of the freestream. This is evident from Fig. 3. The fact that the fluid and particle phase temperatures exceed the freestream temperature for large values of κ is probably due to the work caused by drag between the two phases (last term in Eq. (3b)), which is not present for single-phase flow.

Figure 3 illustrates the influence of the particle loading κ and the temperature inverse Stokes number ε on the wall heat transfer coefficient $\mathring{q}_w$. The dotted lines indicate the limits approached as $\varepsilon \rightarrow \infty$ (thermal equilibrium limit). Increases in the particle loading κ cause G to approach the freestream temperature faster; thus, increasing the fluid-phase temperature gradient at the wall. This is reflected in the increases in $\mathring{q}_w$ caused by increasing κ. Increases in the temperature inverse Stokes number ε cause an increase in the thermal interaction between the two phases. This increases the interphase energy transfer between the two phases, and, therefore, increases the fluid-phase temperature at any position above the plate. This steepening effect produces the same results discussed above; namely, an increase in the wall heat transfer coefficient.

Conclusion

Exact solutions for the thermal aspect of flow of a particulate suspension past an infinite porous flat plate were obtained. Numerical computations of the exact solutions were performed and the computed results were presented graphically to illustrate the properties of this type of flow. It was concluded that the presence of particles causes an augmentation in the wall heat transfer coefficient. This is due to the thermal energy gain by the fluid phase through the thermal interaction between the two phases.

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Transient Combined Conduction and Radiation in Anisotropically Scattering Spherical Media

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Introduction

IN the design of many engineering systems, radiative heat transfer can be an important mode of heat transfer. Much research has been conducted on the analysis of simultaneous radiation and conduction in absorbing, emitting, and scattering one-dimensional planeparallel systems.[1,2] The analysis of radiative transfer in spherical media has been the subject of relatively few investigations, although there are important applications in numerous areas, such as spherical propulsion systems, nuclear energy generation and explosions, astrophysics, and thermal insulation systems.

Many of the previous investigations on radiative transfer in spherically symmetric geometry have been limited to the special case of an absorbing, emitting medium.[3,4] There are

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