



Mixed convective flow of a dusty fluid over a vertical stretching sheet with non-uniform heat source/sink and radiation

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Abstract

Purpose – The purpose of this paper is to study the problem of two-dimensional unsteady mixed convective flow a dusty fluid over a stretching sheet in the presence of thermal radiation and space-dependent heat source/sink.

Design/methodology/approach – The equations governing the fluid flow and temperature fields for both the fluid and dust phases are reduced to coupled non-linear ordinary differential equations by using a suitable set of similarity transformations. Numerical solutions of the resulting equations are obtained using the well known RKF45 method.

Findings – The numerical results are benchmarked with previously published studies and found to be in excellent agreement. Finally, the effects of the pertinent parameters which are of physical and engineering interest on the flow and heat transfer characteristics are presented graphically and in tabulated form.

Originality/value – The problem is relatively original as the dusty fluid works for this type of problem are lacking.

Keywords Unsteady flow and heat transfer, Boundary layer flow, Mixed convection, Stretching porous surface, Dusty fluid, Thermal radiation, Heat source, Heat sink, Numerical solution, Stretched surface, Radiation, Flow, Heat transfer, Convection

Paper type Research paper

1. Introduction

The study of two-dimensional boundary layer flow and mixed convection heat transfer over a porous stretching surface is very important as it finds many practical applications

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in different areas. To be more specific, it may be pointed out that many metallurgical processes involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid and that in the process of drawing these strips, are sometimes stretched. The fluid mechanical properties desired for an outcome of such a process would mainly depend on two aspects, one is the cooling liquid used and the other is the rate of stretching. Liquids of non-Newtonian characteristics, which are electrically conducting, can be opted as a cooling liquid as the flow and the heat transfer can be regulated through some external agency. The rate of stretching is very important as rapid stretching results in sudden solidification, thereby destroying the properties expected for the outcome. The problem mentioned here is a fundamental one and frequently arises in many practical situations such as polymer extrusion process. It is also encountered in other process like drawing, annealing and tinning of copper wires, continuous stretching, rolling and manufacturing of plastic films and artificial fibers, heat treated materials traveling on conveyer belts, glass blowing, crystal growing, paper production and so on.

For a horizontal plate, Sakiadis (1961) initiated the study of the boundary layer flow over a stretched surface moving with a constant velocity and formulated a boundary layer equation for two-dimensional and axisymmetric flows. Tsou *et al.* (1967) analyzed the effect of heat transfer in the boundary layer on a continuous moving surface with a constant velocity and experimentally confirmed the numerical results of Sakiadis (1961). Carragher and Crane (1982) investigated the effect of heat transfer in the flow over a stretching surface in the case when the temperature difference between the surface and the ambient fluid is proportional to a power of distance from the fixed point. Grubka and Bobba (1985) studied the temperature field in the flow over a stretching surface subjected to a uniform heat flux. Vajravelu and Nayfeh (1992) discussed hydromagnetic flow of a dusty fluid over a stretching sheet. Sharidan *et al.* (2006) studied similarity solutions for the unsteady boundary layer flow and heat transfer due to a stretching sheet. Abel *et al.* (2007) and Abel and Mahesha (2008) studied the flow and heat transfer in a viscoelastic boundary layer flow over a stretching sheet with prescribed surface temperature (PST) case and prescribed heat flux (PHF) case, further they discussed heat transfer in MHD viscoelastic fluid flow over a stretching sheet with variable thermal conductivity, non-uniform heat source and radiation. Andersson *et al.* (2000) presented a new similarity solution for the temperature fields is devised, which transforms the time-dependent thermal energy equation to an ordinary differential equation. Recently, Gireesha *et al.* (2011a, b) studied heat transfer characteristics of a continuous stretching surface with variable temperature, further they have studied boundary layer flow and heat transfer of a dusty fluid flow over a stretching sheet with non-uniform heat source/sink. Aziz (2009) obtained a numerical solution for laminar thermal boundary over a flat plate with a convective surface boundary condition using the symbolic algebra software Maple. Ali *et al.* (2010) studied unsteady flow and heat transfer past an axisymmetric permeable shrinking sheet with radiation effect. Sharma and Singh (2009) obtained the numerical results for the effects of variable thermal conductivity and heat source/sink on MHD flow near a stagnation point on a linearly stretching sheet. Sparrow and Gregg (1958) solved the problem of laminar free convection flow on an isothermal vertical plate. The free convection problem on a vertical plate was studied in various ways by Kuiken (1969) and Elbashbeshy (2000). The similarity equation involves Prandtl number, Eckret number; number density and unsteadiness parameter, the effect

of these parameters are discussed graphically. Some physically unrealistic phenomena are encountered for specific values of the unsteadiness parameter.

Motivated by all these works, we contemplate to study the effects of variable non-uniform heat source and thermal radiation on mixed convective flow and heat transfer of dusty fluid over a stretching sheet. Further, both the variable wall temperature (VWT) and variable heat flux (VHF) conditions have been considered. The governing equations are solved numerically using RKF45 method with the help of symbolic algebra software Maple.

2. Mathematical formulation

Consider a two-dimensional unsteady boundary layer flow of dusty viscous and incompressible fluid (with electric conductivity σ_0) past a semi-infinite stretching sheet in the region $y > 0$. It is considered that the flow is generated by stretching of an elastic boundary sheet from a slit with the application of two equal and opposite forces in such way that velocity of boundary sheet is quadratic order of the flow directional coordinate x . A uniform magnetic field B_0 is imposed along the y -axis.

The unsteady two-dimensional boundary layer equations of a dusty fluid in the usual notation are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = v \frac{\partial^2 u}{\partial y^2} + \frac{kN}{\rho} (u_p - u) + g\beta^* (T - T_\infty) - \frac{\sigma B_0^2}{\rho} u, \tag{2}$$

$$\frac{\partial u_p}{\partial t} + \frac{u_p \partial u_p}{\partial x} + \frac{v_p \partial u_p}{\partial y} = \frac{k}{m} (u - u_p) \tag{3}$$

$$\frac{\partial v_p}{\partial t} + \frac{u_p \partial v_p}{\partial x} + \frac{v_p \partial v_p}{\partial y} = \frac{k}{m} (v - v_p) \tag{4}$$

$$\frac{\partial(\rho_p u_p)}{\partial x} + \frac{\partial(\rho_p v_p)}{\partial y} = 0, \tag{5}$$

where t is the time, (u, v) and (u_p, v_p) denote the velocity components of the fluid and particle phase, respectively, along the x -axis and y -axis. Further μ is the coefficient of viscosity of fluid, ρ and ρ_p are the density of the fluid and particle phase, B_0 is the induced magnetic field, g and β^* are acceleration due to gravity and volumetric coefficient of thermal expansion, respectively, and $\tau = (m/K)$ is the relaxation time of particle phase, N is the number density of the particle phase, K the Stoke's resistance co-efficient (for spherical particles of radius r is $6\pi r\mu$), m is the mass concentration of dust particles. In deriving these equations, the Stokesian drag force is considered for the interaction between the fluid and particle phase and the induced magnetic field is neglected. It also is assumed that the external electric field to be zero and the electric field as a result of polarization of charges is negligible.

The boundary conditions applicable to the above problem are defined as:

$$u = U_w(x, t), \quad v = V_w(x, t) \quad \text{at } y = 0, \quad u \rightarrow 0, \quad u_p \rightarrow 0, \quad v_p \rightarrow v, \tag{6}$$

$$N \rightarrow \omega\rho \quad \text{as } y \rightarrow \infty,$$

where $U_w = cx/(1 - \alpha t)$ is the velocity of sheet, $V_w(x, t) = -v_0/\sqrt{(1 - \alpha t)}$ is the suction velocity and c is the initial stretching rate being a positive constant, ω is constant density ratio, α is positive constant which measures the unsteadiness. Whereas the effective stretching rate $c/(1 - \alpha t)$ is increasing with time.

Equations (1)-(5) subjected to boundary condition (6), admits self-similar solution in terms of the similarity function f and the similarity variable η is defined as:

$$\begin{aligned} u &= \frac{cx}{1 - \alpha t} f'(\eta), & v &= -\sqrt{\frac{cv}{(1 - \alpha t)}} f(\eta), & \eta &= \sqrt{\frac{c}{v(1 - \alpha t)}} y, \\ u_p &= \frac{cx}{1 - \alpha t} F(\eta), & v_p &= \sqrt{\frac{cv}{(1 - \alpha t)}} G(\eta), & \rho_r &= H(\eta), \end{aligned} \quad (7)$$

where a prime denotes the differentiation with respect to η . Substituting the equations (7) into equations (1)-(5) one can get:

$$\begin{aligned} f'''(\eta) + f(\eta)f''(\eta) - f'(\eta)^2 - A \left[f'(\eta) + \frac{\eta}{2} f''(\eta) \right] \\ + l\beta H(\eta)[F(\eta) - f'(\eta)] - Mf'(\eta) + \lambda\theta(\eta) = 0, \end{aligned} \quad (8)$$

$$G(\eta)F'(\eta) + F(\eta)^2 - \beta[f'(\eta) - F(\eta)] + A \left[F(\eta) + \frac{\eta}{2} F'(\eta) \right] = 0, \quad (9)$$

$$G(\eta)G'(\eta) + \beta[f(\eta) + G(\eta)] + \frac{A}{2}[G(\eta) + \eta G'(\eta)] = 0, \quad (10)$$

$$F(\eta)H(\eta) + G'(\eta)H(\eta) + H'(\eta)G(\eta) = 0, \quad (11)$$

where $\rho_r = \rho_p/\rho$ is the relative density, $A = \alpha/c$ is the parameter that measures the unsteadiness, $l = mN/\rho_p$ is the mass concentration, $M = (\sigma B_0^2/\rho_b)(1 - \alpha t)$ is the magnetic field parameter, $\lambda = Gr_x/Re_x^2$ is the mixed convection parameter, $Gr_x = (g\beta^*(T_w - T_\infty)x^2)/v^2$ is the local Grashof number, $Re_x = u_w x/v$ is the local Reynolds number and $\beta = (1/\tau b)(1 - \alpha t)$ is the fluid-particle interaction parameter.

The corresponding boundary conditions are transformed to:

$$\begin{aligned} f'(\eta) = 1, & \quad f(\eta) = R \quad \text{at} \quad \eta = 0, \\ f'(\eta) = 0, & \quad F(\eta) = 0, \quad G(\eta) = -f(\eta), \quad H(\eta) = E \quad \text{as} \quad \eta \rightarrow \infty. \end{aligned} \quad (12)$$

where $R = v_0/\sqrt{vc}$ is the suction parameter.

3. Heat transfer analysis

The governing unsteady, dusty boundary layer heat transport equations in the presence of temperature-dependent internal heat generation/absorption for two-dimensional flow are:

$$\rho c_p \left[\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = k^* \frac{\partial^2 T}{\partial y^2} + \frac{Nc_p}{\tau_T} (T_p - T) + \frac{N}{\tau_v} (u_p - u)^2 + q''' - \frac{\partial qr}{\partial y}, \quad (13)$$

$$Nc_m \left[\frac{\partial T_p}{\partial t} + u_p \frac{\partial T_p}{\partial x} + v_p \frac{\partial T_p}{\partial y} \right] = - \frac{Nc_p}{\tau_T} (T_p - T), \quad (14)$$

where T and T_p are the temperature of the fluid and dust particle, c_p and c_m are the specific heat of fluid and dust particles, τ_T is the thermal equilibrium time and is time required by the dust cloud to adjust its temperature to that of fluid, k^* is the thermal conductivity, τ_v is the relaxation time of the of dust particle, i.e. the time required by a dust particle to adjust its velocity relative to the fluid and q''' is the space and temperature-dependent internal heat generation/absorption (non-uniform heat source/sink) (Ali *et al.*, 2010) and which can be expressed in the simplest form as:

$$q''' = \frac{K^* u(x, t)}{xv} [A^* (T_w - T_\infty) f'(\eta) + (T - T_\infty) B^*], \quad (15)$$

where A^* and B^* are the coefficient of space and temperature-dependent heat source/sink, respectively. Note that the case $A^* > 0$ and $B^* > 0$ corresponds to internal heat generation and $A^* < 0$, $B^* < 0$ corresponds to internal heat absorption. From the Rosseland approximation (Elbashbeshy, 2000), the radiative heat flux q_r is modeled as:

$$q_r = - \frac{4\sigma^*}{3k_1} \frac{\partial T^4}{\partial y}, \quad (16)$$

where σ^* is the Stefan-Boltzman constant and k_1 is the mean absorption coefficient. Assuming that the differences in the temperature within the flow are such that T^4 can be expressed as linear combination of the temperature, one can expand T^4 in a Taylor's series about T_∞ as follows:

$$T^4 = T_\infty^4 + 4T_\infty^3(T - T_\infty) + 6T_\infty^2(T - T_\infty)^2 + \dots \quad (17)$$

By neglecting higher order terms beyond the first degree in $(T - T_\infty)$, we get:

$$T^4 \cong -3T_\infty^4 + T_\infty^3 T. \quad (18)$$

Substituting equation (18) in equation (16) we obtain:

$$q_r = - \frac{16T_\infty^4 \sigma^*}{3k_1} \frac{\partial^2 T}{\partial y^2}. \quad (19)$$

The solution of equations (13) and (14) depends on the nature of the prescribed boundary condition. The two types of heating processes are discussed.

Case 1: variable wall temperature (VWT case)

For this heating process, the VWT is assumed to be a quadratic function of x and it is given by:

$$T = T_w = T_\infty + T_0 \left[\frac{cx^2}{v(1 - \alpha t)^2} \right] \quad \text{at } y = 0, \quad (20)$$

$$T \rightarrow T_\infty, \quad T_p \rightarrow T_\infty \quad \text{as } y \rightarrow \infty,$$

where T_w is the surface temperature of the sheet varies with the distance x from the slot and time t , T_0 is a reference temperature such $0 \leq T_0 \leq T_w$ and T_∞ is the temperature far away from the stretching surface with $T_w > T_\infty$.

In order to obtain similarity solution for temperatures $\theta(\eta)$ and $\theta_p(\eta)$ define dimensionless variables as follows:

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \theta_p(\eta) = \frac{T_p - T_\infty}{T_w - T_\infty}, \quad (21)$$

where:

$$T - T_\infty = T_0 \left[\frac{cx^2}{v(1 - \alpha t)^2} \right] \theta(\eta).$$

Using equations (15), (19)-(21) in equations (13) and (14), we get:

$$\begin{aligned} & (1 + Nr)\theta''(\eta) + \text{Pr}[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + a_1N \text{Pr}[\theta_p(\eta) - \theta(\eta)] \\ & + a_2N\text{Pr}Ec[F(\eta) - f'(\eta)]^2 - \frac{A}{2} \text{Pr}[4\theta(\eta) + \eta\theta'(\eta)] + A^*f'(\eta) + B^*\theta(\eta) = 0, \end{aligned} \quad (22)$$

$$G(\eta)\theta'_p(\eta) + 2F(\eta)\theta_p(\eta) + \frac{A}{2} [4\theta_p(\eta) + \eta\theta'_p(\eta)] + b_1[\theta_p(\eta) - \theta(\eta)], \quad (23)$$

where $Pr = \mu c_p/k^*$ is the Prandtl number, $Ec = v^2/C_p T_0$ is the Eckret number, $Nr = 16T_\infty^4 \sigma^*/3k_1 k^*$ is the thermal radiative parameter, $a_1 = (1/\rho \tau_T c)(1 - \alpha t)$ and $b_1 = (c_p/\tau_T c_m c)(1 - \alpha t)$ are the local fluid-particle interaction parameters for temperature, $a_2 = 1/\mu \tau_v(1 - \alpha t)$ is the local fluid-particle interaction parameter of velocity.

Using equations (20) and (21), the corresponding boundary conditions for $\theta(\eta)$ and $\theta_p(\eta)$ reduce to the following form:

$$\theta(\eta) = 1 \quad \text{at} \quad \eta = 0, \quad \theta(\eta) = 0, \quad \theta_p(\eta) = 0 \quad \text{as} \quad \eta \rightarrow \infty. \quad (24)$$

Case 2: variable heat flux (VHF case)

For this heating process, the following VHF boundary condition is employed:

$$\frac{\partial T}{\partial y} = -\frac{q_w(x, t)}{K^*} \quad \text{at} \quad y = 0, \quad T \rightarrow T_\infty, \quad T_p \rightarrow T_\infty \quad \text{as} \quad y \rightarrow \infty, \quad (25)$$

where:

$$q_w(x, t) = q_{w_0} x^2 (c/v)^{3/2} (1 - \alpha t)^{-5/2}.$$

In order to obtain similarity solution for temperature, define dimensionless temperature variables for the VHF case as in equation (21) where:

$$T = T_\infty + \frac{q_{w_0}}{k^*} \left[\frac{bx^2}{v(1 - \alpha t)^2} \right] \theta(\eta).$$

Using the dimensionless variable (equation (21)), the temperature equations (13) and (14) take the form:

$$(1 + Nr)\theta''(\eta) + \text{Pr}[f(\eta)\theta'(\eta) - 2f'(\eta)\theta(\eta)] + a_1 NPr[\theta_p(\eta) - \theta(\eta)] + a_2 NPrEc[F(\eta) - f'(\eta)]^2 - \frac{A}{2} \text{Pr}[4\theta(\eta) + \eta\theta'(\eta)] + A^*f'(\eta) + B^*\theta(\eta) = 0, \quad (26)$$

$$G(\eta)\theta'_p(\eta) + 2F(\eta)\theta_p(\eta) + \frac{A}{2}[4\theta_p(\eta) + \eta\theta'_p(\eta)] + b_1[\theta_p(\eta) - \theta(\eta)] = 0, \quad (27)$$

where Eckert number $Ec = kv^2/c_p q_w$.

The corresponding boundary conditions become:

$$\begin{aligned} \theta'(\eta) &= -1 \quad \text{at} \quad \eta = 0, \\ \theta(\eta) &= 0, \quad \theta_p(\eta) = 0 \quad \text{as} \quad \eta \rightarrow \infty. \end{aligned} \quad (28)$$

The physical quantities of interest are the skin friction coefficient c_f and the local Nusselt number Nu_x , which are defined as:

$$c_f = \frac{\tau_w}{\rho U_w^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)},$$

where the surface shear stress τ_w and the surface heat flux q_w are given by:

$$T_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = k \left(\frac{\partial T}{\partial y} \right)_{y=0},$$

Using the non-dimensional variables, we obtain:

$$c_f Re_x^{1/2} = f''(0), \quad \frac{Nu_x}{Re_x^{1/2}} = -\theta'(0) \quad (\text{VWT}), \quad \frac{Nu_x}{Re_x^{1/2}} = 1/\theta(0) \quad (\text{VHF}).$$

4. Numerical solution

Equations (8)-(11) with the boundary conditions (12) are highly non-linear ordinary differential equations. In order to solve these non-linear equations numerically we adopted symbolic software Maple, it is very efficient in using the well known Runge-Kutta Fehlberg fourth-fifth order method (RKF45 method) (Gireesha *et al.*, 2011b). In accordance with the boundary layer analysis, the boundary condition (12) at $\eta = \infty$ were replaced by $\eta = 5$ The coupled boundary layer equations (8)-(11) and either equations (22) and (23) or (26) and (27) were solved by RKF45 method. The accuracy of this numerical method was validated by direct comparison with the numerical results reported by Grubka and Bobba (1985) and Abel and Mahesha (2008) with ($A = A^* = B^* = Nr = Ec = 0$) If $A = 0$ and $\lambda = 0$, the numerical solution of equations (1)-(5) coincides with the results of Vajravelu and Nayfeh (1992). Table I presents results of this comparison for $-\theta'(0)$. It can be seen from this table that there is a very good agreement between the results.

5. Results and discussion

An unsteady boundary layer flow problem for momentum and heat transfer over a stretching sheet in the presence of non-uniform heat source/sink and thermal radiation is examined in this paper. The boundary layer equations of momentum and heat transfer are solved numerically. The temperature profiles for the VWT and VHF cases are

depicted graphically. The computation through the employed numerical scheme has been carried out for various values of the physical parameters such as the unsteadiness parameter A , magnetic parameter M , mixed convection parameter λ , suction parameter R , fluid-particle interaction parameter β , Prandtl number Pr , number density N , Eckert number Ec , non-uniform heat source (A^* and B^*) and the thermal radiation parameter Nr . From Table I, it is noted that there is a close agreement with the results of previously published work by Grubka and Bobba (1985) and Abel and Mahesha (2008), and thus, verify the accuracy of the method used. The temperature profiles of both phases $\theta(\eta)$ and $\theta_p(\eta)$ for the VWT and VHF cases are depicted graphically.

Numerical values of the fluid phase wall temperature gradient for different values of the governing parameters for the VWT and VHF cases are tabulated in Table II. From this table, it is noticed that $\theta'(0)$ (for the VWT case) and $\theta(0)$ (for the VHF case) increase

Table I.
Comparison of wall temperature gradient $-\theta'(0)$ for several values of Pr with $\alpha = \beta = M = N = Ec = A^* = B^* = 0.0$

Pr	Grubka and Bobba (1985)	Abel and Mahesha (2008)	Present study $-\theta'(0)$
0.72	1.0885	1.0885	1.0885
1.0	1.3333	1.3333	1.3333
10.0	4.7969	4.7968	4.7968

β	A	Pr	Ec	Nr	A^*	B^*	N	λ	$\theta(0)$ (VWT)	$\theta(0)$ (VHF)
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.373407	1.134934
0.5									-1.370000	1.147717
1.0									-1.359684	1.156966
0.1	0	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.223052	1.272788
	0.3								-1.317950	1.183964
	0.5								-1.373407	1.134934
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.373407	1.134934
		1.0							-1.723367	0.939529
		20							-2.895464	0.599388
0.1	0.5	0.72	0.0	1	-0.05	-0.05	0.2	2.0	-1.408676	1.108639
			0.5						-1.391097	1.121643
			2.0						-1.337679	1.162427
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.373407	0.740437
				2					-1.046829	0.958250
				3					-0.871614	1.134934
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.477798	1.060405
					0.0				-1.361288	1.143755
					0.5				-1.232785	1.238633
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	2.0	-1.483207	1.049870
						0.0			-1.360421	1.145654
						-0.05			-1.219230	1.271073
0.1	0.5	0.72	1	1	-0.05	-0.05	0.5	2.0	-1.381983	1.125959
							1.5		-1.409541	1.096846
							30		-1.446400	1.058565
0.1	0.5	0.72	1	1	-0.05	-0.05	0.2	1.0	-0.837940	0.753889
								2.0	-0.871614	0.740437
								3.0	-0.896725	0.730130

Table II.
Values of wall temperature gradient $\theta'(0)$ (for VWT case) and wall temperature function $\theta(0)$ (for VHF case)

with increasing values of the thermal radiation parameter, heat source or sink parameter and the Eckert number. Further, they decrease with increasing values of the Prandtl number, unsteadiness parameter and the mixed convection parameter. The temperature distributions of the VWT and the VHF cases show that the VWT boundary condition succeeds in keeping the liquid warmer than in the case when the VHF boundary condition is applied. Therefore, one can say that the VWT boundary condition is better than the VHF boundary condition in faster cooling of the stretching sheet. From the results of the VWT and VHF cases, we infer that the boundary layer temperature is quantitatively higher in the VWT case compared to the VHF case.

Figure 1(a) and (b) shows typical horizontal velocity profiles of both the fluid and the dust particles for various values of the unsteadiness parameter A and the magnetic parameter M when $Pr = 0.72, R = 2, M = 0.1, N = 0.2, Nr = 1, A^* = B^* = 0.05, \lambda = 2$ and $\beta = 0.1$. From these figures, it is observed that the velocity decreases with the increase of the unsteady parameter A as well as the magnetic parameter M . It is interesting to note that the boundary layer thicknesses of both phases decrease with increasing values of M .

Figure 2(a) and (b) shows the horizontal velocity profiles of both the fluid and dust phases for various values of the mixed convection parameter λ for both the VWT and VHF cases. The velocities of fluid and dust particles increase with increasing values of the mixed convection parameter λ . Physically, $\lambda > 0$ means heating of the fluid or cooling of the boundary surface $\lambda < 0$ means cooling of the fluid or heating of the boundary surface and $\lambda = 0$ corresponds to the absence of free convection current.

Figure 3(a) and (b) shows that the temperature profiles of the fluid and dust phases decrease with increasing values of the mixed convection parameter λ . It is clear that as λ increases, the thermal boundary layer thickness decreases. From these figures, one can observe that the wall temperature gradients decrease as the value of λ increases. The temperature gradient is always negative, which means that the heat is transferred from the sheet to the ambient medium. Hence, the heat transferred rate from the sheet to the ambient medium increases as λ increases.

Figure 4(a) and (b) shows the graphical representation for the temperature distributions for the VWT and VHF cases for different values of unsteady parameter A versus η . It is observed that temperatures profiles of the fluid and the dust phases

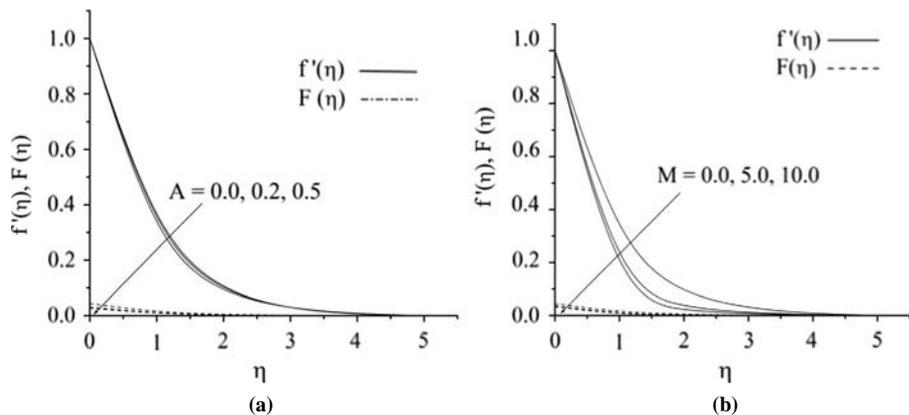


Figure 1.
(a) and (b): Velocity profiles for the effect of unsteadiness parameter (A) and magnetic parameter (M), respectively

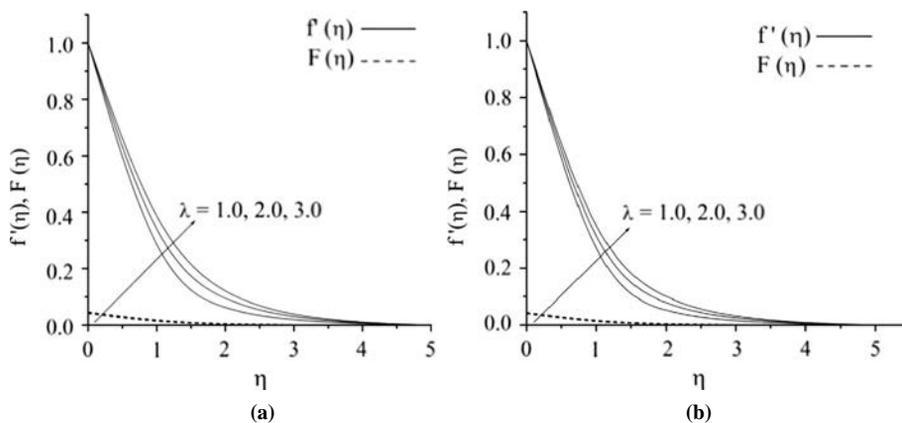


Figure 2.
(a) and (b): Effect of convective parameter λ for fluid and dust velocity for both VWT and VHF cases

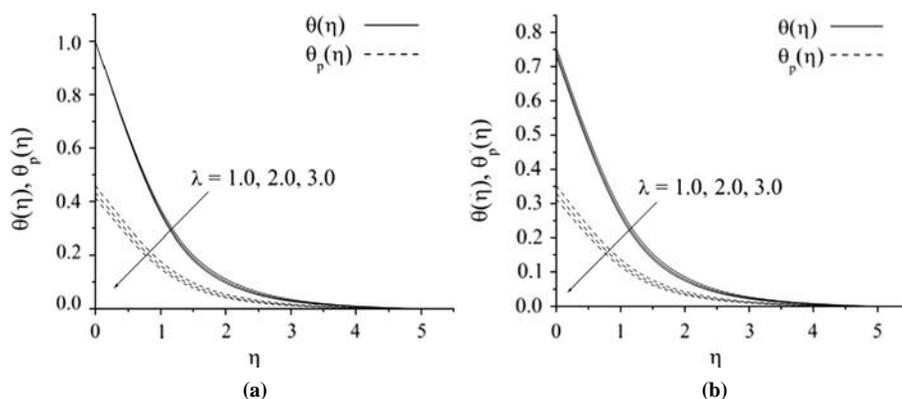


Figure 3.
(a) and (b): Effect of convective parameter λ on temperature distribution for VWT and VHF cases

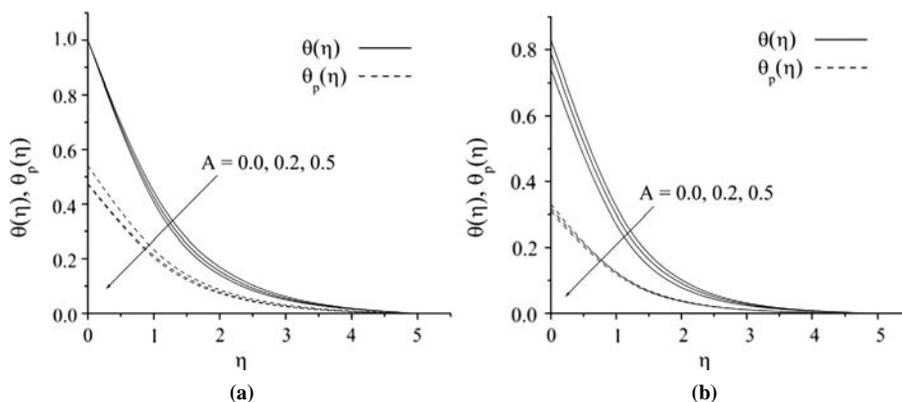


Figure 4.
(a) and (b): Effect of unsteady parameter A on temperature distribution for VWT and VHF cases

decrease with increasing values of the unsteadiness parameter A . It should be noted that we have used throughout our thermal analysis the values of $a_1 = a_2 = b_1 = 2$.

Figure 5(a) and (b) shows the temperature profiles of the fluid phase $\theta(\eta)$ and the dust phase $\theta_p(\eta)$ versus η , for different values of the Prandtl number Pr . We infer from these figures that the temperatures of fluid and dust phases decrease with as Pr increases which imply that the viscous boundary layer is thicker than the thermal boundary layer. As expected the temperatures of both the fluid and dust phases for both the VWT and VHF cases asymptotically approach zero in the free stream region.

Figure 6(a) and (b) shows the effect of the Eckert number Ec on the temperature profiles for the fluid phase $\theta(\eta)$ and the dust phase $\theta_p(\eta)$ versus η , for both the VWT and VHF cases, respectively. It is predicted that increasing the value of Ec enhances the temperature of the fluid and dust phases at any point and that this is true for both the VWT and VHF cases. This is due to fact that the heat energy is stored in the considered liquid due to frictional heating.

Figure 7(a) and (b) shows representative temperature profiles of the fluid phase $\theta(\eta)$ and the dust phase $\theta_p(\eta)$ versus η , for different values of the number density N for both

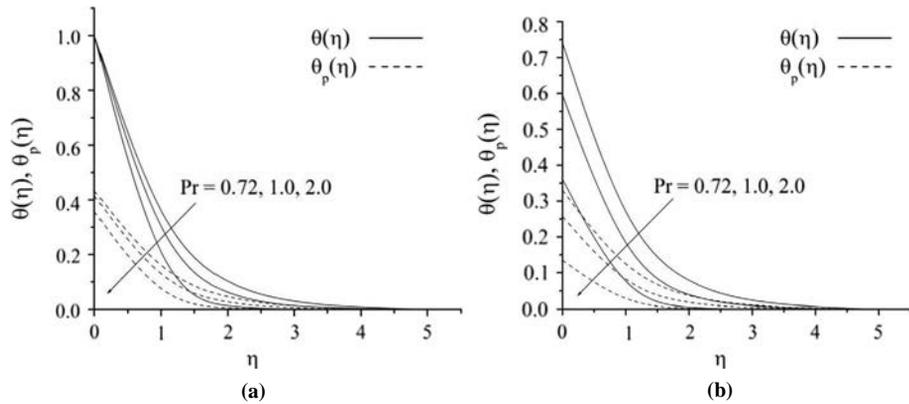


Figure 5.
(a) and (b): Effect of Prandtl number Pr on temperature distribution for VWT and VHF cases

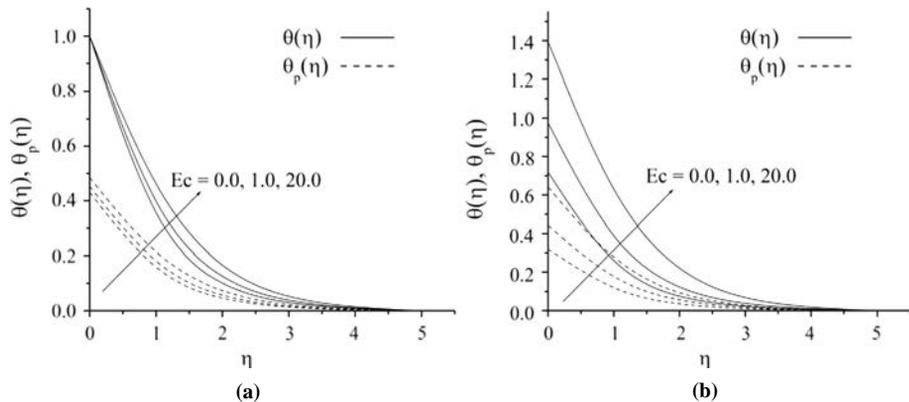


Figure 6.
(a) and (b): Effect of Eckert number Pr on temperature distribution for VWT and VHF cases

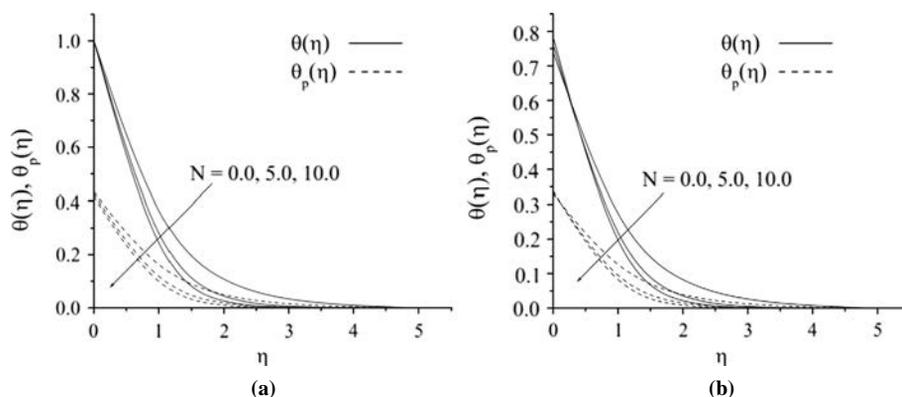


Figure 7.
(a) and (b): Effect of number density N on temperature distribution for VWT and VHF cases

the VWT and VHF cases, respectively. From the figures, it is observed that the temperatures of the fluid and dust phases decrease with increasing values of N .

Figure 8(a) and (b) shows the influence of the thermal radiation parameter Nr on the temperature profiles of the fluid and dust phases for the VWT and VHF cases, respectively. The effect of thermal radiation is observed to intensify the heat transfer. Thus, the radiation should be at its minimum in order to facilitate the cooling process.

Figure 9(a) and (b) shows the effect of the space-dependent heat source/sink parameter A^* on the temperature profiles of both phases for the VWT and VHF cases, respectively. It is observed that the thermal boundary layer generates energy, which causes the temperature profiles of both the fluid and dust phases (for VWT and VHF cases) to increase with increasing values of $A^* > 0$ whereas in the case of $A^* < 0$, the boundary layer absorbs the energy causing the temperatures of both phases to fall considerably. Similar predictions are valid for B^* also and they are shown in Figure 10(a) and (b).

6. Conclusions

A mathematical analysis has been carried out on momentum and heat transfer of an unsteady boundary layer flow of dusty fluid over a stretching sheet in the presence of

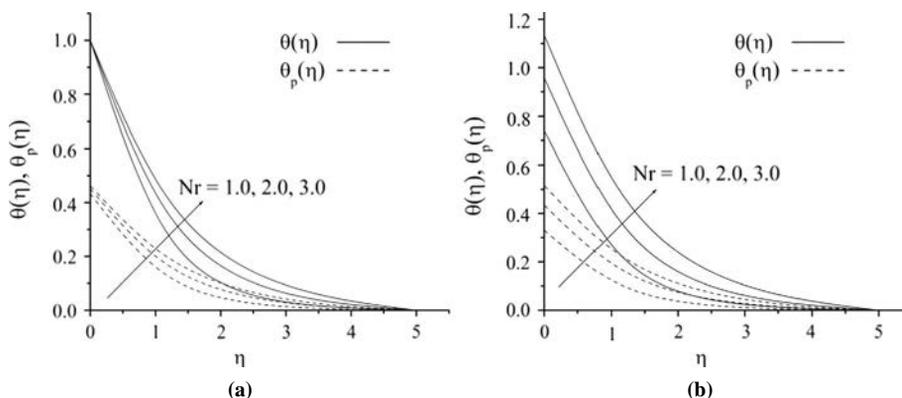


Figure 8.
(a) and (b): Effect of thermal radiation parameter Nr on temperature distribution for VWT and VHF cases

Figure 9.
(a) and (b): Effect of heat source/sink parameter A^* on temperature distribution for VWT and VHF cases

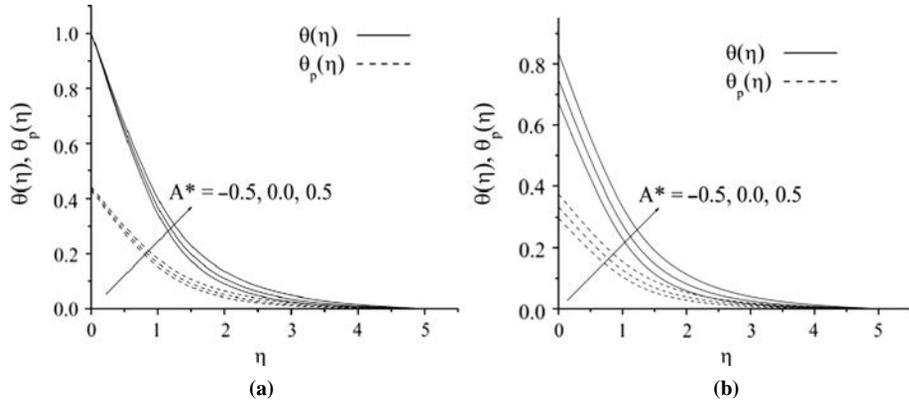
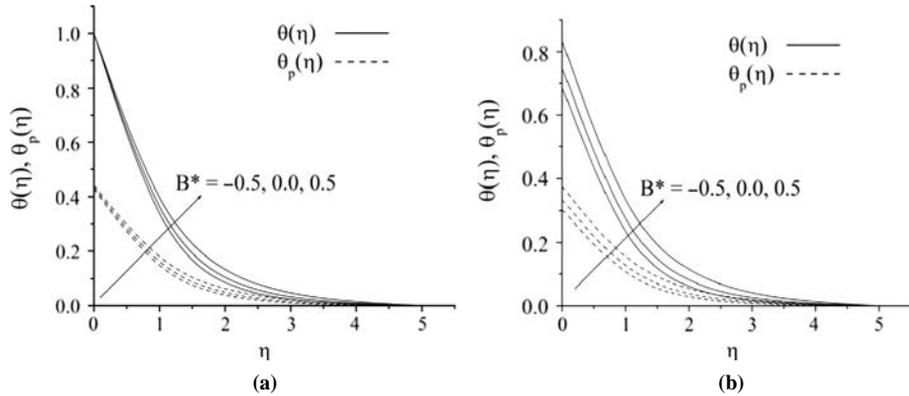


Figure 10.
(a) and (b): Effect of heat source/sink parameter B^* on temperature distribution for VWT and VHF cases



thermal radiation and non-uniform heat source/sink. The highly non-linear governing momentum and energy equations are converted into coupled ordinary differential equations by using similarity transformations. The resultant coupled ordinary differential equations for both the VWT and VHF cases have been solved numerically by the RKF45 method. The obtained numerical results are compared with previously published work and they are found to be in excellent agreement. The effects of various physical parameter like the unsteadiness parameter A , mixed convection parameter λ , Prandtl number Pr , Eckert number Ec , Hartmann number M , number density N , thermal radiation parameter Nr and the non-uniform heat source/sink parameters A^* and B^* on various momentum and heat transfer characteristics are examined. The following observations are listed below:

- The effect of the magnetic field parameter is predicted to decrease the fluid and dust phase velocities.
- Increasing the mixed convection parameter increases the velocity profiles and decreases the temperature profiles of both the fluid and dust phases.
- The effect of the unsteadiness parameter is seen to decrease the temperature profiles of both the fluid and dust phases for both the VWT and VHF cases.

- The effect of increasing the Prandtl number is observed to decrease the thermal boundary layer thicknesses of both phases.
- Increasing the thermal radiation parameter is predicted to increase the temperature profiles of both the fluid and dust phases for the VWT and VHF cases. Also, the temperature profiles of both phases increase as the non-uniform heat source/sink parameter increases.
- The rate of heat transfer $\theta'(0)$ (for the VWT case) is predicted to be negative whereas $\theta(0)$ (for the VHF case) is predicted to be positive.
- The values of $\theta'(0)$ (for the VWT case) and $\theta(0)$ (for the VHF case) are predicted to increase with increases in the values of the thermal radiation parameter, heat source or sink parameter or the Eckert number.

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